HINDU COLLEGE :: GUNTUR

Internal Quality Assurance Cell
(IQAC)

3.3.1- DIGITAL PUBLICATIONS



2017 – 22

HINDU COLLEGE :: GUNTUR

3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
1	*Studies on the distribution of Particulate Trace Metals like Cadmium and Copper in Visakhapatnam Harbour Waters, India*	Dr. T. Umamaheswara Rao	Department of Chemistry	**IOSR Journal of Applied Chemistry (IOSR-JAC)**	**2017**	**e-ISSN: 2278-5736**	https://www.iosrjournals.org/iosr-jac.html	https://www.iosrjournals.org/iosr-jac/papers/vol10-issue9/Version-2/A1009020110.pdf	NA
2	*Distribution of Particulate Trace Metals like Iron and Nickel in the Estuarine Waters of River Godavari, East Coast of India*	Dr. T. Umamaheswara Rao	Department of Chemistry	IOSR Journal of Pharmacy and Biological Sciences (IOSR-JPBS)	2017	e-ISSN:2278-3008, p-ISSN:2319-7676	https://www.iosrjournals.org/iosr-jpbs.html	https://www.iosrjournals.org/iosr-jpbs/papers/Vol12-issue5/Version-3/I1205035159.pdf	NA
3	*Thomas Dekkar's the Shoe-Maker's Holiday – A Reflection of Middle – Class Life*	Smt. V. Beulah Rani Smt Ch. Aruna	Department of English	International Journal on Studies in English Language and Literature (IJSELL)	2017	ISSN 2347-3126 (Print) & ISSN 2347-3134 (Online)	www.arcjournals.org	http://dx.doi.org/10.20431/2347-3134.0503007	NA
4	*Teaching English to Rural Students*	Smt. V. Beulah Rani	Department of English	INTERNATIONAL JOURNAL OFMULTIDISCIPLINARY EDUCATIONAL RESEARCH	2017	ISSN: 2277-7881	www.ijmer	http://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume6/volume6-issue12(1)-2017.pdf	NA
5	*On Trigonometric Moments of the Stereographic Semicircular Gamma Distribution*	Dr. S.V.S. Girija	Department of Mathematics	*European Journal of Pure and Applied Mathematics*	2017	ISSN 1307-5543	www.ejpam.com	https://ejpam.com/index.php/ejpam/article/view/2677/0	NA
6	*On l – axial Chi - Square Distribution*	Dr. S.V.S. Girija	Department of Mathematics	i-Manager's Journal on Mathematics	2017	ISSN: 2277-5129	https://www.sciencegate.app/source/309942	https://www.researchgate.net/publication/323516177_On_L_-_Axial_Chi-Square_Distribution	NA

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7	*Cut – Off Points for Various Tests for Circular Uniformity*	Dr. S.V.S. Girija	Department of Mathematics	i-Manager's Journal on Mathematics	2017	ISSN: 2277-5129	https://www.sciencegate.app/source/309942	https://www.researchgate.net/publication/323517470_Cut-Off_Points_for_Various_Tests_for_Circular_Uniformity	NA
8	*On Stereographic Semicircular New Weibull Pareto Model*	Dr. S.V.S. Girija	Department of Mathematics	IJIRST –International Journal for Innovative Research in Science & Technology	2017	ISSN (online): 2349-6010	http://ijirst.org/index.php	http://www.ijirst.org/articles/IJIRSTV3I11140.pdf	NA
9	*On Wrapping of Exponentiated Inverted Weibull Distribution*	Dr. S.V.S. Girija	Department of Mathematics	IJIRST –International Journal for Innovative Research in Science & Technology	2017	ISSN (online): 2349-6010	http://ijirst.org/index.php	http://ijirst.org/articles/IJIRSTV3I11032.pdf	NA
10	*Generating Function for Ga Spt* $(n)^j$	Dr. K. Hanuma Reddy	Department of Mathematics	International Journal of Latest Technology in Engineering, Management & Applied Science (IJLTEMAS	2017	ISSN 2278-2540	https://oaji.net/journal-detail.html?number=2266	https://www.ijltemas.in/most-viewed-papers/generating-function-ga-sptnj-bar/	NA

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Studies on the Distribution of Particulate Trace Metals like Cadmium and Copper in Visakhapatnam Harbor Waters, India

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I. Introduction

India is a country developing fast industrially and agriculturally. A consequence of the former is urbanization and of the latter, denudation of the forest cover. Both have a telling effect on the quality of the environment. The sea, which represents a regime of the lowest potential energy, receives all types of influxes carrying industrial, agricultural and urban wastes; hence its water quality is affected and thereby its living organisms. Unlike in the case of seas, e.g., the Mediterranean bordering highly industrialized countries, Indian seas are not as much polluted by these effluents, except at certain point locations (a recent review on present status of marine pollution (Qasim & Sen Gupta, 1980). This is a good starting point for evolving and implementing methods of pollution abatement followed by a long-term protection policy of the coastal waters. Usually, many industries are located on river banks, backwaters and harbors etc. , for an easy disposal of their wastes. The harbors in addition to being used as ports are often used as points of discharge of effluents from industries, sewage and agriculture, etc.In recent years there has been a rapidly growing interest on the study of heavy metal content in coastal and harbor, its behavior and the pathways by which they are introduced into the system. Well known that harbors and estuaries of the world are the region heaviest pollution by heavy metals (Forstner and Wittmann, 1981). The physico-chemical processes which act soil of the catchment area normally control the concentration of trace metals in the fluvial and sediments. Due to anthropogenic input, abnormal concentration of heavy metals in both dissolved and particulate phases of the harbor waters. These high inputs can also affect the adjoining coastal waters due to exchange. Addition of these undesirable heavy metals in excess quantities can disrupt the delicate balance which exist between biomass and trace metals. When it exceeds tolerance level, certain species in aquatic organisms will perish. Certain aquatic organisms have the ability to concentrate toxic metals many fold in their body which ultimately passes on to human beings through marine food web causing deleterious effects to human beings.

II. Materials And Methods

Visakhapatnam harbor is divided into 5 stations they are,

Station 1, is located in the centre of the **outer harbor.** The average depth of this station is about 18 meters. A finger type Jetty is situated in this region for loading iron ore into export vessels of as high as 1 lakh DWT.

Station 2, is located in the centre of the **entrance channel**. This is a confluence point between outer harbor and inner harbor. The average depth of this channel is 10.7 m.

Station 3, is located in the centre of the **western arm** (WA, 0.88 km in length, 91 m in width and 10.7 m in depth) of the inner harbor. This is one of the three navigable arms of Visakhapatnam harbor.

Station 4, is located in the centre of the **northwestern arm** (NWA) of the inner harbor, which is the principal source of industrial effluents. A monsoon fed stream. 'Meghadrigedda' opens into this arm on the western side. The average depth of this arm is 10 meters.

Station 5, is located at the end of the **northern arm** (NA) which is the main commercial navigational channel of the inner harbor. The channel is about 152.4 m in width with an average of 10.7 m in depth.

A hired mechanized boat with no laboratory facilities was used for collection of samples. All photometric measurements were carried out on a Shimadzu Spectrophotometer (UV 260) using 1 or 5 cm path length cells. Particulate metals were analyzed on Orion Atomic Absorption Spectrophotometer after proper digestion.

Solvents like acetone, chemicals and acids used in the investigations were all of E-Merck analytical reagent grade. Double distilled water or deionized water was used in the estimation of trace metals and inorganic constituents except for ammonium ion for which ammonia free water was prepared by redistilling distilled water after treatment with potassium persulphate (1.5 g/L) and concentrated sulphuric acid (1 ml/L) was used.

Particulate matter was collected separately on a total of 2 filters at each station and season, one is on 0.45 µm Membrane Millipore HA filter for total suspended particulate metals and other is on GF/F (0.7 µm) for chlorophylls. Millipore filtration apparatus was used for this purpose with a gentle vacuum. The volume of water that was filtered varied from 0.5 to 1.0 liters depending on the turbidity of the sample and its particulate content. All filters were washed with Milli Q water to remove salts and were kept frozen until further analysis. Surface and bottom water samples were collected every month for a period of one year at five different stations in the harbor waters during the year 2012-2013 is shown in the figure 1.Surface waters were collected with a clean plastic bucket and bottom waters were collected Niskin bottom water sampler, they were immediately filtered through Glass fiber GF/F filters papers. The filtered waters are used for the determination of spatial and seasonal distribution of particulate trace metals like Cadmium and Copper in the harbor waters covering all the five stations during the year 2012-2013 for all seasons.



FIG 2.1 VISAKHAPATNAM HARBOUR AND STATION LOCATIONS (St. 1-St. 5)

III. Results And Discussion

Harbor, coastal waters and estuaries act as transition zone in which continental material is trapped and through which some of the material is transported to the open Sea. The chemical composition and variability of suspended particulate matter (SPM) in harbor and coastal water is controlled by complex interplay of physical and biogeochemical processes. Some of the most important geochemical processes influencing variability are mixing of land and riverine suspended matter with marine material, flocculation of colloidal materials (Benoit *et al*., 1994), adsorption and desorption in low salinity zone (Windom *et al*., 1988; Zwolsman*et al*., 1997), production of organic matter by phytoplankton (Balls, 1990; Burton *et al*., 1994), mobilization of Fe-Mn oxides in reducing sediments (Feely *et al*., 1986) and resuspension of bottom sediments (Balls, 1994; Hatje*et al*., 2001). It is not surprising, that studies of metal behavior in polluted harbor and coastal waters have well documented, but there is a substantial difference from one area to another area. Studies of particulate trace metals are usually based on a single, or a small number of surveys. Moreover, the concentration of particulate metals can be variable, both as a result of changing inputs and or seasonal effects involving biological, geochemical and physical interactions.

Numerous studies on the distribution and behavior of trace metals in various coastal and harbor waters in different parts of the world have revealed that individual metals exhibit contrasting behavior between areas. Factors which have been demonstrated to be important in controlling trace metal behavior include flushing time (Morris, 1990; Owens and Balls, 1997).Very few studies have been made on the distribution of trace metals in the Visakhapatnam harbor and coastal waters. Satyanarayana b *et al*., (1985) studied on the distribution of particulate and dissolved metals in Visakhapatnam harbor waters. Sen Gupta *et al*., (1978) studied the trace metal analysis in the Arabian Sea water. Sanzgiri and Mores (1979) Studied the trace metal distribution in the

Laccadive Sea. Braganca and Sanzgiri (1980) studied the concentrations of few trace metals in coastal and offshore regions of the Bay of Bengal. Rajandran *et al*., (1982) studied the dissolved and particulate trace metals in western Bay of Bengal. Jegatheesan and Venugopalan (1973) studied the trace elements in the particulate matter of Porto Novo waters.

Information on the distribution of metals in polluted harbor like Visakhapatnam harbor is essential to assess their involvement in the biogeochemical cycles, their possible accumulations in organisms, and their transfer to human beings through food chain. So far there are single detailed seasonal and spatial studies of trace metal distribution in the Visakhapatnam harbor waters, the author has therefore been taken up the present investigation to assess the spatial and seasonal distribution of particulate trace metals like Cadmium and Copper in the harbor waters along with hydrological and nutrient constituents covering the five stations during the year 2012-2013.

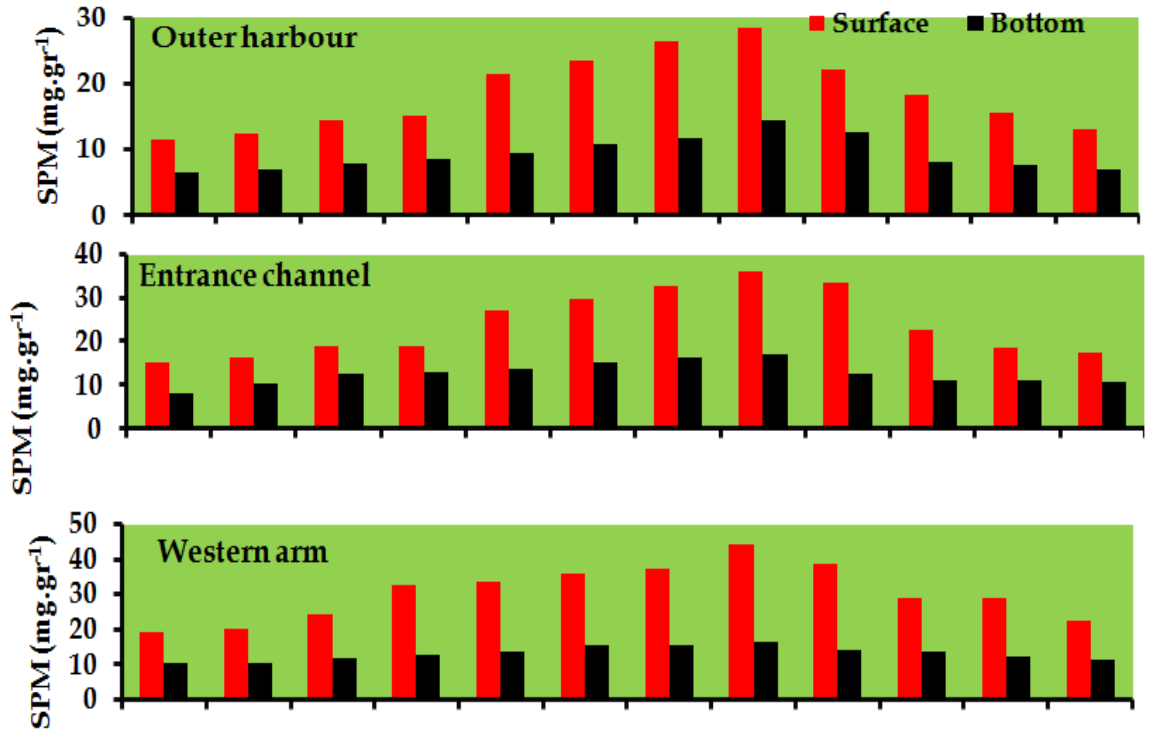
TOTAL SUSPENDED PARTICULATE MATTER

The station-wise summary statistics on total suspended particulate matter (TSPM) in the Visakhapatnam harbor waters during the study period are given in Table 1 The detailed seasonal distribution at five individual all stations are shown in Fig.1.

Table 1 Station – wise summary statistics of SPM (mg.gr^{-1}) in the Visakhapatnam harbor waters during 2012-2013

Station	Surface				Bottom			
	Min.	Max.	Mean	S.D.±	Min.	Max.	Mean	S.D.±
Outer harbor	11.15	28.41	18.63	5.70	6.58	14.65	9.42	2.60
Entrance channel	15.25	36.13	23.94	7.47	8.22	17.25	12.68	2.63
Western arm	19.21	44.29	30.54	7.89	10.23	16.53	13.19	2.07
Northwestern arm	24.57	65.85	40.28	12.96	10.25	21.15	14.52	3.25
Northern arm	21.16	42.41	28.86	6.63	10.91	24.28	16.62	4.67

The TSPM values in the surface waters varied from 11.55 to 65.85mg.gr^{-1} with an average of 28.45mg.gr^{-1} where as in the bottom waters, the TSPM varied from 6.58 to 24.28mg.gr^{-1}with an average of 13.29mg.gr^{-1}.In general, total suspended particulate matter showed relatively higher values in surface when compared with those in bottom waters. This may be due to the land runoff associated with biological and anthropogenic activity. Seasonally higher TSPM values (65.85 mg.gr^{-1}) were observed in the northwestern arm (St.4) during monsoon, which may be due to higher land runoff along with turbulent flow containing higher amount fluvial suspended matter at this station and deceased seaward by order of magnitude.



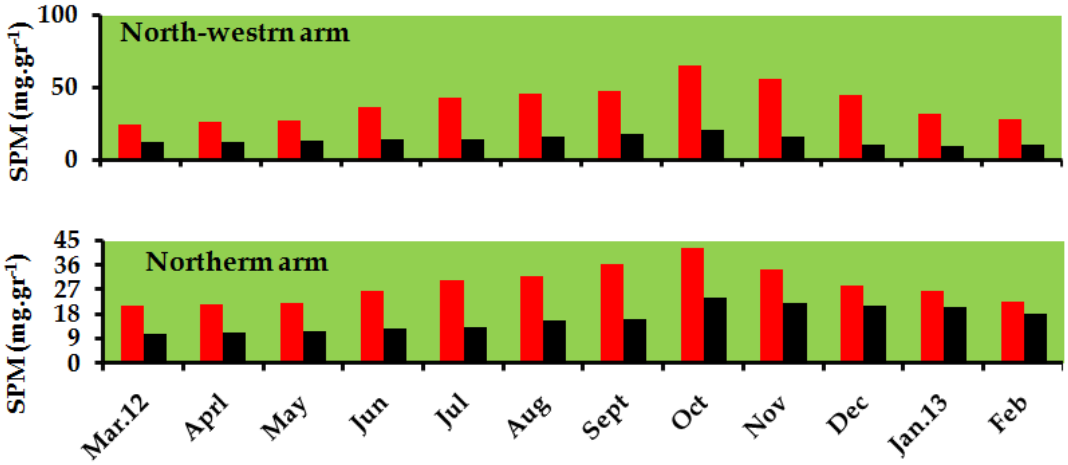


Fig. 1 Monthly distribution of Particulate Suspended Matter in the Visakhapatnam harbor waters during March 2012 to February 2013 (St. 1 - St. 5)

The present study reflects the decrease in suspended particulate matter with increase in salinity and this trend simply denotes the increase in the marine component. It might be argued that the decrease in particulate matter contents is also caused by desorption process. It follows that the decrease in the particulate matter with increasing salinity is primarily caused by mixing of fluvial matter with marine particulates.

Relation of suspended particulate matter with pH:

Significant negative correlations were observed between suspended particulate matter with pH of the harbor waters ($r = -0.87$, $p < 0.001$) is shown in Fig.2 From the above figure, it was observed that the concentration of TSPM decreases with increasing in pH of the harbor waters due to its flocculation at the outer harbor.

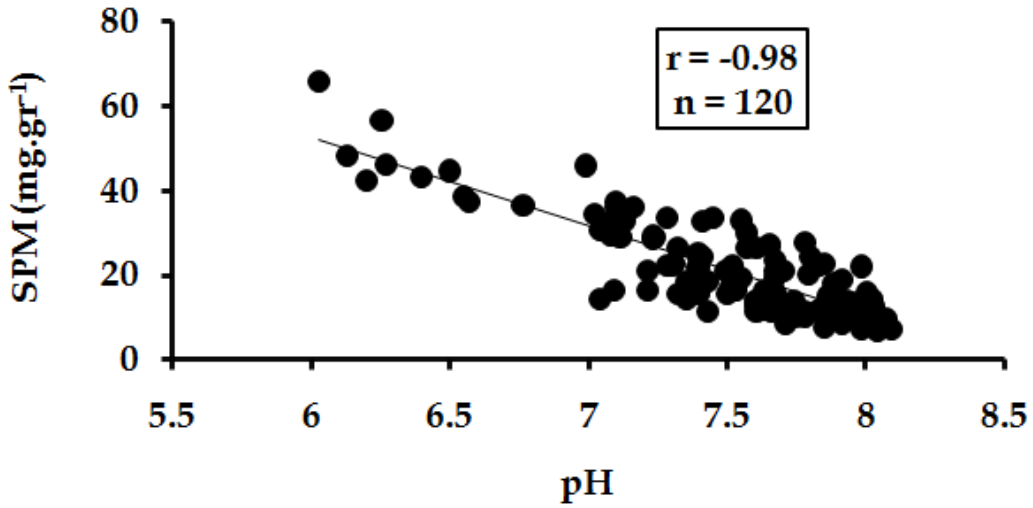


Fig. 2. Relation between TSPM with pH in the Visakhapatnam harbor waters during 2012-2013.

An increase in pH was observed during summer and this may be due to associated with photosynthesis of plank tonic blooms. The lower pH values were observed during monsoon season due to the higher fresh water along with industrial and domestic sewage.

Relation of suspended particulate matter with salinity:

Significant negative relationship was observed between suspended particulate matter with salinity ($r = -0.72$, $p < 0.001$) in the present study is shown in Fig. 3. The decrease of suspended particulate matter with increasing salinity is primarily caused by mixing of fluvial matter with marine waters, and also caused by desorption of suspended particulate matter at higher salinities in the outer harbor area.

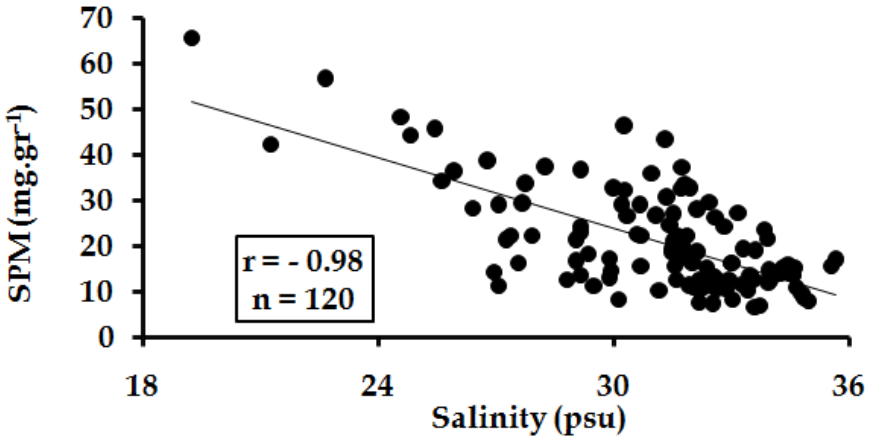


Fig. 3. Relation between TSPM with Salinity in the Visakhapatnam Harbor waters during 2012-2013

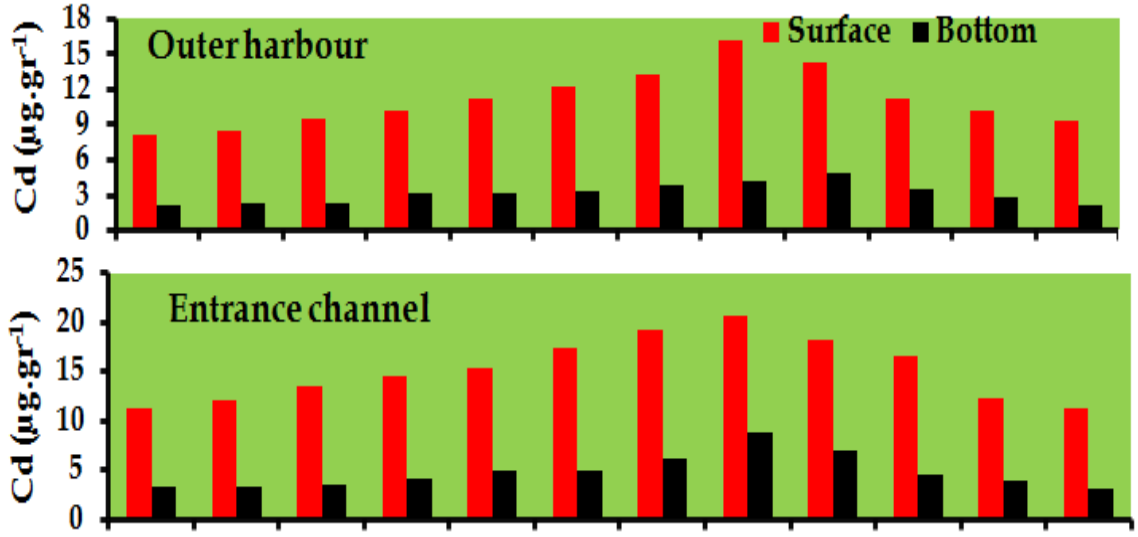
PARTICULATE CADMIUM

The station-wise summary statistics on particulate cadmium in the Visakhapatnam harbor waters during the study period was given in Table 2 The detailed seasonal distribution at five individual stations in both surface and bottom waters are shown in Fig. 4 The particulate cadmium concentrations in the surface waters were in the range of 8.12 to 58.32 µg.gr^{-1} with an average of 21.48 µg.gr^{-1} where as in the bottom waters, its concentrations ranged from 2.15 to 17.85 µg.gr^{-1} with an average of 6.91 µg.gr^{-1}. Higher concentrations of particulate cadmium were observed in the surface waters than that of bottom waters in all five stations.

Table 2 Station – wise summary statistics of particulate cadmium (µg.gr^{-1}) in the Visakhapatnam harbor waters during 2012-2013

Station	Surface				Bottom			
	Min.	Max.	Mean	S.D.±	Min.	Max.	Mean	S.D.±
Outer harbor	8.12	16.25	11.36	2.46	2.15	4.86	3.20	0.89
Entrance channel	11.26	20.56	15.59	3.22	3.20	8.85	4.96	1.76
Western arm	14.22	36.34	23.55	6.67	3.25	12.65	7.74	3.14
Northwestern arm	18.25	58.32	25.23	12.80	8.22	17.85	12.16	3.01
Northern arm	13.11	38.99	23.43	8.42	5.37	10.25	7.33	1.66

Particulate cadmium concentrations in the present study decreased from the north-western arm (St. 4) to outer harbor (St. 1) which may be due to desorption/dilution and solubilization of particulate metals as salinity increases. This process could be expected in harbor/estuaries due to chloride and sulphate complexation and ionic strength effects.



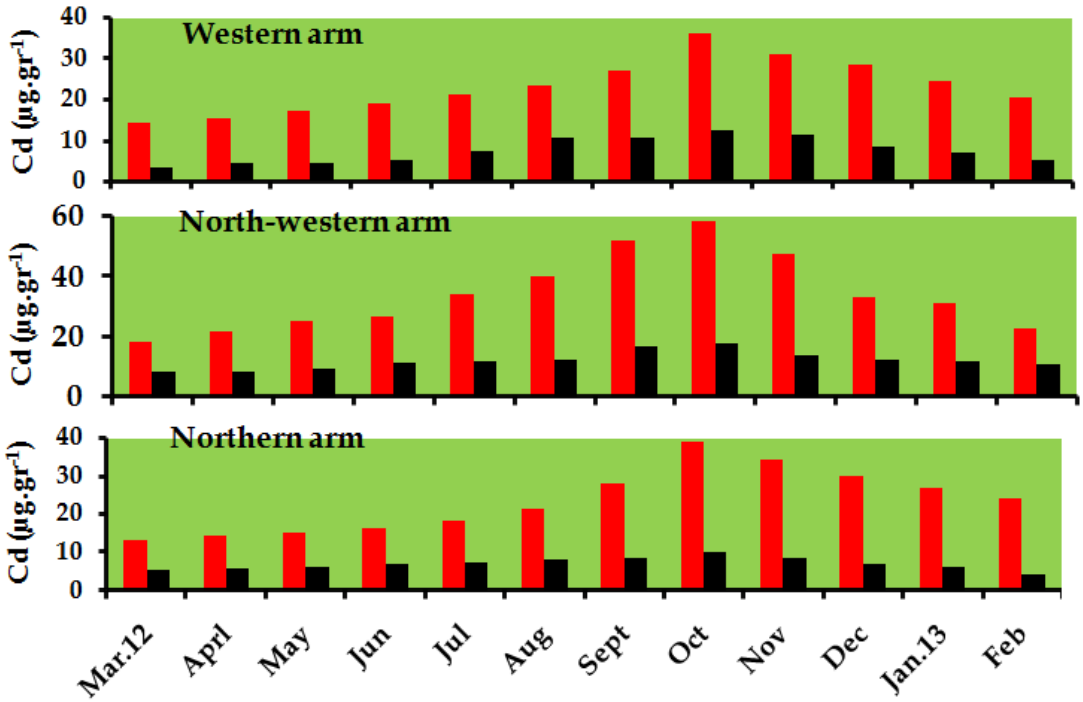


Fig. 4. Monthly distribution of Particulate Cadmium in the Visakhapatnam harbor waters during March 2012 to February 2013 (St.1 - St.5).

Similar behavior of the cadmium was also observed by Coman Van Dijk (1988) during estuarine mixing. Cadmium mobilization is well documented in estuaries in the Amazon plume (Boyle *etal*., 1982); in the ChangJiang estuary (Edmond *et al.*, 1985; in the lower part of the Scheldt estuary (Duinker *etal*., 1982; Salomons and Kerdijk, 1986), in the Gironde and Huanghe estuaries. (Elbaz-poulichet *etal*., 1987), and in the Mississippi estuary (Shiller and Boyle, 1991)

Relation of particulate cadmium with pH

Significant inverse correlations were observed between the particulate cadmium with pH in the Visakhapatnam harbor (r=-0.99, p <0.001), is shown in Fig. 5. It was observed that the particulate cadmium concentrations are decrease with increase in pH due to dilution of inner harbor waters with sea water at the outer harbor waters. Theses outer harbor waters is always contacted with coastal waters of Bay of Bengal.

Similar relationships between pH and particulate cadmium have been observed in the Scheldt estuary (Salomons *et al*., 1981; Duinker *et al.*, 1982. Regnier and Wollast, 1993), and also estuaries in Europe and USA, (Olsen *et al.*, 1989; Schoer 1990; Turner *et al.*, 1991; Paalman and Vander Weijden, 1992).

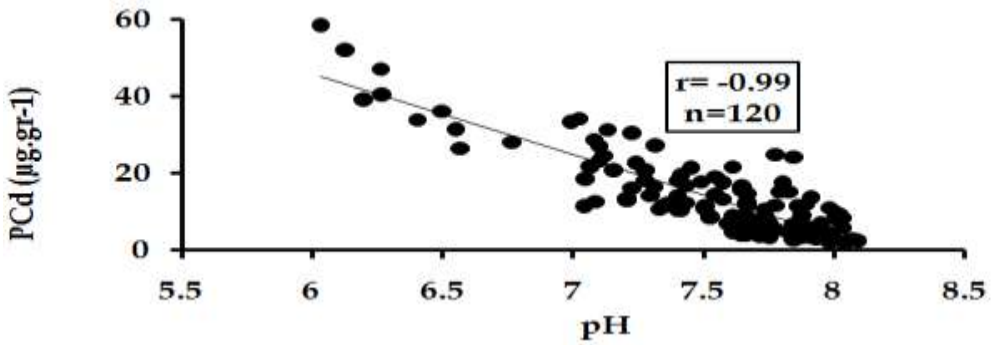


Fig. 5. Relation between pH and Particulate Cadmium in the Visakhapatnam harbor waters during 2012-2013.

Relatively higher pH values in the harbor waters were observed during pre-monsoon (summer) may be due to the influence of marine water intrusion and also by photosynthetic activity by plankton blooms during this period. The decrease of pH was also observed with increasing distance from the outer harbor is due to release of acidic industrial effluents into the inner harbor waters. The influence of ionic strength on the dissociation constants of carbonic acid also contributes to the pH drop in the inner harbor waters which are more acidic in some times.

Relation of particulate cadmium with salinity

Significant inverse correlations were observed between the particulate cadmium with salinity in the harbor waters ($r = -0.99$, $p < 0.001$), is shown in Fig. 6.

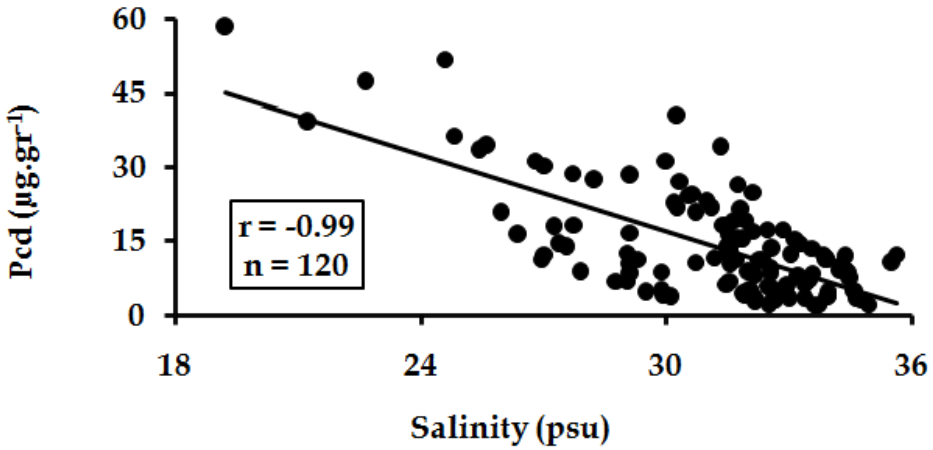


Fig.6. Relation between particulate cadmium with salinity in the Visakhapatnam harbor waters during 2012-2013.

From the above figure it was observed that the concentrations of cadmium decrease with increase in salinity. It is well known that with increase in salinity there is a greater desorption and solubilization of particulate metals. Zwolsman *et a*l., (1993) mentioned that there is overwhelming evidence, both from laboratory experiments and field investigations; cadmium bound onto suspended matter is (partially) desorbed when fresh water mixes with seawater due to formation of chloro-complexes (Turner *et al*., 1981; Comans and Van Dijk, 1988). Several field studies on the chemistry of dissolved cadmium in the Scheldt have confirmed this hypothesis (Kerdijk and Salomons, 1981; Duinker *et al*., 1982; Zwolsman and Van Eck, 1990; Zwolsman *et al*., 1993).

Seasonally higher concentrations particulate cadmium were observed during monsoon and post-monsoon seasons, due to influx of domestic sewage and water along with industrial, effluents. Lower concentrations were observed during pre-monsoon season may be due to desorption and solubilization of particulate cadmium as salinity increased.

PARTICULATE CHROMIUM

Chromium concentration in natural waters is usually very small. Elevated concentrations can result from industrial and mining process (Datar and Vashishtha, 1990). Chromium in water supplies is generally found in the hexavalent form (Thomas, 1963).

The station-wise summary statistics on particulate chromium in the Visakhapatnam harbor waters during the study period was given in Table 4.3. The detailed monthly distribution chromium at five individual stations in both surface and bottom waters are shown in Fig. 7.

Table: 3. Station – wise summary statistics of particulate chromium (µg.gr^{-1}) in the Visakhapatnam harbor waters during 2012-2013

Station	Surface				Bottom			
	Min.	Max.	Mean	S.D.±	Min.	Max.	Mean	S.D.±
Outer harbor	24.12	37.45	31.15	4.96	20.12	28.64	24.07	2.67
Entrance channel	26.81	40.98	33.75	4.24	21.21	31.25	26.01	3.03
Western arm	28.52	43.35	36.16	4.37	22.32	38.21	31.10	5.48
Northwestern arm	35.21	71.58	49.24	11.42	24.32	61.54	41.15	11.66
Northern arm	30.23	59.85	43.36	9.30	23.25	44.25	34.18	6.85

The particulate chromium concentrations in the surface waters of the Visakhapatnam harbor were in the range of 24.12 to 71.58 µg.gr^{-1} with an average of 28.66 µg.gr^{-1} where as in the bottom waters, its concentrations ranged from 20.12 to 61.54 µg.gr^{-1} with an average of 31.30 µg.gr^{-1}. Higher concentrations of particulate chromium were observed in the surface waters than that of bottom waters in all stations.

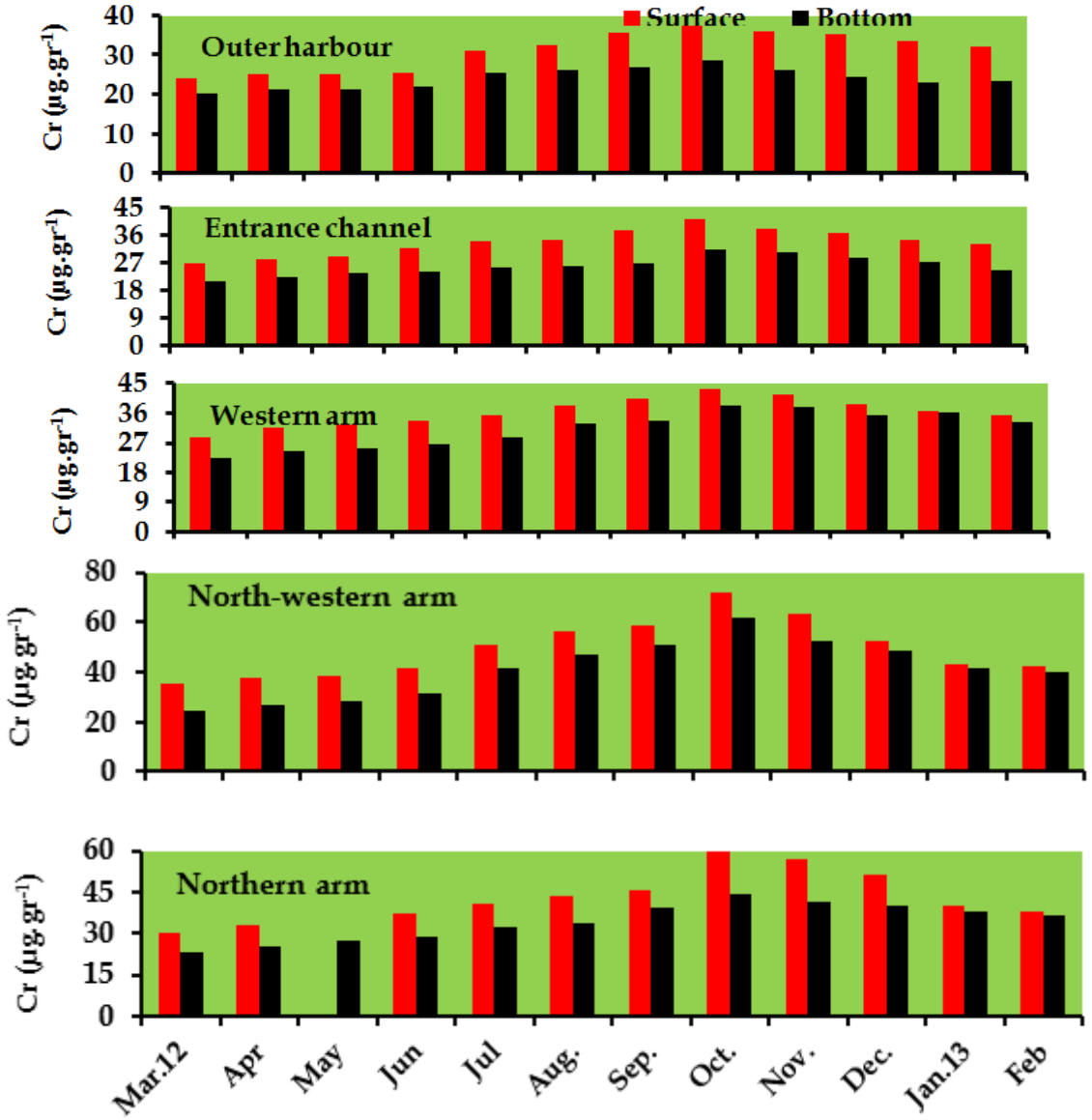


Fig. 7. Monthly distribution of Particulate Chromium in the Visakhapatnam harbor waters during March 2012 to February 2013 St.1 - St.5).

The concentrations of chromium observed in the present study are higher than river, estuarine and coastal waters reported elsewhere. Chromium distribution is well documented in the Amazon plume (Boyle *et al.*, 1982), in the Changjiang (Edmond *et al.*, 1985; Elbaz-Poulichet *et al.*, 1987), in the Gironde and Huanghe estuaries (Kraepiel *et al.*, 1997), in the Mississippi estuary (Shiller and Boyle, 1991) and in the lower part of the Scheldt estuary (Duinker *et al.*, 1982; Salomons and Kerdijk, 1986), in the surface waters off Pondicherry (Solai *et al.*, 2010), in the waters of Cochin estuary, and adjoining Periyar and Muvattupuzha rivers, in the surface waters of Ennore estuary (Rajkumar *et al.*, 2011) and in the estuarine waters of Uppanar, Nagapattinam, India (Sankar *et al.*, 2010).Seasonally, higher values were observed during monsoon season.

Relation of particulate chromium with pH

Significant inverse correlations were observed between the particulate chromium with pH in the Visakhapatnam harbor waters ($r=-0.99$, $p<0.001$), is shown in Fig. 8

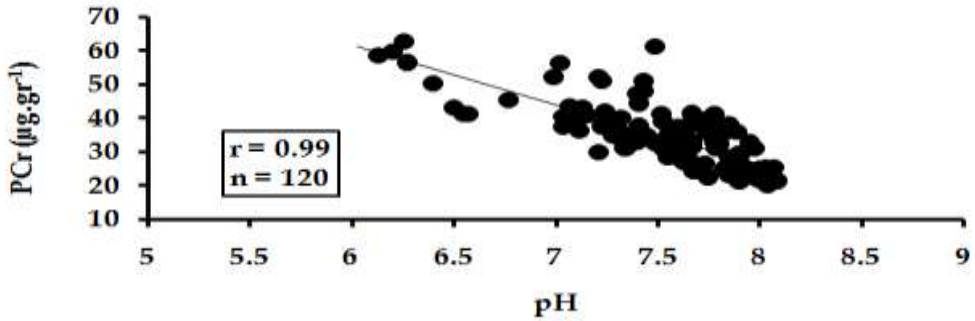


Fig. 8. Relation between pH and Particulate Chromium in the Visakhapatnam harbor waters during 2012-2013.

It was observed that the particulate cadmium concentrations are decrease with increase in pH due to dilution of highly polluted inner harbor waters with less polluted outer harbor waters. Similar relationships between pH and particulate cadmium have been observed in the Scheldt estuary. Relatively higher pH values in the Visakhapatnam harbor waters were observed during pre-monsoon (summer) may be due to the influence of marine water intrusion and also by photosynthetic activity by plankton blooms during this period.

The decrease of pH was also observed with increasing distance from the inner harbor. The influence of ionic strength on the dissociation constants of carbonic acid also contributes to the pH drop in the inner harbor waters and also discharges of acidic industrial effluents into the inner harbor stations.

Relation of particulate chromium with salinity

Significant inverse correlations were observed between the particulate chromium with salinity in the Visakhapatnam harbor waters ($r = --0.99$, $p < 0.001$), is shown in Fig. 9

From the above figure it was observed that the concentrations of chromium decrease with increase in salinity. It is well known that with increase in salinity there is a greater desorption and solubilization of particulate metals

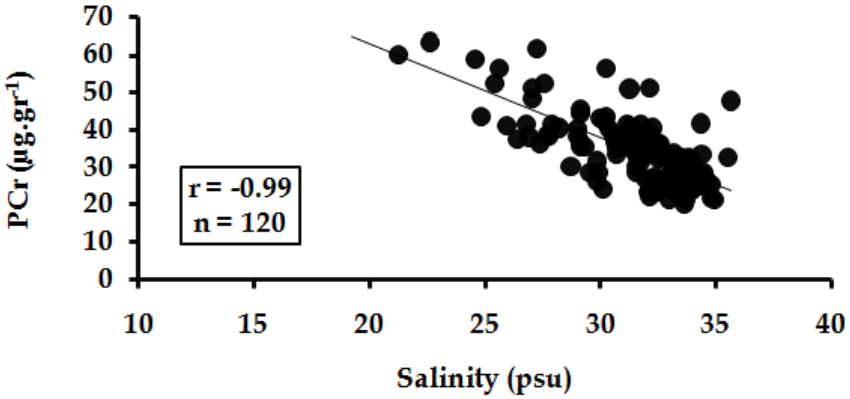


Fig. 9. Relation between particulate chromium with salinity in the Visakhapatnam harbor waters during 2012-2013

Similar relationships between salinity and particulate chromium have also been observed in the estuarine waters of Scheldt (Salomons *et al.*, 1981; Duinker *et al.*, 1982. Regnier and Wollast, 1993) and in many estuaries in Europe and USA, (Olsen *et al.*, 1989; Schoer 1990; Turner *et al.*, 1991; Paalman and Vander Weijden, 1992), in the estuarine waters of Gironde (Krapiel *et al.*, 1997), It is well known that these behaviors are primarily related to physical mixing of fluvial material with marine particulates at the mouth of the estuaries and also caused by flocculation and desorption of particulate metals into the bottom sediments (Zwolsman *et al.*, 1999; Michel *et al.*, 2000; Tang *et al.*, 2002; Abdullah, 2008).

Seasonally higher concentrations particulate chromium were observed during monsoon and post-monsoon seasons, due to influx of high amounts industrial effluents and release of domestic sewage in to the harbor waters are more. Lower concentrations were observed during pre-monsoon season may be due to desorption and solubilization of particulate chromium as salinity increased in the waters of outer harbor is always connected to the coastal waters of Bay of Bengal.

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References

[1]. Abdullah, M. (2008). Trace metal behavior in Mediterranean-Climate coastal Bay: El-Mex Bay, Egypt and its coastal environment. Glob. J. Environ. Res., 2 : 23-29.

[2]. Byrd T. James., Kwang W Lee., Dong S Lee., Ralph G Smith., (1990). The behavior of trace metals in Geum estuary, Korea. Estuaries 13: 8 – 13.

[3]. Caetano, M., Carlos Vale, Manuela Falcao., (2006). Particulate metal distribution in Guadiana estuary punctuated by flood episodes. Estuar., Coast. Shelf. Sci., 70: 109-116.

[4]. Forstner, U. and Wittmann, G.T.W. (eds.) (1983). In: Metal pollution in the aquatic environment. Springer-Verlag, New York, 3-318.

[5]. Forstner, U. (1987). Sediments associated contaminants- an overview of scientific bases for remedial options. Hydrobiol.,149 : 221-246

[6]. Morris, A.W. and Riley, J.P. 1963. Determination of nitrate in seawater. *Anal. Chem. Acta., 29 : 272-279.*

[7]. Murphy, J. and Riley, J.P. 1962. A modified single solution method for the determination of phosphate in natural waters. *Anal, Chem. Acta., 29 : 31-36.*

[8]. Parsons, T.R., Mita, Y. and Lalli, C.M. 1984. A manual of chemicals and biological methods for seawater analysis. Pergamon Press., 101-111.

[9]. Richards FA & Thomson T.G. 1952. The estimation and characterization of plankton population by pigment analysis. II. A spectrophotometric method for the estimation of plant pigment. *J. Mar Res*, 11: 156-172.

[10]. Strickland, J. D.H. and Parsons, T.R., 1963. Discussion of Spectrophotometric determination of marine –plant pigments with revised equation for ascertaining and Carotiniods. *J. Mar. Res*, 21, 155-163.

[11]. Strickland, J.D.H. and Parsons, T.R.,1972. A Practical handbook of seawater analysis. *Fish. Res. Board of Canada Bull., 167 (2nd Edn.), 295-300.*

[12]. Senthilnathan, S., Balasubramanian, T., (1997) Distribution of heavy metals in estuaries of southeast coast of India. Ind. J. Mar. Sci., 26: 95-97.

[13]. Sarma, V.V.S.S., Gupta, S. N. M., Babu, P. V. R., Acharya, T., N. Harikrishnachari, K. Vishnuvardhan, N. S. Rao, N. P. C. Reddy, V. V. Sarma, Y. Sadhuram, T. V. R. Murty, M. D. Kumar. (2009). Influence of river discharge on plankton metabolic rates in the tropical monsoon driven Godavari estuary, India. Cont. Shel. Res., 85 : 515-524.

[14]. Zwolsman, J. J. G. and Van Eck, G.T.M. (1990). The behavior of dissolved Cd, Cu and Zn in the Scheldt estuary. In: Michaelis, W. (ed.), Estuarine Water Quality Management. Springer—Verlag, Berlin, pp. 413 - 420.

[15]. Zwolsman, J. J. G., van Ech, G. T. M., (1993). Dissolved and particulate trace metal geochemistry in the Scheldt estuary , South West Netherlands water column sediments. Aquat. Ecol., 27: 287- 300.

[16]. Zwolsman, J. J. G., Van Eck, B.T. M. and Vander Weijden, C. H. (1997). Geochemistry of dissolved trace metals in the Scheldt Estuary, Southwestern Netherlands: Impact of seasonal variability.

[17]. Geochem. Coschm. Acta 61 : 1635-1652.

[18]. Zwolsman, J. J. G., Van Eck, B.T. M. (1999). Geochemistry of major elements and trace metals in suspended matter of the Scheldt Estuary, Southwestern Netherlands. Mar. Chem., 66 : 91-111.

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Distribution of Particulate Trace Metals like Iron and Nickel in the Estuarine Waters of River Godavari, East Coast of India.

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Abstract: *Godavari is the third largest river on the east coast of India drains in to the Bay of Bengal between the Latitudes 16°.15' to 16°.45' N and longitudes 81°.45' to 82'.25' E, discharging large quantities of fresh water in to the coastal waters of Bay of Bengal particularly during monsoon season. Suspended particulate matter (SPM) showed an increasing trend from pre-monsoon to monsoon followed by post-monsoon season in the study area. High concentration of SPM observed during monsoon can be attributed to the high river discharge and land runoff containing significant quantities of terrestrial material into to the estuary. Distinct spatial and seasonal variations of particulate trace metals were observed. Information on the distribution of metals in the monsoon driven estuary like Gauthami-Godavari is essential to assess their involvement in the biogeochemical cycles, their possible accumulations in organisms, and their transfer to human beings through food chain. So far there are no detailed seasonal and spatial studies of trace metal distribution in the estuarine waters of Godavari; the author has therefore been taken up the present investigation to assess the spatial and seasonal distribution of particulate trace metals like Iron -and Nickel in the estuarine waters of Gauthami-Godavari.*

Keywords*: Godavari Estuarine waters, Particulate matter, trace metals, Flocculation.*

Date of Submission: 08-09-2017 Date of acceptance: 22-09-2017

I. Introduction

The river Godavari receiving the lot of chemicals from the industries located on its banks and also from the agricultural back runoff and from the domestic sewage. All these materials are traveled from the head of the river and settling at the river mouth and dispersed into the sea during flood time of the river. The Godavari is the largest river in south India, largest estuary in the Central East coast of India. The Gauthami-Godavari estuary is a drowned river mouth type of estuary ($16^0 41´$ - $16^0 56´$N, $81^0 45´$ - $82^0 21´$E), and is characterized by numerous islands and creeks that are separated from the main channel by sand bars (Narasimha Rao., 2001). It is having a drainage basin of about 3 x 10^5 Km^2 (Rao 1975) and has a mean annual discharge at the delta apex(Dowlaiswaram) of 1.05 x 10^{14} liters.In recent years there has been a rapidly growing interest on the heavy metallic content in estuaries and on the nature and the pathways by which they are introduced into the system. Due to anthropogenic input, abnormal concentration of heavy metals in both dissolved and particulate phases result. These high inputs can also affect the adjoining coastal waters due to exchange. Addition of these undesirable heavy metals in excess quantities can disrupt the delicate balance which exist between biomass and trace metals. The author has therefore undertaken a systematic study of the distribution of particulate trace metals like Iron and Nickel along with hydrographical and nutrient parameters in the estuarine waters of the Gauthami Godavari over a period of one year during 2013.

II. Materials And Methods

Gauthami-Godawari estuary is divided into 7 stations they are,

Station 1, is located in the centre of the stretch of Gauthami Godavari at **Kotipalle**, head of the estuary. It is 41 km away from the mouth of the estuary and minimum tidal action is there. The water is mostly fresh water (less saline). Both sides of the river have villages and have a extensive cultivated agricultural lands. During monsoon season anthropogenic inputs from these villages and agricultural lands are more in to the river.

Station 2, is located in the center of the river Gauthami Godavari at **Dangeru**, 8 km down to the Kotipalle. Both sides have villages and agricultural lands. Fishing activity is also there. The average depth of the station is 8 m.

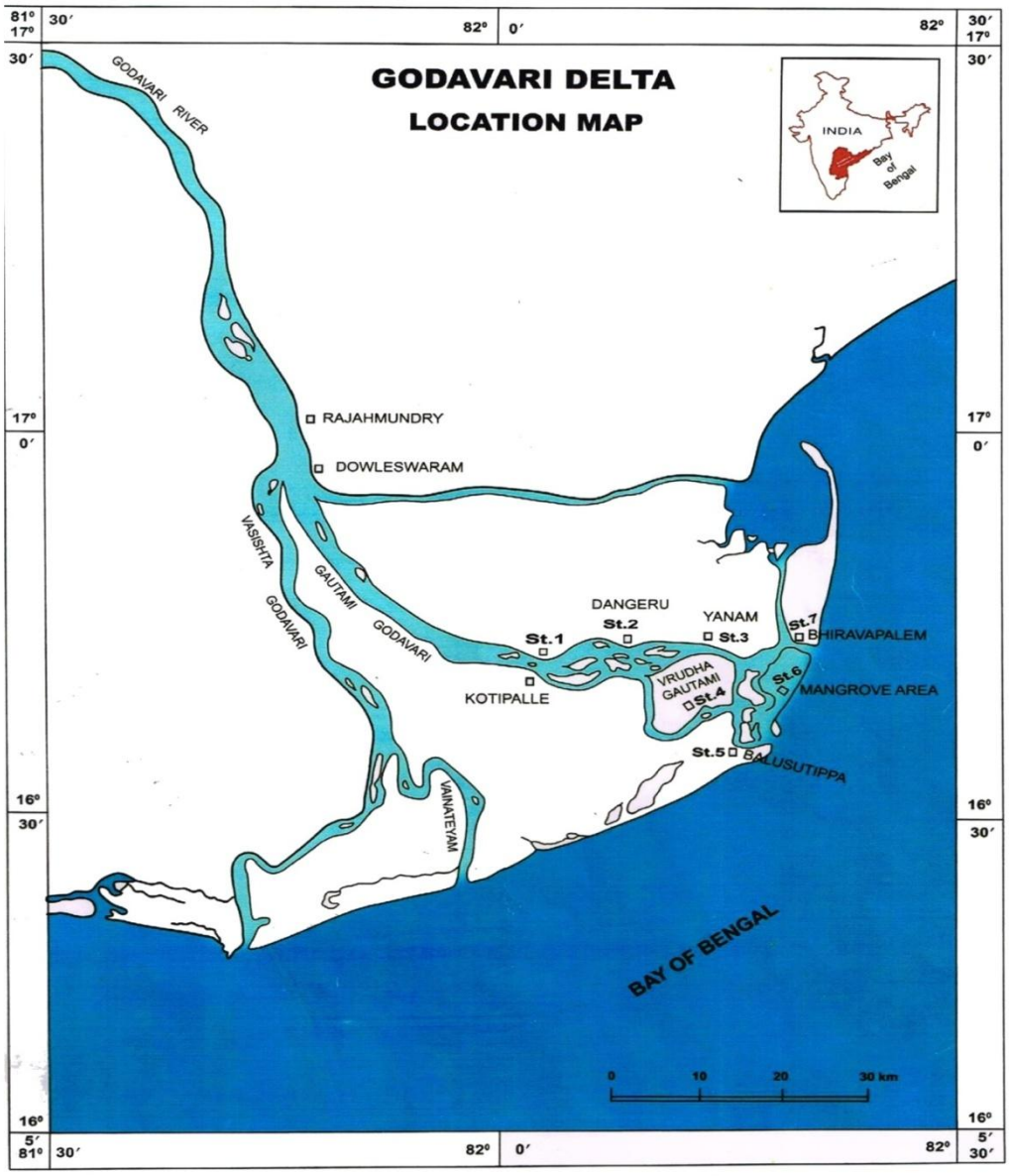
Station 3, is located in the center of the estuary of Gauthami Godavari at **Yanam**, 22 km away from the mouth of the estuary. Yanam is the moderately populated town situatednorthern side of the Gautami Godavari. Most of the domestic effluents are discharging into the estuary. Many fishing boats are anchored as called boat jetty in the estuary. The tidal influence is more and average depth this station is 12 m.

Station 4, is located in the centre of the river at **Vrudha Gautami** of the stretch of Gautami Godavari, 5 km down to Yanam. There are many creeks are enter into this area. The average depth of the station is 6 m.

Station 5, is located at the centre of the river channel at **Balusutippa**. It is 12 km away from the mouth of the estuary. The surrounding areas are fish and shrimp farms, they release appreciable amounts of their effluents into the near creeks and finally come to the main estuary. From this station onwards, mangrove forest are there in both side of the estuary and the average depth of the station is 5 m.

Station 6, is located at the centre of the creek of the **mangrove forest area**. Dense mangrove forest is there at both side of the creek. The average depth of the creek is 4 m.

Station 7, is located at the centre of **the mouth of the estuary**, it is connected to the coastal waters of the Bay of Bengal, relatively very clean waters are prevailed. North side of this station oil industrial activity is there and southern side is mangrove forest. The average depth of this station is 8 m.



III. Results and Discussion

Harbor, coastal waters and estuaries act as transition zone in which continental material is trapped and through which some of the material is transported to the open Sea. The chemical composition and variability of suspended particulate matter (SPM) in harbor and coastal water is controlled by complex interplay of physical and biogeochemical processes. Some of the most important geochemical processes influencing variability are mixing of land and riverine suspended matter with marine material, flocculation of colloidal materials (Benoit *et al*., 1994), adsorption and desorption in low salinity zone (Windom *et al*., 1988; Zwolsman *et al*., 1997), production of organic matter by phytoplankton (Balls, 1990; Burton *et al*., 1994), mobilization of Fe-Mn oxides in reducing sediments (Feely *et al*., 1986) and re-suspension of bottom sediments (Balls, 1994; Hatje*et al*., 2001). It is not surprising, that studies of metal behavior in polluted harbor and coastal waters have well documented, but there is a substantial difference from one area to another area. Studies of particulate trace metals are usually based on a single, or a small number of surveys. Moreover, the concentration of particulate metals can be variable, both as a result of changing inputs and or seasonal effects involving biological, geochemical and physical interactions.

Numerous studies on the distribution and behavior of trace metals in various coastal and harbor waters in different parts of the world have revealed that individual metals exhibit contrasting behavior between areas. Factors which have been demonstrated to be important in controlling trace metal behavior include flushing time (Morris, 1990; Owens and Balls, 1997).Very few studies have been made on the distribution of trace metals in the Visakhapatnam harbor and coastal waters. Satyanarayana b *et al*., (1985) studied on the distribution of particulate and dissolved metals in Visakhapatnam harbor waters. Sen Gupta *et al*., (1978) studied the trace metal analysis in the Arabian Sea water. Sanzgiri and Mores (1979) studied the trace metal distribution in the Laccadive Sea. Braganca and Sanzgiri (1980) studied the concentrations of few trace metals in coastal and offshore regions of the Bay of Bengal. Rajandran *et al*., (1982) studied the dissolved and particulate trace metals in western Bay of Bengal. Jegatheesan and Venugopalan (1973) studied the trace elements in the particulate matter of Porto Novo waters.

3.1 PARTICULATE IRON

The station-wise summery statistics on particulate iron in the estuarine waters of the Gautami Godavari during the study period was given in Table 1. The detailed seasonal distribution at the seven individual stations was shown in Fig. 1

Table 1 Station-wise summary statistics on particulate iron (µg.gr^{-1}) in the estuarine water of Gauthami Godavari during 2013

Station	Surface				Bottom			
	Min.	Max.	Mean	S.D.±	Min.	Max.	Mean	S.D.±
Kotipalli	540.50	4788.50	1944.08	1926.87	423.12	2892.25	1207.52	1135.55
Dangeru	282.25	3816.23	1594.79	1515.91	310.27	2058.26	907.69	788.76
Yanam	482.85	3816.23	1594.79	1515.91	310.27	2058.26	907.69	788.76
V.Godavari	477.26	2265.23	1166.37	784.50	302.65	1987.68	835.01	779.71
Balusutippa	389.25	2823.12	1236.67	1088.73	202.15	1635.29	709.57	639.42
Mangrove area	361.75	2777.27	1199.46	1084.42	200.65	598.85	368.83	186.97
Bhiravapalem	343.25	2013.32	915.525	757.60	186.68	1135.26	504.68	430.22

The particulate iron concentrations in the surface waters were in the range of 343.25 to 4788.80 µg.gr^{-1}with an average of 1442.26 µg.gr^{-1}where as in the bottom waters, its concentrations ranged from 186.68 to 2892.30 µg.gr^{-1} with an average of 843.91 µg.gr^{-1}. In general, higher concentrations of particulate iron were observed during monsoon season and were followed by post-monsoon season and pre-monsoon season. Higher concentrations of iron were observed during monsoon season is due to the combined effect of land and river runoff along with anthropogenic inputs of iron. Generally the pH of rain water is lower in comparison to river water average and at lower pH, the rain water dissolves more iron from the rocks of earth surface (Charlson and Rodhe, 1982); Sundaray, 2007), which contributed more particulate/dissolved iron to the river water. This phenomenon is also observed for the Brahamani River by Rath, (2003). Lower values of particulate iron were observed during premonsoon season due to its utilization by plankton, which are more during this season, and precipitation of iron under alkaline pH condition of the estuary (Stumm and Morgan, (1970) and flocculation of iron with dissolved organic matter may also support the lower concentrations of iron during this season.

Similar seasonal variations of iron were also reported by Fang and Lin (2002) in the Tanshui estuary in Northern Taiwan, Zhou *et al*., (2003) in the waters of Conwy estuary, North Wales, Caetano *et al*., (2006) in Guadiana estuary, Beck *et al*., (2010) in a shallow subterranean estuary, South Bay, New York, Sundaray*et al*., (2011) in Mahanadi river estuarine system, India. Raju *et al*., (2013) in the waters of Cauvery river basin.

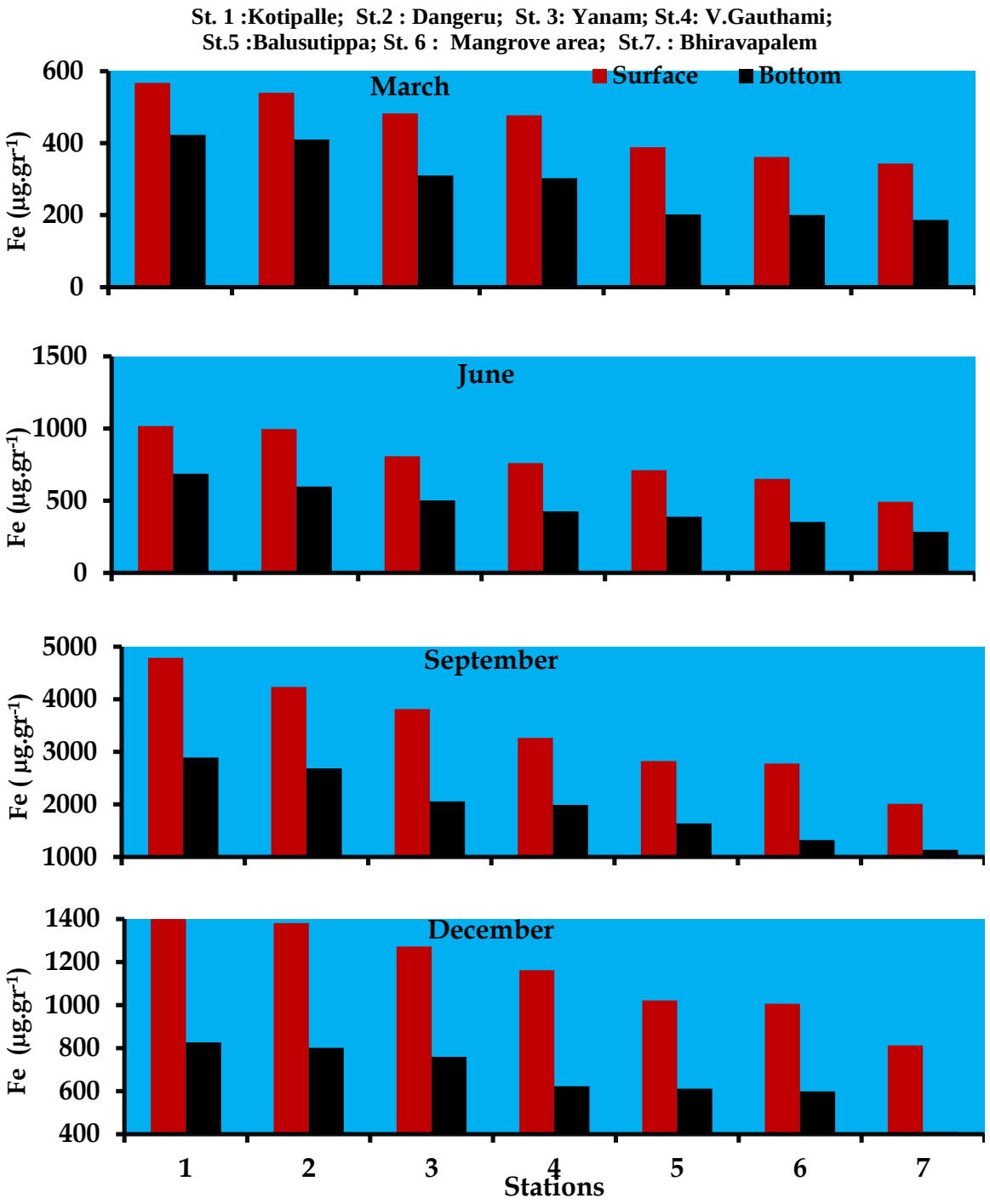


Fig. 1 Seasonal and spatial distribution of particulate Iron in the estuarine waters of the Gauthami Godavari during 2013.

3.2 Relation of particulate Iron with pH:

Significant inverse correlations were observed between the particulate iron with pH in the waters of the Gauthami Godavari ($r = -0.89$, $p < 0.001$) is shown in Fig. 2.Windom *et al*., 1988 stated that iron removed in the Bang Pakong estuary at Thailand is explained by the decrease the iron solubility at higher pH encountered at higher salinities. The concentration of particulate iron in the present study decreases linearly with increase in pH. It may be attributed due to the large scale rapid removal of iron from river water during estuarine mixing. The river-borne iron consists almost entirely of mixed oxide-organic matter which is stabilized by the organic matter. It is colloidal in nature and the particle size is reported to be less than 0.45 µM. Precipitation occurs on estuarine mixing because the cations present in sea water neutralize the negatively charged iron bearing colloids allowing flocculation.

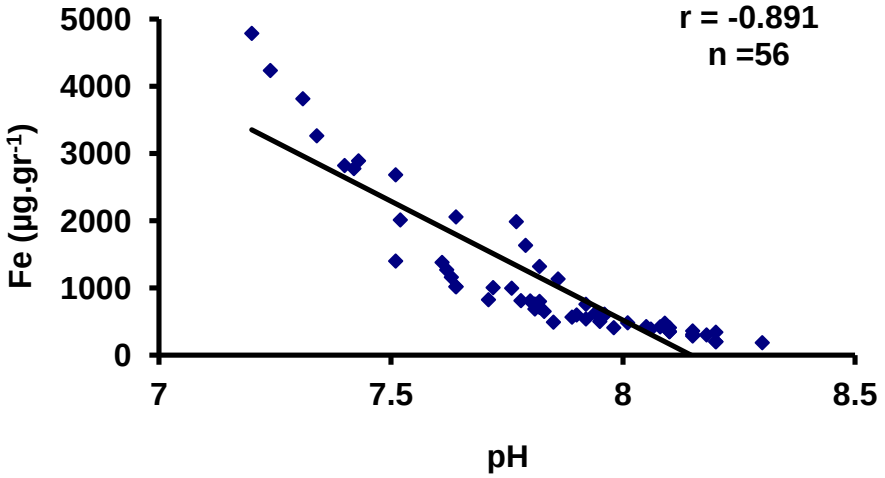


Fig. 2 Relation between pH and particulate iron in the estuarine waters of Gauthami Godavari during 2013

Relation of particulate Iron with salinity:

Significant inverse correlations were observed between the particulate iron with salinity in the waters of the Gauthami Godavari ($r=-0.75$, $p<0.001$) is shown in Fig.3. The particulate iron was observed to be rapidly decreasing in the Gauthami Godavari at higher salinities.

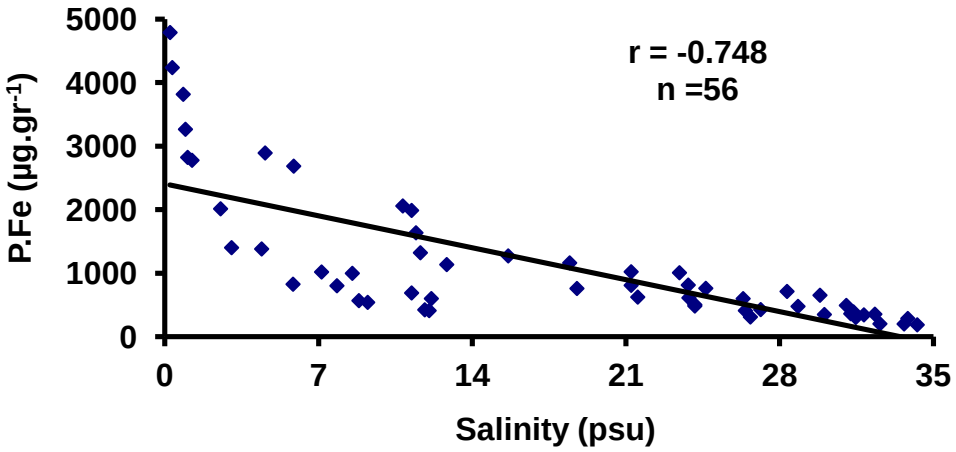


Fig. 3. Relation between particulate iron with salinity in the Estuarine waters of Gauthami Godavari during 2013

In general, the rapid removal of iron from solution is commonly observed in estuarine waters (Byrd *et al*., 1990; Windom *et al.*, 1971, 1988; Yeats and brewers 1976; Boyle *et al.*, 1977) and has been attributed to the flocculation and precipitation of colloidal iron oxy hydroxides with organic matter (Sholkovitz, 1976; Sholkovitz and Cop land, 1981). The dissolved and particulate iron is known to show highly non-conservative behavior in most estuarine environments due to destabilization of humid substances with which it is complexed (Liu, 1996). The removal process of iron is an adsorption on to suspended matter in the estuarine region, enhanced by the pH gradient (Bourg, 1983; Tessier *et al*., 1985). The similar removal process is also observed in Tamar and Beaulieu estuaries in England (Liu *et al*., 1998; Turner *et al*., 1998), Conwy estuary, North Wales (Zhou *et al*., 2003), in the coastal environment of Orissa, India (Nayak *et al*., 2004), in the Mandove and Zuari estuary (Kessarkar*et al*., 2010, 2013) in Mangrove estuary (Shynu *et al*., 2012.

PARTICULATE NICKEL:

The station-wise summery statistics on particulate nickel in the estuarine waters of the Gauthami Godavari during the study period were given in Table 2. The detailed seasonal distributions at seven individual stations are shown in Fig.4. The particulate nickel concentrations in the surface waters were in the range of 3.78 to 34.65 µg.gr^{-1} with an average of 16.70 µg.gr^{-1} where as in the bottom waters, its concentrations ranged from 2.10 to 23.650 µg.gr^{-1} with an average of 10.01 µg.gr^{-1}.

Table. 2 Station-wise summary statistics on particulate nickel (µg.gr^{-1}) in the estuarine water of Gauthami Godavari during 2013

Station	Surface				Bottom			
	Min.	Max.	Mean	S.D.±	Min.	Max.	Mean	S.D.±
Kotipalli	12.96	34.65	21.89	9.34	7.36	23.65	13.86	7.01
Dangeru	11.98	33.12	20.85	9.16	7.01	22.79	13.15	6.86
Yanam	9.67	30.15	18.89	8.79	6.35	21.02	11.32	6.66
V.Godavari	7.25	27.65	16.53	8.55	4.93	19.63	10.12	6.65
Balusutippa	5.38	26.34	14.70	8.86	3.65	17.54	8.38	6.31
Mangrove area	4.15	24.98	13.18	8.89	2.45	15.35	6.86	5.81
Bhiravapalem	3.78	12.42	10.90	6.11	3.10	14.63	6.58	5.43

Nickel occurs principally as Ni^{2+} in surface sea waters, but oxidation states ranging from Ni^{1+} to Ni^{4+} have been reported from time to time (Moore, 1991). Under reducing conditions in surface water, nickel forms insoluble sulphides, provided that sulphur is present in excess. Under aerobic conditions and pH < 9, nickel complexes with hydroxides, carbonates, sulphates, and naturally occurring organic ligands. This has been observed both in fresh waters and estuarine waters (Luther *et al*., 1986). Above pH 9 (rarely found in surface waters), the hydroxide and carbonate complexes precipitate.

Seasonally higher concentrations of particulate nickel were observed during monsoon followed by post-monsoon and pre-monsoon seasons. Higher concentrations were observed during monsoon season due to the combined effect of land and river runoff along with domestic and industrial effluents are more during this season. Lower values of particulate nickel were observed during pre-monsoon season due to its precipitation under alkaline pH condition of the estuary (Stumm and Morgan, 1970), and flocculation with dissolved organic matter.

The removal of nickel by co-precipitation with manganese oxides also observed in the oxic and suboxic interphase in salt marshes of the Scheldt estuary (Zwolsman *et al*., 1993). The concentrations of the particulate metals in estuaries generally decrease seaward due to the mixing of river-borne suspended matter containing higher metal concentrations with marine suspended matter which contains relatively low metal concentrations (Regnier and Wollast, 1993).

Similar variation of nickel were also reported by Fang and Lin (2002) in the Tanshui estuary in Northern Taiwan, Zhou *et al*., (2003) in the waters of Conwy estuary, North Wales, Caetano *et al*., (2006) in Guadiana estuary, Sundaray *et al*., (2011) in Mahanadi river estuarine system, India. Raju*et al*., (2013) in the waters of Cauvery river basin.

Relation of particulate nickel with pH:

Significant inverse correlations were observed between the particulate nickel with pH in the waters of the Gauthami Godavari (r = -0.97, p <0.001) is shown in Fig. 4.14. In the high pH range, particulate nickel concentrations decreased due to dilution with sea water (Abdullah, 2008). The concentration of particulate nickel in the present study decreases linearly with increase of pH. It may be attributed to the large scale rapid removal of nickel from river water and it is a general phenomenon during estuarine mixing. Precipitation of particulate nickel occurs on estuarine mixing because the sea water cations neutralize the negatively charged nickel bearing colloids allowing flocculation.

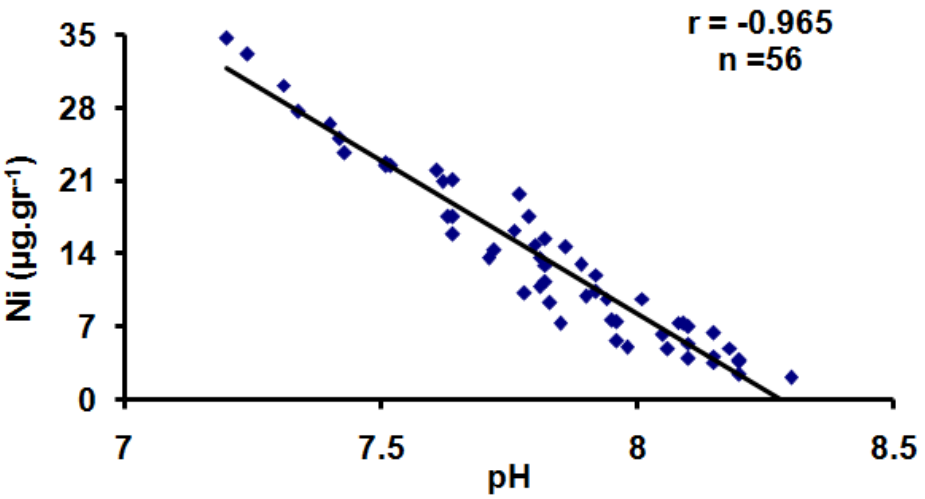


Fig.3. Relation between pH and particulate Nickel in the estuarine waters of Gauthami Godavari during 2013

Relation of particulate nickel with salinity:

Significant inverse correlations were observed between the particulate nickel with salinity in the waters of the Gauthami Godavari ($r = -0.86$, $p < 0.001$) is shown in Fig. 4.15. It indicates that the particulate nickel behaves non-conservatively in Gauthami Godavari estuary. Nickel may be partially associated with sulphides, (Zwolsman *et al.,* 1997). These nickel sulphides are gradually oxidized during restoration of dissolved oxygen with increasing salinity, resulting in releasing of trace metals to the dissolved phase. At the same time Mn hydroxides which are very effective scavengers of trace metals are also formed. The close relationship between Ni, Co and Mn in the suspended matter of the lower estuary indicates that the Ni, mobilized from the lower estuary but it is scavenged back by Mn oxides. Co-precipitation of Ni with Mn oxides was also formed near the oxic-suboxic interface in salt marshes of the Scheldt estuary as reported by (Zwolsman *et al.,* 1999).

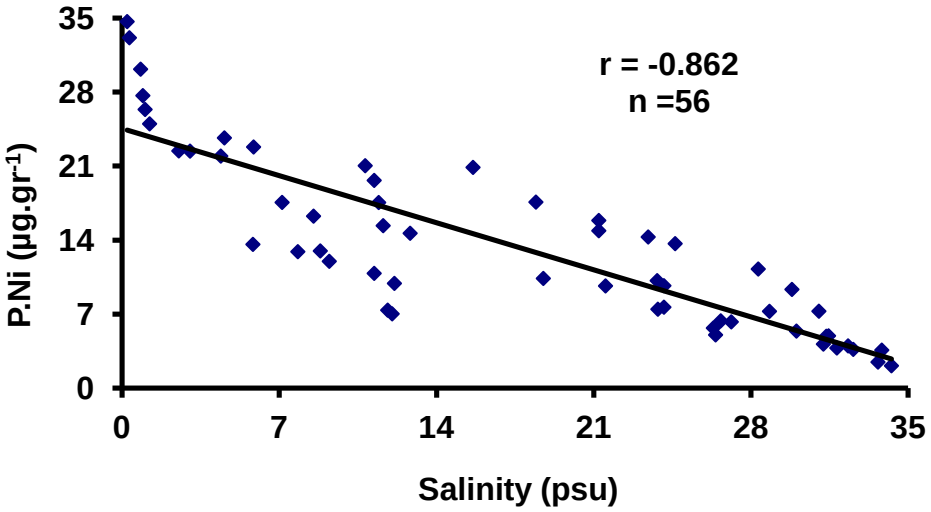


Fig. 4. Relation between particulate nickel with salinity in the Estuarine waters of Gauthami Godavari during 2013

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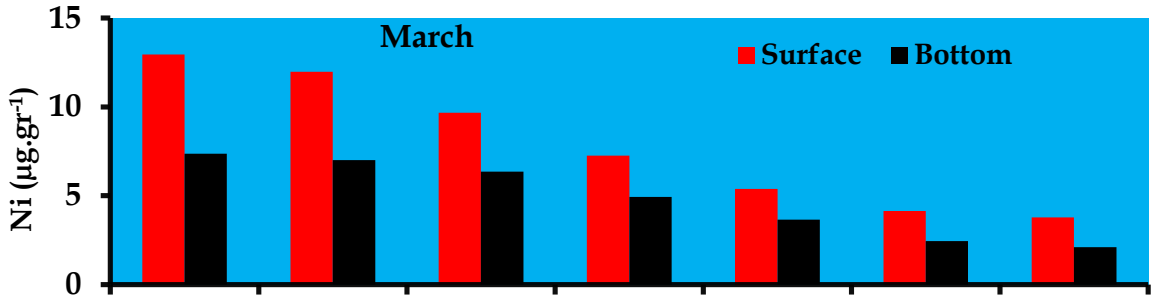
A distinct spatial and seasonal variation of particulate trace metals like iron and nickel were observed in the present study. Relatively higher concentrations of these metals were observed in head of the estuary and low concentrations in mouth of the estuary. Higher concentrations of particulate trace metals like iron and nickel were observed during monsoon season followed by onset of monsoon and post-monsoon seasons, Lower concentrations of these metals were observed during pre-monsoon season. Higher concentrations of all these metals during monsoon season due to the river and land runoff along with domestic and agricultural, industrial effluents are more during this season. Lower concentration during pre-monsoon season due to its utilization by biological activity, or by chemical and physical activity like desorption or flocculation as pH and salinity are increased during this season.

St. 1 :Kotipalle; St.2 : Dangeru; St. 3: Yanam; St.4: V.Gauthami; St.5 :Balusutippa; St. 6 : Mangrove area; St.7. : Bhiravapalem



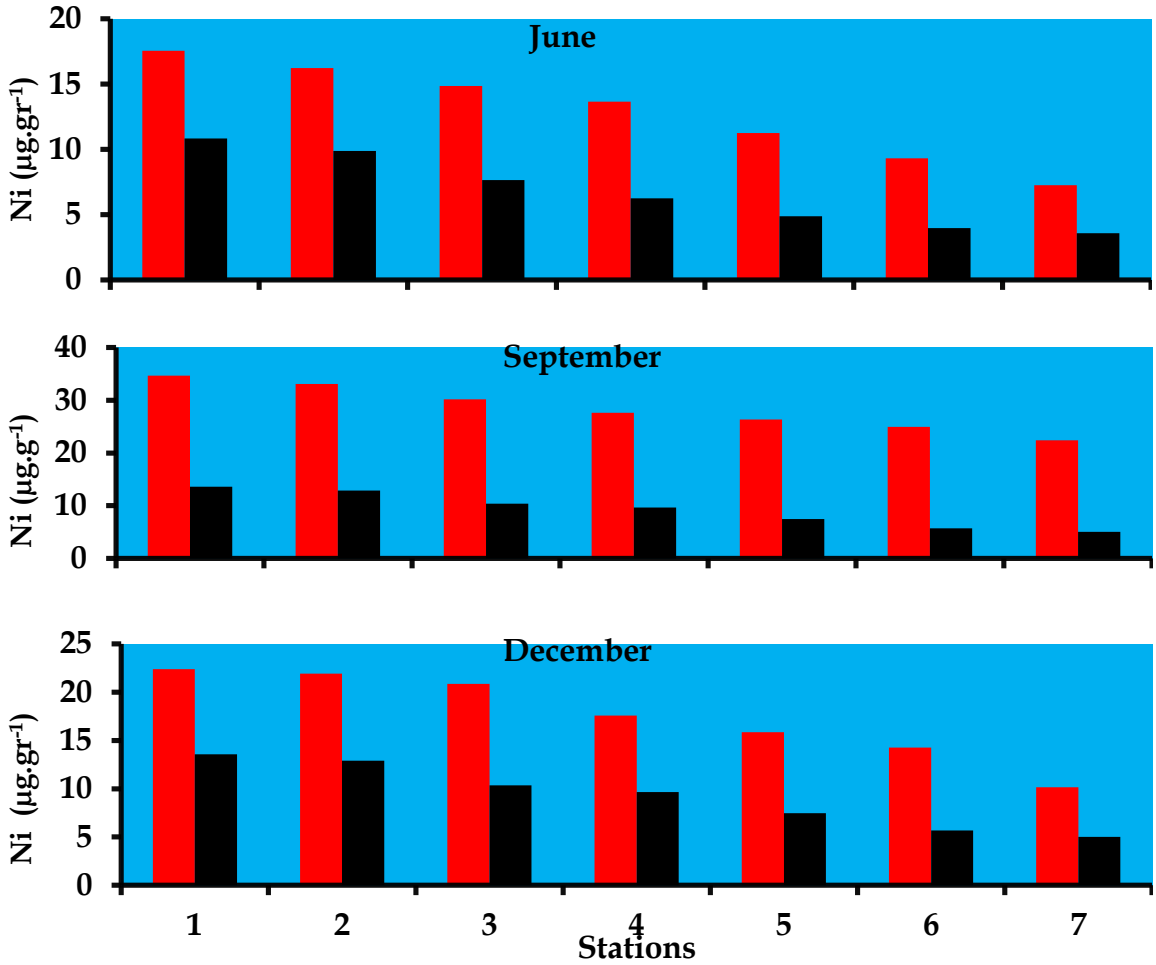


Fig. 4. Seasonal and spatial distribution of particulate nickel in the estuarine waters of Gauthami Godavari during 2013.

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References

[1] Abdallah, M. (2008). Trace metal behaviour in Mediterranean-Climate coastal Bay: El-Mex Bay, Egypt and its coastal environment. Glob.J. Environ. Res., 2 : 23-29.
[2] Balls, P. W., Laslett, R. E. & Price, N. B., (1994). Nutrients and trace metals distributions over a complete semi-diurnal tidal cycle in the forth estuary Scotland. Neth. J. Sea Res., 33 : 1-17.
[3] Benoit, G., Oktay-Marshall, S., Cantu, A., Coleman, E. M., Corapcioglu,
[4] M. O., &Santschi, P. H. (1994). Partitioning of Cu, Pb, Ag, Fe, Al andMn between filter-retained particles, colloids and solution in
[5] six Texas estuaries. Mar. Chem., 45: 307-336.
[6] Brewer, P. G. In Chemical oceanography. Edited by J.P. Riley & G. Skirrow (Academic Press, London) 2nd Ed. 1: 427.
[7] Forstner, U. and Wittmann, G.T.W. (eds.) (1983). In: Metal pollution in the aquatic environment. Springer-Verlag, New York, 3-318.
[8] Raju, K. Somashekar, R. K., Prakash, K. L., (2013). Spatio-temporal variation of heavy metals in Cauvery River basin.Proceedings of the international Academy of Ecology and Environmental Sciences, 3 (1): 59-75.
[9] Rajkumar, J. S. I., John Milton, M. C., and Amborse, T., (2011).Distribution of heavy metal concentrations in surface waters from Ennoreestuary, Tamil Nadu, India. Int.J. Curr. Res., 3 : 237-244.
[10] Salomons, W. and Kerdijk, H. N. (1986).Cadmium in fresh and estuarine waters. In Cadmium in the environment (H. Mislin& O Ravera., eds). (BirkhauserVerlag, Stuttgart, 144 p.
[11] Sankaranarayanan, V. N., Jayalakshmi, K. V., and Thresiamma Joseph, (1998). Particulate trace metals in Cochin backwaters: distribution of seasonal indices. Indian. J. Fish., 45 : 321-329.
[12] Stumm, W. and Morgan, J.J. (eds.) (1996). Chemical equilibria and rates in Natural waters. In: Aquatic chemistry (3rd edition). Wiley-Interscience, New York, 1022-1038.
[13] Sarma, V.V.S.S., Gupta, S. N. M., Babu, P. V. R., Acharya, T., N. Harikrishnachari, K. Vishnuvardhan, N. S. Rao, N. P. C. Reddy, V. V. Sarma, Y. Sadhuram, T. V. R. Murty, M. D. Kumar. (2009).

[14] Sarma, V. V. S. S., Prasad, V. R., Kumar, B. S. K., Rajeev, K., Devi, B. M. M., Reddy, N. P. C., Sarma, V. V., (2010). Inter-annual variability in nutrients in the Godavari estuary, India.Cont. Shelf Res., 30 : 2005-2014.
[15] Taylor, H. E., Antweiler, R. C., Roth, D. A. (2011). Selected Trace Elements in the Sacramento River, California: Occurrence and Distribution.Arch. Environ. Contam.Toxicol., DOI 10. 1007/s 00244-011-9738.
[16] Tripathy, S. C., A. K. Ray, A. K., Patra, S., and Sarma, V. V. (2005). Water quality assessment of Gauthami-Godavari mangrove ecosystem of Andhra Pradesh, India during September 2001.J. Earth Syst. Sci., 114 :, 185-190.
[17] Venugopal, P., Sarala Devi. K., Remani, K. N. andUnnithan, R. V. (1982). Trace metal levels in the sediments of the Cochin backwaters. Bull. Natn. Inst. Oceanogr., 15 : 205-214.
[18] Venugopalan, V.K. and Ramdas (1975). Distribution of dissolved, particulate and sedimentary iron in the Vellar Estuary. Bull. Dept.Mar. Sci., Univ. Cochin.,7:601-608.
[19] Windom, H., Smith, R. and Rawlinson, C. (1988). Trace metal transport in a tropical estuary. Mar. Chem., 24 : 293-305.
[20] Zwolsman, J. J. G. and Van Eck, G.T.M. (1990).The behavior of dissolved Cd, Cu and Zn in the Scheldt estuary. In: Michaelis, W. (ed.), Estuarine Water Quality Management. Springer—Verlag, Berlin, pp.413 - 420.
[21] Zwolsman, J. J. G., van Ech, G. T. M., (1993). Dissolved and particulate trace metal geochemistry in the Scheldt estuary , South West Netherlands water column sediments. Aquat.Ecol., 27: 287-300.

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Thomas Dekkar's the Shoe-Maker's Holiday-A Reflection of Middle-Class Life

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Abstract: *Thomas Dekkar was a professional Elizabethan playwright. He understands the middle-class life with its ups and downs, aspirations and its compromises. The shoe makers's Holiday is his supreme contribution to this class of literature, and of its type it remains an unique example.*

The play consists of two stories, the romantic story with the characters from high class like Row land, Lacy and the story that concerns its ordinary citizens. The shoe makers, like Eyre, Thodge etc. The two stories are closely interwoven in the structure of the play.

Keywords: *Elizabethan comedy – faithful picture of Citizens – Realistic comedy – comedy of London life citizen drama – middleclass life – Elizabethan crafts man – reflects the trading class – aristocratic.*

1. INTRODUCTION

Thomas Dekker was a professional Elizabethan play **wright**. He was not a supreme artist. He is often slip shod, often careless and his eyes are always fixed on the east. The poet in him is irrepressible, but there is no **sunnier writer** than Dekker, nor who understands middle class life with its ups and downs, is aspirations and its compromises better than he. The shoe-makers holiday is his supreme contribution to this of literature, and of its type it remains an unique example.

It is hard to form a final judgement of Dekker as a Dramatist. There is so much in his work to delight us that it is easy to dwell upon his charm and overlook his weakness. On the other hand if one observes his plays closely, one can easily point out his failures. The great fault of his plays is a lack of unity and incoherence of plot. He has written in great haste because of the need for money. Though he is not perfect many of his characters drawn from daily life-like the London shop keeper, the long suffering wife etc., creatures of less of poetic imagination than of sympathetic observation. He is not a great dramatist, but certainly a delightful poet – play wright.

2. ESSAY

"The **shoe-makers's Holiday**" or "**the gentle craft**" is about Simon Eyre, shoe maker and Lord mayor of London. It was acted before tahe **Queens** of England, The idea of this character he got from the Novel on Simon Eyre by Deloney

The play consists of two stories. The romantic story with the characters from high class and the story that concerns ordinary citizens.The shoe makers. The two **stories are closely interwoven in the structure of the play and there** is a blending or **romance and realism**.

Rowland Lacy, nephew of Sir Hugh Lacy, Earl of Lincoln is a high spirited young man. He falls in love with Rose daughter of Sir Roger Oteley a merchant who is Lord Mayor of London as the play begins. The noble man looks down upon the merchant class and does not like his nephew to marry beneath his rank. Sir Roger Oteley also has his prejudice against the aristocracy their extravagance and so opposes the young lovers. The earl wants to pack off his nephew to the war in France and Oteley proposes to send his daughter to his country seat. Both of them fail to realize that, "**love brooks nor barriers**"

Lacy Manages to stay behind in England. He disguises himself as a dutch shoe maker on a new name – Hans. He joins simon Eyre the master – shoe maker as an apprentice. It is Simon Eyre who supplies shoes and leather goods to the mayor's family. Thus Lacy gets several opportunities to meet Rose. Simon and his wife are happy to help the young lovers. They got married. The parents report the matter to the King and request him, to cancel the marriage. But Simon Eyre takes up the case and pleads with the king to forgive them. The king oblizes and rewards him with knight hood. Thus the romantic story ends happily.

The other story is about the household of Simon Eyre – the merry master shoemaker, his assistant and his young wife Jane. Ralph joins army most reluctantly. He does not return from the army. Jane does not want to be a burden to the Simons. So she sets up a seamstress shop and earns her living independently. Master Hammon makes love to Jane. But Jane turns a deaf ear to his courting as she wants to be true to Ralph. After many days he brings her the news of Ralph's death. So she reluctantly agrees to marry him. On the wedding day Ralph returns, seeing the developments he becomes stricken. On the advice of his fellow shoe makers when the party is going to church, he appears on the scene and Jane is restored to Ralph. Thus the second story ends well.

The structure of the play is of real interest. Dekkar exhibits his mastery in contruction. The two stories are very **skillfully and harmoniously inter woven**. Neither the romantic young man, Rowland Lacy, nor the young shoemaker Ralph is the hero of the play. The real hero is Simon Eyre, the merry shoe maker. It is quite obvious that he knits the two stories into a composite whole.

He has his part to play in both the stories and thus serves as the connecting link between the two stories. – S. Banumoorthy.

The shoemaker's holiday is often referred to as a **citizen play** and as a romance of the working class, because of the way in which Dekker potrayed the shoemakers. His attitude to the common man is very much different from that of the other contemporary writers. Most of the dramatists of his age looked down upon the common folk and treated them as the birth of ridicule

The trade's men who carry tools of their trade "had sweaty night cops and emitted their foul and stinking breath.

They are **fickle minded; irrational, incapable of any judgement and often ludicrous and contemptible**. This is the picture of common folk we get in many of the Elizabethan plays. On the other hand, Dekkar attempted to give a faithful and objective picture of the **artisans** the shoemakers. He does not cover their ludicrous aspects, at the same time he is aware of their **sterling qualities** and highlightens them.

In the play Dekker makes fun of their boastful and pretenteous life saying that they are gentlemen pursuing a gentle craft under the protection of **St.Hugh**. He has a good laugh at their affectations and whims Dekkar at the same time projects their enduring qualities. They are a very hard working people. They do not want to live on charity but want to earn their bread by honest work. They have a sense of professional dignity. They are honest and truthful and there is no craft in them. They are friendly lot and they have a sense of oneness. They stand together and help one another. They are perfectly natural, cheerful and gay. They love fun, frolic and a holiday. They have their own simple philosophy of life. Thus Dekkar presents them in a kindly, sympathetic light.

Simon Eyre the chief of the shoemakers is an unforgettable character. He is a very Loud Character fond of swearing. He is never tired of repeating his tall claim **"prince am I none, yet am I princely born"** he is what he is and cannot pretend to be different. He is a jolly type of person given to blunt speaking. He is not refined in his talk. He has real love for his wife which he shows in is own rough unconventional way. He has great concern for the welfare of his workers. But he is not lenient towards sthem. In the shop he is their stern employer. He is kind hearted towards them. That is why he helps Hans (Lacy in disguise) in his love affair.

As a mayor also he does his duties to the approval of all seasons of society. He wins the admiration of the court in the way he conducts himself as a mayor. The king is very eager to meet "**The mad merry mayor**" The noble men describe him to the king as

"One of the merriest mad caps in your land, your grace will think when you behold the man."

He is rather a wild ruffian than a mayor"

Commenting on the character of Eyre **D.J.Palmer** says that he has allusions to "Mephostophilus", "Sultan Soliman", and "Tamburlaine" "**Eyre is something of a play goer himself**"

The other characters also represent their classes with great detail. Sir Hugh Lincoln is a typical aristocrat Sir Rogeroteley bears the stamp of merchant class. Rowlalnd Lacy is a romantic young man in whom love triumphs over his sense of duty. Rose is a young girl of Grace and beauty Doger is a conventional spy and informer. Margery is a fitting wife to Simon. The king who makes his appearance in a **couple** of scenes is noble and fun loving.

Shoemaker's holiday mirrors the feelings of the trading class of the Elizabethen age. They were supposed to be hopelessly materialistic, men deficient in self respect and ready to do anything for money. The women bent on pleasure and ready to be reduced by any gallant who would look at them.

Dekkar sits out to describe the class, the one to which he himself perhaps belonged. They enjoy material prosperity, this natural enjoyment creates an atmosphere of good will and activity. It is not the story, however as it is, that gives this play its perennial charm, it is rather the abounding joy of life in general, of Elizabethen London life in particular that flows from Dekker's pen that attracts us. Simon as lord mayor declares that upon every Shrove Tuesday, at the sound of the pen cake bell shoemakers shall shut up their shops for a holiday. The play taught in a mirthful manner the very much needed lesson of the dignity of Labanto and gave assurance of the highest rise in life even to the lowest of citizen.

The most important feature of Elizabethan comedy is the homely character of its people and scenes. Its theme and its atmosphere, Thomas Dekkar is also a realist like Ben Jonson. Shoemakers holiday is also a realistic comedy. It gives a true and faithful picture of the citizen life of London in those days. In the words of F.S. Schelling "Shoe Makers Holiday" is a typical contribution to domestic comedy and here we have the daily life of the small trades folk of London done to perfection; **their humour, their pleasure, ambitions and hearty good fellowship**" Dekkar also exposes the follies of the age and the play is a comedy of London life. He aims at giving us an accurate picture of the people of his age, particularly those of the gentle craft.

The "**Shoe Maker's Holiday**" is a citizen drama. The citizen element is found in Shakespeare also. But in him it never took deep root, Dekkar and Fletcher concentrated on this line, The citizen drama depend for its effect more on characterization and humour than on situation and plot.

The citizen's drama is a true mirror of contemporary life and no other writer has displayed it with as much faithfulness as Dekkar.

Dekker like **Dickens** with whom he has so much in common, was at his best when writing a homely joys, London life is accurately picturised. The places mentioned in the play are known to every Londoner and the life, character and behavior of the shoemakers is beautifully and accurately portrayed some of the scenes which best represent the true life of London are the following-early morning in Tower street, Eyres election Jane in the seamesters shops and the pan cake beast at the Leaden hall etc.

The best picture of the life of the shoe makers of London in those days come through the characters of Eyre, Hodge, Firk etc. He represents all the glamour and romance of the gentle craft without exaggeration. His character speaks an intelligible language of the tower street workshop. Hodge is the foreman and firk the journeyman, Simon Eyre is their hero and their king.

3. CONCLUSION

In the "**Shoe Maker's Holiday**" a great deal of information is given about the life of Elizabethan Craftsman, their way of living, their feasts, merry making, their drinking breakfasts and jolly chit chats.. From early morning when the Journey man breaks his fast with three post of double ale to deny eve, there is cutting and stitching waxing and paring. In short their life history is given in this play. They are Frank and open-hearted. The "**Shoe Maker's Holiday**" also tell us about the courtiers and middle class people. The courtiers lived an artificial life. They are hypocritical and showy.

The "**Shoe Maker's Holiday**" is a hilarious play. It is a perfect comedy. It is full fun and gaiety. The spirit of the pure comedy is preserved from the beginning to the end of this play. This shows **Dekker's** greatness as a dramatist.

REFERENCES

1. Dekker, Thomas. The shoemaker's Holiday Ed:R.L. Smallwood and Stanley wells. Manchester: Manchester up, 1979. The Revels palys.
2. The Shoemaker's Holiday review – Dekker's 1599 Critique feels tremendously contemporary.
3. Kalpan, Joel H – "Virtue Holiday: Thomas Dekker and simon Eyre". Renaissaince Drama 2 (1969): 103 – 122.
4. Mannhein Michael. "The Construction of the shoemaker's Holiday" studies in English literature, 1500 – 190010 (1970) : 315 – 323.
5. Toliver, Harold E. "The Shoemaker's Holiday. Theme and image". In shakespeare's contemporaries edited by max Bluestone and norman Rabkin. 2d ed Englewood cliffs, N.J. Prentice – Hall, 1970

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Velpula Beulah Rani, have been in service as a Lecturer in English in this August body since 2000. I have enriched my teaching and language skills by participating in various National and International seminars and workshops. Eight of my research articles have been published in different souvenirs. Three of my research articles have been published in international souvenirs.



Chilka Aruna, as a Lecturer in English have been rendering services to the prestigious institution Hindu College, Guntur since 2000. In 2008, I became the Head of the Department. Coming to my personal profile, I have been awarded the A.P.STATE BEST TEACHER AWARD in 2015. Twelve of my research articles have been published in different souvenirs. I feel proud to say that I have been awarded the GURU BRAHMA NATIONAL AWARD-2016 by the MOTHER THERESA SOCIAL WELFARE ORG. (Regd.No:649/2010) in the year 2016. I have participated in various National & International seminars and workshops. Thus I have enriched my language skills as well as teaching skills. Once again I feel proud to say that I have exchanged my views regarding literature and language with popular literary figures and experts in communication skills.

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TEACHING ENGLISH TO RURAL STUDENTS

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Abstract:

Learning English language is highly needed for knowledge, employment and uplift of social status. India is the world's second largest English speaking country, But India lives in villages.

Especially in Andhra Pradesh rural students join in degree colleges without acquiring basic proficiency in English. So, Indian teachers of English find it difficult to teach English particularly in rural colleges where in the student come from poor families.

Therefore, the teachers should be more realistic than idealistic in teaching English dealing with the area of problems like pronunciation, spelling, vocabulary etc.. so the teacher should morally committed to teaching taking always human values into consideration.

Key Words: Proficiency in English – Spelling, Pronunciation – vocabulary – teaching grammer humourous

Introduction:

English is spreading in India used by 125 millions of people with varying degrees of competence in spoken and written forms of 1.32 billions population. This is much more than the English speaking population each in Britian, the U.S. and Canada.

India now claims to be the world's second largest English speaking country. Yet, English is the waterloo as well as the Cinderella of the Indian rural students. India lives in villages , but villages are weeping and cities are laughing just because of English barring the

other reasons. However, in India English is the lingua franca and the associated official language.

Learning English language is highly needed for knowledge, employment and uplift of social status. Especially in Andhra Pradesh rural students join the degree colleges without the basic proficiency in English and leave the college without acquiring the communication skills to use in day to day affairs. Therefore the under privileged students in rural A.P. lie on the teachers of English at the degree colleges.

In India , specially in Andhra Pradesh the classroom is the replica of the society in which we find many clusters of castes, creeds, languages etc. The A.P. state council of Higher Education changed the syllabus of English for degree students stressing the importance of speaking and listening skills. Language labs are established in most of the colleges. English paper has two parts as theory and practicals. In practicals students are tested in speaking and listening skills. Inspite of these there is deficient acquisition of English language skills among the rural students of A.P. in real life situations.

English became the world language born in a small country i.e. , England. It is all because of climatic conditions and International needs. So Indian teachers of English find it difficult to reach English particularly in rural colleges where in the students come from poor families. Since independence we have adopted democracy and been adopting English, the hallmark of development.

After struggling through high school and junior college, the rural students join degree course with more knowledge of their mother tongue Telugu than that of English .Most of them do not buy prescribed English text books. On the contrary , the teachers go to the classes with textbooks and teach through translation method. The use of mother tongue is useful as for as the better understanding of the

content of the text but not in acquiring the basic skills of English as teachers are translating and explaining everything in text in the mother tongue. The use of two languages in the classroom can be done in such a way to provide comprehensible input in the target language. Hence, mother tongue should not be used extensively. The teacher should speak 70% English and 30% Telugu , shuttling between them, should maintain a balance between two. The teacher should deal pleasantly with the importance of English in India as well as in the world. The teachers should also know the familial background of the students.

To teach required skills of English by making the learning interesting , simple stratagies are suggested which include motivating the students by the teachers adopting motivational stratagies.

The teacher should ne more realistic than idealistic in teaching English , He/She should elicit the information from students of how much they have the knowledge of English and their mother tongue with reference to the skills of language , listening , reading, speaking and writing.

The area of problem in teaching English to rural students is pronunciation, which is a Herculean task for Indian English teachers. For that the teacher should depend on the phonetic script and advise the student to follow the same from dictionaries.

The second problem is with spellings while dictating the notes and in speech. The student writes the abbreviation of example as ‘ex ‘ (or) ‘ Ex ’ , but not the correct one e.g. , which is abbreviation of exempligration. It is very difficult to explain them the difference between ’at’ and ‘yet’; ‘our’ and ‘hour’; ‘by’ and ‘buy’; ‘ here’ and ‘hear’ etc…

The third problem is with meaning which is infinite. It is better for students to go for Oxford dictionary for all practical purposes.

Students should be advised to be alert in their every day conversation by giving them the anecdote like the following. It took place at a dinner in England in honour of Winston Churchill, the then great Prime Minister of England. At the dinner a good painter who was also a good friend of the Prime Minister sat beside him and when Churchill finished his dinner he went to the sink to wash his hands (it was the habit of Churchill to keep his kerchief on the chair on which he had sat), and in the meanwhile his painter friend drew the face of a donkey on his kerchief which he saw and got irritated but controlled himself and asked in general air who had cleaned the face with his kerchief . The painter kept silent and Churchill got the upperhand. Really the students do not understand the anecdote as it is dealt with an English as usual and so it needs explanation in Telugu.

While dealing with grammar the examples are localized in a humorous way. Increased use of humour in English class enlivens the students. The group discussions are initiated for the overall improvement of students English language skills. Most of the teachers feel shy to use humour in the class consequently, the English language class becomes dull boring and uninteresting. But they should not. For example , while dealing with comma to show the danger of not using it whenever necessary , ask a student in the class to tell his name, say Srinivas. Then frame the sentence, “Has the dog eaten, Srinivas?” Then the whole class enjoys the example, but the student concerned Srinivas feels a little bit embarrassed. To wipe out his embarrassment and to make them understand that they should use comma perfectly, rewrite the sentence(question) as “ Has the dog eaten Srinivas ? ” . Here , Srinivas is used only to address the person. Grammar is simply taught like mathematical or chemical formulas. Skills can be achieved only through practice, teachers can provide opportunities for such practices.

Pair work and Group Discussions are not often followed in the

English class fearing that class may become noisy. Language experts say, " A noisy language class is a good and effective class" .

Finally a teacher should morally committed to teaching , taking always human values into consideration. English teachers , especially those who teach rural students need to be very patient and systematic. They are expected to do miraculous job under adverse conditions. This is a real challenge. They should not over look the seriousness behind the issue and it would deprive the students opportunities which are mainly attained through communication skills and the specific purpose of teaching English as a communicative tool would get lost.

Bibliography:

1. P.6 " Teaching of English as a second language by J.A. Bright and G.P. Mc Gregor published by English language books society and Longman group limited. "
2. Adrian Doff , (2003) , Teach English , a training course of teachers , Cambridge University Press.
3. Krishna (2008), English language CS Duvvuri Publications, Machilipatnam.
4. Casper R. (1999) . Laughter and humour in the class room . Effects on test performance University of Nebraska . - Lincoln

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On Trigonometric Moments of the Stereographic Semicircular Gamma Distribution

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Abstract. Phani (2013) constructed a good number of circular and semicircular models induced by inverse stereographic projection. Minh and Farnum (2003) and Toshihiro Abe et al (2010) proposed a new method to derive circular distributions from the existing linear models. In this paper, a new semicircular model, which is coined as Stereographic Semicircular Gamma distribution is derived by inducing modified inverse stereographic projection on Gamma distribution. This distribution generalizes Stereographic Semicircular Exponential model (Phani et al (2013)) and the density and distribution functions of proposed model admit closed form. Explicit expressions for trigonometric moments are derived by applying Meijer's G- function and the new semicircular model is extended to construct Stereographic l- axial Gamma distribution.

2010 Mathematics Subject Classifications: 60E05, 62H11

Key Words and Phrases: Circular Model, Directional Data, Meijer's G-function, Stereographic Projection, Trigonometric Moments.

1. Introduction

Directions in two dimensions can be represented as points on the circumference of a unit circle and models for representing such data are called circular distributions. Quite a lot of work was done on circular models defined on the unit circle (Fisher, 1993; Jammalamadaka and Sen Gupta (2001); Mardia and Jupp, (2000)) and recent publications (Dattatreya Rao et al (2007), Girija (2010), Phani et al (2012)). Most of these models are applicable for dealing circular data. To fit/model certain practical data sets, it is not required to go for full circular models but semicircular/arc models are adequate. Some recent

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papers (Guardiola (2004), Byoung et al (2008) and Phani et al (2013)) address this issue and provided some methodology for constructing distributions suitable for modeling these types of data. For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is very much sufficient for modeling such data. Given the angles of initial heading and departure, to trace the debris of aircraft lost problem, semicircular models need to be used. A few more examples of semicircular data is available in Ugai et al (1977). A little attention is paid on the study of semicircular distributions. The aim of the present article is to contribute towards filling this gap. In this paper, the modified inverse stereographic projection is used to define a new semicircular model, coined as the Stereographic Semicircular Gamma distribution which generalizes the Stereographic Semicircular Exponential Model (Phani et al (2013)). Explicit expressions for trigonometric moments are derived in terms of Meijer's G- function and it is extended to the Stereographic l - axial Gamma distribution for modeling axial data.

In section 2, methodology of modified inverse stereographic projection is presented. Section 3 is devoted to introduce the proposed distribution and to present graphs of probability density function for various values of parameters. In section 4, the first four trigonometric moments in terms of Meijer's G- function are derived and the proposed model is extended to l- axial distributions for modeling axial data in section 5.

2. Methodology of Modified Inverse Stereographic Projection (Phani et al (2012))

Modified inverse stereographic projection is defined by a one to one mapping given by $T(\theta) = x = vtan\left(\frac{\theta}{2}\right)$, where $x \in (-\infty, \infty)$, $\theta \in [-\pi, \pi)$, $v > 0$. Suppose x is randomly chosen on the interval $(-\infty, \infty)$. Let $F(x)$ and $f(x)$ denote the cumulative distribution and probability density functions of the random variable X respectively. Then $T^{-1}(x) = \theta = 2tan^{-1}\left(\frac{x}{v}\right)$ by Toshihiro Abe et al (2010) is a random point on the unit circle. Let $G(\theta)$ and $g(\theta)$ denote the cumulative distribution and probability density functions of this random point θ respectively. Then $G(\theta)$ and $g(\theta)$ can be written in terms of $F(x)$ and $f(x)$using the following theorem.

Theorem 2.1. *For $v > 0$,*

i) $G(\theta) = F\left(vtan\left(\frac{\theta}{2}\right)\right)$

ii) $g(\theta) = v\left(\frac{sec^2\left(\frac{\theta}{2}\right)}{2}\right) f\left(vtan\left(\frac{\theta}{2}\right)\right)$

By applying this modified inverse stereographic projection on linear models with support on $R^+(R)$, new distributions mapped onto $[0, \pi)$ $([-\pi, \pi))$ are derived to study semicircular $(circular)$ data.

3. Stereographic Semicircular Gamma Model

Gamma distribution plays a prominent role in actuarial science. Here an attempt is made to construct stereographic version of semicircular Gamma distribution by inducing inverse stereographic projection.

A random variable X on the real line is said to have gamma distribution with index parameter$c > 0$, scale parameter $\lambda > 0$ and location parameter α if the probability density, cumulative distribution and characteristic functions of X are respectively given by

(1) $f(x) = \frac{(x-\alpha)^{c-1}}{\lambda^c \Gamma(c)} \exp\left(\frac{-(x-\alpha)}{\lambda}\right)$ for $\lambda, c > 0,\ x > 0$ and $\alpha > 0$

(2)$F(x) = \frac{\Gamma_{\left(\frac{x}{\lambda}\right)}(c)}{\Gamma(c)}$ for $\lambda, c > 0,\ x > 0$

(3) $\phi_X(t) = \frac{1}{(1-i\lambda t)^c}$ where $t \in R$

By applying inverse stereographic projection defined by a one to one mapping $x = v\ tan\left(\frac{\theta}{2}\right), v > 0, 0 \leq \theta < \pi$ a Stereographic Semicircular Gamma Distribution is obtained

A semicircular random variable θ is said to follow Stereographic Semicircular Gamma Distribution with index parameter $c > 0$, location parameter μ and scale parameter $\sigma > 0$ denoted by SSCG (μ, σ, c) if the probability density and cumulative distribution functions are respectively given by

$$g(\theta) = \frac{1}{2\sigma^c \Gamma(c)} \sec^2\left(\frac{\theta}{2}\right) \left(\tan\left(\frac{\theta}{2}\right) - \mu\right)^{c-1} \exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{\theta}{2}\right) - \mu\right)\right)$$

$$\text{where} \quad 0 \leq \theta < \pi, c > 0, \sigma = \frac{\lambda}{v} > 0 \quad \text{and} \quad \mu = \frac{\alpha}{v} \tag{3.1}$$

$$G(\theta) = \frac{\Gamma_t(c)}{\Gamma(c)} \text{ where } t = \frac{1}{\sigma} \tan\left(\frac{\theta}{2}\right), \theta \in [0, \pi) \tag{3.2}$$

Special case: If $c = 1$ in (3.1) reduces to the density function of Stereographic Semicircular Exponential distribution (Phani et al (2013)).

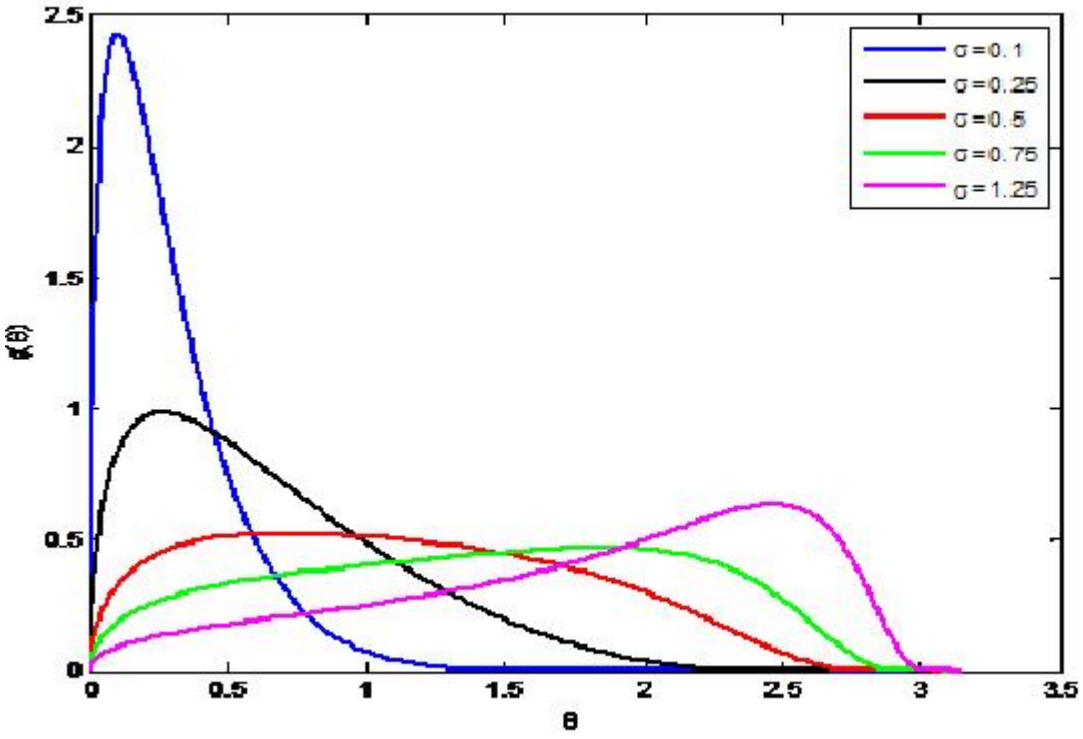
4. Trigonometric moments of Stereographic Semicircular Gamma distribution

The characteristic function of the stereographic semicircular gamma model is

$$\begin{aligned} \phi_\theta(p) &= \int_0^\pi e^{ip\theta} g(\theta) d\theta \\ &= \int_0^\pi e^{ip\theta} \tfrac{1}{2\sigma^c \Gamma(c)} \sec^2\left(\tfrac{\theta}{2}\right) \left(\tan\left(\tfrac{\theta}{2}\right) - \mu\right)^{c-1} \exp\left(-\tfrac{1}{\sigma}\left(\tan\left(\tfrac{\theta}{2}\right) - \mu\right)\right) d\theta \end{aligned}$$

The integration is not tractable. But trigonometric moments can be derived by applying the Meijer G function [Gradshteyn and Ryzhik (2007)]. The trigonometric moments of the distribution are given by $\{\varphi_p : \pm 1, \pm 2, \pm 3, \dots\}$ where $\varphi_p = \alpha_p + i\ \beta_p$ with

Figure 1: **Graphs of probability density function of Stereographic Semicircular gamma distribution for various values of σ and $c = 1.5$**



$\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$ being the pth order cosine and sine moments of the random angle θ, respectively and are required to study the population characteristics.

Theorem 4.1: The trigonometric moments $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$, for $p = 1, 2, 3, 4$ of the stereographic semicircular gamma distribution with $\mu = 0$, are given as follows

$$\alpha_1 = 1 - \frac{1}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2}, 0, \frac{1}{2}\end{matrix}\right)$$

$$\beta_1 = \frac{1}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}\frac{1-c}{2}\\ \frac{1-c}{2}, 0, \frac{1}{2}\end{matrix}\right)$$

$$\alpha_2 = 1 + \frac{4}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}-1\\ -\frac{c}{2}, 0, \frac{1}{2}\end{matrix}\right) - \frac{4}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2}, 0, \frac{1}{2}\end{matrix}\right)$$

$$\beta_2 = \frac{2}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{4}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{(c+1)}{2}\\ -\frac{(1-c)}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$\alpha_3 = 1 - \frac{16}{3\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}-2\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{24}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}-1\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$- \frac{9}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$\beta_3 = \frac{3}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}\frac{1-c}{2}\\ \frac{1-c}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{8}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{(c+1)}{2}\\ -\frac{(1-c)}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$\alpha_4 = 1 + \frac{32}{3\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}-3\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right) + \frac{80}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}-1\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$- \frac{64}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}-2\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{16}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$\beta_4 = \frac{4}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}\frac{1-c}{2}\\ \frac{1-c}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{8}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c+1}{2}\\ \frac{1-c}{2},0,\frac{1}{2}\end{matrix}\right)$$

$$- \frac{16}{\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c+1}{2}\\ \frac{3-c}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{32}{3\sigma^c\sqrt{\pi}\,\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2}\,\middle|\,\begin{matrix}-\frac{c+3}{2}\\ \frac{3-c}{2},0,\frac{1}{2}\end{matrix}\right)$$

where

$$\int_0^\infty x^{2v-1}\left(u^2+x^2\right)^{Q-1} e^{-\mu x}dx = \frac{u^{2v+2Q-2}}{2\sqrt{\pi}\Gamma(1-Q)} G_{13}^{31}\left(\frac{\mu^2u^2}{4}\,\middle|\,\begin{matrix}1-v\\ 1-Q-v,0,\frac{1}{2}\end{matrix}\right) \qquad (4.1)$$

for $|\arg u\pi| < \frac{\pi}{2}, Re\mu > 0$ and $Rev > 0$ and $G_{13}^{31}\left(\frac{\mu^2u^2}{4}\,\middle|\,\begin{matrix}1-v\\ 1-Q-v,0,\frac{1}{2}\end{matrix}\right)$ is called the Meijer's G-function (Gradshteyn and Ryzhik, 2007, formula no. 3.389.2).

Proof.

$$\varphi_p = \int_0^\pi \cos(p\theta)d\theta + i\int_0^\infty \sin(p\theta)g(\theta)d\theta = \alpha_p + i\beta_p$$

where

$$\alpha_p = \frac{1}{2\sigma^c\Gamma(c)} \int_0^\pi \cos(p\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)}d\theta$$

$$\beta_p = \frac{1}{2\sigma^c\Gamma(c)} \int_0^{\pi} \sin(p\theta) \sec^2\left(\frac{\theta}{2}\right) \left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)} d\theta$$

To derive the first order cosine and sine moments

$$\alpha_1 = \frac{1}{2\sigma^c\Gamma(c)} \int_0^{\pi} \cos(\theta) \sec^2\left(\frac{\theta}{2}\right) \left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)} d\theta$$

Consider the transformation $x = \tan\left(\frac{\theta}{2}\right)$, $\cos\theta = 1 - \frac{2x^2}{1+x^2}$ and formula (4.1)

$$\begin{aligned} \alpha_1 &= \tfrac{1}{\sigma^c\Gamma(c)} \int_0^{\pi} \left[1 - \tfrac{2x^2}{1+x^2}\right] x^{c-1} e^{\frac{1}{\sigma}x} dx \\ &= 1 - \tfrac{2}{\sigma^c\Gamma(c)} \int_0^{\pi} x^{c+1} \left(1+x^2\right)^{-1} e^{\frac{1}{\sigma}x} dx \end{aligned}$$

$$\alpha_1 = 1 - \frac{1}{\sigma^c\sqrt{\pi}\ \Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} -\frac{c}{2} \\ -\frac{c}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

$$\beta_1 = \frac{1}{2\sigma^c\Gamma(c)} \int_0^{\pi} \sin(\theta) \sec^2\left(\frac{\theta}{2}\right) \left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)} d\theta$$

Consider the transformation $x = \tan\left(\frac{\theta}{2}\right)$, $\sin\theta = \frac{2x}{1+x^2}$ and formula (4.1)

$$\begin{aligned} \beta_1 &= \tfrac{1}{\sigma^c\Gamma(c)} \int_0^{\pi} \left[\tfrac{2x}{1+x^2}\right] x^{c-1} e^{\frac{1}{\sigma}x} dx \\ &= \tfrac{2}{\sigma^c\Gamma(c)} \int_0^{\pi} x^c \left(1+x^2\right)^{-1} e^{\frac{1}{\sigma}x} dx \end{aligned}$$

$$\beta_1 = \frac{1}{\sigma^c\sqrt{\pi}\ \Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} \frac{1-c}{2} \\ \frac{1-c}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

To derive the second order cosine and sine moments

$$\alpha_2 = \frac{1}{2\sigma^c\Gamma(c)} \int_0^{\pi} \cos(2\theta) \sec^2\left(\frac{\theta}{2}\right) \left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)} d\theta$$

Consider the transformation $x = \tan\left(\frac{\theta}{2}\right)$, $\cos 2\theta = 1 + \frac{8x^4}{(1+x^2)^2} - \frac{8x^2}{(1+x^2)}$ and formula (4.1)

$$\begin{aligned}\alpha_2 &= \tfrac{1}{\sigma^c\Gamma(c)}\int_0^\infty \left[1 + \tfrac{8x^4}{(1+x^2)^2} - \tfrac{8x^2}{(1+x^2)}\right] x^{c-1}e^{-\frac{1}{\sigma}x}dx \\ &= \tfrac{1}{\sigma^c\Gamma(c)}\int_0^\infty x^{c-1}e^{-\frac{1}{\sigma}x}dx + \tfrac{8}{\sigma^c\Gamma(c)}\int_0^\infty \tfrac{x^{c+3}}{(1+x^2)^2}e^{-\frac{1}{\sigma}x}dx - \tfrac{8}{\sigma^c\Gamma(c)}\int_0^\infty \tfrac{x^{c+1}}{(1+x^2)}e^{-\frac{1}{\sigma}x}dx\end{aligned}$$

$$\alpha_2 = 1 + \frac{4}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{c}{2}-1\\ -\frac{c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right) - \frac{4}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)$$

$$\beta_2 = \tfrac{1}{2\sigma^c\Gamma(c)}\int_0^\pi \sin 2\theta \sec^2\left(\tfrac{\theta}{2}\right)\left(\tan\left(\tfrac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)}d\theta$$

Consider the transformation $x = \tan\left(\frac{\theta}{2}\right)$, $\sin 2\theta = \frac{4x}{(1+x^2)} - \frac{8x^3}{(1+x^2)^2}$ and by formula (4.1)

$$\begin{aligned}\beta_2 &= \tfrac{1}{\sigma^c\Gamma(c)}\int_0^\infty \left[\tfrac{4x}{(1+x^2)} - \tfrac{8x^3}{(1+x^2)^2}\right] x^{c-1}e^{-\frac{1}{\sigma}x}dx \\ &= \tfrac{4}{\sigma^c\Gamma(c)}\int_0^\infty \tfrac{x^c}{(1+x^2)}e^{-\frac{1}{\sigma}x}dx - \tfrac{8}{\sigma^c\Gamma(c)}\int_0^\infty \tfrac{x^{c+2}}{(1+x^2)^2}e^{-\frac{1}{\sigma}x}dx\end{aligned}$$

$$\beta_2 = \frac{2}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}\frac{1-c}{2}\\ \frac{1-c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right) - \frac{4}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{(c+1)}{2}\\ \frac{(1-c)}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)$$

To derive the third cosine and sine moments

$$\alpha_3 = \frac{1}{\sigma^c 2\Gamma(c)}\int_0^\pi \cos 3\theta \sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)}d\theta$$

Consider the transformation $x = \tan\left(\frac{\theta}{2}\right)$, $\cos 3\theta = 1 - \frac{32x^6}{(1+x^2)^3} + \frac{48x^4}{(1+x^2)^2} - \frac{18x^2}{(1+x^2)}$

$$\begin{aligned}\alpha_3 &= \tfrac{1}{\sigma^c\Gamma(c)}\int_0^\infty \left[1 - \tfrac{32x^6}{(1+x^2)^3} + \tfrac{48x^4}{(1+x^2)^2} - \tfrac{18x^2}{(1+x^2)}\right] x^{c-1}e^{-\frac{1}{\sigma}x}dx \\ &= 1 - \tfrac{32}{\sigma^c\Gamma(c)}\int_0^\infty x^{c+5}(1+x^2)^{-3}e^{-\frac{1}{\sigma}x}dx + \tfrac{48}{\sigma^c\Gamma(c)}\int_0^\infty x^{c+3}(1+x^2)^{-2}e^{-\frac{1}{\sigma}x}dx \\ &\quad - \tfrac{18}{\sigma^c\Gamma(c)}\int_0^\infty x^{c+1}(1+x^2)^{-1}e^{-\frac{1}{\sigma}x}dx\end{aligned}$$

$$\alpha_3 = 1 - \frac{16}{3\sigma^c\sqrt{\pi}\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} -\frac{c}{2}-2 \\ -\frac{c}{2},\ 0,\ \frac{1}{2} \end{matrix}\right) + \frac{24}{\sigma^c\sqrt{\pi}\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} -\frac{c}{2}-1 \\ -\frac{c}{2},\ 0,\ \frac{1}{2} \end{matrix}\right)$$

$$- \frac{9}{\sqrt{\pi}\sigma^c\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} -\frac{c}{2} \\ -\frac{c}{2},\ 0,\ \frac{1}{2} \end{matrix}\right)$$

$$\beta_3 = \frac{1}{2\sigma^c\Gamma(c)} \int_0^\pi \sin 3\theta \sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)} d\theta$$

Consider the transformation $x = \tan\left(\frac{\theta}{2}\right), \sin 3\theta = \frac{6x}{(1+x^2)} - \frac{32x^3}{(1+x^2)^3}$

$$\beta_3 = \frac{1}{\sigma^c\Gamma(c)} \int_0^\infty \left[\frac{6x}{(1+x^2)} - \frac{32x^3}{(1+x^2)^3}\right] x^{c-1} e^{-\frac{1}{\sigma}x} dx$$

$$= \frac{6}{\sigma^c\Gamma(c)} \int_0^\infty x^c (1+x^2)^{-1} e^{-\sigma x} dx - \frac{32}{\sigma^c\Gamma(c)} \int_0^\infty x^{c+2}(1+x^2)^{-3} e^{-\frac{1}{\sigma}x} dx$$

$$\beta_3 = \frac{3}{\sigma^c\sqrt{\pi}\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} \frac{1-c}{2} \\ \frac{1-c}{2},\ 0,\ \frac{1}{2} \end{matrix}\right) - \frac{8}{\sigma^c\sqrt{\pi}\Gamma(c)} G_{13}^{31}\left(\frac{1}{4\sigma^2} \;\middle|\; \begin{matrix} -\frac{(c+1)}{2} \\ \frac{1-c}{2},\ 0,\ \frac{1}{2} \end{matrix}\right)$$

To derive the fourth cosine and sine moments

$$\alpha_4 = \frac{1}{2\sigma^c\Gamma(c)} \int_0^\pi \cos 4\theta \sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1} e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)} d\theta$$

Consider the transformation

$x = \tan\left(\frac{\theta}{2}\right), \cos 4\theta = 1 + \frac{128x^8}{(1+x^2)^4} + \frac{160x^4}{(1+x^2)^2} - \frac{256x^6}{(1+x^2)^3} - \frac{32x^2}{(1+x^2)}$ and formula (4.1)

$$\alpha_4 = 1 + \frac{128}{\sigma^c\Gamma(c)} \int_0^\infty x^{c+7}(1+x^2)^{-4} e^{-\frac{1}{\sigma}x} dx + \frac{160}{\sigma^c\Gamma(c)} \int_0^\infty x^{c+3}(1+x^2)^{-2} e^{-\frac{1}{\sigma}x} dx$$

$$- \frac{256}{\sigma^c\Gamma(c)} \int_0^\infty x^{c+5}(1+x^2)^{-3} e^{-\frac{1}{\sigma}x} dx - \frac{32}{\sigma^c\Gamma(c)} \int_0^\infty x^{c+1}(1+x^2)^{-1} e^{-\frac{1}{\sigma}x} dx$$

$$= 1 + \frac{128}{\sigma^c\Gamma(c)} \int_0^\infty x^{2\left(\frac{c}{2}+4\right)-1}(1+x^2)^{-3-1} e^{-\frac{1}{\sigma}x} dx + \frac{160}{\sigma^c\Gamma(c)} \int_0^\infty x^{2\left(\frac{c}{2}+2\right)-1}(1+x^2)^{-1-1} e^{-\frac{1}{\sigma}x} dx$$

$$- \frac{256}{\sigma^c\Gamma(c)} \int_0^\infty x^{2\left(\frac{c}{2}+3\right)-1}(1+x^2)^{-2-1} e^{-\frac{1}{\sigma}x} dx - \frac{32}{\sigma^c\Gamma(c)} \int_0^\infty x^{2\left(\frac{c}{2}+1\right)-1}(1+x^2)^{0-1} e^{-\frac{1}{\sigma}x} dx$$

$$\alpha_4 = 1+\frac{32}{3\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{c}{2}-3\\ -\frac{c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)+\frac{80}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{c}{2}-1\\ -c,\ 0,\ \frac{1}{2}\end{matrix}\right)$$

$$-\frac{64}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{c}{2}-2\\ -\frac{c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)-\frac{16}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{c}{2}\\ -\frac{c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)$$

$$\beta_4 = \frac{1}{2\sigma^c\Gamma(c)}\int_0^{\pi}\sin 4\theta\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{c-1}e^{-\frac{1}{\sigma}\tan\left(\frac{\theta}{2}\right)}d\theta$$

$\sin 4\theta = \frac{8x}{(1+x^2)} - \frac{16x^3}{(1+x^2)^2} - \frac{64x^3}{(1+x^2)^3} + \frac{128x^5}{(1+x^2)^4}$ and formula (4.1)

$$\beta_4 = \frac{8}{\sigma^c\Gamma(c)}\int_0^{\infty}x^c(1+x^2)^{-1}e^{-\frac{1}{\sigma}x}dx-\frac{16}{\sigma^{\lambda}\Gamma(c)}\int_0^{\infty}x^{c+2}(1+x^2)^{-2}e^{-\frac{1}{\sigma}x}dx$$

$$-\frac{64}{\sigma^c\Gamma(c)}\int_0^{\infty}x^{c+2}(1+x^2)^{-3}e^{-\frac{1}{\sigma}x}dx+\frac{128}{\sigma^c\Gamma(c)}\int_0^{\infty}x^{c+4}(1+x^2)^{-4}e^{-\frac{1}{\sigma}x}dx$$

$$=\frac{8}{\sigma^c\Gamma(c)}\int_0^{\infty}x^{2\left(\frac{c}{2}+\frac{1}{2}\right)-1}(1+x^2)^{0-1}e^{-\frac{1}{\sigma}x}dx-\frac{16}{\sigma^c\Gamma(c)}\int_0^{\infty}x^{2\left(\frac{c}{2}+\frac{3}{2}\right)-1}(1+x^2)^{-1-1}e^{-\frac{1}{\sigma}x}dx$$

$$-\frac{64}{\sigma^c\Gamma(c)}\int_0^{\infty}x^{2\left(\frac{c}{2}+\frac{3}{2}\right)-1}(1+x^2)^{-2-1}e^{-\frac{1}{\sigma}x}dx+\frac{128}{\sigma^c\Gamma(c)}\int_0^{\infty}x^{2\left(\frac{c}{2}+\frac{5}{2}\right)-1}(1+x^2)^{-3-1}e^{-\frac{1}{\sigma}x}dx$$

$$\beta_4 = \frac{4}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}\frac{1-c}{2}\\ \frac{1-c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)-\frac{8}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{(c+1)}{2}\\ \frac{1-c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)$$

$$-\frac{16}{\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{(c+1)}{2}\\ \frac{3-c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)-\frac{32}{3\sigma^c\sqrt{\pi}\Gamma(c)}G_{13}^{31}\left(\frac{1}{4\sigma^2}\;\middle|\;\begin{matrix}-\frac{(c+3)}{2}\\ \frac{3-c}{2},\ 0,\ \frac{1}{2}\end{matrix}\right)$$

On the similar lines the higher-order moments can be obtained .

5. Stereographic -l-axial Semicircular Gamma Distribution

The above proposed model is extended to the l-axial distribution, which is applicable to any arc of arbitrary length say $\frac{2\pi}{l}$ for $l \in$ N, . It is possible and useful to extend the Stereographic Semicircular Gamma distribution to construct the Stereographic-l-axial Gamma distribution.

The density function of stereographic semicircular gamma distribution by using the transformation $\phi = \frac{2\theta}{l}, l \in$ N. on the probability density function of ϕ is given by

$$g(\phi) = \frac{1}{2\sigma^c\Gamma(c)}\sec^2\left(\frac{l\phi}{4}\right)\left(\tan\left(\frac{l\phi}{4}\right)\right)^{c-1}\exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{l\phi}{4}\right)\right)\right) \qquad (5.1)$$

$0 < \phi < \frac{2\pi}{l}, \sigma > 0, c > 0$ and $l = 1, 2, \ldots$

It is coined as Stereographic -l-axial Gamma Distribution

Case (1) When $l = 1$, in the probability density function (5.1), we get the density function

$$g(\phi) = \frac{1}{2\sigma^c \Gamma(c)} \sec^2\left(\frac{\phi-\mu}{4}\right)\left(\tan\left(\frac{\phi-\mu}{4}\right)\right)^{c-1} \exp\left(-\frac{1}{\sigma}\left(\tan\left(\frac{\phi-\mu}{4}\right)\right)\right) \quad (5.2)$$

$0 < \phi < 2\pi, \sigma > 0$ and $c > 0$

It is named by us as Stereographic Circular Gamma Distribution.

Case (2) When $l = 2$, the probability density function (5.1) is the same as that of Stereographic Semicircular Gamma Distribution.

Case (3) When $l = 2$ and $c = 1$, in the probability density function (5.1) we get the density function of Stereographic Semicircular Exponential Distribution [Phani et al (2013)].

6. Conclusion

In this paper, we derived the semicircular distribution induced by modified inverse stereographic projection on Gamma distribution is discussed and named it by us as Stereographic Semicircular Gamma distribution. The density and distribution function of Stereographic semicircular gamma distribution admit explicit forms, as do trigonometric moments. As this distribution is asymmetric, it is suitable for modeling skewed directional data.

References

[1] Byoung, J.A and Hyoung M.K.,*A New Family of Semicircular Models: The Semicircular Laplace Distributions*, Communications of the Korean Statistical Society Vol.**15**(2008), 775-781.

[2] Dattatreya Rao, A.V., Ramabhadra Sarma, I. and Girija, S.V.S., *On Wrapped Version of Some Life Testing Models*, Comm Statist,- Theor.Meth. , Vol.**36(11)**(2007), 2027-2035.

[3] Fisher, N. I.,*Statistical Analysis of Circular Data*. Cambridge University Press, Cambridge (1993).

[4] Girija, S.V.S., *Construction of New Circular Models*, VDM VERLAG, Germany. ISBN 978-3-639-27939-9,(2010).

[5] Gradshteyn and Ryzhik *Table of Integrals, series and products*, 7th edition, Academic Press(2007).

[6] Guardiola, J.H. , *The Semicircular Normal Distribution*, Ph.D Dissertation, Baylor University, Institute of Statistics(2004).

[7] Jammalamadaka S. Rao and Sen Gupta, A., *Topics in Circular Statistics*, World Scientific Press, Singapore(2001).

[8] Mardia, K.V. and Jupp, P.E. , *Directional Statistics*, John Wiley, Chichester(2000).

[9] Minh, Do Le and Farnum, Nicholas R., *Using Bilinear Transformations to Induce Probability Distributions*, Communication in Statistics - Theory and Methods, Vol.**32 (1)**(2003), 1 - 9.

[10] Phani, Y., Girija S.V.S. and Dattatreya Rao A.V., *Circular Model Induced by Inverse Stereographic Projection On Extreme-Value Distribution*, IRACST - Engineering Science and Technology: An International Journal (ESTIJ), ISSN: 2250-3498, Vol.**2(5)**(2012),881 - 888.

[11] Phani, Y., Girija S.V.S. and Dattatreya Rao A.V.,*On Construction of Stereographic Semicircular Models, Journal of Applied Probability and Statistics*, Vol. **8 (1)**(2013), pp. 75-90.

[12] Phani Y., *On Stereographic Circular and Semicircular Models*, Thesis submitted to Acharya Nagarjuna University for the award of Ph.D.(2013).

[13] Ugai. S.K., Nishijima, M. and Kan, T. , *Characteristics of raindrop size and raindrop shape*, Open symposium URSI Commission F,(1977), 225-230.

[14] Toshihiro Abe, Kunio Shimizu and Arthur Pewsey,*Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection*, J. Japan Statist. Soc., Vol. **40 (1)**(2010), 45-61.

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ABSTRACT

Glancing the literature, semicircular, arc and skewed angular data is observed in the applications such as Feldspar laths data (Fisher, 1993, p. 240), Face - cleat in a coal seam data (Fisher, 1993, p. 254), Fallen trees data (Toshihiro, et al., 2012), face recognition problem, etc., and not much was done to model such angular data. Moreover Chi – Square distribution can be used as a quick test of significance in most situations, especially using machine learning algorithms. This finer pointer has become a motivating factor to work on l - axial (arc) models, and they can be viewed as the most general angular models from which Circular, Semicircular and other kinds of models could be deduced as particular cases and also to derive new angular model from Chi-Square distribution. In this paper, the authors introduce a new semicircular model, induced by Modified Inverse Stereographic Projection on Chi-Square distribution for modeling semicircular data. The authors extend it to the l- axial Chi-Square distribution for modeling axial data and also they derive the first two trigonometric moments for the proposed distribution in closed forms.

Keywords: Circular Models, Inverse Stereographic Projection, Trigonometric Moments, l- Axial Model.

INTRODUCTION

Angular data/ circular data are very common in the areas of biology, geology, meteorology, earth science, political science, Economics, computer science, etc. Full circular models are prevalent at most of the textbooks (Fisher, 1993; Jammalamadaka and Sengupta, 2001; Mardia and Jupp, 2000). In some of the cases the angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004), Jones (1968), Ahn and Kim (2008) and Yedlapalli et al. (2013). For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost, but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data are available in Ugai et al. (1977).

Guardiola (2004) obtained the semicircular normal distribution by using a simple projection and Ahn and Kim (2008) developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Yedalapalli et al. (2013) constructed some semicircular distributions by applying modified inverse Stereographic projection. In this paper, the authors derive a new semicircular Chi-square distribution by applying modified inverse stereographic projection on linear Chi-square distribution and they also construct l-axial chi-square distribution for modeling l-axial data. The first two trigonometric moments of the proposed model are derived.

This paper is organized as follows. Section 1 describes the methodology of Modified Inverse Stereographic Projection. Section 2 defines the proposed semicircular distribution, draw the graphs of density and distribution functions for various degrees of freedom, and in section 3 the first two trigonometric moments are derived, which are sufficient for studying

population characteristics. In section 4 the proposed model is extended to I-axial data.

1. Methodology of Modified Inverse Stereographic Projection

Modified Inverse Stereographic Projection is defined by a one to one mapping given by $T(\theta)=x=v\tan(\theta/2)$ where $x\in(-\infty, \infty)$, $(-\pi, \pi)$, $v\in\mathbb{R}^+$. Suppose x is randomly chosen on the interval $(-\infty, \infty)$. Let F(x) and f(x) denote the Cumulative distribution and probability density functions of the random variable X, respectively. Then $T^{-1}(x)=\theta=2\tan^{-1}(x/v)$ by Toshihiro Abe et al. (2010) is a random point on the unit circle. Let $G(\theta)$ and $g(\theta)$ denote the Cumulative distribution and probability density functions of this random point, respectively. Then $G(\theta)$ and $g(\theta)$ can be written in terms of F(x) and f(x) using the following Theorem.

1.1 Theorem

For $v>0$,

- $G(\theta)=F\left(v\tan\left(\frac{\theta}{2}\right)\right)=F(x(\theta))$
- $g(\theta)=v\left(\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{2}\right)f\left(v\tan\left(\frac{\theta}{2}\right)\right)$

If a linear random variable X has a support on, $\mathbb{R}$ then θ has a support on $(-\pi, \pi)$, and if X has a support on, $\mathbb{R}^+$ then θ has a support on $(0, \pi)$. These means that, after the Inverse Stereographic Projection is applied, we can deal circular data if the support of X is on $\mathbb{R}$ and we can handle semicircular data if the support of X is on $\mathbb{R}^+$

2. Stereographic Semicircular χ^2 (Chi-Square) Distribution

2.1 Definition

A random variable X on the real line is said to have Chi-square Distribution with v degree of freedom, if the probability density and cumulative distribution functions of X are respectively given by,

$$f(x)=\frac{x^{\frac{v}{2}-1}}{2^{\frac{v}{2}}\Gamma\left(\frac{v}{2}\right)}\exp\left(-\frac{x}{2}\right),\quad 0<x<\infty \tag{1}$$

and

$$F(x,v)=\frac{\Gamma_x\left(\frac{v}{2}\right)}{\Gamma\left(\frac{v}{2}\right)},\ 0<x<\infty\ \text{ and }\Gamma_x\left(\frac{v}{2}\right)=\int_0^x y^{\left(\frac{v}{2}-1\right)}e^{-y}dy \tag{2}$$

Then by applying modified inverse Stereographic projection defined by a one to one mapping $x=v\tan(\theta/2)$, $v\in R^+$, which leads to a Semicircular Model on unit semicircle. This model is called as Stereographic Semicircular Chi-Square Distribution.

2.2 Definition

A random variable X_{SC} on the Semicircle is said to have the Stereographic Semicircular Quasi Lindley distribution with v degree of freedom, denoted by SSC2D (σ, v), if the probability density and the cumulative distribution functions are respectively given by,

$$g(\theta;v,\sigma)=\frac{\sigma^{\frac{v}{2}}\sec^2\left(\frac{\theta}{2}\right)}{2\Gamma\left(\frac{v}{2}\right)}\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{v}{2}-1}\exp\left(-\sigma\tan\left(\frac{\theta}{2}\right)\right), \tag{3}$$

where, $0 \le \theta < \pi, \sigma = \frac{v}{2} > 0$

$$G(\theta, \nu) = \frac{\Gamma_{\left(\nu \tan\left(\frac{\theta}{2}\right)\right)}\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}, \quad 0 < \theta < \pi \quad \text{and} \quad \Gamma_{\nu \tan\left(\frac{\theta}{2}\right)}\left(\frac{\nu}{2}\right) = \int_0^{\nu \tan\left(\frac{\theta}{2}\right)} y^{\left(\frac{\nu}{2}-1\right)} e^{-y} dy \tag{4}$$

Clearly

- $g(\theta) \ge 0, \quad \forall\ 0 \le \theta < \pi$
- $\int_0^{\pi} g(\theta) d\theta = 1$
- $g(\theta + 2k\pi) = g(\theta), \ for\ k \in Z$

Figure 1 shows the Graph of pdf stereographic semicircular chi-square model for $\sigma = 5$

3. Trigonometric Moments of Stereographic Semicircular χ^2 (Chi-Square) Distribution

It is customary to derive the trigonometric moments when a new distribution is proposed.

The trigonometric moments of the distribution are given by $\{\varphi_p: p=0, \pm1, \pm2, \pm3, ...\}$, where $\varphi_p = \alpha_p + \beta i_p$ with $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$ being the pth order cosine and sine moments of the random angle θ, respectively.

3.1 Theorem

Under the pdf of Stereographic Semicircular χ^2 (Chi-Square) Distribution, the first two $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$, $p = 1, 2$ are given as follows:

$$\alpha_1 = 1 - \frac{\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle| \begin{matrix} -\frac{\nu}{2} \\ -\frac{\nu}{4}, 0, \frac{1}{2} \end{matrix}\right)$$

$$\beta_1 = \frac{\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle| \begin{matrix} 1-\frac{\nu}{2} \\ 1-\frac{\nu}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

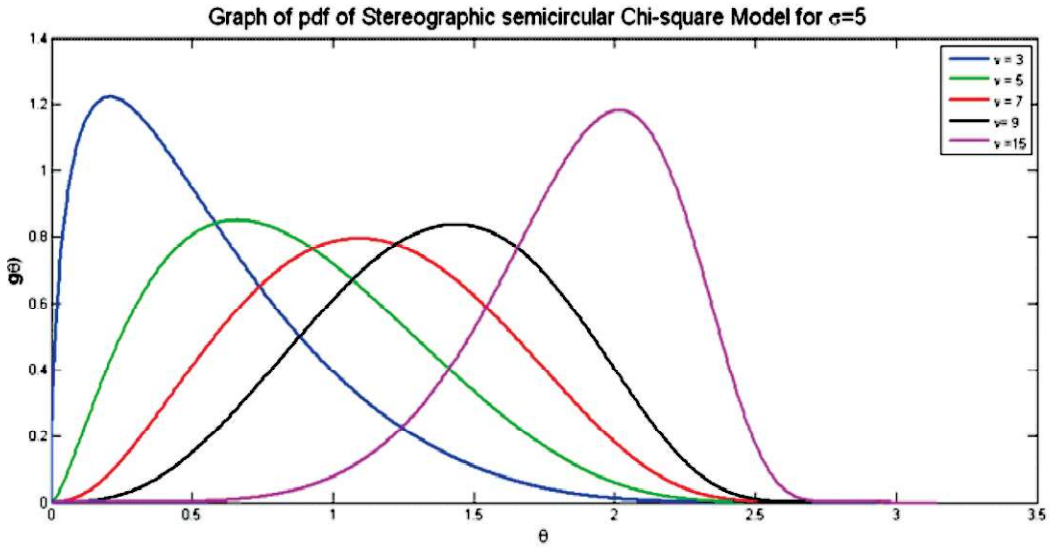

Figure 1. Graphs of Probability Density and Distribution Functions of Stereographic Semicircular Chi-Square Distribution for Various v

$$\alpha_2 = 1 + \frac{4\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left[G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{\nu}{4}-1\\ -\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right) - G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{\nu}{4}\\ -\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right)\right]$$

$$\beta_2 = \frac{2\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left[G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{1}{2}-\frac{\nu}{4}\\ 1-\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right) - 2G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}-\frac{\nu}{4}\\ \frac{1}{2}-\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right)\right]$$

Proof:

$$\varphi_p = \int_0^{\pi} e^{ip\theta} g(\theta)d\theta = \int_0^{\pi} \cos(p\theta)g(\theta)d\theta + i\int_0^{\pi} \sin(p\theta)g(\theta)d\theta$$

$$= E(\cos(p\theta)) + iE(\sin(p\theta)) = \alpha_p + i\beta_p \quad \text{for } p = 0,\pm1,\pm2,\pm3,...,$$

For the first cosine and sine moments, use the transformations $x = \tan\left(\frac{\theta}{2}\right)$, $\cos\theta = 1 - \frac{2x^2}{1+x^2}$, and $\sin\theta = \frac{2x}{1+x^2}$ the results α_1 and β_1 follows by the integral formula 3.389.2 (Gradshteyn and Ryzhik, 2007).

Now, $E(\cos(p\theta)) = \frac{\sigma^{\frac{\nu}{2}}}{2\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\pi} \cos(p\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{\nu}{2}-1} e^{-\sigma\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$ and

$$E(\sin(p\theta)) = \frac{\sigma^{\frac{\nu}{2}}}{2\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\pi} \sin(p\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{\nu}{2}-1} e^{-\sigma\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$\alpha_1 = \frac{\sigma^{\frac{\nu}{2}}}{2\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\pi} \cos\theta\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{\nu}{2}-1} e^{-\sigma\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$= \frac{\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty}\left[1 - \frac{2x^2}{1+x^2}\right](x)^{\frac{\nu}{2}-1} e^{-\sigma x}dx$$

$$= 1 - \frac{2\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{2\left(\frac{\nu+4}{4}\right)-1}\left(1+x^2\right)^{0-1} e^{-\sigma x}dx$$

$$\alpha_1 = 1 - \frac{\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{\nu}{2}\\ -\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right)$$

$$\beta_1 = \frac{\sigma^{\frac{\nu}{2}}}{2\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\pi} \sin\theta \sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{\nu}{2}-1} e^{-\sigma\left(\tan\left(\frac{\theta}{2}\right)\right)} d\theta$$

$$= \frac{\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty}\left(\frac{x}{1+x^2}\right)(x)^{\frac{\nu}{2}-1} e^{-\sigma x} dx$$

$$= \frac{2\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{\frac{\nu}{2}}\left(1+x^2\right)^{0-1} e^{-\sigma x} dx$$

$$= \frac{2\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{2\left(\frac{\nu+2}{4}\right)-1}\left(1+x^2\right)^{0-1} e^{-\sigma x} dx$$

$$\beta_1 = \frac{\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)} G_{13}^{31}\left(\frac{\sigma^2}{4}\middle| \begin{matrix} 1-\frac{\nu}{2} \\ 1-\frac{\nu}{2}, 0, \frac{1}{2} \end{matrix}\right)$$

To obtain second cosine and sine moments α_2 and β_2, we use the transformations $x = \tan\left(\frac{\theta}{2}\right)$, $\cos 2\theta = 1 + \frac{8x^4}{\left(1+x^2\right)^2} - \frac{8x^2}{\left(1+x^2\right)}$ and $\sin 2\theta = \frac{4x}{\left(1+x^2\right)} - \frac{8x^3}{\left(1+x^2\right)^2}$, the results of α_2 and β_2 follow by the same integral formula of α_1.

$$\alpha_2 = \frac{\sigma^{\frac{\nu}{2}}}{2\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\pi} \cos(2\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{\nu}{2}-1} e^{-\sigma\left(\tan\left(\frac{\theta}{2}\right)\right)} d\theta$$

$$= \frac{\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty}\left[1 + \frac{8x^4}{\left(1+x^2\right)^2} - \frac{8x^2}{\left(1+x^2\right)}\right](x)^{\frac{\nu}{2}-1} e^{-\sigma x} dx$$

$$= 1 + \frac{8\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{\frac{\nu}{2}+3}\left(1+x^2\right)^{-1-1} e^{-\sigma x} dx - \frac{8\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{\frac{\nu}{2}+1}\left(1+x^2\right)^{-0-1} e^{-\sigma x} dx$$

$$= 1 + \frac{8\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{2\left(\frac{\nu+8}{4}\right)-1}\left(1+x^2\right)^{-1-1} e^{-\sigma x} dx - \frac{8\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{2\left(\frac{\nu+4}{4}\right)-1}\left(1+x^2\right)^{-0-1} e^{-\sigma x} dx$$

$$\alpha_2 = 1 + \frac{4\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left[G_{13}^{31}\left(\frac{\sigma^2}{4}\middle| \begin{matrix} -\frac{\nu}{4}-1 \\ -\frac{\nu}{4}, 0, \frac{1}{2} \end{matrix}\right) - G_{13}^{31}\left(\frac{\sigma^2}{4}\middle| \begin{matrix} -\frac{\nu}{4} \\ -\frac{\nu}{4}, 0, \frac{1}{2} \end{matrix}\right)\right]$$

$$\beta_2 = \frac{\sigma^{\frac{\nu}{2}}}{2\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\pi} \sin(2\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\frac{\nu}{2}-1} e^{-\sigma\left(\tan\left(\frac{\theta}{2}\right)\right)} d\theta$$

$$= \frac{\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty}\left[\frac{4x}{(1+x^2)} - \frac{8x^3}{(1+x^2)^2}\right](x)^{\frac{\nu}{2}-1} e^{-\sigma x} dx$$

$$= \frac{4\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{\frac{\nu}{2}}(1+x^2)^{-1} e^{-\sigma x} dx - \frac{8\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{\frac{\nu}{2}+2}(1+x^2)^{-1-1} e^{-\sigma x} dx$$

$$= \frac{4\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{2\left(\frac{\nu+2}{4}\right)-1}(1+x^2)^{0-1} e^{-\sigma x} dx - \frac{8\sigma^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\int_0^{\infty} x^{2\left(\frac{\nu+6}{4}\right)-1}(1+x^2)^{-1-1} e^{-\sigma x} dx$$

$$\beta_2 = \frac{2\sigma^{\frac{\nu}{2}}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)}\left[G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}\frac{1}{2}-\frac{\nu}{4}\\ 1-\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right) - 2G_{13}^{31}\left(\frac{\sigma^2}{4}\middle|\begin{matrix}-\frac{1}{2}-\frac{\nu}{4}\\ \frac{1}{2}-\frac{\nu}{4},0,\frac{1}{2}\end{matrix}\right)\right]$$

Higher-order moments can be obtained similarly.

The first two trigonometric moments are sufficient for calculating population characteristics .

4. Stereographic -l-axial χ^2 (Chi-Square) Distribution

The authors extend the above Stereographic Semicircular model to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1, 2,....$, so it is desirable to extend the Stereographic Semicircular χ^2 (Chi-Square) distribution to construct the Stereographic- l-axial χ^2 (Chi-Square) distribution, we consider the density function of Stereographic Semicircular χ^2 (Chi-Square) distribution and use the transformation $\varphi = 2\theta/l$, $l=1,2,.....$ The probability density function of φ is given by,

$$g(\theta;\nu,\sigma) = \frac{l\sigma^{\frac{\nu}{2}}\sec^2\left(\frac{l\theta}{4}\right)}{4\Gamma\left(\frac{\nu}{2}\right)}\left(\tan\left(\frac{l\theta}{4}\right)\right)^{\frac{\nu}{2}-1}\exp\left(-\sigma\tan\left(\frac{l\theta}{4}\right)\right),\ 0<\theta<\frac{2\pi}{l},\ l=1,2,..\ \text{and}\ \nu\ dof \qquad (5)$$

We call it as Stereographic -l-axial χ^2 (Chi-Square) distribution.

Case (1) When $l=1$, in the probability density function (5), we get the density function

$$g(\theta;\nu,\sigma) = \frac{\sigma^{\frac{\nu}{2}}\sec^2\left(\frac{\theta}{4}\right)}{4\Gamma\left(\frac{\nu}{2}\right)}\left(\tan\left(\frac{\theta}{4}\right)\right)^{\frac{\nu}{2}-1}\exp\left(-\sigma\tan\left(\frac{\theta}{4}\right)\right),\ 0<\theta<2\pi,\quad \nu\quad dof \qquad (6)$$

We call it as Stereographic Circular χ^2 (Chi-Square) distribution.

Case (2) When $l=2$, the probability density function (5) is the same as that of Stereographic Semicircular χ^2 (Chi-Square) Distribution. Figure 2 shows the Graph of pdf stereographic semicircular chi-square model for $\sigma = 5$

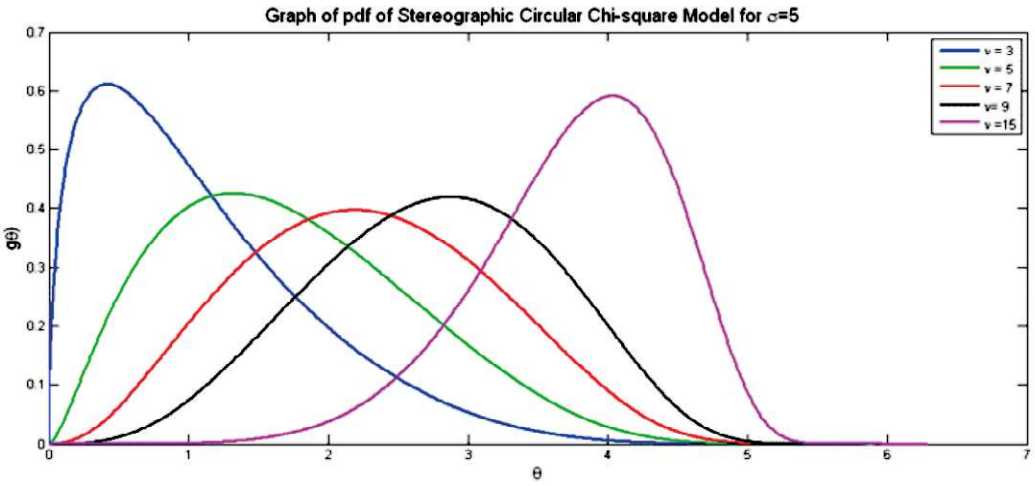

Figure 2. Graphs of Probability Density and Distribution Function of Sterographic Circular Chi-Square Distribution for various V

Conclusion

In this paper, the authors discussed the significance of l - axial / semicircular / angular data which we come across in many practical situations. Concentrating on this aspect, in this paper semicircular distribution induced by modified inverse stereographic projection on Chi-Square Distribution is derived. The density and distribution function of Stereographic semicircular Chi-Square distribution admitted explicit forms, as did trigonometric moments. In deriving trigonometric moments, Meijer - G function from the integral formula 3.389.2 (Gradshteyn and Ryzhik, 2007) is adopted. As this distribution is asymmetric, promising for modeling asymmetrical directional data.

References

[1]. Abe, T., Shimizu, K., & Pewsey, A. (2010). Symmetric unimodal models for directional data motivated by inverse stereographic projection. *Journal of the Japan Statistical Society,* 40(1), 045-061.

[2]. Ahn, B. J. & Kim, H. M. (2008). A New Family of Semicircular Models: The Semicircular Laplace Distributions. *Communications for Statistical Applications and Methods,* 15(5), 775-781.

[3]. Fisher, N. I. (1995). *Statistical Analysis of Circular Data*. Cambridge University Press.

[4]. Girija, S. V. S., Rao, A. D., & Yedlapalli, P. (2013). On stereographic lognormal distribution. *International Journal of Advances in Applied Sciences,* 2(3), 125-132.

[5]. Gradshteyn, I. S. & Ryzhik, I. M. (2007). *Table of Integrals, Series and Products* 7th Edn., Edited by A Jeffrey and D. Zwillinger (New York: Academic).

[6]. Jammalamadaka, S. R. & Sengupta, A. (2001). *Topics in Circular Statistics* (Vol. 5). World Scientific.

[7]. Kim, H. M. (2010). A Projected Exponential Family for Modeling Semicircular Data. *Korean Journal of Applied Statistics,* 23(6), 1125-1145.

[8]. Mardia, K. V. & Jupp, P. (2000). Directional Statistics. John Wiley. Hoboken, NJ.

[9]. Minh, D. L. & Farnum, N. R. (2003). Using bilinear transformations to induce probability distributions. *Communications in Statistics-Theory and Methods,* 32(1), 1-9.

[10]. Phani, Y., Girija, S. V. S., & Dattatreya Rao, A. V. (2012). Circular model induced by inverse stereographic projection on extreme-value distribution. *Engineering Science and Technology*, 2(5), 881-888.

[11]. Yedlapalli, P., Girija, S. V. S., & Rao, A. D. (2013). On construction of stereographic semicircular models. *Journal of Applied Probability,* 8(1), 75-90.

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CUT-OFF POINTS FOR VARIOUS TESTS FOR CIRCULAR UNIFORMITY

By

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ABSTRACT

Because of central role played by Circular Uniform Distribution, the most important hypothesis about a distribution on the Circle is that of Uniformity. Though several tests were proposed, cut-off points for various sample sizes and level of significance were not made available in the literature. Hence, an attempt is made here to present tables of cut-off points and their utility in discussing goodness of fit of a model is presented.

Keywords: Circular Model, Cut-off Points, Goodness of Fit, Wrapped Lognormal, Wrapped Logistic, Wrapped Extreme-Value Models.

INTRODUCTION

Given a sample $\theta_1, \theta_2, \ldots, \theta_n$ from a distribution and in order to verify goodness of fit we need to make use of the tests for circular uniformity for a particular sample size n and since the cut off points for various test procedures for different sample sizes n, here an attempt is made to find out these cut off points for various sample sizes and level of significance. Some of the following important tests procedures for circular uniformity including Graphical assessment are reviewed here for ready reference. Several authors suggested various test procedures. Some of them are Rayleigh Test (Cordeiroand Ferrari (1991)), Kuiper's Test (1960), Watson's U^2 - Test (1967), Hodges-Ajne Test (Hodges (1955), Ajne (1968)), Range Test (Laubscher and Rudolph (1968)and Rao (1969), Rao's Equal Spacing Test (1969) and Ajne Test (1968). These are discussed in Mardia and Jupp (2000). Marrero (1983) discussed the Performance of Several Statistical Tests for Seasonality in Monthly Data. Fernandez-Duran and Gregorio-Dominguez (2010) studied a Likelihood Ratio Test for Homogeneity in Circular Data.

Graphical Test for Uniformity

Assuming the given datais a set of angles, ordered as $0 \le \theta_{(1)} \le \theta_{(2)} \le \ldots\ldots \le \theta_{(n)} \le 2\pi$ with respect to some initial direction then $\theta_{(i)}/(2\pi)$ is plotted against $i/(n+1)$. If $\theta_1, \theta_2, \ldots, \theta_n$ is a random sample from the Circular Uniform distribution then the points should lie near a straight line of slope 45^0 passing through the origin.

Note that the plot depends on the choice initial direction; one way of guarding against misleading impressions arising out of wrong choice of initial direction is to extend the plot by adding the points,

$$1+\frac{i}{n+1}, \qquad 1+\frac{\theta_{(i)}}{2\pi} \text{ for (say) } 1 \le i \le 0.2n \tag{1}$$

and

$$-1+\frac{i}{n+1}, \qquad -1+\frac{\theta_{(i)}}{2\pi} \text{ for (say) } 0.8n \le i \le n \tag{2}$$

The Rayleigh Test

Let P_i be a point on the circumference of the unit circle corresponding to the angle θ_i, i=1(1)n. Then the mean direction θ_0 of $\theta_1, \theta_2, \ldots, \theta_n$ is defined to be the direction of the resultant of the unit vectors $\overline{OP_1}, \overline{OP_2}, \ldots, \overline{OP_n}$. The Cartesian coordinates of P_i are

$(\cos\theta_i, \sin\theta_i)$ so that the centre of gravity of these points is (C, S) where,

$$C = \frac{1}{n}\sum \cos\theta_i \text{ and } S = \frac{1}{n}\sum \sin\theta_i$$

then, $R = \sqrt{C^2 + S^2}$ gives the length of the resultant vector. The direction θ_0 of the resultant vector,

$$\theta_0 = \begin{cases} \tan^{-1}(S/C) & \text{if } S > 0,\ C > 0 \\ \tan^{-1}(S/C) + \pi & \text{if } C < 0 \\ \tan^{-1}(S/C) + 2\pi & \text{if } S < 0,\ C > 0 \end{cases}$$

is called the mean direction.

$$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} \cos(\theta_{(i)}), \bar{S} = \frac{1}{n}\sum_{i=1}^{n} \sin(\theta_{(i)}) \text{ and } \bar{R} = \sqrt{\bar{C}^2 + \bar{S}^2}$$

$$2(1 - \bar{R}) = \frac{2}{n}\sum_{i=1}^{n} [1 - \cos(\theta_i - \theta_0)]$$

leads to define circular variance

The simplest of all the test procedures is Rayleigh Test. Because $E[(\cos\theta, \sin\theta)]^T = 0$ when θ has Uniform Distribution, it is by intuition to reject uniformity when the vector of sample mean $[\bar{C}, \bar{S}]$ is far from 0 or when R is large.

It follows from the general theory of score tests [Cox and Hinkley (1974)] that the large sample asymptotic distribution of $2n\bar{R}^2$ under Uniformity follows χ_2^2

$$2n\bar{R}^2 \approx \chi_2^2 \tag{3}$$

where, $\bar{C} = \frac{1}{n}\sum_{i=1}^{n} \cos(\theta_{(i)})$

$$\bar{S} = \frac{1}{n}\sum_{i=1}^{n} \sin(\theta_{(i)})$$

Kuiper's Test

Various tests for distributions on the line are on measuring the deviation between the empirical and hypothesized cumulative distribution functions (example Kolmogrov one sample test). This test procedure is an analogue for distributions on Circle.The procedure is as follows.

Given the angular data, the ordered observation $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(n)}$ are augmented by $\theta_{(0)} = 0$, and $\theta_{(n+1)} = 2\pi$

Define $U_i = \frac{\theta_{(i)}}{2\pi}$ $i = 1,2,\ldots,n+1$ (4)

$$D_n^+ = \max_{1 \le i \le n}\left\{\frac{i}{n} - U_i\right\} \tag{5}$$

$$D_n^- = \max_{1 \le i \le n}\left\{U_i - \frac{i-1}{n}\right\} \tag{6}$$

$V_n = D_n^+ + D_n^-$ is Kuiper's Statistics (7)

It was modified by Stephens (1970) as,

$$V_n^* = \sqrt{n}\ V_n\left(1 + \frac{0.155}{\sqrt{n}} + \frac{0.24}{n}\right) \tag{8}$$

Watson's U^2 - Test

Instead of measuring the discrepancy between the empirical distribution function and the cumulative distribution function of a Uniform distribution by a variant of the maximum deviation as Kuiper's test, Watson (1961) considered corrected mean

square deviation to construct this test statistic which is as follows

$$\text{Compute } U^2 = \sum_{i=1}^{n}\left[U_i - \overline{U} - \frac{i-1/2}{n} + \frac{1}{2}\right]^2 + \frac{1}{12n} \tag{9}$$

where $\overline{U}$ is mean of U_i's

A modified version of the Statistics suggested by Stephens (1970) is given by,

$$U^{*2} = \left(U^2 - \frac{0.1}{n} + \frac{0.1}{n^2}\right)\left(1 + \frac{0.8}{n}\right) \tag{10}$$

The Hodges -Ajne Test

An alternative strategy for testing Uniformity on the circle is to reject if the observed number of sample points which fall in an arc defers greatly from the expected number. One of the natural measures of the discrepancy between observed and expected numbers is the maximum difference. The description of the test is as follows.

Let N() be the number of observations in the semicircle centered on θ_i i.e., in the arc (θ-π/2, $\theta+\pi$/2). The Hodges- Ajne test rejects Uniformity for large values of

$$M = \max_{\theta} N(\theta), \text{ or equivalently for small values of} \tag{11}$$

$$m = \min_{\theta} N(\theta) \tag{12}$$

Range Test

The circular sample range w is defined as the length of smallest arc which contains all observations. The description is as follows

$$\text{Define } T_i = \theta_{(i)} - \theta_{(i-1)} \quad i=1,\ldots,n-1 \tag{13}$$

$$T_n = 2\pi - (\theta_{(n)} - \theta_{(1)}) \tag{14}$$

$$T = \max_{1\le i\le n} T_i \tag{15}$$

then the Range Statistics w is given by,

$$w = 2\pi - T \tag{16}$$

Since small values of w indicates clustering of the observations, hence uniformity assumption is rejected. It can be seen that w is invariant under rotations.

Test of Equal Spacings

Under Uniformity $E[T_i] = 2\pi/n$. Hence it is intuitively reasonable to reject Uniformity for large values of

$$L = \frac{1}{2}\sum_{i=1}^{n}\left|T_i - \frac{2\pi}{n}\right| \tag{17}$$

This was suggested by Rao (1969)

Ajne's Test

Let $N(\theta)$denote the number of observations in the semicircle centered on θ, i.e., in the arc (θ-π/2, $\theta+\pi$/2). Since the expected value of $N(\theta)$under Uniformity is n/2, it is intuitively reasonable to reject Uniformity for large values of)

$$A_n = \frac{1}{2\pi n}\int_0^{2\Pi}\left\{N(\theta) - \frac{n}{2}\right\}^2 d\theta \tag{18}$$

as proposed by Ajne (1968).

The following is the computational formula for

$$A_n = \frac{n}{4} - \frac{1}{n\pi}\sum_{j=2}^{n}\sum_{i=1}^{j-1} \min\left\{\theta_{(j)} - \theta_{(i)}, 2\pi - [\theta_{(j)} - \theta_{(i)}]\right\} \quad (19)$$

1. Methodology of Obtaining Cut-off Points

In this section an attempt is made to find cut-off points for the following Tests,

- Rayleigh Test
- Kuiper's Test
- Watson's U^2- Test
- Hodges-Ajne Test
- Range Test
- Rao's Equal Spacing Test
- Ajne Test

For various values of sample size n = 5, 10, 15, 20, 25 and 30 in addition to n= 21, 13 and 76 and for various values of Level of significance (LOS)α = 1%, 5%, 10%, the methodology of obtaining cut-off points in the form of algorithm is as follows,

- For a specified value of n, angular data $\theta_1, \theta_2, \ldots, \theta_n$, as 2π*U where U$\in$[0,1].
- The vector is sorted in ascending order and the ordered statistics $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(n)}$ are obtained.
- The following are computed

  (i) $\bar{C} = \frac{1}{n}\sum_{i=1}^{n}\cos(\theta_{(i)})$ (ii) $\bar{S} = \frac{1}{n}\sum_{i=1}^{n}\sin(\theta_{(i)})$ (iii) $\bar{R} = \sqrt{\bar{C}^2 + \bar{S}^2}$

- Then the Rayleigh statistic R is calculated as R= $2n\bar{R}^2$
- Similarly the other statistics as given in equations 8, 10, 12, 16, 17, and 19 are computed.

The above steps are repeated for 2000 cycles and in each case depending on left tail or right tail, lower α% cut-off points or upper α% cut-off points are noted. In case of two-tailed test lower $\frac{\alpha}{2}$ and upper $\frac{\alpha}{2}$ cut-off points are recorded and these cut-off points for all these tests are presented in Table 1.

Remarks on the Table of Cut-off Points

- Interpretation of two-tail cut-off points is obvious and useful in testing the hypothesis whether data exhibit the evidence of Circular Uniformity or not.
- Left or Right tail cut-off points may be used in testing whether μ (the mean direction) $< \mu_0$ or $\mu \geq \mu_0$ (μ_0 is specified values).

2. Goodness of Fit for Live Data

For the purpose of verifying goodness of fit the following "Live" data sets were considered.

Data set- 1: Cross - bed Azimuths of palaeocurrents [Fisher (1993), p.242]

Set of cross - bed azimuths of palaeocurrents measured in degrees in the Belford Anticline (New South Wales) [Fisher & Powell (1989)].

284	311	334	320	294	137	123	166	143	127
244	243	152	242	143	186	263	234	209	267
315	329	235	38	241	319	308	127	217	245
169	161	263	209	228	168	98	278	154	279

Data set - 2: Pigeon homing Experimental data [Mardia and Jupp (2000), p.108]

In a pigeon-homing experiment, 13birds were released singly in the Toggenburg Valley. Their vanishing angles were

		1%				5%				10%			
	LOS	One TAIL		Two Tail		One TAIL		Two Tail		One TAIL		Two Tail	
n	Test	L	R	L	R	L	R	L	R	L	R	L	R
5	Test-1	0.023	7.944	0.012	8.570	0.132	5.802	0.066	6.905	0.244	4.500	0.132	5.802
	Test-2	0.720	2.025	0.687	2.127	0.847	1.766	0.788	1.875	0.926	1.635	0.847	1.766
	Test-3	0.007	0.264	0.006	0.298	0.018	0.186	0.012	0.215	0.024	0.154	0.018	0.186
	Test-4	-2.236	1.342	-2.236	1.342	-0.447	1.342	-1.342	1.342	0.447	1.342	-0.447	1.342
	Test-5	1.459	5.173	1.235	5.313	2.050	4.772	1.760	4.978	2.437	4.544	2.050	4.772
	Test-6	0.743	4.095	0.624	4.546	1.061	3.310	0.924	3.687	1.256	3.012	1.061	3.310
	Test-7	0.050	0.826	0.050	0.861	0.050	0.625	0.050	0.724	0.064	0.516	0.050	0.625
10	Test-1	0.018	8.752	0.008	9.487	0.087	5.863	0.044	7.293	0.192	4.770	0.087	5.863
	Test-2	0.723	1.990	0.687	2.066	0.837	1.732	0.781	1.847	0.906	1.605	0.837	1.732
	Test-3	0.013	0.268	0.011	0.296	0.021	0.198	0.017	0.230	0.027	0.158	0.021	0.198
	Test-4	0.000	2.530	0.000	2.530	0.632	2.530	0.632	2.530	0.632	2.530	0.632	2.530
	Test-5	2.952	5.361	2.637	5.441	3.424	5.170	3.210	5.257	3.739	5.071	3.424	5.170
	Test-6	1.167	3.499	1.020	3.620	1.411	3.130	1.294	3.315	1.564	2.885	1.411	3.130
	Test-7	0.028	0.878	0.024	0.940	0.051	0.625	0.039	0.736	0.064	0.507	0.051	0.625
13	Test-1	0.0254	8.8917	0.01	10.01	0.1118	5.9786	0.0611	6.8871	0.2096	4.6744	0.1118	5.9786
	Test-2	0.7434	2.0119	0.695	2.13	0.8268	1.7709	0.7871	1.8691	0.903	1.63	0.8268	1.7709
	Test-3	0.0156	0.2917	0.013	0.3334	0.0229	0.1932	0.0187	0.23	0.0279	0.1507	0.0229	0.1932
	Test-41	0.8321	3.0509	0.277	3.0509	0.8321	2.4962	0.8321	3.0509	1.3868	2.4962	0.8321	2.4962
	Test-42	1.0000	5.0000	1.0000	6.0000	2.0000	5.0000	1.0000	5.0000	2.0000	4.0000	2.0000	5.0000
	Test-5	3.4723	5.4669	3.363	5.4925	3.993	5.3135	3.7381	5.3957	4.2509	5.2456	3.993	5.3135
	Test-6	1.2693	3.3479	1.161	3.494	1.5084	3.0175	1.3933	3.1687	1.6509	2.8072	1.5084	3.0175
	Test-7	0.032	0.962	0.029	1.0436	0.0503	0.6263	0.0396	0.7465	0.0643	0.5039	0.0503	0.6263
15	Test-1	0.018	8.110	0.009	9.663	0.111	5.699	0.052	6.836	0.222	4.447	0.111	5.699
	Test-2	0.725	1.983	0.693	2.053	0.837	1.721	0.763	1.829	0.895	1.605	0.837	1.721
	Test-3	0.015	0.280	0.012	0.295	0.023	0.186	0.019	0.229	0.029	0.151	0.023	0.186
	Test-4	0.775	3.357	0.775	3.357	1.291	2.840	0.775	3.357	1.291	2.840	1.291	2.840
	Test-5	3.649	5.531	3.531	5.564	4.185	5.413	3.984	5.469	4.406	5.349	4.185	5.413
	Test-6	1.365	3.417	1.287	3.631	1.599	2.975	1.483	3.143	1.727	2.799	1.599	2.975
	Test-7	0.034	0.975	0.028	1.140	0.050	0.643	0.043	0.789	0.064	0.506	0.050	0.643
20	Test-1	0.022	9.267	0.011	11.573	0.110	6.139	0.062	7.152	0.229	4.682	0.110	6.139
	Test-2	0.726	2.043	0.688	2.176	0.829	1.733	0.775	1.877	0.904	1.587	0.829	1.733
	Test-3	0.016	0.261	0.013	0.286	0.024	0.178	0.019	0.218	0.030	0.146	0.024	0.178
	Test-4	1.342	3.578	0.894	3.578	1.342	3.130	1.342	3.578	1.789	3.130	1.342	3.130
	Test-5	4.219	5.651	4.048	5.679	4.618	5.561	4.457	5.593	4.772	5.500	4.618	5.561
	Test-6	1.460	3.194	1.393	3.369	1.690	2.871	1.588	3.035	1.801	2.722	1.690	2.871
	Test-7	0.032	0.994	0.029	1.087	0.051	0.701	0.041	0.840	0.068	0.549	0.051	0.701
21	Test-1	0.0182	8.8258	0.009	9.9306	0.0979	5.916	0.0449	7.6058	0.2108	4.6618	0.0979	5.916
	Test-2	0.735	1.9839	0.699	2.12	0.8288	1.7453	0.7893	1.8521	0.898	1.6116	0.8288	1.7453
	Test-3	0.0166	0.2773	0.015	0.3048	0.0242	0.1814	0.0203	0.2132	0.0296	0.1488	0.0242	0.1814
	Test-41	1.0911	3.7097	1.091	3.7097	1.5275	3.2733	1.5275	3.7097	1.964	3.2733	1.5275	3.2733
	Test-42	2.0000	8.0000	2.0000	8.0000	3.0000	7.0000	3.0000	7.00000	3.0000	6.0000	3.0000	7.0000
	Test-5	4.2562	5.6635	3.998	5.694	4.6404	5.5687	4.4727	5.6122	4.8277	5.5204	4.6404	5.5687
	Test-6	1.4806	3.2266	1.408	3.5034	1.7343	2.875	1.6172	3.0112	1.8172	2.7299	1.7343	2.875
	Test-7	0.0315	1.0071	0.028	1.0639	0.05	0.6368	0.0398	0.8167	0.0636	0.4917	0.05	0.6368
25	Test-1	0.030	9.222	0.016	10.045	0.111	6.189	0.058	7.622	0.235	4.709	0.111	6.189
	Test-2	0.739	2.002	0.686	2.086	0.848	1.743	0.796	1.858	0.914	1.621	0.848	1.743
	Test-3	0.017	0.279	0.015	0.315	0.025	0.191	0.020	0.230	0.030	0.156	0.025	0.191
	Test-4	1.400	4.200	1.400	4.200	1.800	3.800	1.800	3.800	1.800	3.400	1.800	3.800
	Test-5	4.455	5.725	4.342	5.741	4.810	5.660	4.663	5.684	4.986	5.609	4.810	5.660
	Test-6	1.552	3.019	1.469	3.088	1.740	2.803	1.642	2.912	1.859	2.682	1.740	2.803
	Test-7	0.033	0.969	0.027	1.034	0.052	0.674	0.041	0.803	0.068	0.528	0.052	0.674

30	Test-1	0.017	8.971	0.008	10.691	0.100	5.797	0.037	7.172	0.208	4.496	0.100	5.797
	Test-2	0.716	2.045	0.697	2.174	0.832	1.725	0.779	1.846	0.891	1.581	0.832	1.725
	Test-3	0.017	0.284	0.015	0.313	0.025	0.192	0.021	0.228	0.030	0.152	0.025	0.192
	Test-4	1.826	4.382	1.461	4.382	2.191	4.017	1.826	4.017	2.191	3.651	2.191	4.017
	Test-5	4.779	5.793	4.689	5.814	5.032	5.733	4.893	5.764	5.162	5.691	5.032	5.733
	Test-6	1.656	3.010	1.611	3.148	1.805	2.781	1.740	2.872	1.905	2.646	1.805	2.781
	Test-7	0.031	0.926	0.026	1.103	0.051	0.668	0.042	0.773	0.065	0.525	0.051	0.668
40	Test-1	0.0138	9.5305	0.005	10.671	0.0852	5.7499	0.0347	7.1035	0.209	4.4112	0.0852	5.7499
	Test-2	0.7363	2.0501	0.696	2.1568	0.8305	1.7314	0.7804	1.8647	0.8965	1.6112	0.8305	1.7314
	Test-3	0.0167	0.2714	0.015	0.3092	0.0246	0.1800	0.0209	0.2318	0.0305	0.1473	0.0246	0.18
	Test-41	2.2136	4.7434	1.897	5.0596	2.5298	4.4272	2.2136	4.7434	2.5298	4.111	2.5298	4.4272
	Test-42	5.0000	13.000	4.0000	14.000	6.0000	12.000	5.0000	13.000	7.0000	12.000	6.0000	12.000
	Test-5	5.0765	5.8789	4.97	5.9058	5.2894	5.8344	5.186	5.8597	5.3839	5.8002	5.2894	5.8344
	Test-6	1.7206	2.8882	1.603	2.9408	1.8881	2.7037	1.788	2.7913	1.9605	2.6089	1.8881	2.7037
	Test-7	0.0308	0.9828	0.027	1.1936	0.0492	0.6814	0.0398	0.8136	0.0641	0.5341	0.0492	0.6814
76	Test-1	0.0225	9.3082	0.013	11.605	0.093	6.0220	0.0525	7.6470	0.1889	4.6642	0.093	6.022
	Test-2	0.7338	1.9562	0.69	2.0576	0.8391	1.7297	0.7806	1.8307	0.9126	1.614	0.8391	1.7297
	Test-3	0.0193	0.2729	0.017	0.3131	0.027	0.1871	0.0234	0.2246	0.0332	0.1535	0.027	0.1871
	Test-41	3.2118	6.1942	3.212	6.4236	3.6707	5.7354	3.4412	5.9648	3.9001	5.506	3.6707	5.7354
	Test-42	11.000	24.000	10.000	24.000	13.000	22.000	12.000	23.000	14.000	21.0000	13.000	22.000
	Test-5	5.5847	6.0264	5.509	6.0341	5.6987	6.0006	5.6467	6.0132	5.7555	5.9817	5.6987	6.0006
	Test-6	1.8857	2.7285	1.817	2.8022	2.0011	2.6138	1.9424	2.6573	2.0598	2.5357	2.0011	2.6138
	Test-7	0.0309	0.9613	0.027	1.043	0.0484	0.6521	0.0383	0.7951	0.0639	0.5105	0.0484	0.6521

Note: n- sample size, Test-1, Test-2, Test-3, Test-4, Test-5, Test-6, Test-7 are respectively denote Rayleigh, Kuiper's, Watson's U^2, Hodges –Ajne,Range, Rao's Equal Spacing and Ajne Tests.

Table 1. Cut-off Points for Various Tests for Circular Uniformity

20^0, 135^0, 145^0, 165^0, 170^0, 200^0, 300^0, 325^0, 335^0, 350^0, 350^0, 350^0, 355^0.

Data Set – 3: Movements of Turtles

Orientations of 76 turtles after laying eggs [RaoJ ammalamadaka andS en Gupta (2001) p.5] Direction (in degrees) clockwise from north

8	9	13	13	14	18	22	27	30	34
38	38	40	44	45	47	48	48	48	48
50	53	56	57	58	58	61	63	64	64
64	65	65	68	70	73	78	78	78	83
83	88	88	88	90	92	92	93	95	96
98	100	103	106	113	118	138	153	153	155
204	215	223	226	237	238	243	244	250	251
257	268	285	319	343	350				

The above data sets are used respectively to verify goodness of fit of Wrapped Lognormal, Wrapped Logistic and Wrapped Extreme- value models.

Considering the data, the mean direction (μ) and the concentration parameter (ρ) were estimated. These are tabulated as follows Table 2.

Methodology (Using Tests of Uniformity)

Substituting data sets - 1, 2 and 3 respectively in (20), (21) and (22) corresponding uniform [0, 2π] variates denoted by $\theta_1, \theta_2, \ldots, \theta_n$ are obtained. Using these θ_is, the tests statistics of Rayleigh Test, Kuiper's Test, Watson's U^2 - Test, Hodges - Ajne Test, Range Test, Rao's Equal Spacing Test and Ajne Test are computed and are tabulated in Table 3.

	μ	ρ
Wrapped Lognormal	3.9804	0.4049
Wrapped Logistic	-0.2735	0.2156
Wrapped Extreme-Value	1.12	0.4971

Table 2. Estimates of μ and ρ

Tests	Wrapped Lognormal Sample Size (n=21)	Wrapped Logistic Sample Size (n=13)	Wrapped Extreme-Value Sample Size (n=76)
Rayleigh Test	10.7540	3.2381	11.0364
Kuiper's Test	2.4313	2.2079	2.9130
Watson's U^2-Test	0.0056	0.0649	0.0016
Hodges-Ajne Test	12.000	9.0000	49.000
Range Test	5.0448	3.4732	5.9132
Rao's Equal Spacing Test	2.7560	5.1342	4.4689
Ajne Test	3.8664	0.5495	1.3800

Remark: In goodness of fit, the cut-off points under two tails corresponding to the sample sizes n=21, 13 and 76 are reproduced here for ready reference

Table 3. Statistic Values of Various Test Procedures

LOS Tests	1%	5%	10%
Rayleigh Test	0.0090 - 9.9306	0.0449 - 7.6058	0.0979 - 5.9160
Kuiper's Test	0.6990 - 2.1200	0.7893 - 1.8521	0.8288 - 1.7453
Watson's U^2-Test	0.0150 - 0.3048	0.0203 - 0.2132	0.0242 - 0.1814
Hodges-Ajne Test	2.0000 - 8.0000	3.0000 - 7.0000	3.0000 - 7.0000
Range Test	3.9980 - 5.6940	4.4727 - 5.6122	4.6404 - 5.5687
Rao's Equal Spacing Test	1.4080 - 3.5034	1.6172 - 3.0112	1.7343 - 2.8750
Ajne Test	0.0280 - 1.0639	0.0398 - 0.8167	0.0500 - 0.6368

Remark: Wrapped Lognormal model appears to be good fit at 1%, 5% and 10% based on Rayleigh test and Rao's Equal Spacing test.

Table 4. Cut-off Points for Wrapped lognormal Sample size (n=21)

LOS Tests	1%	5%	10%
Rayleigh Test	0.0100 - 0.0100	0.0611 - 6.8871	0.1118 - 5.9786
Kuiper's Test	0.6950 - 2.1300	0.7871 - 1.8691	0.8268 - 1.7709
Watson's U^2-Test	0.0130 - 0.3334	0.0187 - 0.2300	0.0229 - 0.1932
Hodges-Ajne Test	1.0000 - 6.0000	1.0000 - 5.0000	2.0000 - 5.0000
Range Test	3.3630 - 5.4925	3.7381 - 5.3957	3.9930 - 5.3135
Rao's Equal Spacing Test	1.1610 - 3.4940	1.3933 - 3.1687	1.5084 - 3.0175
Ajne Test	0.0290 - 1.0436	0.0396 - 0.7465	0.0503 - 0.6263

Remark: Wrapped Logistic model appears to be good fit at 1%, 5% and 10% based on Rayleigh test, Watson's

Table 5. Cut-off Points for Wrapped Logistic Sample Size (n=13)

LOS Tests	1%	5%	10%
Rayleigh Test	0.0130 - 11.605	0.0525 - 7.6470	0.0930 - 6.0220
Kuiper's Test	0.6900 - 2.0576	0.7806 - 1.8307	0.8391 - 1.7297
Watson's U^2-Test	0.0170 - 0.3131	0.0234 - 0.2246	0.0270 - 0.1871
Hodges-Ajne Test	10.000 - 24.000	12.000 – 23.000	13.000 – 22.000
Range Test	5.5090 - 6.0341	5.6467 - 6.0132	5.6987 - 6.0006
Rao's Equal Spacing Test	1.8170 - 2.8022	1.9424 - 2.6573	2.0011 - 2.6138
Ajne Test	0.0270 - 1.0430	0.0383 - 0.7951	0.0484 - 0.6521

Remark: Wrapped Extreme Value model appears to be good fit at 1%, 5% and 10% based on Range test. It is also good fit at 1% with respect to Rayleigh test.

Table 6. Cut-off Points for Wrapped Extreme- Value Sample Size (n=76)

To write comments on goodness of fit, the cut-off points under two tails corresponding to the sample sizes n=21, 13 and 76 are reproduced here for ready reference.

Conclusion

Utility of cut- off points via goodness of fit is demonstrated.

Reference

[1]. **Ajne, B. (1968).** A simple test for uniformity of a circular distribution. *Biometrika*, 55(2), 343-354.

[2]. **Corcleiro, G. Y..Paula, G. A. & c Rotter, D. A. (1994).** Irnproved likelihoocl ratio tests for dispersion models. *Internat. Statist. Rew.,* 62, 257-274.

[3]. **Cox, D. R. & Hinkley, D. V. (1974).** Theoretical Statistics. Chapman & Hall.

[4]. **Fernandez-Duran, J. J., Gregorio-Dominguez, M. M. (2010).** A Likelihood Ratio Test for Homogeneity in Circular Data. *J Biomet Biostat* 1:107.

[5]. **Fisher, N. I. (1993).** *Statistical Analysis of Circular Data.* Cambridge University Press, Cambridge.

[6]. **Fisher, N. I., & Powell, C. M. (1989).** Statistical analysis of two-dimensional palaeocurrent data: methods and examples. *Journal of the Geological Society of Australia,* 36(1), 91-107.

[7]. **Hodges J. L. (1955).** *A bivariate sign trst. Ann. Math. Statisst.*, 26, 523-527.

[8]. **Kuiper, N. H. (1960).** Tests conccrning random points on a circle. *Ned. Akad. Wet.Pmc. Ser. A*, 63, 38-47.

[9]. **Laubscher, N. F. 8~ Rudolpli, G. J. (1M8).** A ddvtribution arising f r o r i b rnndonipants on the circrcmference of a circle. Technical Report 268, CSIR, Pretoria. London.

[10]. **Mardia, K. V., & Jupp, P. (2000).** Directional statistics. John Wiley. Hoboken, NJ.

[11]. **Marrero, O. (1983).** The performance of several statistical tests for seasonality in monthly data. *Journal of Statistical Computation and Simulation,* 17(4), 275-296.

[12]. **Rao J. S. and Sen Gupta, A. (2001).** Topics in Circular Statistics, World Scientific Press, Singapore.

[13]. **Rao, J. S. (1969).** Some contributions to the analysis of circular data (Doctoral dissertation, Indian Statistical Institute, Kolkata).

[14]. **Stephens, M. A. (1970).** Use of the Kolmogorov-Smirnov, Cramér-Von Mises and related statistics without extensive tables. *Journal of the Royal Statistical Society. Series B (Methodological),* 115-122.

[15]. **Watson, G. S. (1961).** Goodness-of-fit tests on a circle. *Biometrika,* 48(½), 109-114.

[16]. **Watson, G. S. (1962).** Goodness-of-fit tests on a circle. *II. Biometrika,* 49(½), 57-63.

[17]. **Watson, G. S. (1967).** Another test for the uniformity of a circular distribution. *Biometrika,* 54(3-4), 675-677.

Appendix

The density and distribution functions of Wrapped Lognormal Distribution Wrapped Logistic Distribution Wrapped Extreme-Value Distribution are

Wrapped Lognormal Distribution

The distribution function (cdf) of the Wrapped Lognormal distribution is given by,

$$G(\theta) = \sum_{k=0}^{\infty} \frac{1}{2}\left[\operatorname{erf}\left(\frac{\log(\theta + 2k\pi - \mu)}{\sqrt{2}\sigma} \right) - \operatorname{erf}\left(\frac{\log(2k\pi)}{\sqrt{2}\sigma} \right) \right] \qquad (20)$$

where $\theta,\mu\in[0,2\pi)$, $\theta>\mu$, $\sigma>0$

The Wrapped Lognormal density admits the Fourier representation, hence

$$g(\theta)=\frac{1}{2\pi}\left[1+2\sum_{p=1}^{\infty}\left\{\alpha_p\cos p\theta+\beta_p\sin p\theta\right\}\right]$$

where $\theta,\mu[0,2\pi)$, $\theta>\mu$, $\sigma>0$, $p\in\mathbb{Z}$

Wrapped Logistic Distribution

The distribution function cdf of the Wrapped Logistic distribution is

$$G(\theta)=\sum_{k=-\infty}^{\infty}\frac{1}{2}\left[\tanh\left\{\frac{1}{2}\left(\frac{\theta+2k\pi-\mu}{\sigma}\right)\right\}-\tanh\left\{\frac{1}{2}\left(\frac{2k\pi}{\sigma}\right)\right\}\right] \tag{21}$$

$\theta>\mu$ and $\theta,\mu[0,2\pi)$, $\sigma>0$

The Wrapped Logistic density admits the Fourier representation

$$g(\theta)=\frac{1}{2\pi}\left[1+2\sum_{p=1}^{\infty}\frac{\pi\sigma p}{\sinh\pi\sigma p}\cos p(\theta-\mu)\right]$$

Wrapped Extreme-Value Distribution

The cdf of the Wrapped Extreme-Value distribution is

$$G(\theta)=\sum_{k=-\infty}^{\infty}\left[\exp\left(-\exp\left(\frac{-(\theta+2k\pi)}{\sigma}\right)\right)-\exp\left(-\exp\left(\frac{-(2k\pi)}{\sigma}\right)\right)\right] \tag{22}$$

The Wrapped Extreme - Value density admits the Fourier representation

$$g(\theta)=\frac{1}{2\pi}\left[1+2\sum_{p=1}^{\infty}(\alpha_p\cos p\theta+\beta_p\sin p\theta)\right],\quad \theta\in[0,2\pi)$$

where $\alpha_p=\mathrm{Re}(\Gamma(1-ip\sigma)),\beta_p=\mathrm{Im}(\Gamma(1-ip\sigma)),p\in\mathbb{Z}$

Re- Real part and Im Imaginary part.

In conclusion these test procedures asymptotic conditions of various test procedures are already derived by the respective researchers, whereas for small samples the cut off points suggested in this paper are useful for practitioners.

ABOUT THE AUTHOR

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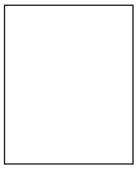

On Stereographic Semicircular New Weibull Pareto Model

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Abstract

A very good number of Circular and Semicircular models have been constructed for the important and widely used life testing linear distributions by employing techniques like wrapping, inverse stereographic projections, rising sun function etc. Taking cue from this work an attempt is made in this paper to develop a new Semicircular model called Stereographic Semicircular New Weibull Pareto Distribution using the method of inverse stereographic projection. The Probability density function, Cumulative density function, Characteristic function, Trigonometric moments and population characteristics for the new Stereographic Semicircular New Weibull Pareto Distribution are studied.

Keywords: Circular Distribution, Stereographic projection, Semicircular Models, Trigonometric Moments, New Weibull Pareto distribution

I. INTRODUCTION

In the literature of Directional Statistics we can find a significant number of Circular models defined on unit circle, developed from the existing linear distributions, both continuous and discrete by Fisher (1993), Jammalamadaka and Sengupta (2001), Dattatreya Rao et al (2007). A new method called Stereographic Projection was employed for developing the probability distributions of angular models. Minh and Farnum (2003) used a bilinear transformations to map points on the unit circle in the complex plane into points x on the real line and for a density function g(α) on the interval (−π, π), and demonstrated how a corresponding density function f(x) on (−∞, ∞) is induced. Toshihiro Abe et al (2010) developed symmetric circular models applying the Inverse Stereographic Projection. Dattatreya Rao et al (2011) developed Cauchy type models by inducing Stereographic Projection on circular Cardioid distribution. Phani et al (2012) developed circular model induced by Inverse Stereographic Projection on Extreme-Value distribution.

Taking cue from these works, an attempt is made in this paper to develop a Semicircular model for the New Weibull Pareto distribution called Stereographic Semicircular New Weibull Pareto Distribution using the method of inverse stereographic projection.

This paper has been organised in to four sections. Section 2 deals with the methodology of inverse stereographic projection on a linear probability distribution. In the Section 3 the proposed new Stereographic Semicircular New Weibull Pareto Distribution is presented with relevant graphs of probability density, distribution and characteristic functions at different values of parameters. Important population characteristics for the new Stereographic Semicircular New Weibull Pareto Distribution are also tabulated. Section 4 presents a brief summary on the findings of this study.

For this paper software MATLAB is used for all the computations and for plotting of graphs.

II. INVERSE STEREOGRAPHIC PROJECTION METHOD

For analysing angular data new models can be constructed by applying inverse stereographic projection on the linear probability distributions. Inverse stereographic projection when applied on a linear model yields a circular model.

Stereographic projection creates a one to one relationship between the points on the unit circle and those on the real line. Circular as well as linear Probability distributions can be obtained by applying stereographic projection. Inverse stereographic projection is defined by a one to one mapping given by $T(\theta) = x = u + v\tan\left(\frac{\theta}{2}\right)$, where $x \in (-\infty, \infty)$, $\theta \in [-\pi, \pi)$, $u \in \square$, $and\ \ v > 0$.

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If x is random variable defined on the interval $(-\infty, \infty)$ with probability density functions $f(x)$ and cumulative distribution as $F(x)$, then $T^{-1}(x) = \theta = 2\tan^{-1}\left\{\frac{(x-u)}{v}\right\}$ is a random point on a unit circle.

If $G(\theta)$ and $g(\theta)$ are respectively the cumulative distribution and the probability density functions of this random point θ, then $G(\theta)$ and $g(\theta)$ can be derived in terms of $F(x)$ and $f(x)$ using the following transformation

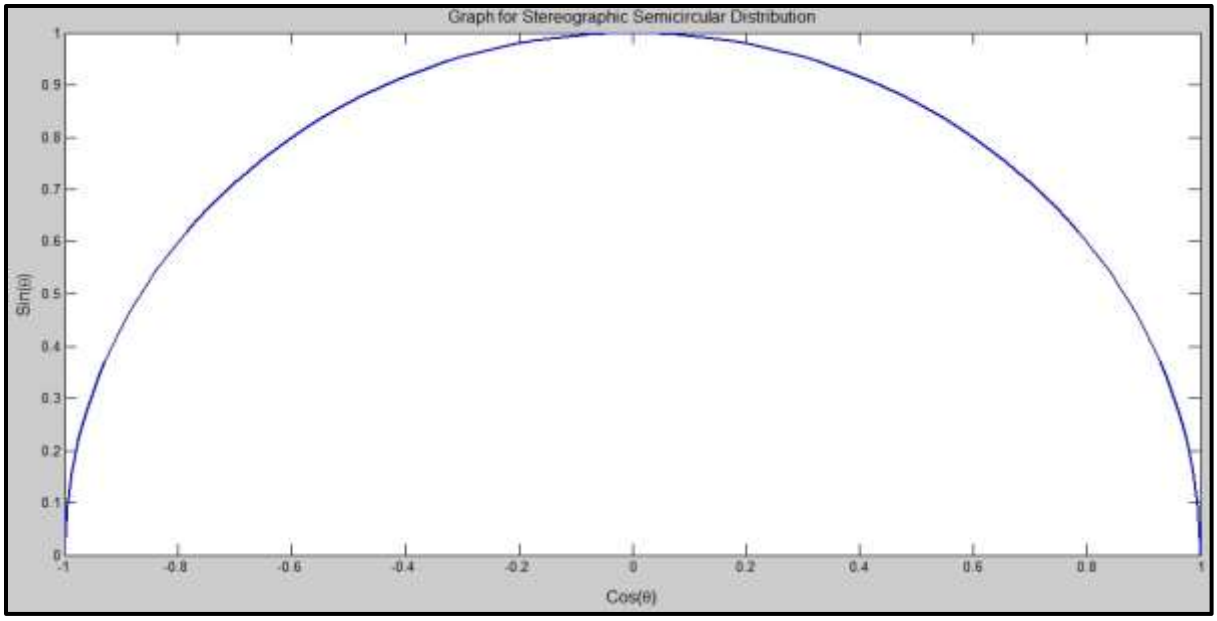
For $v > 0$

$$G(\theta) = F\left(u + v\tan\left(\frac{\theta}{2}\right)\right) = F(x(\theta)) \quad \ldots(1)$$

$$g(\theta) = v\left\{\frac{\sec^2\left(\frac{\theta}{2}\right)}{2}\right\} f\left(u + v\tan\left(\frac{\theta}{2}\right)\right) \quad \ldots(2)$$

Without loss of generality u is considered as zero in this paper.

If Inverse Stereographic Projection is applied on a linear model whose probability density function is defined on $(0,\infty)$ then the resultant distribution is a Stereographic Semicircular models automatically mapped onto $(0,\pi)$. The graph depicted below demonstrates this fact.



III. STEREOGRAPHIC SEMICIRCULAR NEW WEIBULL PARETO DISTRIBUTION

A linear random variable X is said to follow a three parameter New Weibull Pareto (NWP) Distribution if the distribution function of X is given by

$$F(X) = 1 - e^{-\delta\left(\frac{x}{\lambda}\right)^{C}}$$

Where **c** is the shape parameter and λ and δ are the scale parameters and
$0 < x < \infty$ and $c > 0$, $\lambda > 0$ and $\delta > 0$ the pdf of New Weibull Pareto Distribution is

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$$f(x) = \frac{c.\delta}{\lambda}\left(\frac{x}{\lambda}\right)^{c-1} e^{-\delta\left(\frac{x}{\lambda}\right)^{c}}$$

Where $0 < x < \infty$ and $c > 0$, $\lambda > 0$ and $\delta > 0$

Here if $\delta = 1$, this NWP distribution reduces to Weibull distribution and if $\delta = 1$ and $c = 1$ this distribution reduces to Exponential distribution. Also when $c=2$ and $\delta = 1$ the New Weibull Pareto distribution becomes Rayleigh distribution.

A1. Probability density function for Stereographic Semicircular New Weibull Pareto (SSNWP) distribution:

As discussed in the previous section, by applying the inverse stereographic projection on the New Weibull Pareto distribution, the one to one mapping

$$T(\theta) = x = v\tan\left(\frac{\theta}{2}\right);\ where\ \ x \in (0,\infty),\theta \in [0,\pi)\ \text{and}\ v > 0$$

will yield the Stereographic Semicircular New Weibull Pareto distribution with probability density function g(θ) and cumulative distribution function G(θ) as below:

$$g(\theta) = \frac{v}{2}\mathrm{Sec}^2\left(\frac{\theta}{2}\right)\frac{c.\delta}{\lambda}\left(\frac{v\tan\left(\frac{\theta}{2}\right)}{\lambda}\right)^{c-1} e^{-\delta\left(\frac{v\tan\left(\frac{\theta}{2}\right)}{\lambda}\right)^{c}} \quad \ldots (3)$$

and

$$G(\theta) = 1 - e^{-\delta\left(\frac{v\tan\left(\frac{\theta}{2}\right)}{\lambda}\right)^{c}} \quad \ldots (4)$$

$where\ \theta \in (0,\pi),c,\lambda,\delta > 0, and\ v \in \mathbb{R}^{+}$ The graph depicting the linear representation of the pdf of SSNWP distribution for different values of **c** keeping the value for the parameter **λ** and δ at 2.0 is as follows:

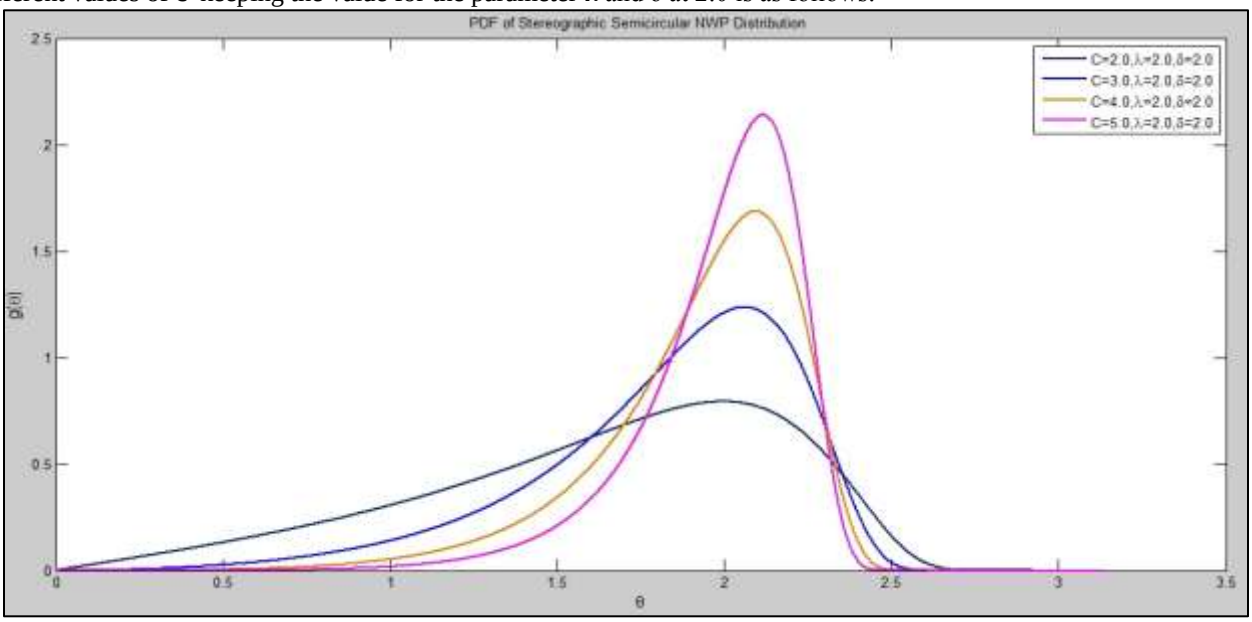

Fig. 1: PDF of SSNWP distribution (Linear Representation)

The linear representation of pdf for different values of **λ** keeping the values for parameters **c** and δ at 2.0 is obtained as below

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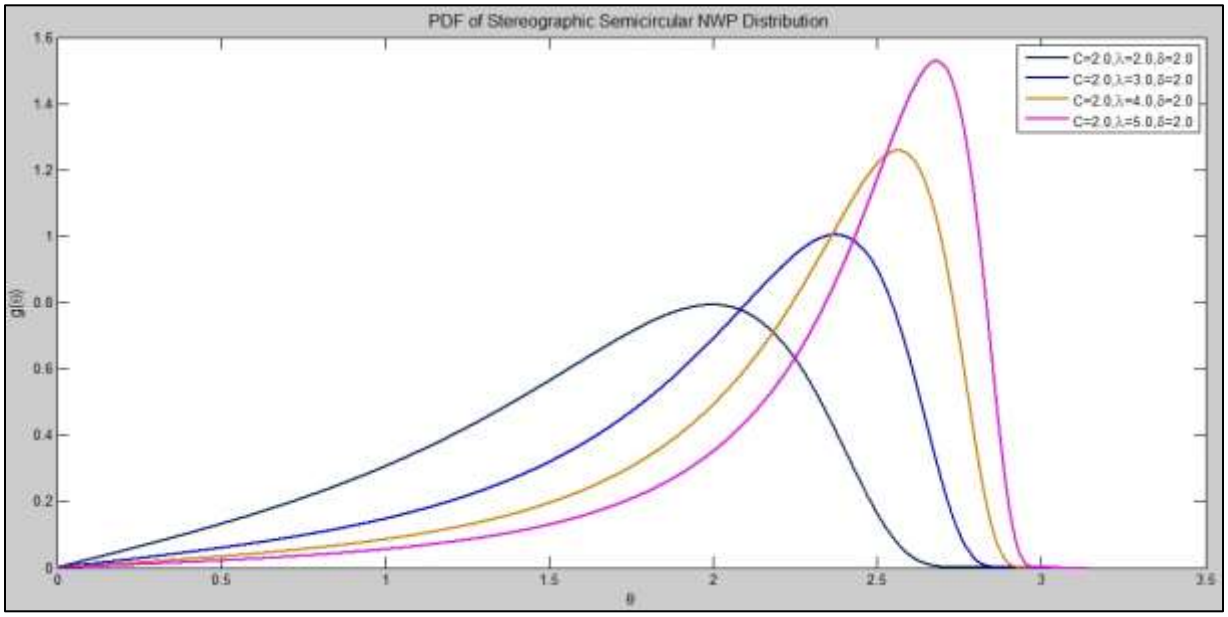

Fig. 2: PDF of SSNWP distribution (Linear Representation)

Also the linear representation of pdf for different values of δ keeping the values for parameters **c** and **λ** at 2.0 is obtained as below

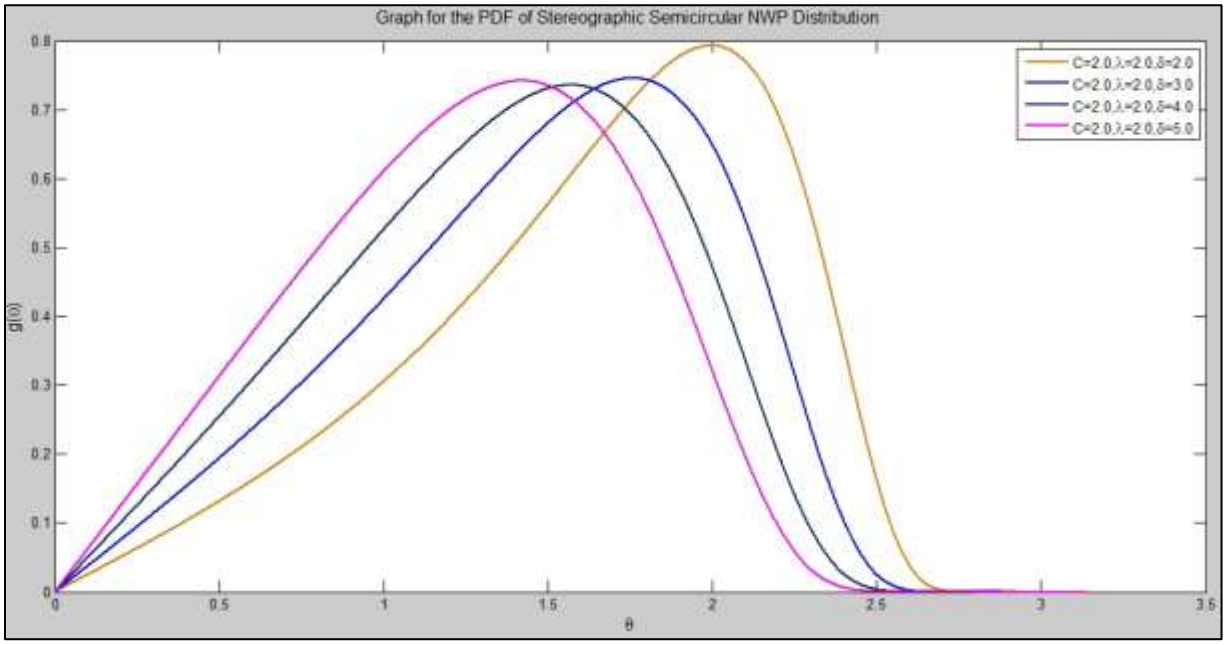

Fig. 3: PDF of SSNWP distribution (Linear Representation)

The graph depicting the circular representation of the pdf of SSNWP Distribution for different values of **c** keeping the value for the parameter and **λ** and δ at 2.0 is shown below:

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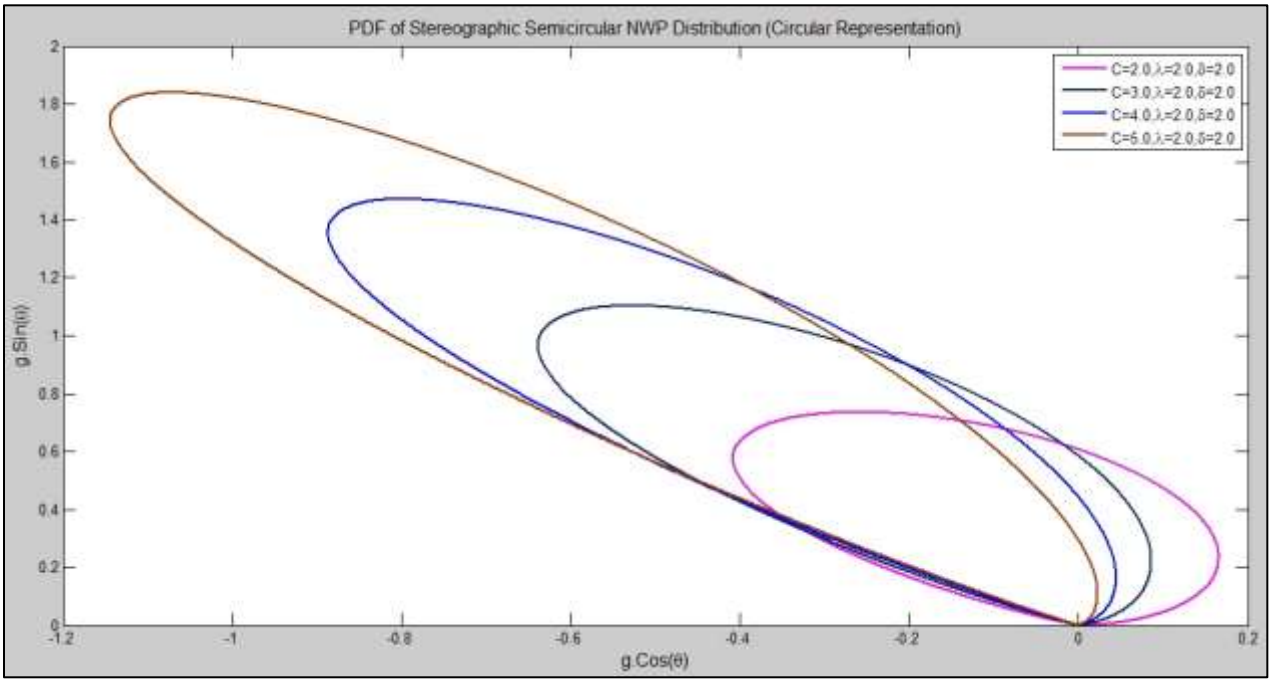

Fig. 4: PDF of SSNWP distribution (Circular Representation)

The circular representation for the pdf for the different values of λ keeping the values for parameters **c** and δ at 2.0 is obtained as below:

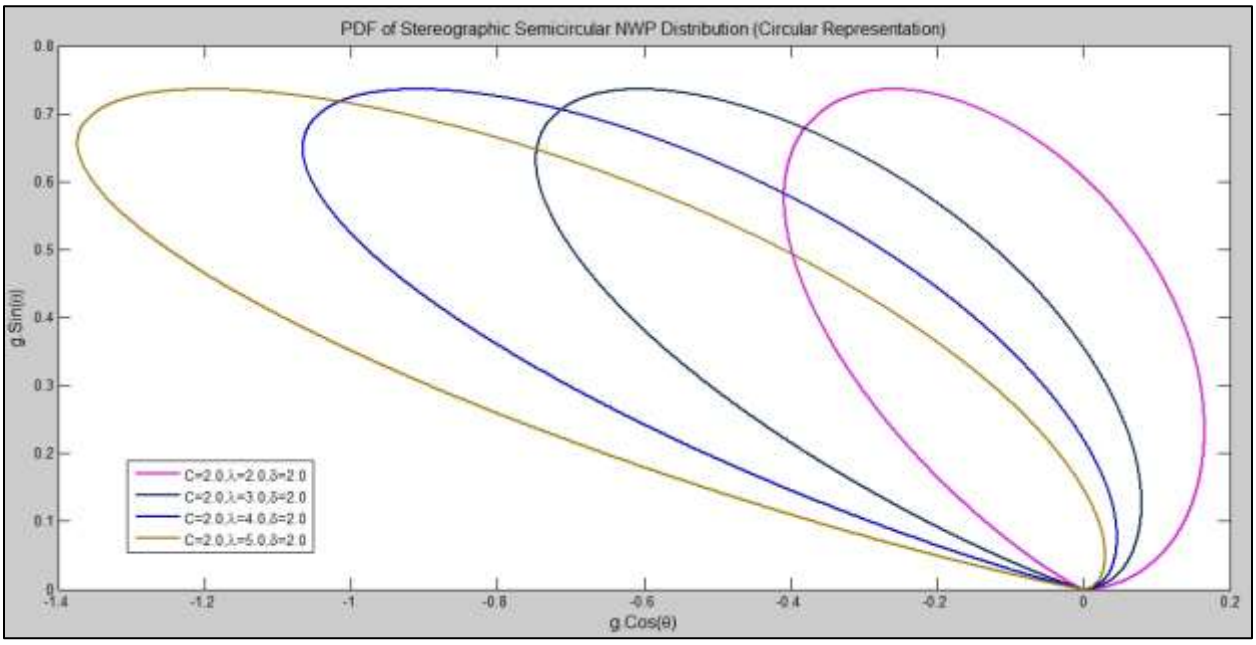

Fig. 5: PDF of SSNWP distribution (Circular Representation)

Also the circular representation for the pdf for different values of δ keeping the values for parameters **c** and **λ** at 2.0 is obtained as below

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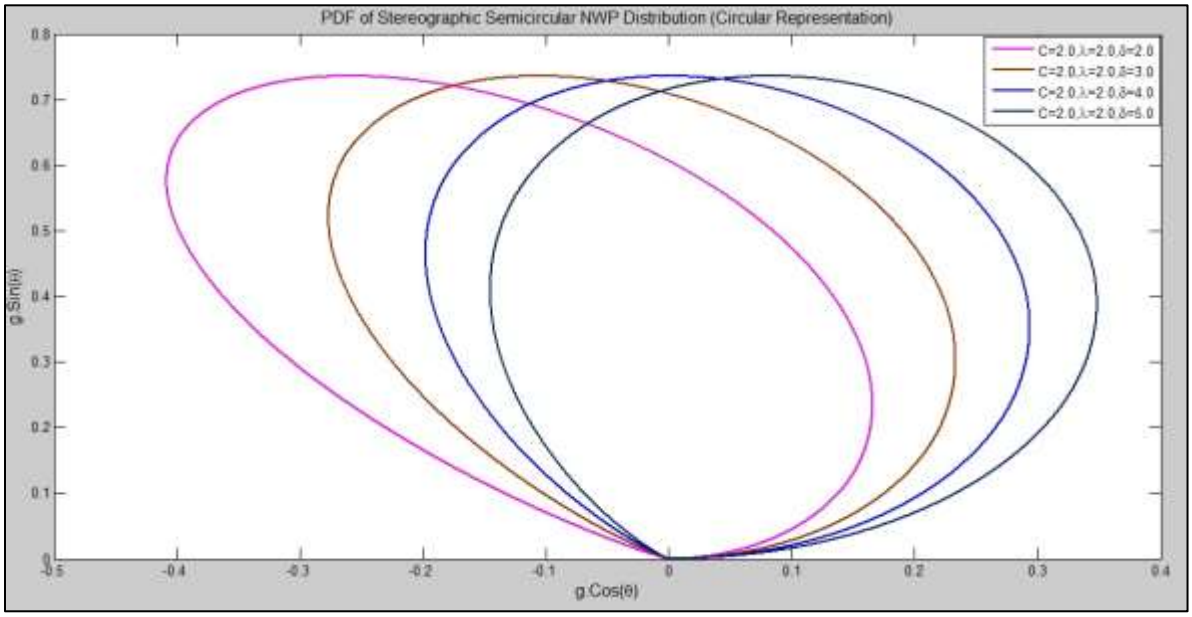

Fig. 6: PDF of SSNWP distribution (Circular Representation)

B. Cumulative Distribution Function for SSNWP distribution:

The graph for the CDF, G (θ) for SSNWP is obtained as below:

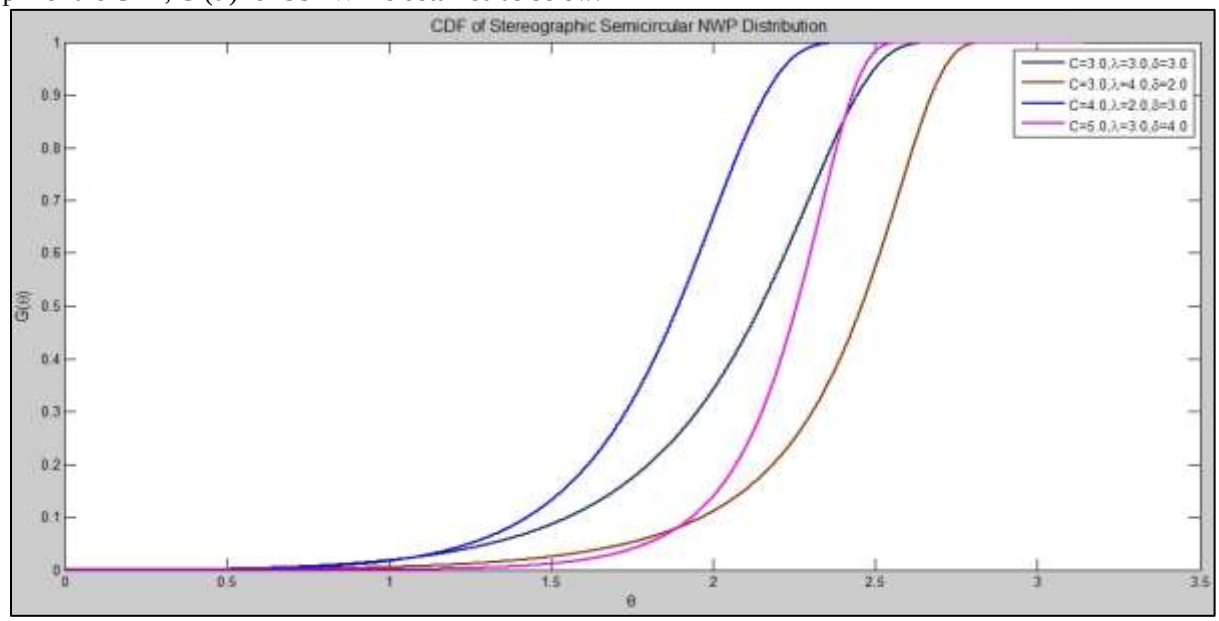

Fig. 7: CDF of SSNWP distribution

C. Characteristic function for SSNWP distribution:

1) The characteristic function of a Stereographic Semicircular model

If the cumulative distribution function and probability density function for a Stereographic Semicircular model are $G(\theta)$ and $g(\theta)$ respectively and if $F(x)$ and $f(x)$ are the corresponding cumulative distribution function and probability density function of the linear model, then for the Stereographic Semicircular model the characteristic function can obtained as

$$\varphi_{X_{Sc}}(p) = \int_0^{\pi} e^{ip\theta} g(\theta)\, d\theta \ , \ p \in \square$$

Without loss of generality, considering v =1, the characteristic function for the Stereographic Semicircular New Weibull Pareto distribution can be obtained from the following equation.

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$$\varphi_{X_{Sc}}(p) = \int_0^{\pi} e^{ip\theta} \frac{v}{2} \mathrm{Sec}^2\left(\frac{\theta}{2}\right) \frac{c.\delta}{\lambda} \left(\frac{v\tan\left(\frac{\theta}{2}\right)}{\lambda}\right)^{c-1} e^{-\delta\left(\frac{v\tan\left(\frac{\theta}{2}\right)}{\lambda}\right)^{c}} d\theta \quad \ldots (5)$$

Taking $u = \left(\frac{\tan\left(\frac{\theta}{2}\right)}{\lambda}\right)^{-c}$ and transforming (5) we get

$$\varphi_{X_{Sc}}(p) = \int_0^{\infty} e^{ip\left(2\tan^{-1}\left(\lambda.u^{\left(\frac{1}{c}\right)}\right)\right)} \delta e^{-\delta u} du \quad \ldots (6)$$

Again considering k= u.λ and making necessary transformations for (6) the characteristic function for the Stereographic Semicircular New Weibull Pareto distribution can be obtained as below

$$\varphi_{X_{Sc}}(p) = \int_0^{\infty} e^{ip\left(2\tan^{-1}\left(\lambda.\left(\frac{k}{\delta}\right)^{\frac{1}{c}}\right)\right)} e^{-k} dk \quad \ldots . (7)$$

It can be noticed that the integral obtained above cannot be evaluated analytically in its general form, for evaluating the characteristic function to obtain the trigonometric moments for the new Stereographic Semicircular New Weibull Pareto distribution, the n – point Gauss – Laguerre quadrature formula for numerical integration as given in Rao et al, (1975) is applied on the equation (7) above.

The real and imaginary parts α_p and β_p respectively are obtained from the characteristic function of the SSNWP distribution.

The following are the graphs for the characteristic function of the SSNWP distribution showing the real part and imaginary part separately for the parameters c = 2, λ=3 δ=4.0 in Fig 8, and for c = 5, λ=4 δ=3.0 in Fig 9.

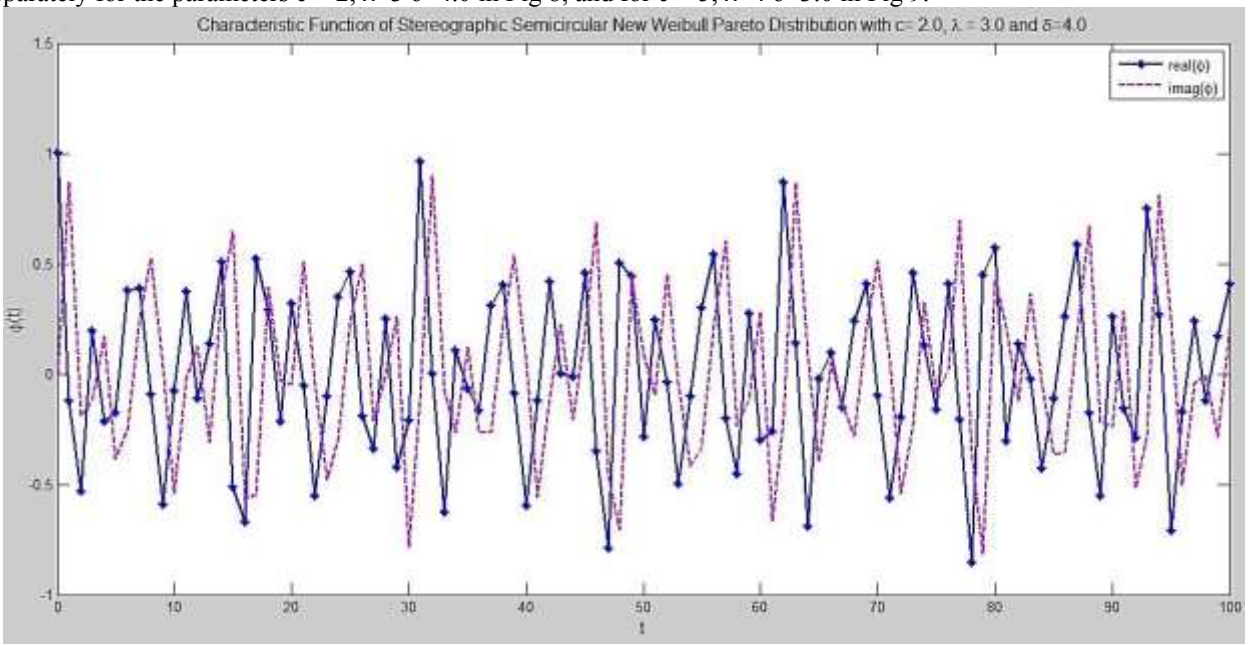

Fig. 8: Characteristic Function of SSNWP distribution

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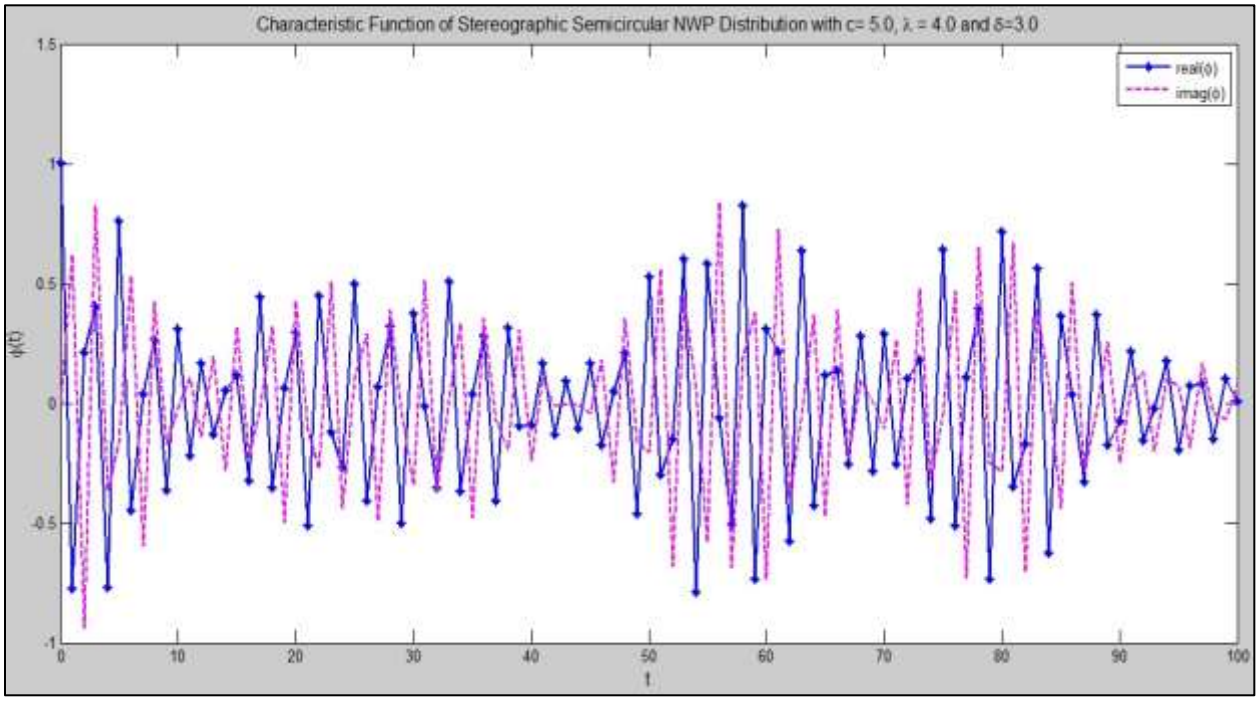

Fig. 9: Characteristic Function of SSNWP distribution

D. Population Characteristics:

Mardia and Jupp (2000) had derived expressions for mean direction μ_o resultant length ρ_1, Circular variance V_0, Central Trigonometric Moments α_p^*, β_p^*, Skewness γ_1^o and Kurtosis γ_2^o from the values of the characteristic function φ_p at p=1 and 2.

The Population Characteristics for the Stereographic Semicircular New Weibull Pareto distribution for different values of the parameters c, λ and δ are computed and tabulated here under.

Table – 1

Characteristics of Stereographic Semicircular New Weibull Pareto distribution at λ = 3.0 and δ=3.0

At different values of c	*c=2.0*	*c=3.0*	*c=4.0*	*c=5.0*	*c=6.0*
Trigonometric Moments					
α_1	*-0.2318*	*-0.4640*	*-0.5740*	*-0.6328*	*-0.6683*
α_2	*-0.4753*	*-0.4181*	*-0.2757*	*-0.1649*	*-0.0862*
β_1	*0.8510*	*0.8311*	*0.7894*	*0.7561*	*0.7316*
β_2	*-0.3288*	*-0.7000*	*-0.8630*	*-0.9303*	*-0.9603*
Resultant Length					
ρ_1	*0.8820*	*0.9518*	*0.9760*	*0.9860*	*0.9910*
ρ_2	*0.5780*	*0.8154*	*0.9060*	*0.9448*	*0.9641*
Mean Direction					
μ_0	*1.8367*	*2.0799*	*2.1995*	*2.2676*	*2.3110*
Circular Variance					
V_0	*0.1180*	*0.0482*	*0.0240*	*0.0140*	*0.0090*
Circular Standard Deviation					
σ_0	*0.5010*	*0.3142*	*0.2205*	*0.1678*	*0.1348*
	1.0471	*0.6389*	*0.4445*	*0.3371*	*0.2703*
Central Trigonometric Moments					
α_1^*	*0.8820*	*0.9518*	*0.9760*	*0.9860*	*0.9910*
α_2^*	*0.5764*	*0.8153*	*0.9059*	*0.9448*	*0.9641*
β_1^*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*
β_2^*	*0.0423*	*0.0114*	*0.0038*	*0.0016*	*0.0008*
Skewness					
Υ_1^0	*1.0444*	*1.0828*	*1.0184*	*0.9532*	*0.8989*
Kurtosis					
Υ_2^0	*-2.0747*	*-2.3482*	*-2.4262*	*-2.4460*	*-2.4476*

Table – 2

Characteristics of Stereographic Semicircular New Weibull Pareto distribution at c = 3.0 and δ = 3.0

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At different values of λ	λ *=2.0*	λ *=3.0*	λ *=4.0*	λ *=5.0*	λ *=6.0*
Trigonometric Moments					
α_1	*-0.1395*	*-0.4640*	*-0.6443*	*-0.7506*	*-0.8170*
α_2	*-0.7376*	*-0.4181*	*-0.0837*	*0.1751*	*0.3631*
β_1	*0.9302*	*0.8311*	*0.7173*	*0.6197*	*0.5408*
β_2	*-0.2377*	*-0.7000*	*-0.8569*	*-0.8785*	*-0.8459*
Resultant Length					
ρ_1	*0.9406*	*0.9518*	*0.9641*	*0.9733*	*0.9798*
ρ_2	*0.7749*	*0.8154*	*0.8610*	*0.8958*	*0.9205*
Mean Direction					
μ_0	*1.7196*	*2.0799*	*2.3027*	*2.4514*	*2.5568*
Circular Variance					
V_0	*0.0594*	*0.0482*	*0.0359*	*0.0267*	*0.0202*
Circular Standard Deviation					
σ_0	*0.3500*	*0.3142*	*0.2702*	*0.2325*	*0.2021*
	0.7141	*0.6389*	*0.5471*	*0.4691*	*0.4070*
Central Trigonometric Moments					
α_1^*	*0.9406*	*0.9518*	*0.9641*	*0.9733*	*0.9798*
α_2^*	*0.7749*	*0.8153*	*0.8610*	*0.8958*	*0.9205*
β_1^*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*
β_2^*	*0.0110*	*0.0114*	*0.0084*	*0.0057*	*0.0039*
Skewness					
Υ_1^0	*0.7564*	*1.0828*	*1.2329*	*1.3110*	*1.3561*
Kurtosis					
Υ_2^0	*-2.2216*	*-2.3482*	*-2.4269*	*-2.4786*	*-2.5131*

Table – 3

Characteristics of Stereographic Semicircular New Weibull Pareto distribution at c = 3.0 and λ = 3.0

At different values of δ	δ*=2.0*	δ*=3.0*	δ*=4.0*	δ*=5.0*	δ*=6.0*
Trigonometric Moments					
α_1	*-0.5547*	*-0.4640*	*-0.3936*	*-0.3359*	*-0.2869*
α_2	*-0.2653*	*-0.4181*	*-0.5166*	*-0.5849*	*-0.6344*
β_1	*0.7804*	*0.8311*	*0.8627*	*0.8841*	*0.8993*
β_2	*-0.7931*	*-0.7000*	*-0.6136*	*-0.5357*	*-0.4656*
Resultant Length					
ρ_1	*0.9575*	*0.9518*	*0.9482*	*0.9457*	*0.9439*
ρ_2	*0.8363*	*0.8154*	*0.8022*	*0.7932*	*0.7869*
Mean Direction					
μ_0	*2.1887*	*2.0799*	*1.9988*	*1.9339*	*1.8797*
Circular Variance					
V_0	*0.0425*	*0.0482*	*0.0518*	*0.0543*	*0.0561*
Circular Standard Deviation					
σ_0	*0.2948*	*0.3142*	*0.3262*	*0.3342*	*0.3397*
	0.5980	*0.6389*	*0.6640*	*0.6808*	*0.6924*
Central Trigonometric Moments					
α_1^*	*0.9575*	*0.9518*	*0.9482*	*0.9457*	*0.9439*
α_2^*	*0.8362*	*0.8153*	*0.8021*	*0.7931*	*0.7868*
β_1^*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*
β_2^*	*0.0102*	*0.0114*	*0.0120*	*0.0122*	*0.0121*
Skewness					
Υ_1^0	*1.1611*	*1.0828*	*1.0181*	*0.9625*	*0.9137*
Kurtosis					
Υ_2^0	*-2.3865*	*-2.3482*	*-2.3198*	*-2.2972*	*-2.2784*

IV. CONCLUSION

It can be observed that the Stereographic Semicircular New Weibull Pareto distribution becomes Stereographic Semicircular Exponential distribution when c = 1 and δ =1 and when δ =1, it reduces to Stereographic Semicircular Weibull distribution. Also when c=2 and δ =1 the Stereographic Semicircular New Weibull Pareto distribution becomes Stereographic Semicircular Rayleigh distribution.

From the population characteristics of Stereographic Semicircular New Weibull Pareto distribution tabulated in the last section, we can observe in all the different cases the distribution remained platykurtic. Circular Variance got reduced with incrementing the value of parameter **c,** when **λ** and δ kept constant at 3.0, and also with incrementing value of λ when **c** and δ are

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kept constant. The Skewness of the distribution also got reduced with higher value of c and δ when the other parameters kept constant.

REFERENCES

[1] Dattatreya Rao, A.V., Ramabhadra Sarma, I., Girija S.V.S., (2007). "On wrapped version of some life testing models". Communication in Statistics - Theory and Methods, 36, 2027-2035
[2] Dattatreya Rao A.V, Girija S.V.S, Phani. Y. (2011) "Differential Approach to Cardioid Distribution", Computer Engineering and Intelligent Systems, Vol/Issue: 2(8). Pp. 1-6, 2011.
[3] Fisher, N. I. (1993). "Statistical Analysis of Circular Data". Cambridge University Press, Cambridge.
[4] Girija, S.V.S., 2010. "Construction of New Circular Models". VDM - VERLAG, Germany.
[5] Jammalamadaka S. Rao, Sen Gupta, A., (2001). "Topics in Circular Statistics", World Scientific Press, Singapore.
[6] Mardia, K.V. and Jupp, P.E. (2000), "Directional Statistics", John Wiley, Chichester.
[7] Minh, Do Le and Farnum, Nicholas R. (2003), "Using Bilinear Transformations to Induce Probability Distributions", Communication in Statistics – Theory and Methods, 32, 1, pp. 1 – 9.
[8] Phani. Y., Girija S.V.S and Dattatreya Rao A.V. (2012), "Circular Model Induced by Inverse Stereographic Projection On Extreme-Value Distribution", IRACST – Engineering Science and Technology: An International Journal (ESTIJ), Vol.2, No. 5, 2250-3498
[9] Ramabhadra Sarma, I., Dattatreya Rao, A.V. and Girija S.V.S., (2009). "On Characteristic Functions of Wrapped Half Logistic and Binormal Distributions", International Journal of Statistics and Systems, Vol 4(1), pp. 33–45.
[10] Rao, C.R. & Mitra, S.K. (1975), "Formulae and Tables for Statistical Work", Statistical Publishing Society.
[11] Tahir M. H, Gauss M. Cordeiro, Ayman Alzaatreh, M. Mansoor, and M. Zubair, (2016), "A New Weibull-Pareto Distribution: Properties and Applications", Communications in Statistics - Simulation and Computation Vol. 45, Iss. 10,2016
[12] Toshihiro Abe, Kunio Shimizu, Arthur Pewsey (2010). "Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection", J. Japan Statist. Soc., Vol /Issue: 40(1). Pp 45-61.

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On Wrapping of Exponentiated Inverted Weibull Distribution

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Abstract

In many life testing experiments 'directions' are the observations. Directional data have many new and distinctive characteristics and challenges in terms of its modelling as well as in conducting statistical analysis. To draw more meaningful inferences, many circular models were developed from the existing linear distributions using variety of techniques like wrapping, inverse stereographic projections etc. In this article an attempt is made to construct a circular model for the Exponentiated Inverted Weibull Distribution by using the method of wrapping. Exponentiated Inverted Weibull is considered to be the most frequently used probability distribution for analyzing the life time data with some monotone failure rates. In this paper the probability density function, distribution function and characteristic function are derived for this Wrapped Exponentiated Inverted Weibull Distribution. The Trigonometric moments and important population characteristics for this wrapped EIW Distribution are computed.

Keywords: Circular Statistics, Wrapping, Exponentiated Inverted Weibull, Trigonometric Moments

I. INTRODUCTION

Dattatreya Rao et al (2007) constructed Wrapped Lognormal, Wrapped Logistic, Wrapped Weibull, and Wrapped Extreme Value Distributions. Ramabhadra Sarma et al (2009) derived characteristic function of Wrapped Half Logistic and Wrapped Binormal Distribution. Mardia and Jupp (2000) gave expressions for population characteristics such as variance, standard deviation, skewness, kurtosis etc. for circular distributions. Girija et al (2010) introduced new construction procedures of constructing Circular models calling Rising Sun Circular models and studied M L estimation parameters of Cardioid distribution from complete samples.

Contributing to this work, an attempt is made here to derive a new circular model for the Exponentiated Inverted Weibull distribution using the method of wrapping. Wrapping is a technique which reduces a linear variable to it's modulo 2π. The density, distribution function, characteristic function for the wrapped Exponentiated Inverted Weibull distribution are derived and using the trigonometric moments, important population characteristics for the proposed wrapped EIW distribution are also computed.

This paper is organised as follows. Section 2 describes the Circular probability distribution and the methodology of wrapping a linear probability distribution. Section 3 defines the proposed wrapped Exponentiated Inverted Weibull distribution, and presents the graphs of density, distribution and characteristic functions for various values of parameters. Important population characteristics for the wrapped Exponentiated Inverted Weibull distribution are computed. Section 4 summarises the findings of this study.

For this paper software MATLAB is used for all the computations and for plotting of graphs.

II. CIRCULAR PROBABILITY DISTRIBUTION

A circular random variable in a continuous circular distribution $g:[0,\ 2\pi) \to \mathbb{R}$ is said to be following a circular probability density function of g (θ) if and only if g has the following basic properties

$$g(\theta) \geq 0, \quad \forall\ \theta \qquad \ldots (1)$$

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$$\int_0^{2\pi} g(\theta)\, d\theta = 1 \qquad \ldots (2)$$

$g(\theta) = g(\theta + 2k\pi)$ g is periodic , for any integer k (Mardia,2000) … (3)

A. Method of Wrapping

If X is a random variable defined on $\mathbb{R}$, then the corresponding circular random variable X_w is defined by the modulo 2π reduction.

$$X_W \equiv X \left(\text{mod } 2\pi\right)$$

If f (x) is the probability density function (pdf) of the linear random variable X then for the circular random variable X_w, the corresponding pdf, g(θ) is defined as,

$$g(\theta) = \sum_{k=-\infty}^{\infty} f(\theta + 2k\pi) \text{, where } \theta \in [0, 2\pi)$$

It can be verified that $g(\theta)$ with total probability concentrated on the unit circle $\{(\cos\theta, \sin\theta) / 0 \le \theta < 2\pi \}$ in the plane and satisfies the properties (1) to (3) above.
Also the characteristic function for X_w given its distribution function $F(\theta)$ is given by

$$\phi_\theta(t) = E\left(e^{it\theta}\right) = \int_0^{2\pi} e^{it\theta} dF(\theta) = \rho_t e^{i\mu_t} \qquad t \in \square$$

It is clear from the above that whenever $\phi(t) \neq 0$, $e^{2\pi it} = 1$ (Mardia 2000). Implies $\phi(t)$ can only be defined for integer values of t. Also the characteristic function for the wrapped distribution is $\phi(p) = \phi_p$ and is defined as

$$\phi_\theta(p) = E\left(e^{ip\theta}\right) = \int_0^{2\pi} e^{ip\theta} dF(\theta) = \rho_p e^{i\mu_p} \text{ , } p \in \mathbf{Z} \quad \text{also } \phi_0 = 1, \quad \bar{\phi}_p = \phi_{-p},$$

B. Trigonometric moments

ϕ_p, the pth trigonometric moment is value of the characteristic function ϕ_t at t = p. The real part and the imaginary part of ϕ_p are trigonometric moments α_p and β_p respectively and are denoted as

$$\alpha_p = E(\cos p\theta), \ \beta_p = E(\sin p\theta) \quad \text{Where} \quad p \in \mathbf{Z}$$

III. WRAPPED EXPONENTIATED INVERTED WEIBULL DISTRIBUTION(WEIW)

The Exponentiated Inverted Weibull distribution is a generalization to the inverted Weibull distribution through adding a new shape parameter $\lambda \in R^+$ by exponentiation to Inverted Weibull distribution function. A linear random variable X is said to follow a two parameter Exponentiated Inverted Weibull distribution, if the distribution function of X takes the following form

$$F(x) = \left(e^{-x^{-c}}\right)^{\lambda}$$

Where c and λ are shape parameters and $0 < x < \infty$ and $c > 0$, $\lambda > 0$
Hence the probability density function of Exponentiated Inverted Weibull distribution is

$$f(x) = \lambda . c . x^{-(c+1)} \left(e^{-x^{-c}}\right)^{\lambda}$$

where $0 < x < \infty$ and $c > 0$, $\lambda > 0$

Here if $\lambda = 1$, this EIW distribution becomes the standard Inverted Weibull distribution and if $c = 1$ this distribution represents standard Inverted Exponential distribution.

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A. *Probability density function for WEIW distribution*

Applying the method wrapping the pdf for WEIW distribution $g(\theta)$ can be written as

$$g(\theta)=\sum_{k=0}^{\infty} \lambda . c(\theta+2\pi k)^{-(c+1)}\left(e^{-(\theta+2\pi k)^{-c}}\right)^{\lambda} \quad \text{where} \quad \theta \in [0,2\pi) \text{ and } c>0, \lambda>0$$

The graph depicting the linear representation of the pdf of WEIW distribution for different values of c keeping the value for the parameter λ at 2.0 is as follows:

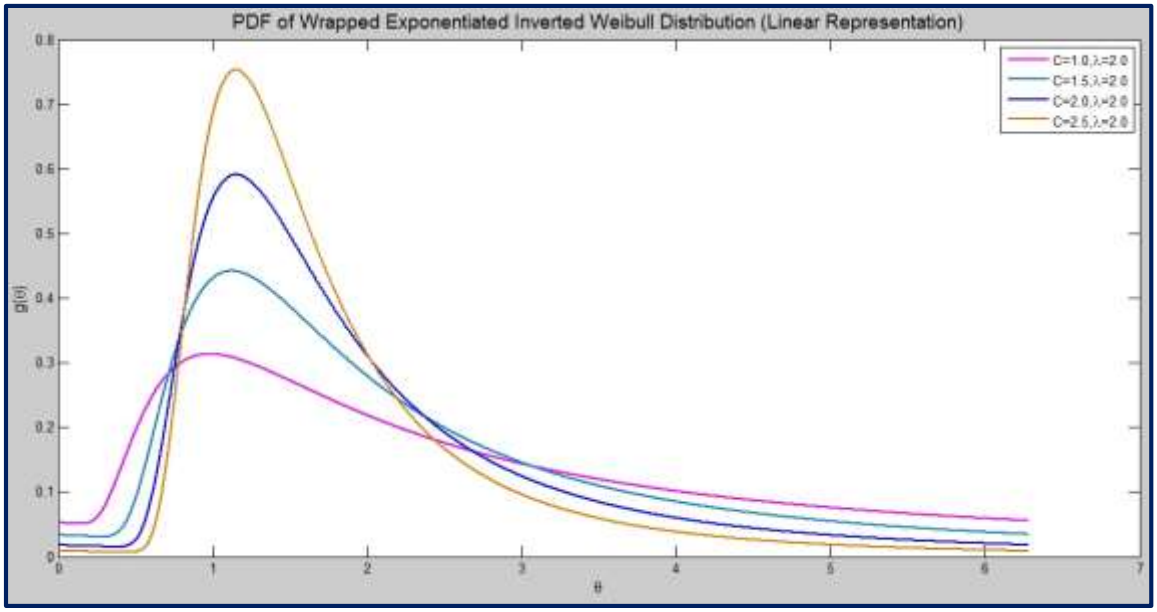

Fig. 1: PDF of WEIW distribution (Linear Representation)

The same linear representation of pdf for different values of λ keeping the values for parameters c at 2.0 is obtained as below

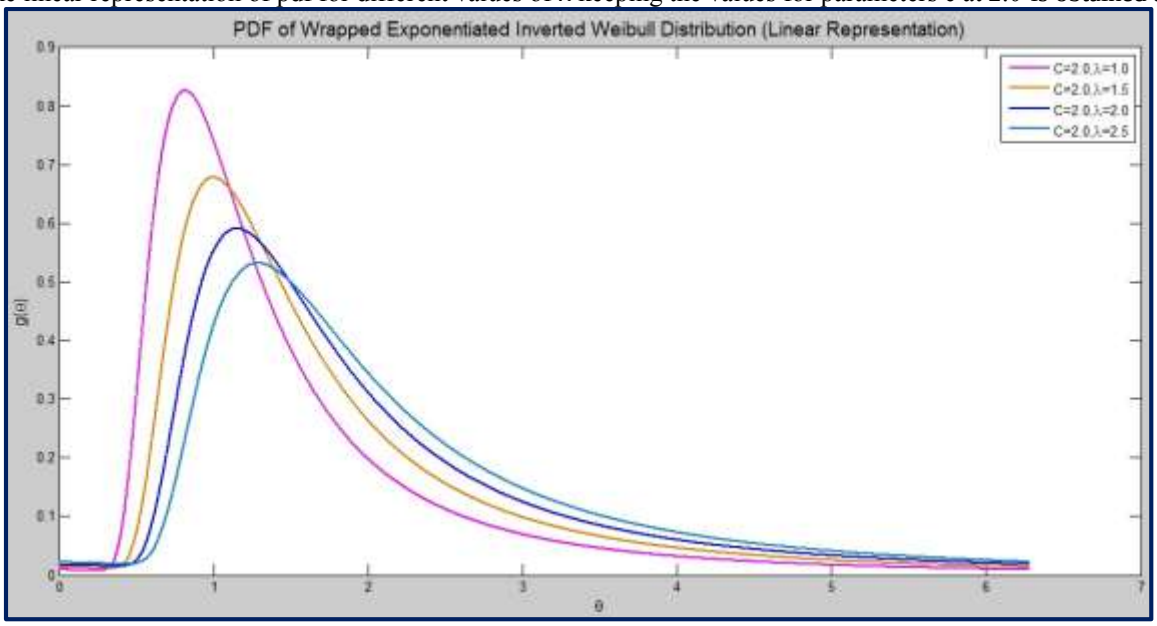

Fig. 2: PDF of WEIW distribution (Linear Representation)

Now the graph depicting the circular representation of the pdf of WEIW Distribution for different values of c keeping the value for the parameter λ at 2.0 is shown below:

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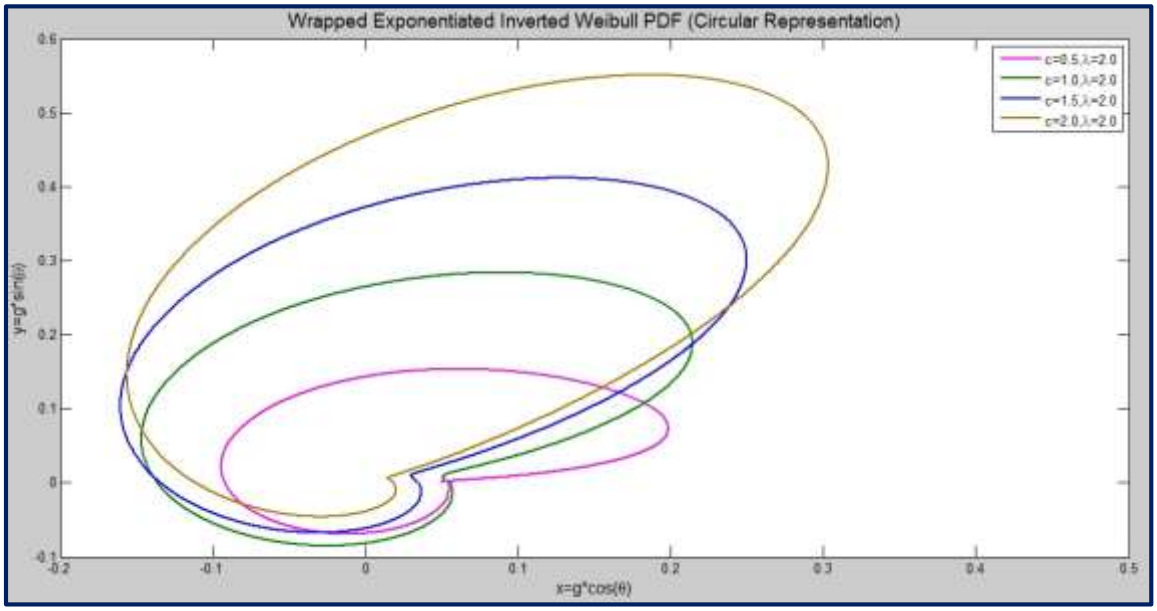

Fig. 3: PDF of WEIW distribution (Circular Representation)

Same circular representation for the pdf now for the different values of λ keeping the values for parameters c at 2.0 is obtained as below:

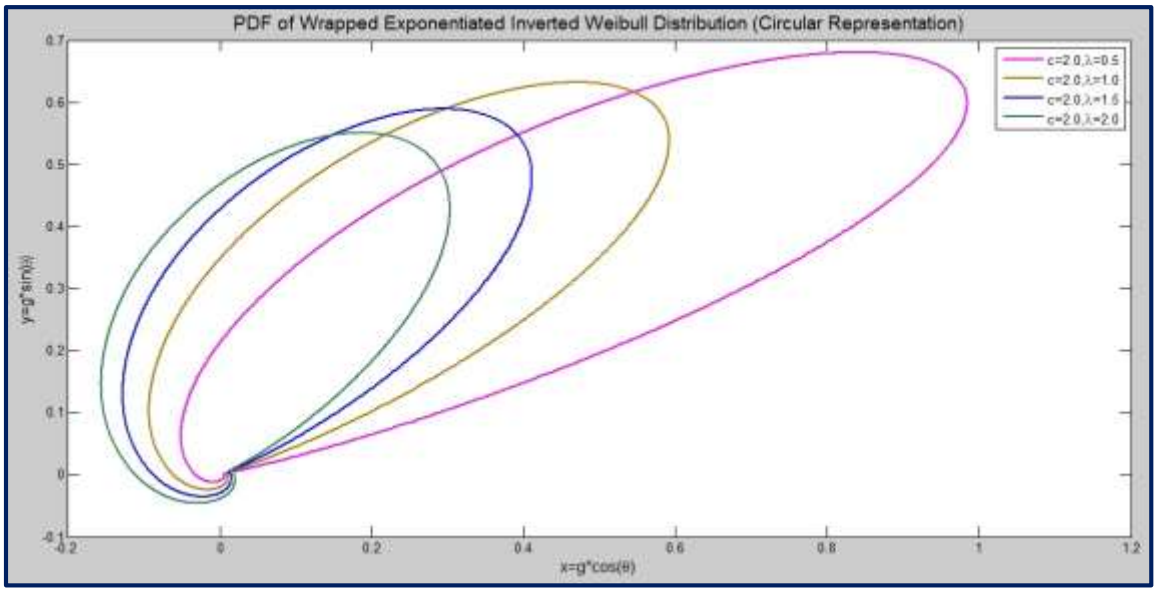

Fig. 4: PDF of WEIW distribution (Circular Representation)

Cumulative Distribution Function for WEIW distribution:
The Distribution function of the WEIW distribution can be derived as

$$G(\theta) = \int_0^{\theta} \sum_{k=0}^{\infty} \lambda . c(\theta + 2\pi k)^{-(c+1)} \left(e^{-(\theta+2\pi k)^{-c}} \right)^{\lambda} d\theta$$

Taking $m = (\theta + 2k\pi)^{-c}$ and solving the integral we get

$$G(\theta) = \sum_{k=0}^{\infty} (e^{-\lambda.(\theta+2\pi k)^{-c}} - e^{-\lambda.(2\pi k)^{-c}})$$ where $\theta \in [0, 2\pi)$ and c >0, λ >0

The graph for the CDF, G (θ) for WEIW distribution is obtained as below

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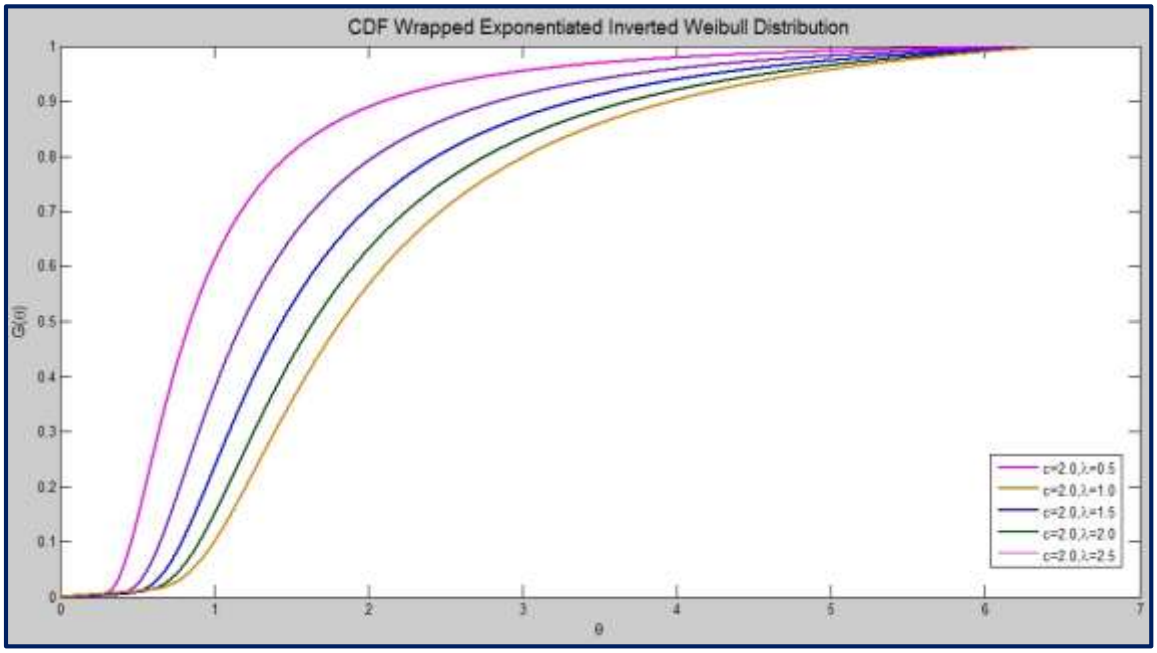

Fig. 5: CDF of WEIW distribution

Characteristic function for WEIW distribution:
As discussed in the previous section the characteristic function of EIW distribution is

$$\phi(t) = \int_0^\infty e^{itx} \lambda.c.x^{-(c+1)} (e^{-x^{-c}})^{\lambda} dx \qquad \ldots (4)$$

taking $x^{-c} = U$ we get

$$\phi(t) = \int_0^\infty e^{-u\lambda} e^{it(u)^{(-1/c)}} \lambda du \qquad \ldots (5)$$

considering $v = u\lambda$ then (5) can be reduced to

$$\phi(t) = \int_0^\infty e^{it\left(\frac{v}{\lambda}\right)^{(-1/c)}} e^{-v} dv \qquad \ldots (6)$$

Now Equation (6) can be written as

$$\phi(t) = \int_0^\infty e^{-v} \sum_{k=0}^{\infty} \frac{\left(it\left(\frac{v}{\lambda}\right)^{(-1/c)}\right)^k}{k!} dv \qquad \ldots. (7)$$

$$= \sum_{k=0}^{\infty} \frac{\left(it\lambda^{-1/c}\right)^k}{k!} \int_0^\infty e^{-v} v^{-k/c} dv$$

$$= \sum_{k=0}^{\infty} \frac{\left(it\lambda^{-1/c}\right)^k}{k!} \Gamma\left(1-k/c\right) \qquad c>0 \text{ and } \lambda>0 \qquad \ldots (8)$$

The convergence of the series in (8) fails at least for some values of c for example when c takes values between 0 and 1 and k ≠0. To solve this for obtaining the trigonometric moments, the n – point Gauss – Laguerre quadrature formula for numerical

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integration as given in Rao et al, (1975) is applied for equation (6). For $p \in$ the characteristic function of the wrapped Exponentiated Inverted Weibull distribution is hence given by

$$\phi(p) = \int_0^{2\pi} e^{ipx} g(\theta) \, d\theta$$

The real and imaginary parts α_p and β_p respectively are obtained from the characteristic function of the WEIW distribution.

The following are the graphs for the characteristic function of the WEIW distribution showing the real part and imaginary part separately for different values of c and λ

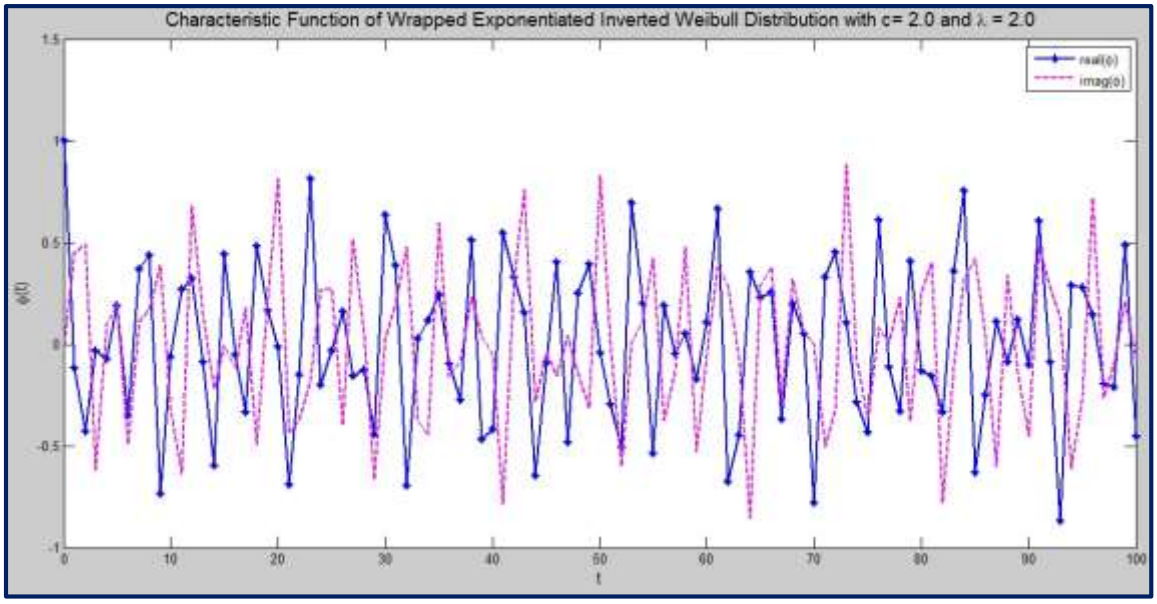

Fig. 6: Characteristic Function of WEIW distribution (1)

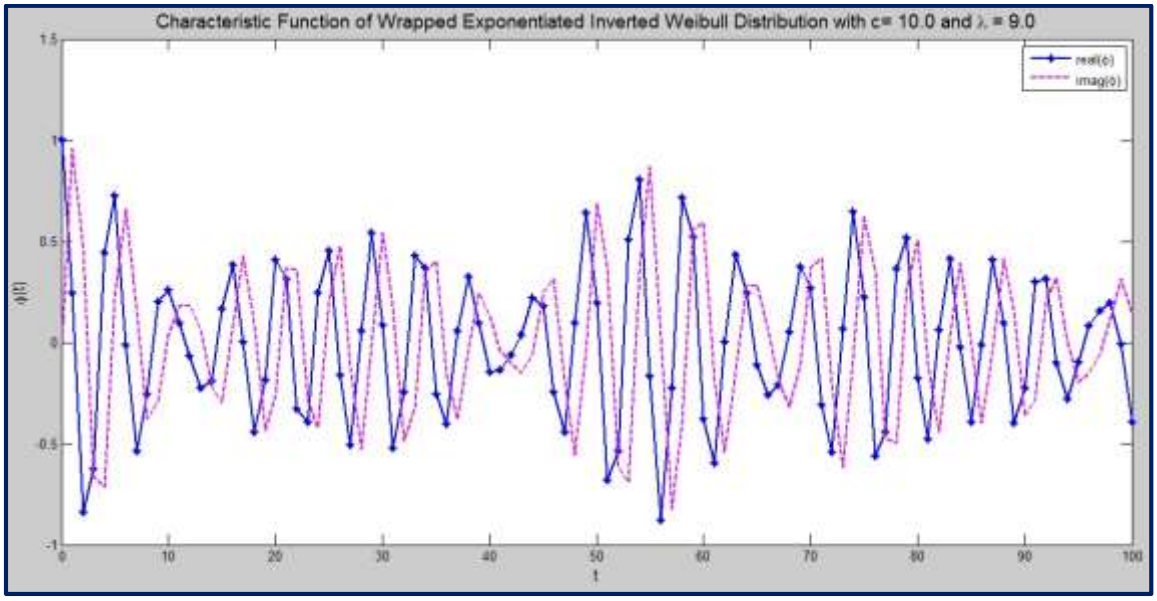

Fig. 7: Characteristic Function of WEIW distribution (2)

B. Population Characteristics:

Given a Circular distribution, Mardia (2000) had derived expressions for mean direction μ_o resultant length ρ_1, Circular variance V_0, Central Trigonometric Moments α_p^*, β_p^*, Skewness γ_1^o and Kurtosis γ_2^o. Using these expressions the Population Characteristics for the Exponentiated Inverted Weibull distribution for different values of the parameters c and λ are computed and tabulated here under.

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Table – 1
Characteristics of Exponentiated Inverted Weibull Distribution at λ= 2.0

c	*c=0.5*	*c=1.5*	*c=2.0*	*c=2.5*	*c=3.0*
Trigonometric Moments					
α_1	*-0.0279*	*0.3009*	*-0.1147*	*-0.1098*	*-0.0063*
α_2	*-0.1353*	*-0.1451*	*-0.4282*	*-0.2263*	*-0.4357*
β_1	*0.5497*	*0.5067*	*0.4467*	*0.7085*	*0.8347*
β_2	*0.5425*	*-0.2179*	*0.4915*	*0.1864*	*0.0940*
Resultant Length					
ρ_1	*0.5504*	*0.5893*	*0.4612*	*0.7170*	*0.8347*
ρ_2	*0.5591*	*0.2618*	*0.6519*	*0.2932*	*0.4457*
Mean Direction					
μ_0	*1.6215*	*1.0349*	*1.8222*	*1.7246*	*1.5783*
Variance					
V_0	*0.4496*	*0.4107*	*0.5388*	*0.2830*	*0.1653*
Circular Standard Deviation					
σ_0	*1.0928*	*1.0284*	*1.2441*	*0.8157*	*0.6011*
	1.0783	*1.6371*	*0.9250*	*1.5665*	*1.2712*
Circular Trigonometric Moments					
α_1^*	*0.5504*	*0.5893*	*0.4612*	*0.7170*	*0.8347*
α_2^*	*0.0797*	*-0.1220*	*0.1383*	*0.1593*	*0.4343*
β_1^*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*
β_2^*	*-0.5534*	*0.2317*	*-0.6371*	*-0.2461*	*-0.1005*
Skewness					
Υ_1^0	*-1.8358*	*0.8803*	*-1.6109*	*-1.6349*	*-1.4960*
Kurtosis					
Υ_2^0	*-0.0596*	*-1.4384*	*0.3207*	*-1.3110*	*-1.8765*

Table – 2
Characteristics of Exponentiated Inverted Weibull Distribution at c= 2.0

λ	*λ=0.5*	*λ=1.5*	*λ=2.0*	*λ=2.5*	*λ=3.0*
Trigonometric Moments					
α_1	*0.4262*	*-0.0583*	*-0.1147*	*-0.1133*	*-0.0862*
α_2	*-0.1147*	*-0.1268*	*-0.4282*	*-0.6898*	*-0.7536*
β_1	*0.7225*	*0.5645*	*0.4467*	*0.3626*	*0.3181*
β_2	*0.4467*	*0.4844*	*0.4915*	*0.2556*	*-0.1030*
Resultant Length					
ρ_1	*0.8388*	*0.5675*	*0.4612*	*0.3799*	*0.3296*
ρ_2	*0.4612*	*0.5007*	*0.6519*	*0.7357*	*0.7606*
Mean Direction					
μ_0	*1.0378*	*1.6738*	*1.8222*	*1.8737*	*1.8353*
Variance					
V_0	*0.1612*	*0.4325*	*0.5388*	*0.6201*	*0.6704*
Circular Standard Deviation					
σ_0	*0.5928*	*1.0645*	*1.2441*	*1.3912*	*1.4899*
	1.2441	*1.1762*	*0.9250*	*0.7835*	*0.7398*
Circular Trigonometric Moments					
α_1^*	*0.8388*	*0.5675*	*0.4612*	*0.3799*	*0.3296*
α_2^*	*0.4465*	*0.0251*	*0.1383*	*0.4215*	*0.7026*
β_1^*	*0.0000*	*0.0000*	*0.0000*	*0.0000*	*0.0000*
β_2^*	*-0.1156*	*-0.5001*	*-0.6371*	*-0.6030*	*-0.2914*
Skewness					
Υ_1^0	*-1.7875*	*-1.7582*	*-1.6109*	*-1.2349*	*-0.5308*
Kurtosis					
Υ_2^0	*-1.8729*	*-0.4203*	*0.3207*	*1.0420*	*1.5370*

IV. CONCLUSION

It can be observed that the Wrapped Exponentiated Inverted Weibull distribution becomes Wrapped model for standard Inverted Weibull distribution when λ = 1 and when c =1, Wrapped Exponentiated Inverted Weibull distribution becomes wrapped model for Exponentiated Inverted Exponential distribution.

From the population characteristics for the Wrapped Exponentiated Inverted Weibull distribution tabulated above in the last section, we can observe that with increasing value of shape parameter c, keeping other shape parameters λ = 2.0, the Circular variance gradually decreased, the distribution is negatively skewed and remained platykurtic. With increasing value of the scale

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parameter λ keeping other scale parameter at $c = 2.0$ the Circular variance gradually increased, the distribution started shifting from negatively skewed to near symmetric and from platykurtic to mesokurtic.

REFERENCES

[1] Dattatreya Rao, A.V., Ramabhadra Sarma, I., Girija S.V.S., (2007). On wrapped version of some life testing models. Communication in Statistics - Theory and Methods, 36, 2027-2035.
[2] Girija, S.V.S., 2010. Construction of New Circular Models. VDM - VERLAG, Germany.
[3] A. Flaih, H. Elsalloukh, E. Mendi and M. Milanova (2012), The Exponentiated Inverted Weibull Distribution, Appl. Math. Inf. Sci. 6, No. 2, 167-171
[4] Jammalamadaka S. Rao, Sen Gupta, A., 2001. Topics in Circular Statistics, World Scientific Press, Singapore.
[5] Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, John Wiley, Chichester.
[6] Ramabhadra Sarma, I., Dattatreya Rao, A.V. and Girija S.V.S., (2009). On Characteristic Functions of Wrapped Half Logistic and Binormal Distributions, International Journal of Statistics and Systems, Vol 4(1), pp. 33–45.
[7] Ramabhadra Sarma, I., Dattatreya Rao, A.V. and Girija, S.V.S., (2011). On Characteristic Functions of Wrapped Lognormal and Weibull Distributions, Journal of Statistical Computation and Simulation Vol. 81(5), 579–589.
[8] Rao, C.R. & Mitra, S.K. (1975), Formulae and Tables for Statistical Work, Statistical Publishing Society.

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Generating Function for $\overline{Ga\ spt}(n)^j$

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Abstract: In this section, we derive a formula for the generating function the number of smallest parts including repetitions in all j^{th} over *Ga* partition of *n* .

I. INTRODUCTION

Corteal and Love joy [1] initiated the study of over partitions. Hanuma Reddy and Janakamma [3] derived a formula for the number of ith over Ga partition of n

When the parts are in A.P. In this paper, we obtain a formula for the generating function for the number of smallest parts including repetitions in all j^{th} over *Ga* partition of n .

1.1 Definitions and notation:

1) A r-partition of n is a non-deceasing sequence of positive integers whose sum is =n. Each number is called partition
2) A jth over *r – partition of n is r-partition of n in which a part is over lined j times at its first* appearances.
3) The cardinality of the set of jth over *r – partition*s of *n is* denoted by $\overline{p_r(n)}_j$.
4) $\overline{p_r(s,n)}_j$: The number of jth over *partitions* of n with least part greater than or equal to s is defined by $\overline{p_r(s,n)}_j$.
5) *Ga* partition : A partition of *n* is a *Ga partition* if smallest parts are of the form a^{k-1}, $k \in N$.
6) over *Ga* partition Is a *Ga* partition in which first (equivalently, the final) occurrence of a part is over lined up to j times successively.
7) $\overline{Ga\ spt}(n)^j$: denotes the number of smallest parts including repetitions in all j^{th} Over *r – partition* of *n*
8) $(a,q)_\infty : \prod_{n=1}(1-aq^n)$
9) $(q)_\infty : (1,q)_\infty$
10) $d(a,n)$ is the number of divisors of n of the form? a^{k-1}, $k \in N$

II. GENARATING FUNCTIONS:

In this section, we obtain a formula for the generating function for the number of smallest parts including repetitions in all j^{th} over *Ga* partitions of *n*

2.1 Proposition: The number of partitions of n *into* r *parts each* $> a^{k-1}$ *= number of partitions of* $n - a^{k-1}r$ *into* r *parts.*

$$i.e\ \overline{p_r\left(a_{k-1}+1, n\right)}_j = \overline{p_r\left(n - a_{k-1}r\right)}_j \qquad (2.3(i))$$

Proof: Let $n = (\lambda_1, \lambda_2, \ldots, \lambda_r), \lambda > a^{k-1}$, write

$n - ra^{k-1} = (\mu_1, \mu_2, \ldots\ldots \mu_r)$ where $\mu_i = \lambda_i - a^{k-1}$ .Futher if $n - ra^{k-1} = (\mu_1, \mu_2, \ldots\ldots \mu_r)$ *and* $\lambda_i = \mu_i + a^{k-1}$

then $n = (\lambda_1, \ldots\ldots \lambda_r)$ cleary the correspondence $(\lambda_1, \ldots\ldots \lambda_r) \leftrightarrow (\mu_1, \ldots\ldots \mu_r)$

is one one and onto from $r - partitions\ of\ n$ with smallest part greater than a^{k-1} and all

$r - partitions\ of\ n - a^{k-1}$ .this one- one correspondence yields the required equality.

*2.2 Corollary***:** For $j = 1$ the number of r-*over partitions* of *n* with parts,

greater than or equal to $a^{k-1}+1$ is the number of $r - overpartitions$ of $n - a^{k-1}r$ parts.

We now derive the generating function for the number of smallest parts of all $j^{th}\ overGa\ partitions$ of *n* with the help of $r - j^{th}\ overGa\ partitions$ of n.

*2.3 Theorem***: T**he generating function for $\overline{Ga\ spt(n)}_j$ is

$$\sum_{n=1}^{\infty} \overline{Ga\ spt(n)}_j^{\,j} q^n = \frac{(-j, q)_\infty}{(q)_\infty} \sum_{n=1}^{\infty} \frac{(j+1)q^{a^{n-1}}}{\left(1 - q_{a^{n-1}}\right)} \frac{(q)_{a^{n-1}-1}}{(-j, q)_{a^{n-1}+1}} \qquad \text{.....(A)}$$

Proof: From theorem (3.1) of [3] we have

$$\overline{Ga\ spt(n)}_{k=1}^{\infty}{}^{j} = \sum_{t=1}^{\infty}\sum \overline{p\left(a_{k-1}, n - ta_{k-1}\right)}^{\infty}{}^{j} + j\sum_{k=1}^{\infty}\sum_{t=1} \overline{p\left(a_{k-1}+1, n - ta_{k-1}\right)}^{\,j} + (j+1)d(a, n)$$

Replace $a^{k-1}+1$ by a^{k-1}, n by $n - ta^{k-1}$ for first part and n by $n - ta^{k-1}$ for second part in (2.5) From

$$\overline{Ga\ spt(n)}^{\,j} = \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty} p_r \overline{\left(n - ta_{k-1} - r\left(a_{k-1}-1\right)\right)}^{\,j} + j\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty} p_r \overline{\left(n - ta_{k-1}k - ra_{k-1}\right)}^{\,j} + (j+1)d(a, n)$$

$$\sum_{n=1}^{\infty} \overline{Ga\ spt(n)q^n}_j$$

$$= \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty} \frac{q^{r+ta^{k-1}+r\left(a^{k-1}-1\right)}(-j, q)_r}{(q)_r} + j\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty} \frac{q^{r+ta^{k-1}+ra^{k-1}}(-j, q)_r}{(q)_r} + \sum_{k=1}^{\infty} \frac{(j+1)q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$= \sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty} \frac{q^{ta^{k-1}+ra^{k-1}}}{(q)_r}{}^{r} + j\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty} \frac{q^{r+ta^{k-1}+ra^{k-1}}}{(q)_r}{}^{r} + \sum_{k=1}^{\infty} \frac{(j+1)q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$= \sum_{k=1}^{\infty}\sum_{t=1}^{\infty} q^{ta^{k-1}} \left[\sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}}\right)^r (-j,q)_r}{(q)_r} \right] + j \sum_{k=1}^{\infty}\sum_{t=1}^{\infty} q^{ta^{k-1}} \left[\sum_{r=1}^{\infty} \frac{q^r \left(q^{a^{k-1}}\right)^r (-j,q)_r}{(q)_r} \right] + \sum_{k=1}^{\infty} \frac{(j+1)\, q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$= \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \left[\left(1 + \sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}}\right)^r (-j,q)_r}{(q)_r} \right) - 1 \right]$$

$$+ j \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \left[\left(1 + \sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}+1}\right)^r (-j,q)_r}{(q)_r} \right) - 1 \right] + \sum_{r=1}^{\infty} \frac{(j+1)\, q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$= \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \left(1 + \sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}}\right)^r (-j,q)_r}{(q)_r} \right) + j \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \left(1 + \sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}+1}\right)^r (-j,q)_r}{(q)_r} \right)$$

$$= \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \sum_{r=0}^{\infty} \frac{\left(1+jq^r q^{a^{k-1}}\right)}{\left(1-q^{a^{k-1}}\right)} + j \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \sum_{r=0}^{\infty} \frac{\left(1+jq^r q^{a^{k-1}+1}\right)}{\left(1-q^{a^{k-1}+1}\right)} \quad \text{from [3]}$$

$$= \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \sum_{r=0}^{\infty} \frac{\left(1+jq^{r+a^{k-1}}\right)}{\left(1-q^{a^{k-1}}\right)} + j \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \sum_{r=0}^{\infty} \frac{\left(1+jq^{r+a^{k-1}+1}\right)}{\left(1-q^{a^{k-1}+1}\right)}$$

$$= \frac{(-j,q)_\infty}{(q)_\infty} \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \frac{(q)_{a^{k-1}-1}}{(-j,q)_{a^{k-1}}} + j \frac{(-j,q)_\infty}{(q)_\infty} \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \frac{(q)_{a^{k-1}}}{(-j,q)_{a^{k-1}+1}}$$

$$= \frac{(-j,q)_\infty}{(q)_\infty} \sum_{k=1}^{\infty} \frac{q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \frac{(q)_{a^{k-1}-1}}{(-j,q)_{a^{k-1}}} \left[1 + j \frac{\left(1-q^{a^{k-1}}\right)}{\left(1+jq^{a^{k-1}}\right)} \right]$$

$$= \frac{(-j,q)_\infty}{(q)_\infty} \sum_{k=1}^{\infty} \frac{(j+1)\, q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)} \frac{(q)_{a^{k-1}-1}}{(-j,q)_{a^{k-1}+1}}$$

$$= \frac{(-j,q)_\infty}{(q)_\infty} \sum_{n=1}^{\infty} \frac{(j+1)\, q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)} \frac{(q)_{a^{n-1}-1}}{(-j,q)_{a^{n-1}+1}}$$

2.3.4 Illustration: We explain our theorem by an illustration. In this context we often come across with terms of the form

$\frac{1}{1 \pm x}$. we replace these terms by the power series expansions

$$\frac{1}{1 \pm x} = 1 \pm x \pm x^2 \pm \ldots \pm x^n \pm \ldots$$

(With alternate $-$ *and* $+$ signs for $1+x$ and $+$ signs for $1-x$ in the denominator.)

The coefficient of x^n in this context can be found with the help of Math lab.

When j = 2 and a = 3 in theorem 2.3.3 we have

$$\sum_{n=1}^{\infty} \overline{G\,3spt(n)}\, q^n = \frac{(-2,q)_\infty}{(q)_\infty} \sum_{n=1}^{\infty} \left[\frac{(2+1)q^{3^{n-1}}}{3^{n-1}} \cdot \frac{(q)_{3^{n-1}-1}}{\left(1-q^{(2+1)q^{n-1}}\right)(-2,q)_{3^{n-1}+1}} \right]$$

$$= \frac{(2+1)q^3(1-q)(1-q^2)}{(q)_\infty \left[(1-q)(1+2)(1+2q) \right]} + \frac{(2+1)q^9(1-q)(1-q^2)(1-q^3)(1-q^4)(1-q^5)(1-q^6)(1-q^7)(1-q^8)}{(1-q^3)(2+1)(1+2q)(1+2q^2)(1+2q^3)} + \ldots$$

$$= \frac{(-2,q)_\infty}{(q)_\infty} \left[q - q^2 + 4q^3 - 8q^4 + 14q^5 - 25q^6 + 58q^7 - 118q^8 + 226q^9 - 448q^{10} + \ldots \right]$$

$$= 3\left(1 + 3q + 6q^2 + 15q^3 + 27q^4 + 51q^5 + 93q^6 + \ldots\right)$$

$$\left(q - q^2 + 4q^3 - 8q^4 + 14q^5 - 25q^6 + 58q^7 - \ldots\right)$$

$$= 3\left(q + 2q^2 + 7q^3 + 13q^4 + 26q^5 + 53q^6 + 97q^7 + \ldots\right)$$

$$= 3q + 6q^2 + 21q^3 + 39q^4 + 78q^5 + 159q^6 + 291q^7 + \ldots$$

2.3.5 Corollary: The generating function for $\overline{Ga\,A_c(n)}^{\,j}$, the number of smallest parts of the jth *overGa partitions* of n which are multiples of c is

$$\sum_{n=1}^{\infty} \overline{Ga\,A(n)q^n}^{\,j} = \frac{(-j,q)_\infty}{(q)_\infty} \sum_{n=1}^{\infty} \frac{(j+1)q^{ca^{n-1}}}{\left(1-q^{ca^{n-1}}\right)} \frac{(q)_{ca^{n-1}-1}}{(-j,q)_{ca^{n-1}+1}}$$

2.3.6 Corollary: The generating function for $\overline{Ga\,A_c(n)}$, the number of smallest parts of the *overGa partitions* of n which are multiples of c is

$$\sum_{n=1}^{\infty} \overline{Ga\,A(n)q^{n}}_{c} = \frac{(-1,q)_\infty}{(q)_\infty} \sum_{n=1}^{\infty} \frac{2q^{ca^{n-1}}}{\left(1-q^{ca^{n-1}}\right)} \frac{(q)_{ca^{n-1}-1}}{(-1,q)_{ca^{n-1}+1}}$$

To evaluate the sum of smallest parts of $r-j^{th} overGa\ partitions$ of n by applying the concept of $r-j^{th} overGa\ partitions$ of *n*, we propose the following theorem.

*2.3.7 Theorem***:** The generating function for the sum of smallest parts of $j^{th}\ overGa\ partitions$ of n is

$$\sum_{n=1}^{\infty} \overline{sum\ Ga\ spt(n)q^{n}}_{j} = \frac{(-j,q)_\infty}{(q)_\infty} \sum_{n=t_1}^{\infty} \frac{(j+1)a^{n-1}q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)} \frac{(q)_{a^{n-1}-1}}{(-j,q)_{a^{n-1}+1}}$$

Proof: **T**he sum of smallest parts of $j^{th}\ overGa\ partitions$ of a positive integer$n$ is

$$\overline{sum\ Ga\ spt(n)}_{j} = \sum_{t=1}^{\infty}\sum_{k=1}^{\infty} a^{k-1}\ \overline{p\left(a^{k-1}, n-ta^{k-1}\right)}_{j} + j\sum_{t=1}^{\infty}\sum_{k=1}^{\infty} a^{k-1}\ \overline{p\left(a^{k-1}+1, n-ta^{k-1}\right)}_{j}$$
$$+(j+1)d(a,n)$$
$$= \sum_{r=1}^{\infty}\sum_{t=1}^{\infty}\sum_{k=1}^{\infty} a^{k-1}\ \overline{p_r\left(a^{k-1}, n-ta^{k-1}\right)^{j}} + j\sum_{r=1}^{\infty}\sum_{t=1}^{\infty}\sum_{k=1}^{\infty} a^{k-1}\ \overline{p_r\left(a^{k-1}+1, n-ta^{k-1}\right)^{j}}$$
$$+(j+1)d(a,n)$$

First replace $a^{k-1}+1$ by a^{k-1}, then replace n by $n-ta^{k-1}$ in (2.1.1)

$$= \sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\sum_{k=t_1}^{\infty} \overline{a^{k-1}.p_r\left(n-ta^{k-1}-r\left(a^{k-1}-1\right)\right)^{j}}$$
$$+ j\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\sum_{k=t_1}^{\infty} \overline{a^{k-1}.p_r\left(n-ta^{k-1}-ra^{k-1}\right)^{j}}$$
$$+(j+1)d(a,n)$$

Hence From (2.2(i)) the generating function for the sum of smallest parts of the $r-j^{th} overGa\ partitions$ of n.

$$= \sum_{k=1}^{\infty} \frac{a^{k-1}q^{a^{k-1}}}{1-q^{a^{k-1}}} \left[\sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}}\right)^{r}(-j,q)_r}{(q)_r} \right]$$
$$+ j\sum_{k=1}^{\infty} \frac{a^{k-1}q^{a^{k-1}}}{1-q^{a^{k-1}}} \left[\sum_{r=1}^{\infty} \frac{q^{r}\left(q^{1+a^{k-1}}\right)^{r}(-j,q)_r}{(q)_r} \right] + \sum_{k=1}^{\infty} \frac{j+1\,a^{k-1}q^{a^{k-1}}}{1-q^{a^{k-1}}}$$

$$= \sum_{k=1}^{\infty} \frac{a^{k-1}q^{a^{k-1}}}{1-q^{a^{k-1}}} \left[1 + \sum_{r=1}^{\infty} \frac{\left(q^{a^{k-1}}\right)^{r}(-j,q)_r}{(q)_r} \right]$$
$$+ j\sum_{k=1}^{\infty} \frac{a^{k-1}q^{a^{k-1}}}{1-q^{a^{k-1}}} \left[1 + \sum_{r=1}^{\infty} \frac{q^{r\left(1+a^{k-1}\right)^{r}}(-j,q)_r}{(q)_r} \right]$$

$$= \sum_{k=1}^{\infty} \frac{a^{k-1} q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)_{r=0}} \left(\prod_{r=0}^{\infty} \frac{1+jq^{r} q^{a^{k-1}}}{1-q^{a^{k-1}} q^{r}} \right) + j \sum_{k=1}^{\infty} \frac{a^{k-1} q^{a^{k-1}}}{\left(1-q^{a^{k-1}}\right)_{r=0}} \left(\prod_{r=0}^{\infty} \frac{1+jq^{r} q^{a^{k-1}+1}}{1-q^{r} q^{a^{k-1}+1}} \right) \quad \text{(from [3])}$$

$$= \frac{(-j,q)_{\infty}}{(q)_{\infty}} \sum_{n=1}^{\infty} \frac{(j+1) a^{n-1} q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)} \frac{(q)_{a^{n-1}-1}}{(-j,q)_{a^{n-1}+1}}$$

$$\sum_{n=1}^{\infty} \overline{sum\ G\ a\ spt\ (n)} q^{n\ j} = \frac{(-j,q)_{\infty}}{(q)_{\infty}} \sum_{n=1}^{\infty} \frac{(j+1) a^{n-1} q^{a^{n-1}}}{\left(1-q^{a^{n-1}}\right)} \frac{(q)_{a^{n-1}-1}}{(-j,q)_{a^{n-1}+1}}$$

2.8 Illustration: When $j = 2$, $a = 2$ by 2.7 we have

$$\sum_{n=1}^{\infty} \overline{sum\ G\ 2\ spt\ (n)} q^{n\ j} = \frac{(-2,q)_{\infty}}{(q)_{\infty}} \sum_{n=1}^{\infty} \frac{(2+1)\, 2^{n-1} q^{2^{n-1}}}{\left(1-q^{2^{n-1}}\right)} \frac{(q)_{2^{n-1}-1}}{(-j,q)_{2^{n-1}+1}}$$

By using math lab the R.H.S can be simplified into

$$3q + 12q^{2} + 18q^{3} + 63q^{4} + 96q^{5} + 189q^{6} + 336q^{7} + 645q^{8} + 1053q^{9} + 1725q^{10} + \ldots$$

The sum of smallest parts of second *overG* 2 *partitions* of 6 is 225 and is verified from the following

$5+\underline{1}$, $\overline{5}+\underline{1}$, $\overline{5}+\underline{1}$, $5+\overline{\underline{1}}$, $\overline{5}+\overline{\underline{1}}$, $\overline{\overline{5}}+\underline{1}$,
$5+\overline{\overline{\underline{1}}}$, $\overline{5}+\overline{\overline{\underline{1}}}$, $\overline{5}+\overline{\overline{\underline{1}}}$, $4+\underline{2}$, $\overline{4}+\underline{2}$, $\overline{4}+\underline{2}$,
$4+\overline{\underline{2}}$, $\overline{4}+\overline{\underline{2}}$, $\overline{4}+\overline{\underline{2}}$, $4+\overline{\underline{2}}$, $\overline{4}+\overline{\underline{2}}$, $\overline{\overline{4}}+\overline{\underline{2}}$,

$4+\underline{1}+\underline{1}$, $\overline{4}+\underline{1}+\underline{1}$, $\overline{4}+\underline{1}+\underline{1}$, $4+\overline{\underline{1}}+\underline{1}$, $\overline{4}+\overline{\underline{1}}+\underline{1}$, $\overline{4}+\overline{\underline{1}}+\overline{\underline{1}}$,
$4+\overline{\overline{\underline{1}}}+\underline{1}$, $\overline{4}+\overline{\overline{\underline{1}}}+\underline{1}$, $\overline{4}+\overline{\overline{\underline{1}}}+\underline{1}$, $3+2+\underline{1}$, $\overline{3}+2+\underline{1}$, $\overline{3}+2+\underline{1}$,
$3+\overline{2}+1$, $\overline{3}+\overline{2}+\underline{1}$, $\overline{3}+\overline{2}+\underline{1}$, $3+\overline{2}+\underline{1}$, $\overline{3}+\overline{2}+\underline{1}$, $\overline{3}+\overline{2}+\underline{1}$,

$3+2+1$, $\overline{3}+2+\overline{1}$, $\overline{3}+2+\overline{\underline{1}}$, $3+\overline{2}+\overline{\underline{1}}$, $\overline{3}+\overline{2}+\overline{\underline{1}}$, $\overline{3}+\overline{2}+\overline{\underline{1}}$,
$3+\overline{\overline{2}}+1$, $\overline{3}+\overline{\overline{2}}+\overline{\underline{1}}$, $\overline{3}+\overline{\overline{2}}+\overline{\underline{1}}$, $3+2+\overline{\underline{1}}$, $\overline{3}+2+\overline{\overline{\underline{1}}}$, $\overline{3}+2+\overline{\overline{\underline{1}}}$,
$3+\overline{2}+1$, $\overline{3}+\overline{2}+\underline{1}$, $\overline{3}+\overline{2}+\overline{\underline{1}}$, $3+\overline{2}+\overline{\underline{1}}$, $\overline{3}+\overline{2}+\overline{\underline{1}}$, $\overline{3}+\overline{2}+\underline{1}$,

$\underline{2}+\underline{2}+\underline{2}$, $\overline{2}+\underline{2}+\underline{2}$, $\overline{\overline{2}}+\underline{2}+\underline{2}$, $3+\underline{1}+\underline{1}+\underline{1}$, $\overline{3}+\underline{1}+\underline{1}+\underline{1}$,
$3+\underline{1}+\underline{1}+\underline{1}$, $3+\overline{\underline{1}}+1+\underline{1}$, $\overline{3}+\overline{\underline{1}}+\underline{1}+\underline{1}$, $3+\overline{\underline{1}}+\underline{1}+\underline{1}$, $3+\overline{\overline{\underline{1}}}+\underline{1}+\underline{1}$,
$\overline{3}+\overline{\overline{\underline{1}}}+\underline{1}+\underline{1}$, $\overline{\overline{3}}+\overline{\overline{\underline{1}}}+\underline{1}+\underline{1}$, $2+2+\underline{1}+\underline{1}$, $\overline{2}+2+\underline{1}+\underline{1}$, $\overline{\overline{2}}+2+\underline{1}+\underline{1}$,

$2+2+1+\underline{1}, 2+2+1+\underline{1}, 2+2+1+\underline{1}, 2+2+1+\underline{1}, 2+2+1+\underline{1},$
$2+2+1+\underline{1}, 2+1+1+1+\underline{1}, 2+1+1+1+\underline{1}, 2+1+1+1+\underline{1}, 2+1+1+1+\underline{1},$
$2+1+1+1+\underline{1}, 2+1+1+1+\underline{1}, 2+1+1+1+\underline{1}, 2+1+1+1+\underline{1}, 2+1+1+1+\underline{1},$
$1+1+1+1+1+\underline{1}, 1+1+1+1+\underline{1}, 1+1+1+1+1.$

In the above table, 1 is under lined in each partition to specify its least property. The over partitions have to be counted taking into consideration the over lines on the parts. The number of such over partition is 225and therefore the sum of the least parts (each being 1) is 225.

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REFERENCES

[1]. Sylve Corteel and Jeremy Love joy: Over partitions, Trans.Amer.Math.Soc 356(4), 1623-1635.
[2]. Hanumareddy .K (2010). A study of r-partitions, thesis submitted to Acharya Nagarjuna university for award of Ph.D in Mathematics
[3]. Hanuma Reddy .K, K. Janakamma "Generating Function for Partitions with Parts in A.P" International Journal of Latest Technology in Engineering, Management & Applied Science-IJLTEMAS vol.6 issue 8, pp.134-139 2017
[4]. Segar G.V.R.K: A study of M2 partitions. Thesis submitted to Acharya Nagarjuna University for award of PhD in Mathematics (2017)

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3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal

2018-19

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link to the recognition in UGC enlistment of the Journal		
							Link to website of the Journal	Link to article / paper / abstract of the article	Is it listed in UGC Care list
1	*Biochemical and elemental analysis of syzygium samarangense Roots.*	Dr. M. Madhavi	Department of Botany	**Journal of Pharmacognosy and Phytochemistry**	**2019**	**E-ISSN: 2278-4136 P-ISSN: 2349-8234**	www.phytojournal.com	https://www.phytojournal.com/archives/2019/vol8issue6/PartF/8-6-24-852.pdf	NO
2	*Advanced Collaborative Framework Model to Improve Software Process Improvement*	Sri Ch. Subbarao	Department of Computer Science (PG)	International Journal of Scientific Research in Computer Science Applications and Management Studies	2018	ISSN 2319 – 1953	www.ijsrcsams.com	http://www.ijsrcsams.com/images/stories/Past_Issue_Docs/ijsrcsamsv7i3p81.pdf	NO
3	*The Impact of Feminism on Sujatha Bhatt's Poetry*	Smt. Ch. Aruna	Department of English	International Journal on Studies in English Language and Literature (IJSELL)	2019	ISSN 2347-3126 (Print) & ISSN 2347-3134 (Online)	www.arcjournals.org	http://dx.doi.org/10.20431/2347-3134.0703002	NO
4	*Multiculturalism in Indian English Literature*	Smt. V. Beulah Rani	Department of English	International Journal on Studies in English Language and Literature (IJSELL)	2019	ISSN 2347-3126 (Print) & ISSN 2347-3134 (Online)	www.arcjournals.org	http://dx.doi.org/10.20431/2347-3134.0703004	NO
5	*महादेवी वर्मा साहित्यमसोसकरन*	Dr. G. Vijayaratna Kumar	Department of Hindi	INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH	2019	ISSN: 2277-7881;	http://ijmer.in/index.aspx	http://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume8/volume8-issue1(2)-2019.pdf	NO
6	*आधुनिक साहित्य - प्रकृति का वर्णन*	Smt Sk. Baji	Department of Hindi	INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH	2019	ISSN: 2277-7881;	http://ijmer.in/index.aspx	http://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume8/volume8-issue1(1)-2019.pdf	NO

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S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
7	*Magneto Hydro Dynamics Convective Flow Past a Vertical Porous Surface in Slip – Flow Regime*	Dr. Y. Uday Kumar	Department of Mathematics	International Journal of Theoretical and Applied Mechanics	2017	ISSN 0973-6085	https://www.ripublication.com/ijtam.htm	http://www.ripublication.com/ijtam17/ijtamv12n1_07.pdf	NO
8	*On Discrete Wrapped Exponential distribution-Characteristics*	Dr. S.V.S. Girija	Department of Mathematics	International Journal of Scientific Research in Mathematical and Statistical Sciences	2018	E-ISSN: 2348-4519	https://www.isroset.org/journal/IJSRMSS/index.php	https://www.isroset.org/pub_paper/IJSRMSS/8-IJSRMSS-0826.pdf	NO
9	*On Wrapped Negative Binomial Model*	Dr. S.V.S. Girija	Department of Mathematics	International Journal of Mathematical Archive	2018	ISSN 2229 – 5046	http://www.ijma.info/index.php/ijma	http://www.ijma.info/index.php/ijma/article/view/5639	NO
10	*Stereographic l – Axial Half Logistic Distribution*	Dr. S.V.S. Girija Sri Y. Sreekanth	Department of Mathematics	International Journal of Applied Engineering Research	2018	ISSN 0973-4562	https://www.ripublication.com/ijaer.htm	https://www.ripublication.com/ijaer18/ijaerv13n12_72.pdf	NO
11	*Semicircular Logistic Distribution induced by Simple Projection Method*	Dr. S.V.S. Girija Sri Y. Sreekanth	Department of Mathematics	Mathematical Theory and Modeling	2018	ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)	https://iiste.org/Journals/index.php/MTM	https://iiste.org/Journals/index.php/MTM/article/viewFile/42970/44263	NO
12	*Stereographic l – axial Reflected Log – logistic distribution*	Dr. S.V.S. Girija Sri Y. Sreekanth	Department of Mathematics	International Journal of Mathematical Archive	2018	ISSN 2229 – 5046	http://www.ijma.info/index.php/ijma	http://www.ijma.info/index.php/ijma/article/view/5745	NO
13	*On Construction of Discrete Wrapped Lindley Distribution*	Dr. S.V.S. Girija Sri R. Srinivas	Department of Mathematics	i-manager's Journal on Mathematics	2018	ISSN Print: 2277-5129 ISSN Online: 2277-5137	https://imanagerpublications.com/journalsfulldetails/27/JournalonMathematics	https://imanagerpublications.com/index.php/article/15874/	NO
14	*Correlation between Bond Energy and Electronegativity Differences in a Few Semiconductors*	Dr. D.V. Raghuram Sri P.M. Prasad	Department of Physics	International Journal for Research in Applied Science & Engineering Technology (IJRASET)	2019	ISSN: 2321-9653	https://www.ijraset.com/	https://www.ijraset.com/fileserve.php?FID=21063	NO

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S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
15	*A Correlation between Hardness and Bond Ionicity in Crystals*	Dr. D.V. Raghuram Sri P.M. Prasad Smt V. Manikumari	Department of Physics Department of Chemistry	International Journal for Research in Applied Science & Engineering Technology (IJRASET)	2019	ISSN: 2321-9653	https://www.ijraset.com/	https://www.ijraset.com/fileserve.php?FID=21211	NO
16	*Employees' Attitude towards Information Technology*	Dr. N. Viswam	Department of Statistics	International Journal of Academic Research	2018	ISSN: 2348-7666	http://ijar.org.in/#gsc.tab=0	http://ijar.org.in/stuff/issues/v5-i4/v5-i4-a015.pdf	NO
17	*Tobacco Product Users in Kavali Town – A Sample Survey*	Dr. N. Viswam	Department of Statistics	International Journal of Academic Research	2018	ISSN: 2348-7666	http://ijar.org.in/#gsc.tab=0	http://ijar.org.in/stuff/issues/v5-i3/v5-i3-a019.pdf	NO
18	*Stock market prediction using time series analysis*	Dr. N. Viswam	Department of Statistics	International Journal of Statistics and Applied Mathematics	2018	ISSN: 2456-1452	https://www.mathsjournal.com/	https://www.mathsjournal.com/pdf/2018/vol3issue1/PartF/3-3-24-433.pdf	NO
19	*Logistic regression model applications in retail industry*	Dr. N. Viswam	Department of Statistics	International Journal of Statistics and Applied Mathematics	2018	ISSN: 2456-1452	https://www.mathsjournal.com/	https://www.mathsjournal.com/pdf/2018/vol3issue1/PartF/3-3-26-252.pdf	NO
20	*Control charts and cusum charts under linear trend with log-logistic distribution*	Dr. N. Viswam	Department of Statistics	International Journal of Statistics and Applied Mathematics	2018	ISSN: 2456-1452	https://www.mathsjournal.com/	https://www.mathsjournal.com/pdf/2018/vol3issue3/PartB/3-3-10-317.pdf	NO

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Biochemical and elemental analysis of *Syzygium samarangense* Roots

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Abstract
The use of plants and phytochemicals, both with known biological properties can be of great significance in treatment. The present study includes biochemical and elemental analysis of root extract of *Syzygium samarangense*. Biochemical analysis includes the estimation of amino acids, proteins, soluble sugars, starch, crude fiber, crude fat, crude protein, total nitrogen, phosphorus, also analyzed dry matter, moisture, and ash. Elements and metals present in the roots were extracted using acid digestion method and the level of calcium, magnesium, iron, manganese, zinc, copper, and cobalt were estimated by using Atomic Absorption Spectrophotometer. Sodium and Potassium were estimated flame photometrically following the standard method of flame photometer. The results, confirm the presence of aforesaid biochemical compounds and elements in the selected plant. This report may be helpful for ethnomedicine practitioners, researchers, and pharmacognosists for further validation of *S. samarangense* in depth.

Keywords: *Syzygium samarangense*, roots, biochemical analysis, moisture analysis, mineral analysis

Introduction
Since ancient times, about 80% of individuals use traditional medicine, which has chemical compounds derived from medicinal plants. Several hundred plant species and herbs in the form of whole plant, crude extract or purification, purified constituents are used in indigenous system of medicines and are of great importance to the health of individuals and communities, which have ultimately evaluated into the modern therapeutic science. Medicinal plants are an important source of life saving drugs for majority of the world population (Madhavi and Ram, 2015) [1].

Syzygium samarangense (common name - wax jambu) is a plant species in the family Myrtaceae that is widely cultivated in the tropics [2-10]. There are many traditional claims that have been reported of leaves, root, bark, fruits of the plant [11-14]. Various pharmacological activities like antidiarrhoeal activity, anticholinesterase activity, immunopharmacological activity, cytotoxic activity, anti hyperglycaemic activity, analgesic and anti-inflammatory activity are reported with various parts of the plant [15-22]. And also Investigators have found their principal constituent to be tannins, Quercetin glycosides, monoterpenes secondary metabolites those involved in pharmacological properties [23, 24]. Traditionally the root bark decoction of the *S. samarangense* is used in dysentery and amenorrhea and used as an abortifacient. Root is used as a diuretic and is given to alleviate edema. Malayans use powdered dried root preparations for itching. As plant root has significant therapeutic uses, this study is aimed to screen the phytochemicals of the root and study of its biological activities

Materials and methods
Chemicals: The solvents used for root extraction are Methanol and Ethyl Acetate. Distilled water has been used for aqueous extraction.

Apparatus: Denver electronic balance, TECHCOMP – UV 2301 Double Bean UV Visible Spectrophotometer with HITACHI 2.2 software, Tech-comp UV visible spectrophotometer, soxhlet extraction apparatus, heating mantle, incubator, autoclave.

Sample collection: Root material of *S. samarangense* plants were collected from farms in various places of East Godavari district, Andhra Pradesh, India. The roots are separated and allowed to shade dry. The root sample was ground and powdered for solvent extraction.

Solvent extraction: The phytochemicals present in the roots of the collected plants were isolated using different solvents like ethyl acetate, methanol and water in a series of extraction method from low polarity to high polarity using soxhlet extraction method.

Biochemical and elemental analysis of root

Estimation of Amino acids

Trichloroacetic acid extraction (TCA) was performed on fish samples, by following the procedure set up by Boland *et al.* 1971 [25]. 200mg of sample was added to 10mL of 7.5% (w/w) TCA and minced by means of a vertical homogenizer. The resulting product was filtered with filter paper (No. 4) from Whatman filter paper. The pH of a 1 mL aliquot was adjusted to 7.8 using 9 M KOH in an Eppendorf microfuge tube and centrifuged at 14 K rpm for 5 min in order to remove potassium trichloroacetate precipitate. The so obtained supernatant was stored at −80 °C.

A free amino acid level was estimated by the ninhydrin method as described by *Moore and Stein* [26]. To 1mLof supernatant, 2.0 mL of ninhydrin reagent was added and the contents were boiled for exactly 5min. They were cooled under tap water and the volume was made to 10mL with distilled water. The optical density of the color developed was measured using a spectrophotometer at a wavelength of 570 nm. A blank using distilled water and amino acid standards were also run similarly.

Estimation of Proteins

Total proteins were estimated following the method of Lowry *et al*. [27]. In this method protein was estimated in trichloroacetic acid precipitates, which had been treated, to remove contaminants [28]. Bovine serum albumin (Sigma) was used as standard. All determinations were carried out 3 times to permit standard evaluation.

To the sample extract, 4mL of 0.1N sodium hydroxide and to 1mL of each of these solutions, 5mL of reagent -D (mixture of 2% sodium carbonate and 0.5% copper sulphate in 50:1 ratio) was added. The samples were allowed to stand for 10 min, at the end of which 0.4mL of folin-phenol reagent(diluted with double distilled water in 1:1 ratio before use) was added. Finally volume of the solution was makeup to 10ml and then the optical density of the color developed was measured using a spectrophotometer at a wavelength of 660nm.

Estimation of Soluble Sugars

Plant root sample was extracted with 5ml of methanol. The residue was extracted again with 5 ml of methanol and the combined extract was reduced to dryness. This was resuspended in 5ml of distilled water. An aliquot 0.1 to 0.5 ml of this solution was employed for the determination of soluble sugars by Anthrone Calorimetric method [29].

The reaction was carried out under conditions similar to those used by Trevelyan & Harrison [30]. The anthrone reagent (5ml**)** was pippeted into thick walled pyrex tubes (150 x 25 mm) and chilled in ice water. The solution under test (1ml) was layered on the acid, cooled for a further 5min, and then thoroughly mixed while still immersed in ice water. The tubes were loosely fitted with corks, heated as required in vigorously boiling, constant level water bath and then cooled in water for 5min. then it was make up to 10ml with water and Absorption spectra was determined in a spectrophotometer with a 1cm cell.

Estimation of Starch

Starch was extracted from the residue of the sample after extracting the soluble sugar in methanol. This residue was boiled in water to extract the starch and estimated by I_2 KI method [29].

Dry matter and Moisture

The dry matter of the sample represents the amount of material left after the complete removal of moisture from it. The moisture of the sample was lost by volatilization caused by heat. The amount of material left after the removal of the moisture was the dry matter. Dry matter and moisture of the material were determined by following the method by AOAC (1990) [31]. Dishes were washed with detergents and then were dried at 105 ^{0}C in oven for overnight. Then dishes were removed from oven and then kept in desiccator for cooling and weights. Two gm fruit sample was taken in dishes and placed in oven at 105 ^{0}C overnight. The dry matter and moisture were calculated by using the following formulae:

$$\text{Dry matter (\%)} = \frac{(\text{Weight of dish + Weight of dried sample}) - \text{Weight of dish}}{\text{Weight of sample before drying}} \times 100$$

$$\text{Moisture content (\%)} = \frac{(\text{Weight of fresh sample} - \text{Weight of dry sample})}{\text{Weight of fresh sample}} \times 100$$

Ash

Ash value was determined by following the method of AOAC (1990) [31]. For this crucible were kept in a muffle furnace at 600 ^{0}C for 1h. Then they were transferred crucible from furnace to a desiccator and cooled to room temperature and weighed as quickly as possible to prevent moisture absorption. Two gram dry fruit powder was taken in tared silica crucible and placed in a muffle furnace at 600 ^{0}C for 6h. Then crucible was transferred to a desiccator and cooled to room temperature, crucible was transferred as quickly as possible to avoid moisture absorption. The percentage of ash was calculated by using the following formula:-

$$\text{Ash (\%)} = \frac{\text{Weight of Ash}}{\text{Weight of sample}} \times 100$$

Crude fiber

Crude fiber content was determined by following the method of *Sadasivam* and *Manikam* (1992) [32]. Two gm of dried sample of fruit was boiled with 200ml H_2SO_4 for 30 minutes with bumping chips. Then it was filtered through muslins cloth and washed with boiling water until washings were free of acid. Then the residue was boiled with 200 ml of NaOH for 30 minutes. Again it was filtered through muslin cloth and washed with 25 ml of boiling H2SO4 three 50 ml portion of water and 25 ml of alcohol. Then residue was removed and transferred to pre-weighed ashing dish (W1 g). The residue was analyzed for 2 hour at 130 ^{0}C, cooled in desiccator and weighed (W2 g). It was ignited for 30 minutes at 600 ^{0}C. After cooling in desiccator it was again reweighed (W3 g). The percentage of crude fiber was calculated by using following formula

$$\text{Crude fiber content (\%)} = \frac{\text{Loss in weight on ignition (W2 - W1)} - \text{(W3 - W1)}}{\text{Weight of sample}} \times 100$$

Where, W1= Pre-weighted ashing dish; W2= Ashing dish with dry residue; W3= Ashing dish with ash.

Crude fat

The Crude fat content was determined by following the method of *Sadasivam* and *Manikam* (1992) [302]. Two gram dried sample was taken in a thimble (prepared from Whatman No.41 filter paper) and kept it in the soxhlet apparatus. A dry pre-weighted solvent flask (‘a’ g) was connected beneath the apparatus and to it was added the required volume of solvent (petroleum ether) and then was connected to the condenser.

Then the heating was adjusted to give a condensation rate of 2-3 drops and extracted for 16 h. After the completion of extraction thimble was removed and ether was retained from the apparatus. The excess of ether was evaporate from the solvent flask on a hot water bath and dried the flask. Then it was cooled in desiccator and weighed ('b' g). Crude fat was calculated by using the following formula:

$$\text{Crude Fat content (\%)} = \frac{(b - a)}{\text{Weight of sample}} \times 100$$

Crude Protein

The sample was subjected to micro Kjeldahl method to obtain its nitrogen content. Oven dried 0.5 g root powder was taken in Kjeldahl's flask and to it added10 ml 1:1 H_2SO_4. A pinch of microsalt (200g K2SO4 +5 g dehydrated CuSO4) and a few glass beads were added to accelerate the digestion and to avoid bumping of solution in flask, respectively. Digestion was carried out till a clear solution was obtained. After cooling to room temperature, it was transferred quantitatively to 100 ml volumetric flask and the volume was made with distilled water and then stored overnight at room temperature. Next day, it was filtered through Whatman No.1 filter paper and from the filtrate; nitrogen was estimated following the method by Hawk *et al.* (1948) [33]. For this, the assay mixture contained, 1ml plant extract, a drop of 8% KHSO4 and 15 ml Nessler's reagent (Reagent A-7 g KI + 10 g HgI2 in 40 ml distilled water, Reagent B- 10 g NaOH in 50 ml distilled water. Reagents A and B were mixed in the proportion of 4:5, diluted to 50 ml with distilled water. In place of any extract, distilled water along with other assay mixture served as blank. The absorbance was recorded at 520 nm on a double beam spectrophotometer (Shimadzu UV 190). A standard curve of Ammonium sulphate (0.05 mg N.ml-1) was prepared and the nitrogen content was calculated. The total nitrogen content of the sample was multiplied by factor 6.25 to calculate the crude protein content.

Total nitrogen

Total nitrogen content in fruits was estimated according to the method given by Hawk *et al.* (1948) [33]. Oven dried five hundred milligram powder of fruit samples was taken in Kjeldahl's flask with pinch of microsalt (200g K_2SO_4 +5 g dehydrated CuSO4) and to it 5 ml $H2SO_4$ (1:1) were carefully added. A few glass beads were added to accelerate the digestion and to avoid bumping of solution in flask, respectively. Digestion was carried out till a clear solution was obtained. After cooling to room temperature, it was transferred quantitatively to 100 ml volumetric flask and the volume was made with distilled water and then stored overnight at room temperature. Next day, it was filtered through Whatman No.1 filter paper and used for the estimation of nitrogen. For this, the assay mixture contained, 1ml plant extract, a drop of 8% $KHSO_4$ and 15 ml Nessler's reagent (Reagent A-7 g KI + 10 g HgI_2 in 40 ml distilled water, Reagent B- 10 g NaOH in 50 ml distilled water. Reagent A and B was mixed in the proportion of 4:5), diluted to 50 ml with distilled water. In place of any extract, distilled water along with other assay mixture served as blank. The absorbance was recorded at 520 nm on a double beam spectrophotometer (Shimadzu UV 190). A standard curve of Ammonium sulphate (0.05 mg N.ml-1) was prepared and the nitrogen content was calculated

Estimation of Phosphorus

Phosphorus was estimated from the same acid digest by following the method described by Sekine *et al.* (1965) [34]. Two mls of acid digest were pipett out in a test tube, to which 2 ml of 2 N HNO3 were added followed by 1 ml of Molybdate-Vanadate reagent (Reagent A: 1.25 g ammonium vanadate dissolved in 1 N HNO3 and volume was made to 500 ml with 1 N HNO3. Reagent B: 25 g ammonium molybdate dissolved in distilled water and volume was made to 500 ml. Then reagent A and B were mixed in equal volumes). The volume was made to 10 ml with distilled water. The ingredients were mixed well and allowed to react for 20 minutes. After 20 minutes, yellow colour intensity was measured at 420nm by using a reaction mixture blank containing no phosphorus. The color developed by standards of known concentration of phosphorus in KH_2PO_4 solution (0.110 g KH_2PO_4 per liter = 0.025 mg P.ml-1) with Molybdate - Vanadate reagent was used for plotting the standard curve. With the help of standard curve, the concentration of phosphorus in the plant material was expressed in mg 100 g-1on dry weight basis.

Mineral Analysis

Preparation of acid digests

The acid digestion method of Toth *et al.* (1948) [35] has been followed for the analysis of inorganic constituents. Fresh fruits were washed with water. Blotted to dry and then kept in oven at 60 ^{0}C till a constant weight was obtained. The oven dried plant material was randomly mixed and powdered. Five hundred mg oven dried powder of fruits was transferred to 150 ml clean borosil beaker and to that 10 ml concentrated HNO_3 were added. It was covered with watch glass and kept for an hour till the primary reactions subsided. Then, it was then heated on hot plate till all the material was completely dissolved. It was allowed to cool to room temperature and then 10 ml of Perchloric acid (60%) was added to it and mixed thoroughly. Then, it was then heated strongly on the hot plate until the solution became colorless and reduced to about 2-3 ml. While heating, the solution was not allowed to dry. After cooling, it was transferred quantitatively to 100 ml capacity volumetric flask, diluted to 100 ml with distilled water and kept overnight. Next day the extract was filtered through Whatman No. 44 (Ash less) filter paper. The filtrate was stored properly and used for analysis of inorganic constituents.

The level of Calcium, Magnesium, Iron, Manganese, Zinc, Copper and Cobalt were estimated by using Atomic Absorption Spectrophotometer. In case needed, appropriate dilution of plant extract was made with distilled water. Sodium and Potassium were estimated flame photometrically following the standard method of flame photometer

Results & Discussion

Biochemical and Mineral analysis

The Biochemical compounds like Amino Acids, Proteins, Soluble Sugars, Crude fat and Crude Protein etc. present in the root of *S. samarangense* was studied based on the methods reported previously. Amino acids present in the root were analyzed using ninhydrin reagent method as described by *Moore and Stein* (1954). Folin-phenol reagent method as described by *Khanna et al*., 1969 was adopted for the estimation of Proteins. Anthrone Calorimetric method was used for the estimation of sugars. Traditional methods were for the determination of ash, dry matter, moisture etc. The results of the Biochemical Analysis were given in table 1.

Table 1: Biochemical analysis of root of *S. samarangense*

S. No	Parameter	Amount present* (% w/w)
1	Amino Acids	4.25±0.28
2	Proteins	2.69±0.17
3	Soluble Sugars	1.11±0.09
4	Crude fiber	0.41±0.12
5	Crude fat	3.36±0.23
6	Crude Protein	5.81±0.36
7	Total nitrogen	2.71±0.14
8	Dry matter	82.17±1.20
9	Moisture	4.25±0.31
10	Ash	6.49±0.26
11	Acid insoluble ash	0.37±0.17

* Results are expressed as Mean± Standard deviation of three replicate measurements

Elements and metals present in the roots were extracted using acid digestion method and the level of Calcium, Magnesium, Iron, Manganese, Zinc, Copper and Cobalt were estimated by using Atomic Absorption Spectrophotometer. Sodium and Potassium were estimated flame photometrically following the standard method of flame photometer. The results of the Mineral Analysis were given in table 2.

Table 2: Mineral Analysis results of root of *S. samarangense*

S. No	Element	Amount present*
1	Sodium	9.86±0.71
2	Potassium	8.14±0.53
3	Calcium	3.29±0.29
4	Magnesium	0.52±0.21
5	Iron	0.69±0.32
6	Manganese	0.11±0.03
7	Zinc	0.10±0.02
8	Copper	0.09±0.01
9	Cobalt	BDL
10	Cadmium	BDL
11	Mercury	BDL
12	Silver	BDL

BDL: Below detectable levels

* Results are expressed as Mean± Standard deviation of three replicate measurements

Conclusion

In the present study, biochemical and mineral analysis of root powder was evaluated and biochemical compounds like amino acids, proteins, soluble sugars, crude fat and crude protein etc. were present. Elements and metals present in the roots were extracted using acid digestion method and the level of calcium, magnesium, iron, manganese, zinc, copper and cobalt were estimated by using Atomic Absorption Spectrophotometer. Sodium and Potassium were estimated flame photometrically following the standard method of flame photometer. Sodium, potassium were found in high quantity.

References

1. Madhavi M, Raghu Ram M. Phytochemical screening and evaluation of biological activity of root extracts of *Syzygium samarangense*. International journal of research in pharmacy and chemistry. 2015; 5(4):753-763.
2. Whitman, William F. Five Decades with Tropical Fruit, a Personal Journey. Stuart, Florida: Quisqualis Books in cooperation with Fairchild Tropical Garden, 2001.
3. "*Syzygium samarangense*, *Syzygium javanicum*, *Eugenia javanica*". toptropicals.com. N.d. Web. 9 Dec, 2014.
4. "*Syzygium samarangense*." worldagroforestrycenter.org. Web. 1 Mar. 2015.
5. Whitman, William F. CSIR. The Wealth of India: Raw materials. Vol X Sp-W. CSIR, 1976.
6. Jensen M. Trees commonly cultivated in South-East Asia: An illustrated field guide. RAP publication: 1995/38.Bangkok, Thailand, 1995, 229pp.
7. Martin FW, Campbell CW, Ruberte RM. Perennial edible fruits of tropics: an inventory. US Department of Agriculture, Agriculture Handbook No. 642. 1987, pp 252.
8. Okuda TT, Yoshida, Hatamo T, Yazaki K, Ashida M. Ellagitannins of the Casuarinaceae, Stachyuraceae and Myrtaceae. Phytochemistry. 1982; 21(12):2871-2874.
9. Panggabean G. Syzygium aqueum (Burm.f.) Alst., *Syzygium malaccense* (L.) M. & P, and *Syzygium samarangense* (Blume) M. & P. In Coronel, R.E., et al. (Eds.): PROSEA. No. 2: Edible fruits and nuts. Prosea Foundation, Bogor, Indonesia, 1992, pp. 292-294.
10. Walter A, Sam C. Fruits of Oceania. ACIAR Monograph No. 85. Canberra, 2002, 329 pp.
11. Al-Saif AM, Sharif H, Rosna MT, Moneruzzaman KM. African Journal of Agricultural Research. 2011; 6(15):3623.
12. Ghayur MN, Gilani AH, Khan A, Amor EC, Villaseñor IM, Choudhary MI. Phytother Res. 2006; 20(1):49.
13. Reynertson KA, Basile MJ, Kennelly EJ. Ethnobotany Research & Applications. 2005, 3, 25.
14. Whistler AW, Arthur W. Species Profiles for Pacific Island., Agroforestrywww.traditionaltree.org, 2010.
15. Amor EC, Villaseñor IM, Nawaz SA, Hussain MS, Choudhar I. A dihydrochalcone from *Syzygium samarangense* with anticholinesterase activity. Philippine Journal of Science. 2005; 134(2):105.
16. Kuo YC, Yang LM, Lin LC. Isolation and immunomodulatory effect of flavonoids from *Syzygium samarangense*. Planta medica. 2004; 70(12):1237-1239.
17. Amor EC, Villaseñor IM, Antemano R, Perveen Z, Concepcion GP, Choudhary MI. Cytotoxic C-Methylated Chalcones from *Syzygium samarangense*. Pharmaceutical biology. 2007; 45(10):777-783.
18. Resurreccion-Magno MHC, Villaseñor IM, Harada N, Monde K. Antihyperglycaemic flavonoids from *Syzygium samarangense* (Blume) merr. And perry. Phytotherapy Research: An International Journal Devoted to Pharmacological and Toxicological Evaluation of Natural Product Derivatives. 2005; 19(3):246-251.
19. Ratnam KV, Raju RV. In vitro antimicrobial screening of the fruit extracts of two *Syzygium* species (Myrtaceae). Advances in Biological Research. 2008; 2(1-2):17-20.
20. Simirgiotis MJ, Adachi S, To S, Yang H, Reynertson KA, Basile MJ, Kennelly EJ. Cytotoxic chalcones and antioxidants from the fruits of *Syzygium samarangense* (Wax Jambu). Food chemistry. 2008; 107(2):813-819.
21. Soubir T. Antioxidant activities of some local bangladeshi fruits (*Artocarpus heterophyllus*, *Annona squamosa*, *Terminalia bellirica*, *Syzygium samarangense*, *Averrhoa carambola* and *Olea europa*). Chinese journal of biotechnology. 2007; 23(2):257-261.
22. Reynertson KA, Yang H, Jiang B, Basile MJ, Kennelly EJ. Quantitative analysis of antiradical phenolic constituents from fourteen edible Myrtaceae fruits. Food chemistry. 2008; 109(4):883-890.
23. Evangeline CA, Irene MV, Amsha Y, Iqbal C. Prolyl Endopeptidase Inhibitors from *Syzygium samarangense* (Blume) Merr. & L. M. Perry. Z. Naturforsch. 2004; 59c:86.

24. Nair AG, Krishnan S, Ravikrishna C, Madhusudanan KP. Fitoterapia, 1999, 70, 148.
25. Boland FE. Trimethylammine nitrogen in seadfood method colorimetric. Journal - Association of Official Analytical Chemists, 1971, 54, 9.
26. Moore S, Stein WH. A modified ninhydrin reagent for the photometric determination of amino acids and treated compounds. Journal of Biological Chemistry. 1954; 211:907-913.
27. Lowry OH, Rosebrough NJ, Farr AL, Randal RJ. Protein measurement with folin - phenol reagent, Journal of Biological Chemistry. 1951; 193:266-275.
28. Khanna SK, Mattoo RL, Viswanatha~i PN, Tewari CP, Sanwall GG. Colorimetric determination of prote~n and orthophosphates in plant tissues rich in phenolics. Journal of Biological Chemistry. 1969; 6:21-25.
29. Mc Cready RM, Guddolz J, Silviera V, Ovwens HS. Determination of starch and amylase in vegetables. Analytical Chemistry. 1950; 22:1156-1158.
30. Trevelyan WE, Forrest RS, Harrison JS. Determination of Yeast Carbohydrates with the Anthrone Reagent, Nature. 1952; 170:626-627.
31. AOAC. Official Methods of Analysis. Association of Official Analytical Chemists, Washington. DC, 1990.
32. Sathishkumar T, Baskar R, Shanmugam S, Rajasekaran P, Sadasivam S, Manikandan V. Optimization of flavonoids extraction from the leaves of *Tabernaemontana heyneana* Wall. Using L16 Orthogonal design, Nature and Science. 2008; 6(3):10-21.
33. Hawk PB, Oser BL, Summerson WH. Practical physiological chemistry (Publ.). The Blockiston Co. USA., 1948.
34. Sekine T, Sasakawa T, Morita S, Kimura T, Kuratom K. (1965). cf. labrotory manual for physiological studies of Rice (Eds.) Yoshida, S., Forno, D., Cook, J.B. and Gomez, K.A. Pub. International Rice Research institute, Manila, India, 1972.
35. Toth SJ, Prince AL, Wallace A, Mikkenlsen DS. Rapid quantitative determination of eight mineral elements in plant tissue systematic procedure involving use of a flame photometer. Soil Science. 1948; 66:459-466.

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Advanced Collaborative Framework Model to Improve Software Process Improvement

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Abstract: Improvement of software products with continuous development of software process is an emerging concept in real time application development. To release efficient software product into outside environment, maintain software process improvement is crucial task in terms of distributed implementation of software development in product release into outside environment. To achieve these directions in software development, traditionally, introduce enhanced fuzzy approach to explore client requirement based software services. After took special research to study software process improvement on different multinational enterprise applications in distributed software product implemented strategy. So that, we introduce Collaborative Software Process Improved Framework (CSPIF), it is extension of enhanced fuzzy oriented approach to support continuous software development. Our proposed approach sub sequentially support to software process improvement [SPI] framework. Basic ideas related proposed approach is to define different grow thing opinions of technical applications present in software process improvement. Our experimental evaluation gives best and definitive procedure to improve software quality assurance with relative similarities.

Keywords: Software Requests, Software Process Improvement, Software Services, Software Engineering, Control of Process, Distributed Software Development.

I. INTRODUCTION

Enhancing quality in broad daylight area is some of the time hazardous as a result of the particular attribute conditions. Attributes are characterized in [1,2] with need of different solidly situated in different choices, culture, numerous partners for some procedures. Different information technologies (IT) to be possibly enhance the nature of legislative administrations, existing procedures ought to be enhanced heretofore [3]. Mechanization rehearses in legislative foundations have not given the normal proficiency enhancements in Turkey, since the mechanization of procedures are done with existing procedure surrenders [4]. As described in [5;6], Architecture related to Enterprises (EA) in the general population area must be changed from IT-driven to business-driven. However just a couple of papers manage the fundamental implementations in different business administration space [7]. Open associations are not profit-oriented what's more, encounter unrestricted request while their procedures are oftentimes unstructured, and rely upon worker judgment. In this way, low profitability and process execution, high deformity rates, worker and subject disappointment and high expenses emerge. There are different very much acknowledged non specific Software Process Improvements (Capability Maturity Model Integration for Development) show [9]. These models are utilized as an evaluative and near reason for process change as well as appraisal, expecting that higher procedure ability or authoritative development is related with better execution. Because of watched advantages of these models, numerous procedure ability models are created in view of different approaches and so on.

Software process improvement (SPI) keeps on being the centerpiece of value administration activities attempted by programming firms to create items with high caliber in a practical manner. SPI alludes to the arrangement of moves made to enhance the abilities of a product association. Research related to SPI in software application development. Based on this motivated work available to increase the quality of software products, there are two basic gaps in software process improvement traditionally; firstly, we implement SPI procedure deal with chocolate software development in real time environment. In that coordination and administration are most challenging tasks to manage different social network activities related with required technical aspects to develop software product implementation. Secondly, analyze software product analysis for individual development of different real time organizations. And it is not possible to access different scenarios to improve flexibility and predictability in distributed environment.

In this document we present SPI writing by researching the plan, usage, and evaluation of a conveyed improvement process system at a main undertaking programming item advancement firm (named as Super soft in this paper for classification).

Our research was done as activity inquires about in a joint effort with Super soft specialists. Activity examine utilizes mediation into hazardous social circumstances as a way to progress logical information and has increased wide acknowledgment among explore researchers. In this manner, a collective exertion was attempted to build up a procedure development structure that incorporated both the institutionalization and customization parts of advancement forms with an expanded interest from Super-soft product offering administrators and designers. This brought about a three-advance (multi-layered) developmental process development system that had the upfront investment of key administrators and engineers. The principal activity look into cycle was finished with the portrayal of how the recently created process development system was executed for the ventures embraced by eight improvement groups also, hence assessed. Main contribution of this developed work is as follows

1. We address the process improvement in distributed software product development with ream work progression for all products.
2. We define fundamental characteristics of distributed environment for different software oriented services relates to collaboration, readiness, technology and relation of software applications.
3. We define key process areas for custom development to increase the control of software product improvements.
4. We define interrelation between levels of project implementation process to border level procedure to satisfy the preferable capabilities to governor software development.

II. IMPROVEMENT OF SOFTWARE PROCESS

Software process covers an extensive different type of exercises, from essential task administration teaches, for example, venture arranging and following to complex constant change of improvement forms. Noteworthy driver represents worldview has been the world's biggest customer & maker of programming, the U.S. Defense. Looked with expanded dependence on programming providers, the Defense set up software improvement to direct programming creating associations towards practices. One trademark that recognizes software improvement from prior change standards is that endeavors quite often are started by an evaluation of present technique implements. The object is to discover where upgrades are required most & can be connected to the best impact. In the change cycle, such an evaluation is rehashed each 12 to year and a half. Another trademark is the utilization of a hidden approach. The most powerful and prevalent of these models is the CMM, which was produced at SEI. In CMM, more elevated amounts demonstrate higher development of the association's capacity to create programming. Each stage is described by an arrangement of key process regions (e.g., venture administration, arrangement administration and software quality assurance) that an association should rehearse satisfactorily at each stage. The appraisal will portray the stage of development and prescribe different procedures to make strides. Implemented procedures typically identify inside the model. The outcomes from the evaluation are utilized to produce a technique for enhancing a few or the greater part of the regions distinguished in the appraisal. The system regularly has the specified time allotment of 12 to year and a half and it includes shaping a few SPI groups—one for every change zone. With some coordination among the groups, and association of whatever remains of the association, the enhancements will be guided and, if observed to be sufficient for the association, executed and organized all through the association. The encounters from this whole exertion are dissected and taken after by another change cycle. This general way to deal with SPI is all around portrayed in the IDEAL model created at SEI because of issues experienced by associations engaged with software improvement. IDEAL approach gives a repeating procedure to SPI that is portrayed in five stages.

1. Starting the SPI exertion. This includes defining objectives, acquiring responsibility, and setting up a change foundation.
2. Current approaches through a development appraisal. This commonly depends on a development demonstrate, which is utilized to describe the present state and create and organize proposals for enhancements.
3. Setting up particular, centered change activities. A SPI group is set up to manage each of the suggested change zones from stage 2.
4. Showcasing these activities. The SPI groups create and execute answers for every change zone.
5. Learning in view of the outcomes and encounters from the activities. Information on the upgrades are gathered and arrangements are made for another development evaluation.

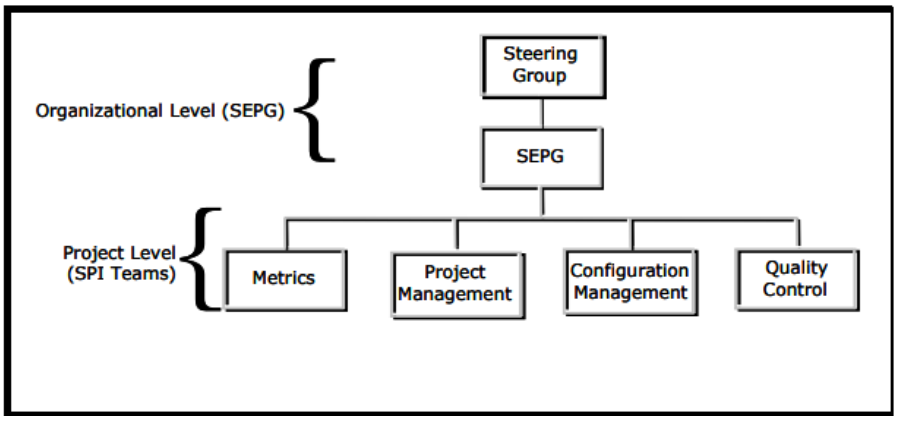

Figure 1. Software process improvement with respect to different project levels.

In many associations, SPI endeavors subsequently are sorted out 2 levels (Grady et.al 1997; McFeeley et.al 1996; Zahran et.al 1998): The hierarchical step with the product building process gathering (SEPG) (Fowler et.al and Rifkin et.al 1990) & particular venture level with devoted SPI groups. At hierarchical level, SPI is composed as a long haul exertion went for transformative changes to the development of the product association. The fundamental duties of the SEPG incorporate directing development appraisals, sorting out and planning SPI groups, and guaranteeing administration association supports with software improvement. At the venture level, the software improvement groups are accused of completing exercises to enhance a particular procedure inside the association, as appeared in Figure 1.

III. BACKGROUND WORK

There are distinctive CSPIF models show up in industry and also in the scholarly world. We alluded to some of them; some of the prominent condition of workmanship has been talked about in the accompanying area:

The V-Model

V-approach model [5] embraced the conventional programming improvement model to combine a framework from reusability programming parts. The advancement exercises appeared in figure 2. It comprises of a few stages & gives the subtle elements are the outline stages. The primary accentuation of V-Development is part advancement lifecycle. Part improvement of software improvement life cycle various procedure. Choice stage gets contribution from the different framework that generally finds and assesses the appropriate parts to be made into the framework. The V approach model is an adjustment of unbending customary line model demonstrates for measured framework improvement adaptability.

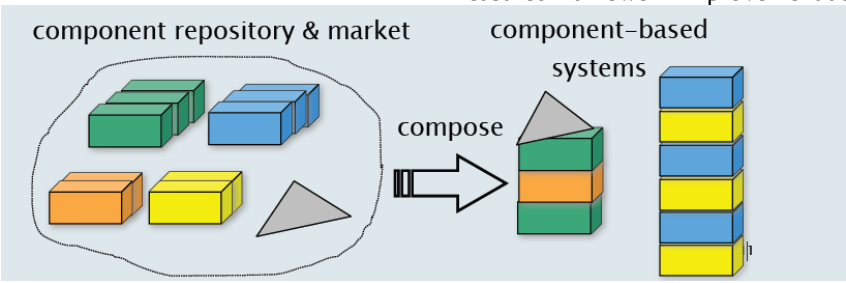

Figure 2. Basic design relates software process improvement

Each stage consists following stages, testing is access related with waterfall model demonstration to define life cycle before doing different strategies before usage of reusability. Life cycle displays waterfall show implementations, based on system framework design is madder. Software advancement is begun with different design models and centers

meetings consists reusability gathering representations.

The X-Model

Tamar & Gill [12] defines X approach in the procedures began in the standard route by necessity building and prerequisite determination as appeared in the below Figure. This product life cycle display for the most part centers on the reusability where software is worked by combined reusability operations for software improvement and programming advancement for reusable and other parameter segments. In programming advancement, there are two noteworthy methodologies, construct nonexclusive programming parts or create programming segment reusability & programming improvement alteration in reusability segment.

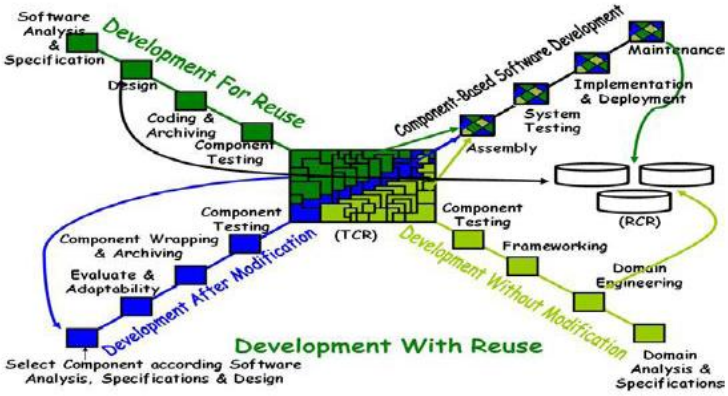

Figure 3. Software quality assurance for X-Data relations

Above diagram shows life cycle with different segments based on software improvements as appeared in figure 3. It consists main cases relate to software advancement which happens in programming improvement. Development of reusability operations, different exchange parameters and segment relations.

IV. COLLABORATIVE FRAMEWORK FOR SPI

From the effects of above sections with conclusion of proposed approach CSPIF have some delimitations and other representations with different software life cycle. It is the part of software advancement development shown in figure 4 with classical model implementation, Reusability operations are excesses with software component development. This implementation consists Undertaking with Feasibility study, Requirements & analysis and basic design and other parameter sequences appeared in figure 4. In detail about the proposed implementation as follows:

System Requirement Specification

The main step to develop client side application, system requirements to analyze or explicit software services. In our presented proposed model, our procedure done to access available services, requirements present in real time software applications.

Once the prerequisites are completely gathered, necessity investigation process begins to distinguish basic necessities of the framework and sub related systems, to recognize & discover conceivable re-using programming models. The significant results of this stage are : detail framework prerequisites, distinguishing proof of the parts that can be reuse on the normal necessities as framework investigators knows about accessible segments in the in-house vault. Primary relations and non-utilitarian necessities of the framework is additionally distinguished in this stage, a framework design demonstrate is taken according to partner's prerequisites and necessities which must be approved and confirmed with the partners for any equivocalness or logical inconsistency while necessities are gathered and reported. Framework prerequisites creation is deteriorated into sub-necessities and hunting down reusable part began to actualize the necessity. This procedure might be rehashed a few times until there is an assertion amongst partners and framework examiner to proceed from next stage in the life cycle.

Framework building administration design must be utilized to record all techniques which characterize framework engineering and programming. Assessment through reviews must be led as often as possible and frequently by a free program association. Procedure of the proposed approach is as shown below:

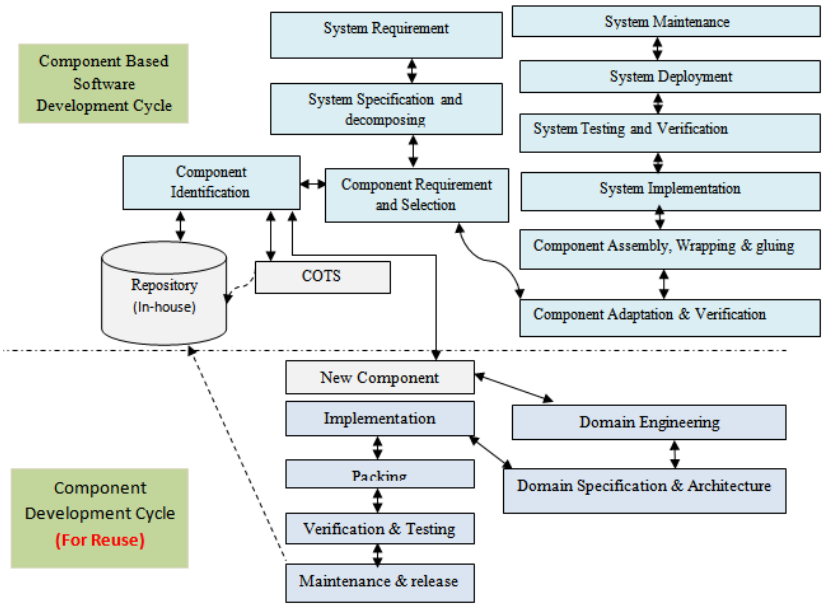

Figure 4. Overview of the proposed approach with different scenarios.

Selection & Requirement of Parts

Main basic frameworks necessities are gathered a framework engineering approaches were outlined in view of the coordinating prerequisite. The product group decides from the framework prerequisites which of the product necessities can be considered to structure as opposed to building them from the scratch. An entire money saving advantage investigation is accessed to define different cost included is embracing the segment like (cycle representations, upkeep costs, refresh necessity cost, authorizing and generated costs). These advantages investigation helps in settling on a choice to reuse a segment or to procure COTS.

On the off chance that the competitor segment isn't accessible in store, different commercial the shelf (COTS) segment is sought with reliable data relations, which is an outsider instrument and should be obtained. In the event that the applicant part isn't accessible either in the inside kept up store or in the market, a choice is being made to build up the scratch according to the prerequisite as appeared in above figure.

Segment Adaptation and Verification

When competitor part is chosen its capacity and wellness in the engineering issues should be tended to, the chosen segment should be fit in the framework engineering outline, an investigation normally being done to know how far they chose segment is perfect with the framework design. Confirmation of the part is being made through the product measurements and money saving advantage examination methods. The product engineering must be versatile, so that, parts can be effectively incorporated into the framework.

Assemble parts, Wrapper Class

Based on application development with respect to segments present in co-related parts or interfaces. Skilled coding parts are controlled with different segments exchanged from useful data and extracted with different attribute relations. In that wrapper class depends on collaborative frame work presentations, mapping relations with portraits interface relations expressed with compressed or accomplished data relations present in source data

Verification Framework Testing

In research development, testing is application and progressed with software quality based on given or predefined representations. In our proposed approach, source code consists unrelated data parts and co-ordinates basic issues utilized in testing scenario. Dynamic testing of different predefined parameters and other segments are co-ordinate difficulties of developed source code based on application programming interface statistics present in client side with different testing implementations with basic relations.

V. EXPERIMENTAL EVALUATION

Our assessment of the moves made amid execution assessment comprised of two phases. In the main stage, we broke down if the issues recognized amid the finding step were settled after the activities of

exploratory assessment. We say that, reconstruction of software quality assurance is circulated with different advanced groups, and different architects and other special individual's process controlled by conveyed and combined with different groups. These definitions are altogether based on quality service present in developed re-active applications based on independent data relations. They led month to month survey sessions with the circulated item advancement groups, and furthermore upheld venture organization as far as giving help with venture exertion and size estimation, monitoring venture advance, and sanctioning procedure control. The information gathered and confirmed by the quality architects were passed on to the focal SEPG storehouse and added to the improvement of benchmarks particular to the dispersed advancement ventures.

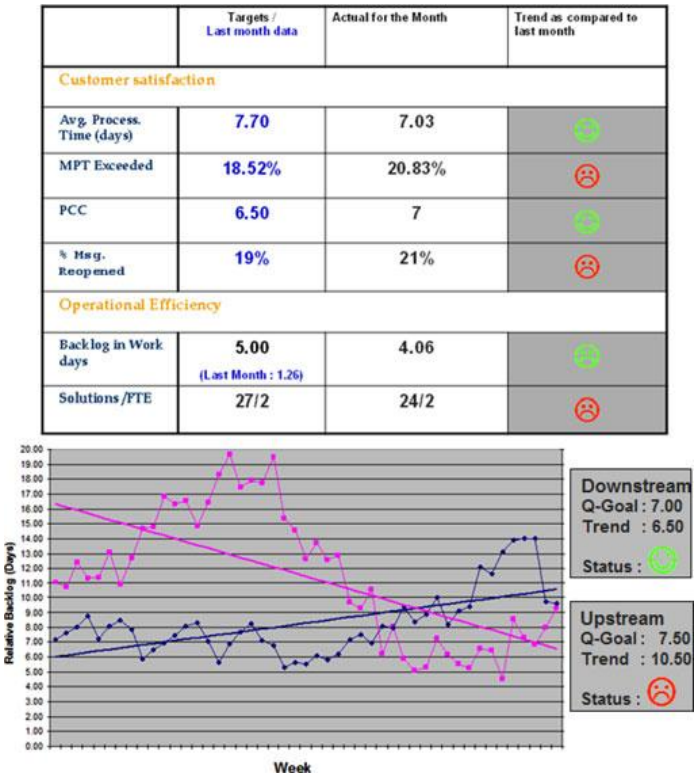

Figure 5. Example of extensive results of the proposed approach.

Our appraisal of the undertaking control modes utilized as a part of the ventures after the reception of the extended procedure development structure showed that the principle upgrades were reflected in the expanded utilization of quantitative and target information to quantify and assess results. Undertaking chiefs could promptly get to quantitative information at different assigned exercises in the task and extract patterns after some time. Fig.5 demonstrates an example quality report, showing the expanded utilization of pertinent target quantitative measurements. Along these lines, the requirement for control models and venture administration and organization bolster, to be recognized as an issue region previously, was effectively satisfied through the rebuilding of the quality capacity. The second phase of our assessment included a quantitative evaluation of the effects of the procedure change endeavors attempted by the eight item advancement groups. We evaluated 62 ventures finished by the eight product offering groups utilizing the new procedure development system; two years after the groups had received the structure for their tasks. Experiments from this evaluation were utilized to foresee the execution parameters of the dispersed activities implemented with 8 different groups utilizing factual relapse models. This helped us to comprehend if process change in the circulated groups contributed to better extend execution.

VI. CONCLUSION

In this paper, we extensively improve systematic software improvement with refused collaborative framework successfully based on different application programming procedures developed in our work. Traditional software implemented

applications based on fuzzy oriented applications do not consider or provide software reuse property externally and they were not support to reusable concept with advanced software application. So to achieve software reusability operations in software product implement in real time applications. We present collaborative software framework to support reusability for software components development. The major different between traditional and proposed approach is to separation of software application development with respect to developed components. In this paper, we discuss different software development model to support reusability in software component definition. Further extension of our proposed model is to support quality of service in development software process improvement to develop extensive software applications. So we plan to develop advanced models for software services.

REFERENCES

[1] Narayan Ramasubbu," Governing Software Process Improvements in Globally Distributed Product Development", IEEE TRANSACTIONS ON SOFTWARE ENGINEERING, VOL. 40, NO. 3, MARCH 2014.

[2] Asif Irshad Khan1,*, Noor-ul-Qayyum2, Usman Ali Khan, " An Improved Model for Component Based Software Development", Software Engineering 2012, 2(4): 138-146 DOI: 10.5923/j.se.20120204.07.

[3] Murat güneştaş , "A study on component based software engineering", a Master's thesis in Computer Engineering, Atılım University, JANUARY 2005.

[4] Luiz Fernando Capretz, "Y: A New Component-based software life cycle model", Journal of Computer Science 1 (1): 76-82, 2005, ISSN 1549-3636 © Science Publications, 2005.

[5] K.Kaur;H Singh, "Candidate process models for component based software development", Journal of Software Engineering 4 (1):16-29, Academic Journal Inc, India 2010.

[6] Syed Ahsan Fahmi; Ho-Jin Choi, "Life Cycles for Component-Based Software Development", IEEE 8th International Conference on Computer and Information Technology Workshops 2008.

[7] DSDM, http://www.dsdm.org/dsdm-atern/, June 2013.

[8] N. Ramasubbu, S. Mithas, M.S. Krishnan, and C.F. Kemerer, "Work Dispersion, Process-Based Learning, and Offshore Software Development Performance," MIS Quarterly, vol. 32, no. 2, pp. 437-458, 2008.

[9] M.B. O'Leary and J.N. Cummings, "The Spatial, Temporal and Configurational Characteristics of Geographic Dispersion in Teams," MIS Quarterly, vol. 31, no. 3, pp. 433-452, 2007.

[10] D. Vogel, M. Genuchten, D. Lou, S. Verveen, M. Eekout, and A. Adams, "Exploratory Research on the Role of National and Professional Cultures in a Distributed Learning Project," IEEE Trans. Professional Comm., vol. 44, no. 2, pp. 114-125, 2001.

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The Impact of Feminism on Sujatha Bhatt's Poetry

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Abstract: *The very emergence of women writing in English was of great significance, for their works; Scanty in output as they were, who wrote before and after them. The battle for emancipation was taken over by few educated women themselves who in their effort to communicate to the world their own bitter experiences as women as well as their ideas of social reform turned writers. Women writers struggled to give form and shape to their autobiographical accounts, which attracted publishers both in India and abroad.*

Many women fail to understand the real spirit of feminism. The movement of feminism has many facets. It doesn't mean getting of equal rights which men & women for the recognition of their creative talent. In fact, it is more than that knowing the reality, feminism is not definitely some kind of chauvinistic or aggressive way of living, dressed up like man. A definition like feminism is therefore, a mode of existence in which the woman is free of the dependence syndrome."

Keywords: *Feminism, Renaissance, Reformation, Motivation, Enlightenment, Injustice, Chauvinistic, Male Domination*

1. INTRODUCTION

Each age has its own historical spatiality either in the social life of in the political sphere or in the religious field or in the sphere of art and literature, and reformation in the religious world. But towards the later part of the twentieth century a movement was developing all over the world, called, feminism. In this paper an attempt has been made to expose what is Feminism, Following the definitions and understanding of feminism, Indian trend in its relationship as well as difference with the general concepts of it. Taking into consideration of the general understanding of the concept and the dimensions of the same in Indian situation the conclusion has been drawn.

In fact Literature has always been an effective instrument in the hands of mankind since time immemorial and contributed for the all round development of the human beings and for bringing harmony among different cultures and societies. There are several forms of Literature but Drama, Poetry, Short story and novel occupies the major part of literature. What so ever be the form all literary works can be placed in a particular context. In other words, all literary works are written at a particular time amidst many social changes. So when we study a literary text it would help us to know more about these contexts or when and where it was written and what was happening in the society then. Myths and histories are written about them and a strong sense of national and cultural identity is constructed through writing.

The very emergence of women writing in English was of great significance, for their works; scanty in output as they were, were who wrote before and after them. This claim should be accepted as perfectly justified at least in respect of a few writers like Toru Dutt, Mrs Ghosal, Sarojini, Cornelia and Krupabai Sathianathan and so on.

The battle for emancipation was taken over by a few educated women themselves who, in their effort to communicate to the world their own bitter experiences as women as well as their ideas of social reform, turned writers. Women writers struggled to give form and shape to their autobiographical accounts, which attracted publishers both in India and abroad.

Themes of almost all the 'Poetic-out put' of the early women poets was the Indian woman, the new woman as the writers saw her emerge in the fast changing social milieu.

A careful glance of the poetry of the earliest women writers of Indian poetry in English as well as of the Post-Independence era helps us see a district continuity of trends and tendencies. There is in the

poems of all the women poets, old or new a naked pre-occupation with nostalgia, dream and introspection, frequent turning away from the Zeitgeist and not an infrequent flower and music.

The major motivation of the poetry from its birth had always been a projection of the social situation and social consciousness. The reflection of poetry has always been a social comment of social criticism. Women writers particularly shared the experiences of Indian women in general and transmuted them into fictional form. While reading them it becomes necessary to understand the situation which provides the material and motivation for their fictional writings. The early poetry by Indian women in all Indian languages was an effort to give expression to their agonies and suffering of which they never had any dearth.

The ability to write in English opened new vistas for young Indian women in the latter half of the 19th century. The precocious and sensitive Bengali girl, Toru Dutt, wrote Binaca, or the young Spanish Maiden, pub posthumously in 1878. Binaca moves from innocent girlhood to romantic love. Like Emily Bronte, Toru Dutt attempts an imaginative and powerful projection of young love in their literary exploration.

The women poets seem to be increasingly concerned mainly with women wallowing in the mire of lust with the rich, the powerful and the corrupt. The poems tend to reflect the contingent reality in an artistic way.

Poetry being the most characteristic and powerful form of literacy expression today, has acquired a prestigious position in the Indo – English Literature. As a distinct literacy form, the novel is undoubtedly a recent birth. It is in fact, the latest of literacy forms to be evolved and the most dominant and commonly agreed that the novel is the readiest and most acceptable way of embodying experiences and ideas in the context of our times.

Poetry by women poets constitutes a major segment of the contemporary Indian writing in English. It provides insights, a wealth of understanding a reservoir of meanings and a basis of discussion. Through women poets eyes we can seek it realize the potential of human achievement. In many appraisal of the Indian English literature, an appreciation of the writing of its women is essential.

Many of the Indian women poets focus on women's issues; they have a women's perspective on the world. It involves the writing of the materials and literature that deal, in a direct of implied fashion, women's improvements and their general enlightenment. One of the reasons that women have, in such large number, taken up their pen is because it has allowed them to create their own world. It has allowed them to set the conditions of existence, free from the direct interference of men. Similarly, so many women have taken to reading women's writings because it allows them to a 'safe place' from which they can explore a wide range of experience of the world, from which they can identify of existences. Women's writing has occupied a significant and central place in women's lives. In male-dominated Indian society, woman is still a "second person". By and large, women do not enjoy an equal status with men.

Indians have been writing in English for more than a century now. Contrary to the western notion of retiring, veiled, tradition encrusted, dumb race, some Indian women have proved to be quite vocal. Seen in its totality, literature has several thought provoking facets and the most visual of them is of course 'feminism' Sujatha Bhatts poetry is also of no exception. Sujatha's fabric of poetry women not only with the elements of feminism but also with different outlook and approach.

It is necessary to arrive at a meaningful definition of the term feminism before how the women writer responds to the movement of feminism which has so far been variously defined. Feminism as many people view, does not mean 'femalism or femaleness' – the qualities which all the women possess. It includes personal courage. Feminism is not ' anti-sitaism' in Indian context as some are fond of saying because Sita is often portrayed as symbol of submission.

Sita indeed, must have been very courageous to have been able to resist and fight Ravana for full one year. As to the question of individuality and self-respect, by no means is she lacking them. After uniting here sons with Rama, it is to her mother, the earth Goddess that she goes. Feminism at the same time is not living outside the marriage. Many women fail to understand the real spirit of feminism. The movement of feminism has many facets. It doesn't mean getting of equal rights which men or women for the recognition of their creative talents. In fact, it is more than that. No doubt, women raising their voice against the injustice, after having gone through agony for centuries, it led to

feminism. According to the French models of Feminism, as has been expressed in Beauvoir's 'The Second Sex' and other work, feminism implies grater sexual expression. If we take into account the British models, all feminists slowly become respectable, or reclaimed into the male world order. The American models are more outspoken in which we find an assertion of self-expression.

A woman in the past never gave a full throated expression to her creative talents. Though we see a lot of improvement in the social and conditions of life, and the freedom woman enjoys in writing and in exploring the hither to prohibited regions of experience, women is still suffering from physical and emotional harassment. The pain of woman is that of deep wounded feelings. Sujatha is also no exception to it. Despite, their feeble voice, majority of the poets on the seen can be read, criticism and analysis and they deserve critical attention. Mention may be made of Bhatt Sujatha,Tarapatel, Makarandparanjape,. TabishKhair, Anna Sujathamathaibhibupadhi, SumanJossan, Raja ram and others. This new generation is emerging with renewed spirit and honest efforts. They saw to crave niche for themselves. They write poetry with sincerity and devotion frankness and emotion and that is why an humble sense of intimate and intensity circumscribes their poetry. Moments, events situations, experiences serve as the subject, attar of their poetry. And their use of languages sparkles with directness and simplicity. There is no ironical stance, no castigation, and no complicate mode of device in expressing the smaller realities of life fidelity to experience and honest of expression remain keynote of these poets. Their use of technique is impressionistic if not imagistic.

Even when we talk of this new generation of poets we can divide them into two groups one group has been present for more than two years. This includes poets like raja ram, carmine D.Souja, Bhatta, Sujatha, Tara Patel and Anna SujathaMathai. The other group is relative 'young both in age nad exposure' all these poets irrespective of sex sect and age maintain distinctive features about their generation.

Although this generation is familiar with the works of the older poets, they never seen to imitate or reveal their direct influence. They are definitely moving in their own direction and that is something respectable if not admirable.

Poets like SujathaBatta, Tara patel, Jeet, SujathaMathai and charmanyne use all elementary observations in their poetry. And they are so alert in responding to modern theoretical development such as post colonical studies, feminism, neo-marxusm, psychoanalysis and soon. The feminism of SujathaBatta, Tara patel, SujathaMathai and are among those who practice the modern theoretical development. Something even they are also glib and critics complain that today's poetry is not flourishingas it blooming in the post. It is limited to public reading, poetry workshops and the small presses it is not a genuine observation. We have more poetry than ever before. But it is not being exposed to many. It is not blooming in vacuume. Frankly speaking our trouble is not with poetry, but with poetic perception. The most visible fact is that it is less allusive, less comprehensive and direct. We can't say that there is no good poetry at all. Having seen progress in recent Indo-English poetry Kaki N. Daruwalla viewed "Not that the poetic mills were ever slow in churning away. It was the publishers who could not folerate anything that rhymed of scanned on their.

Painting presses: There is good poetry around. All it needs in publishers"10. They have brought up very good collections of poems in volumes by different newly forwarded. It gives no safe, cautions, unornamented and assured. It gives no aesthetics pleasure or values, no philosophy ideas, no splendor of thought. Previously Indo-Anglian native is ornamental, sublime, over mind. They use language for sublime themes and mysterious ideas. Compares to pious generation of poetry today's poetry is shapeless, inartistic, smeared with rare splendor and delicious of touch. Yet undoubtedly they are worth enjoyable and readable.

For example markarandparanjape's poetry is a fine ramp through matrimonial love. It is a request of thirty love poems between a husband and wife. Irony and a quickly, self-deprecating hanour abounds these poems. There is a passion in many poems which very compelling poems. A fine uneven poet, she mixes her tenses now and then. "Birthday notes" "Basics" and "To be alive" are fine poems which reveal bare facts of life. In "birthday notes" she moves from the present to he, and back to the present. There is no single line without an acknowledge. Among the poems included in "Single women" "to be alive" marks a ponder able utterances in a refined manner. As she acknowledged the 'life-flow' has lost its decency life in this earth unless they found some new reason everyday "to be leave.'.

Tabishkhair's poetry could well bethe 'pick of the pack' the hardly ever falter in his collection, my lord. He has an eye for landscape and gives a fresh angle to the themes he tackles. His line has a

whipcord tension about it and even though it may be unusually long on occasion, it is never 'limp or lucid' and there is finality about Calcutta is a miniature classic. For them Calcutta is already on its death bed

We are sure he expects Calcutta to live on forever, he uses the language awkwardly at times,' as if they feared the figures of the breeze' of mink coat happiness.

Bidhupadhi's "a wound elsewhere" reminds us of worthy predecessors in Philip larkin and A.K.Ramanjun, Padhi's poems deal with everyday, common place things such as listening to rain, a power cut a dead sparrow and a winter after noon, Certain motifs are re current and obsessive. There is no grand statement about life no sense of exuberant in the romantic senses. The romantic urge, as in Larkin, id always under cut. Constantly tempered by the realization of life's fatal flaws, Padh's poems remain subdued and low-keyed. A somber of awareness of decay and death comes as a frequent remainder not just in poems like 'Taking care of the dead", "burial in summer", "among the ruins" and "celebration". The treatment of death, however, is far from uniform. The poem "Rest" for instance, narrates the news paper report of the aged Danish father being frozen alive in snow in the act of providing warmth to his baby daughter the season's "ultimate snow" guarantees the journeys end. But has this death been of any consequence? Doesn't this prosaic event uncovered the perennial truth of sacrificing love.

Bhatt Sujatha's poems are like sparks struck from a whirring flint, they did not add up to a sustained illumination. Many of others poems spring from her personal life what it may be remorse loss or mere loneliness. Her personal utterances are used mark to conceal her true tragic personality. They are individual poems showing great moral insight, pity, concern, sweetness and moral philosophy in her life.

Her poetry is essentially poetry of moods and occasions of colours tone and stance. As a result her poetry becomes a mystic industry rather than anesthetic pleaure. She is not a poet of reformist not sings the brighter side of life. She honestly surveys. On the surface of realities of life illuminative poetry remains us of T.S.Eliot's the waste land. She wishes to make a home happy with faith hope and charity.

Her poetry sometimes questions the fundamental things of human lives. She questions can we go beyond love? A sense of loss, sorrow, a deep human suffering rules her mind off and on. She deals with everyday reality but it is deeply reflective and meditative, seeking to pierce the dazzling on humdrum surface realities to arrive at core of experience.

SujathaBhatta is a skilful and accomplished poet who brings her task great sensitivity an awareness of oth the possibility and limitations of her poetic vocabulary and a performance for sharply defined images and metaphors. There is always the danger that lyric poetry will degenerated into verified sentimentally. Sujatha's small lyrics seldom do and this in itself is no small achievement.

At first reading, she seems to be an illustrator of common truth and a story teller using verse as her medium and appeals to a wider audience. No doubt her poetry is directly in touch with the spoken language of the day. She is not of course merely a reporter of happening situations or events. In fault her poems do much of memory, genetic memory. At a more human universal level her poems form the colour of unresolved emotions type of mar me life prefer of remain quickly into the rockery of reflection. What is Sid in a line or a half line suggests the whole.

Today there is an increasing progress and a heightened awareness of actual Indian experience. Recent poetry is richer in its sense of location and range of subject matter. However modern Indian English poetry has to play an integral part in the modernization which includes urbanization, industrialization, independency, mobility, social change, more easy access to communication in the form of films, television, radios, journals and newspapers, national and international transportation networks, mass education and the resulting paradox that as an in independent national modern, usually westernized world. Then only more than a century and half old history of Indo-Anglia poets gets its total significance. It is time for current day caught in storms of dull standards.

Poets should manage to time their own cynical range and retain their creative fire. The writers of today can no more offered to line in their archaic ivory tower wither tried to respond to the various political, cultural and philosophical crises. To their societies our crises have been different and we will have to evoke our own concepts and theories. It is ultimate challenge of the writer to stay a truth letter inspire of reorganization or want of it to 'sing his own god'.

"Dear K After all our talks about the meaning" of a sari – the colour, the cloth, the style – we used words like 'symbol', 'blessing','curse', 'power', 'personality;. After all our talks about the pros and cons of wearing a saree – 1 find my heart has other memories."1"

Men are expected to express themselves through physical and other acts of heroism and adventure through their forays into the outer world because they are seen as natural heirs to authority just as women are viewed as naturally passive and susceptible to being objects. Clarity, beauty which we recognize the quintessential femine and masculine elements.

Romance in the Indian context is necessarily shrouded in secrecy and guilt. It includes desire, lust and longing but ultimately refuses to grant these emotions any validity unless they are expressed in the context of future conjugality. Thus the experience of marriage becomes central to female sexual discovery and often this proves to be traumatic.

Women's writing has relevance and validity for reasons more than one. Not only does it project the observations situations responses and struggles of half of humanity. It also reflects a consciousness constructed by gender the being which is defined traditionally by frame works of kinship, marriage and procreation. At another level it questions values and structures hitherto considered axiomatic. Women's writing throws up all kinds of queries related to oppression and colonization. It has helped both to build and express the idea of a female "self" and dismantle the concept of the all-inclusive male "1".

Working through radical movements and silent changes through legal and political battles and psychological barriers women are learning to know and discover themselves. "True all women writing is not feminist equally true that even when gender identification cross cultures.

Traditionally the work of women poets has been undervalued due to patriarchal assumptions about the superior worth of make experience. One factor contributing to this prejudice is the fact that most of these women write about the enclosed domestic space and women's perceptions of their experience within it. Consequently it is assumed that their work will automatically rank below the works of male writers who deal with 'weightier' themes. Additionally, Indian women writers in English are victims of a second prejudice that is their regional counterparts. Since proficiency in English is available only to the writers of the intellectual, affluent,' educated classless, a frequent' judgment is made that the writers and their works belong to a high social strata and are cut off from the reality of Indian life. The majority of these novels depict the psychological suffering of the frustrated housewife this subject matter often being considered superficial compared to the depiction of them depressed and the oppressed lives of women of lower classes that we find in regional authors writing in Hindi, Bengali, Malayalam, Urdu, Tamil, Telugu and other native languages.

Sujatha Bhatt is one of the most distinctive and original of Indian poets writing in English. Most of her poems deal with the themes of "Interpersonal relations", "Inter cultural relations" and "The element of feminism". But the element of feminism dealt by her is not so impressive and within the frame work of established fabric of established feminism. Her experiences were limited and so also is her range. At the same time she is not Auto-biographical. She did not recognize her limitations as Kamala Das did. Her strength as well as her weakness as a poet consist in the fact that she is most herself and cannot carried away by any literary influences. If this gives her an individuality and personal meaning. It also leads her to lapses of fact and a General looseness in verse structure. Otherwise in her poetry we have an expression of female sensibility at its best. For example in the poem "This room is part of the NYS Subway system". Expresses her female sensibility in the following lines.

"We sleep in

A room filled with Sylvia Schister's drawings.

A room filled with point dark heads – stern,

Pensive. They take up all the space on the walls.

There is such beauty in the shape of their skulls:

An ideal geometry of saw bones and cheek bones".

Not only Bhatt, for that matter except kamala Das no other Indian women poet aggressively incolved in the job of taking feministic flavors to the readers in their poetry. In fact Kamala Das poetry is unconventional shocking to the Othodoxical people her treatment of sexual love is a kind of rebellion against male dominant world. As a result her poetry is sharp, punching and untraditional. Though looks Unpretentional she sticks to her confessions. Sujatha Bhatt along with other set of contemporary poets are aware of female sensibility in security feelings of women in social fabric as well as domestic setup. Kamala Das attacks with the male dominated world in the following words.

"Then … I wore a shirt and my Brother's trousers cut my hair short and ignored my womanliness. Drest in sarees be girl, be wife, they said. Be embroiderer, be cook, Be a quarreler with sentiments. Fit in. Oh, Belong cried the categorizers.

Sujatha Bhatt expressed the same but in a gentle and meak voice. No doubt she speaks of the common women and her basic need for love and security with inside knowledge. She likes to be a social rebel, sometimes to go against social norms, frustration and disillusionment with her were lot in life. Yet herself expression in her poetry is not Herpetic. Her poetry doesn't reflect the mirror of women life though they suffer under horrors and rare joys of love. In one of her poems "Honey moon" she views,

"I remember your power –

Your distance : triumphant

And the lack of any horizon on your face.

But I always wanted to know, grandmother,

What had you been denied?

What great bitterness was it

That made you decide.

Your twelve year old daughter, my father's sister,

Had to accompany my parents

On their honeymoon?

In fact Indo-English poetry originated from the quest of educated Indians to find a suitable medium for expression. Sujatha Bhatt familiarity with English Education system made her to express ideas encouragingly. As others use she also use English as it was her mother tongue. Being a member of a family with much literary heritage her attempts to projects her views were encouraged. She gives mixual impressions of unhappiness and disinterestedness in life.Happy or unhappy she continuous to live with them. Unable to bare some bitter realities of the world of course caused by male dominant, society, she wants to live in an imaginary world. " In my mothers way of wearing a saree", she opens windows to real and imagine land and cityscapes. Though primarily she is associated with India past and present remains 'a necessary obsession'. She takes her readers to various parts of the world introduces new science, language, history not because she is familiar with them but wants to get a kind of relief she tells,

"The part I like best, the part I ask her

To show me again and again is the way she

Makes the pleats – snapping them into existence –

As the neem tree in our garden starts snapping at

The elegant bluish eloquent encalyptus leaves".

She always imagines a dreamy land and finds place in that imaginary land In the beginning the main task of the Indo-Anglian poet is to present Indian life and environment in their varied colours without loosing the spirit of English meter. Toru Dutt Sarojini Naidu followed this native sensibility mastered the English language expressed their impressions, emotions and beliefs. At the same time they never fail to grasp the beauty of language and attaining the ingredients of poetry. But Bhatt Sujatha and other emerging new poets failed in this direction. They are very conscious bold and mystic. Their ideas endowed with beyond the physical elements. As a result they use to attain the renaissance in the

social fabric attitude but also a ray of spiritual growth. They are philosophical, symbolic, imagery, imaginative, transcendental, conscious and so on. They expected a lot from the universe itself instead of waiting for somebody to change the world. Sarojini Naidu states.

"Children ye have not lived to you it seems life is a lovely stalactite of dreams or carnival of careless joys that leap about your hearts like billows on the deep in flames of amber and of amethyst. Children ye have not lived, ye but exist till some resistless hour shall rise and move your hearts to wake and hunger after love and third with passionate longing for the things that burn your brows with blood re sufferings. Till ye have battled with great grief and fears and borne the conflict of dreams – shattering years wounded with fierce desire and worn with strike childred ye have not lived for this is life'.

2. Conclusion

Sujatha Bhatt is not impulsive towards the male dominated world. She is thinkitive and reflective but not impulsive. She uses symbols, images to explore the cry of 'women problems' at the same time she did not give any brand to her claim. She is neither specific in women problems nor in general in her attitude. She is evident from every bit of her poetry. In fact this is the plus as well minus in her poetry as well as in the emerging set of poets.

References

[1] M.K.Naik, 'The achievement of Indo-English poetry' 'Perspectives on Indian poetry in English, ed

[2] KekiN.Daruwalla, ed 'Two decades pf Indian poetry' 1960-80 (Delhi: Vikas pub) p:xvii

[3] M.K.Naik,op.cit. pp 219-220

[4] Bijay Kumar Das, "Aspects of modern Indian poetry in English' The Indian journal of English studies. (New series vol. x.x.1980: new series vol.1) p.39

[5] D.Premapathi, 'contemporary Indian poetry' "Cracks in the walls" ed. P.V.Dhanija (New delhi. Prestigr books 1989) p.16

[6] P.v.dhamija,op.cit,p.18,

[7] Adil jassuwalla, 'The New Poetry', Reading in commonwealth Literature. Ed.William Walsh (Oxford University pre: Ely house,1973). P.39

[8] Bijay Kumar Das "aspects of modern Indian poetry in English" JIWE, vol, xx.1980,p.39

[9] K.R.Srinicas I year 'The New Poets' 'Indian writing in English' (Sterling pvt. Ltd, 1972) p.653.

[10] Keki N.Daruwala, 'Poetry Review good, Better and verse' Recent publications of Indian poetry in English, Sunday, May4, 1991, p.95

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Multiculturalism in Indian English Literature

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Abstract: *India has several sophisticated literary languages. The languages have long standing literary traditions. English is assumed as the main literary language as it has been given the status of "official language" by Indian Constitution. Multiculturalism in Indian literature is different from other post-colonial literatures in English. The main objective of this paper is to probe the role of Multiculturalism in Indian English Literature.*

An Indian is inevitably bicultural and lives within a bilingual and multilingual cultural idiom. He is born with a skill to switch his cultural-code according to the needs of his social situation. He even lives within an organic and native social context. The traditional terms used in Indian languages that institutionalised these diverse pulls were "Marga" and "Desi".

In India it is not possible to have a truly autonomous Indian English style. Metophorically speaking the body of modern Indian culture is draped in ancient shoes, a medieval gown and a modern hat. Multiculturalism should bring a new strength to a literature.

Keywords: *Multiculturalism, Intimate Enemy, Elite Culture, Middle Class Culture, Marga, Desi, Tripartite Relationship, Four Predictable Styles, Parampara, Openness, Electrism*

1. INTRODUCTION

Role of English in Indian literature is a curious cultural phenomenon, but English is not an Indian language. It is the language of England, Ireland, Australia, Canada, and the USA. India has several sophisticated literary languages. The languages have long –standing literary traditions. Many students of other Countries think that India has no literature other than that written in English. It is assumed English is the main literary language in India because it has been given the status of "official language" by the Indian Constitution. In reality, Indian English literature is the newest developed branch of Indian literature. For an Indian writing in English involves a conflict between the mother tongue and English, this other tongue which has been our intimate enemy during the last two centuries.

English was introduced as a medium of instruction into the Indian education system in 1835 by Lord Macaulay. The British rules of of India saw it as an intellectual tool to be used to civilize the natives. When English literature was included in school and University Courses, India already had a long and rich tradition of literature at the juncture of this encounter was being written in more than a dozen living languages, but since in English literature was produced in the country of the rulers, and was praised by them, it immediately acquired a high cultural status in India. Indian culture has been eclectic in the extreme. It has had a long tradition of receiving foreign influences. The English term culture is not sufficiently large to cover the semantic scope of the cultural situation in India. The term pervasive traits of a class or a section in a given society. Terms like Elite culture or Middle – class culture are efficient and adequate to describe the true cultural character groups.

Speaking of multicultural context with reference to Indian literature is different in some important ways from speaking of it in the context of other post-colonial literatures in English. In India there has been such a complex inter relationship between various social, linguistic, racial and religious cultures, and these distinguishable sub cultures are so numerous, that it is virtually impossible to fit them in a common formula of sociological linguistic or ethnic cultural structure. We should not compare simple multicultural contexts and complex multicultural contexts. It is rather to compare complex and confusingly complicated contexts. Such complications are inevitable in a society that speaks nearly 80 languages, writes it's literature in nearly 20 different languages, has a history of large-scale social

migrations for every 3000 years. Indian literature in English is India's youngest literary tradition with a very short history of about 100 years. Multiculturalism on the other hand is the oldest and the most durable feature of the Indian way of life.

An Indian, however is inevitably bicultural and lives within a bilingual or a multilingual cultural idiom. He is born with a skill to switch his cultural-code according to the needs of his social situation. And even he lives within an organic and native social context, with many mainstream cultural traditions and several sub stream cultural currents. The traditional terms used in Indian languages that institutionalised these diverse pulls were Marga and Desi. The Marga being the cultures pervading the entire subcontinent horizontally and the Desi, being the vertical, local, fast-changing features of one's cultural identity. Traditionally, the social mobility brought about by economic upheavals, religious convensions and linguistic affinity. Perhaps it is possible to perceive this complicated social fragmentation and formation in terms of a tripartite relationship in the marga traditions the alien traditions and the desi traditions. The alien traditions in India imitated in part the Marga traditions and in part Desi traditions. These were at once stylised, formalised and given a dominating place in Indian culture. And at the same time, used pragmatically and in close correspondence with life as it was like the traditions of the desi types. English literature produced in India has acquired this double identity, at once stylised and pragmatic.

Therefore there are some problems for Indian literature in English which are unique in the entire gamut of commonwealth writings. The first is a split in its personality. The mainstream being considered inherently superior. By imitating both, the alien traditions in India develop a compulsion to be a pan-Indian as well as true to the local nuances and the colour. The styles of Indian literature in English show clear signs of this compulsion. For example, even after writing for more than half a century both R.K. Narayan and Mulk Raj Anand still have to keep translating their regional concepts and terms into neutral English. The same problem is faced by a dialect writer in England or by a South American writer. The compulsion to write two-tonal, text-neutral and coloured does not come merely from the fact of the Indian writer's being bilingual/ bicultural. V.S. Naipaul and Salman Rushdie, the non-typical Indian writers are bicultural too, but they have evolved a complete stlye after escaping the cultural split that a typical Indian writer has to suffer. It is because of this split that excessive stress has to be given on irony and humour, myth and rhetoric and verbal features of characters.

The English language in India is treated as a superior language in comparision with Indian languages. The Indian writer likes to claim it as his own 'Indian' language, at the same time he writes for the readership belonging to that social class. With this split, the Indian English style can barely become a discourse. The other problem of a serious nature by conflicting cultural demands on English literature in India is related to growth. The Indian nation has accepted the English language as one of its two national languages, the other being Hindi. But whereas Hindi is a regional language and has a specific geographical base but English does not. The result is that it is used, on the one hand, as a language of status and on the other as a static language. Because of an international language it is called as status language and not being an Indian language it is more learnt than spoken in India, so it is a static language. It enjoys as an enormously resourceful language that it has been elsewhere. The inevitable consequences of this attitude, in order to keep in with the international status of the language most writers assume a learned ironic tone at the same time; to rid it of its stasis, burden it with excessive local colour. A good example of these attitudes is" Nissim Eziekel's poems ", in which ambivalence pervades the reproduction of somewhat comical but truly Indian use of English. The other example is Aurobindo's epic "Savitri" written in Miltonic diction replete with references to the stylized filklore of the ancient past. Thus, the English language in India is legitimated as a national language but treated only as an international language and used for conveying the local colour. It carries the burden of the tripartite relation between the marga, the alien, and the desi, like other cultural institutions in India.

The tripartite cultural relationship can be seen in a large number of writers particularly those given to self-examination. For example R. Partha Sarathy, whose poem. "Rough Passage" is an eloquent of this relationship. The poem is divided into three sections, Exile, Trail and Homecoming. The first section shows him in an alien British setting stunned with the discovery of being an Indian. The idiom of the poem and the quality of emotion in this section one comparable to the western standard of alienation. The second section is about poet's journey, and his relationships, the final sections is about

his personal growth, through philosophic disillusionments. The poet brings together the western, the Marga and the Desi expectations to bear upon the subject. The other major poems in Indian English literature- Sri Aurobindo's "Savitri" and Jayanta Mahapatra's "Relationship- from this perspective". Thus, English as a medium of creative writing in India spreads the writers canvas too wide but dilutes his colours rather too much. R.K.Narayan's "The painter of signs" or "The Talkative man" seem to portray the prototype of the Indian writing in English.

The entire history of Indian English literature of various patterns of conflict and collaboration between the western, the Marga and the Desi cultural traditions, such a fate seems to have been inscribed in the being of Indian English literature from the moment of its birth. Briefly refering to the context of its birth Indian writers had acquired the skill of writing in alien languages much before the English language arrived in India. Indian writers wrote using Arabic or Percian languages of alien extraction. Amir Khosrau of 13th century, and the sanskrit poet Jagannath pandit were bilingual, to mention just two of the numerous avilable examples, who wrote in languages other than their own. When English literature was introduced into Indian educational system there was no senseof outrage in contemporary Indian intellectuals. Thinkers like Raja Ram Mohan Roy, some Britsh ideologists inspired by Sir William Jones demanded education through the established medium of Arabic and Sanskrit. The debate now known as the Anglicist and the orientalist debate. But there was a conflict between the western and the Marga cultures. There were already well established literary traditions in these languages such as Gujarathi, Tamil, nad Marathi. These were the Desi traditions. The final outcome of the debate was that the British administration in India decided to incorporate all the three components in the education system. Even today the official language policy of the Indian Government is known as the three language policy. Because of the consistency of this policy from 1835 to this day, Indian writings in English remains confined to all three cultural strands, and works simultanously within them all.

The multicultural context of Indian literature in English produces the following four predictable styles of writing:

- Style in which internationalism and nationalism meet and collaborate, where the alien cultural features work together with the marga cultural feature. In fiction the ideal example is Raja Rao, who combines an International sort of existentialism with national sort of sentimentality.
- Style in which nationalism or the Marga cultural features merge with the local or the discultural features. In fiction, a good example is Prathap Sarma's novel on Punjab, "The day of the Turban ".
- Styles in which the International cultural features conflict with the national or the marga features were irony and sentimentality constantly thwart each other. Mulk Raj Anand's usual style belongs to this type.
- Style in which the national and the regional, the marga and the desi cultural features are at cross purposes. R.K.Narayan at times seems to be facing this problem.

In poetry, A.K.Ramanujan, Jayantha Mahapatra, Sri Aurobindo and R. Partha Sarathy exemplify the four types respectively. Following this line of argument, one could maintain that in India it is not possible to have a truly autonomous Indian English style. The style varieties of Indian English literature are linked to the multicultural context of it. The tension between the three cultural streams in contemporary India could have become vigorously creative. The alien culture embodied in the English language is of recent origin and has a short history of a century or so. The desi culture emerged in India around the beginning of 20th century and have acquired and retained the sociological dents of all those centuries. The marga traditions emerged even earlier, and many are as old as Indian civilisation itself. Thus the three cultures originate in an agrarian, a feudal and an industrialised society, respectively. Metaphorically speaking the body of modern Indian culture is draped in ancient shoes, a medieval gown and a modern hat. The problem is further complicated because the Indian sesibility invests disproportionate value in what is ancient even when it is unusable, and also in what is young even when it is not fully mature. Thus sanskrit and the culture based in it, as well as English and the culture surrounding it are treated with reverence. It is in fact the Indian sense of tradition as Parampara that is at play behind such attitudes to language and culture. Indian history has as similaited languages well – preserved in India's cultural memory. And Indians learn foreigner's

language very quickly. But these learned languages are always maintained as essentially foreign and imperfectly assimilated. The literature produced in India is traditionalised at a slow pace.

Indian literature in English also has the context of colonialism in its origin. Colonialism had two major consequences for Indian culture, one was an upsurge of a revivalist tendency, the other was the growth of a desire to imitate the British. The revavalistic tendency fanned by the Indological admiration for the ancient Indian culture created excessive respect for the remote Indian past. The desire to imitate it created undue sense of awe for English language. Socialogists describe these two tedencies with the terms 'sanskritisation' and 'westernisation'. These two trends have been institutionalised in modern Indian culture and literature. The implications of such a process for a growing literary tradition are the serious aberrations, are created in its self-awareness, and the growth of its own tradition becomes tortuous.

2. Conclusion

Literature becomes a great literature when it develops from a language that is rooted in the soil that grows organically from people's experiences piled together for generations. The English language was grafted onto India's linguistic banyan tree. It has been a successful grafting only to a limited extent, for the language has still not taken firm root in the soil. As a result the literature produced in it has to operate within a severely limited social space. Hence, Multiculturalism has not been a sufficiently fruitful context for its survival and growth.

Having outlined the specific cultural pressures on Indian English literature, it is necessary however, to state that the role that it can play lies within the marga tradition, as a national literature of India rather than as a regional literature of any desi tradition. Whether we like it or not the fact remains that at the moment it is the only national literature in India. And for depicting the marga side of modern Indian life it is the only vehicle. Its growth will depend on its realisation of this role, on its quickness to rid itself of its multicultural context.

References

[1] M. Rajagopalachary , K. DamodarRao, Multiculturalism in Indian tradition and literature.

[2] Lakshmi, Vijay " Of Immigrant's writing " Dislocation and Multiculturalisms, Ed. Jasbir Jain, (New Delhi : Rawat Publication, 2004) P:52

[3] Ashok Bhaskar "Multiculturalism in Indian fiction in English".

[4] Luniano, W.(1996), " Like being Mugged by a Metaphor : Multiculturalism and state Narratives". In Mapping Multicularalism, eds. A.F. Gortdon and C. New field. Minneapolis: University of Minnesota Press.

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महादेवी वर्मा-साहित्यमसोसकरन

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महादेवी वर्मा (२६ मार्च १९०७ – ११ सितंबर १९८७) हिन्दी की सर्वाधिक प्रतिभावान कवयित्रियों में से हैं। वे हिन्दी साहित्य में छायावादी युग के चार प्रमुख स्तंभों[क] में से एक मानी जाती हैं। आधुनिक हिन्दी की सबसे सशक्त कवयित्रियों में से एक होने के कारण उन्हें *आधुनिक मीरा* के नाम से भी जाना जाता है। कवि निराला ने उन्हें "हिन्दी के विशाल मन्दिर की सरस्वती" भी कहा है।[ख] महादेवी ने स्वतंत्रता के पहले का भारत भी देखा और उसके बाद का भी। वे उन कवियों में से एक हैं जिन्होंने व्यापक समाज में काम करते हुए भारत के भीतर विद्यमान हाहाकार, रुदन को देखा, परखा और करुण होकर अन्धकार को दूर करने वाली दृष्टि देने की कोशिश की। न केवल उनका काव्य बल्कि उनके सामाजसुधार के कार्य और महिलाओं के प्रति चेतना भावना भी इस दृष्टि से प्रभावित रहे। उन्होंने मन की पीड़ा को इतने स्नेह और शृंगार से सजाया कि *दीपशिखा* में वह जन-जन की पीड़ा के रूप में स्थापित हुई और उसने केवल पाठकों को ही नहीं समीक्षकों को भी गहराई तक प्रभावित किया।

उन्होंने खड़ी बोली हिन्दी की कविता में उस कोमल शब्दावली का विकास किया जो अभी तक केवल बृजभाषामें ही संभव मानी जाती थी। इसके लिए उन्होंने अपने समय के अनुकूल संस्कृत और बांग्ला के कोमल शब्दों को चुनकर हिन्दी का जामा पहनाया। संगीत की जानकार होने के कारण उनके गीतों का नाद-सौंदर्य और पैनी उक्तियों की व्यंजना शैली अन्यत्र दुर्लभ है। उन्होंने अध्यापन से अपने कार्यजीवन की शुरूआत की और अंतिम समय तक वे प्रयाग महिला विद्यापीठ की प्रधानाचार्या बनी रहीं। उनका बाल-विवाह हुआ परंतु उन्होंने अविवाहित की भांति जीवन-यापन किया। प्रतिभावान कवयित्री और गद्य लेखिका महादेवी वर्मा साहित्य और संगीत में निपुण होने के साथ-साथ कुशल चित्रकार और सृजनात्मक अनुवादक भी थीं। उन्हें हिन्दी साहित्य के सभी महत्त्वपूर्ण पुरस्कार प्राप्त करने का गौरव प्राप्त है। भारत के साहित्य आकाश में महादेवी वर्मा का नाम ध्रुव तारे की भांति प्रकाशमान है। गत शताब्दी की सर्वाधिक लोकप्रिय महिला साहित्यकार के रूप में वे जीवन भर पूजनीय बनी रहीं। वर्ष २००७ उनकी जन्म शताब्दी के रूप में मनाया गया।२७ अप्रैल १९८२ को भारतीय साहित्य में अतुलनीय योगदान के लिए ज्ञानपीठ पुरस्कार से इन्हें सम्मानित किया गया था। गूगल ने इस दिवस की याद में वर्ष २०१८ में गूगल डूडल के माध्यम से मनाया।

जन्म और परिवार

महादेवी का जन्म २६ मार्च १९०७ को प्रातः ८ बजे[7]फ़र्रुख़ाबाद उत्तर प्रदेश, भारत में हुआ। उनके परिवार में लगभग २०० वर्षों या सात पीढ़ियों के बाद पहली बार पुत्री का जन्म हुआ था। अतः बाबा बाबू बाँके विहारी जी हर्ष से झूम उठे और इन्हें घर की देवी – महादेवी मानते हुए पुत्री का नाम महादेवी रखा। उनके पिता श्री गोविंद प्रसाद

वर्मा भागलपुर के एक कॉलेज में प्राध्यापक थे। उनकी माता का नाम हेमरानी देवी था। हेमरानी देवी बड़ी धर्म परायण, कर्मनिष्ठ, भावुक एवं शाकाहारी महिला थीं। विवाह के समय अपने साथ सिंहासनासीन भगवान की मूर्ति भी लायी थीं वे प्रतिदिन कई घंटे पूजा-पाठ तथा रामायण, गीता एवं विनय पत्रिका का पारायण करती थीं और संगीत में भी उनकी अत्यधिक रुचि थी। इसके बिल्कुल विपरीत उनके पिता गोविन्द प्रसाद वर्मा सुन्दर, विद्वान, संगीत प्रेमी, नास्तिक, शिकार करने एवं घूमने के शौकीन, मांसाहारी तथा हँसमुख व्यक्ति थे। महादेवी वर्मा के मानस बंधुओं में सुमित्रानंदन पंत एवं निराला का नाम लिया जा सकता है, जो उनसे जीवन पर्यन्त राखी बँधवाते रहे। निराला जी से उनकी अत्यधिक निकटता थी, [9] उनकी पुष्ट कलाइयों में महादेवी जी लगभग चालीस वर्षों तक राखी बाँधती रहीं।

शिक्षा

महादेवी जी की शिक्षा इंदौर में मिशन स्कूल से प्रारम्भ हुई साथ ही संस्कृत, अंग्रेज़ी, संगीत तथा चित्रकला की शिक्षा अध्यापकों द्वारा घर पर ही दी जाती रही। बीच में विवाह जैसी बाधा पड़ जाने के कारण कुछ दिन शिक्षा स्थगित रही। विवाहोपरान्त महादेवी जी ने १९१९ में क्रास्थवेट कॉलेजइलाहाबाद में प्रवेश लिया और कॉलेज के छात्रावास में रहने लगीं। १९२१ में महादेवी जी ने आठवीं कक्षा में प्रान्त भर में प्रथम स्थान प्राप्त किया। यहीं पर उन्होंने अपने काव्य जीवन की शुरुआत की। वे सात वर्ष की अवस्था से ही कविता लिखने लगी थीं और १९२५ तक जब उन्होंने मैट्रिक की परीक्षा उत्तीर्ण की, वे एक सफल कवयित्री के रूप में प्रसिद्ध हो चुकी थीं। विभिन्न पत्र-पत्रिकाओं में आपकी कविताओं का प्रकाशन होने लगा था। कालेज में सुभद्रा कुमारी चौहानके साथ उनकी घनिष्ठ मित्रता हो गई। सुभद्रा कुमारी चौहान महादेवी जी का हाथ पकड़ कर सखियों के बीच में ले जाती और कहतीं—"सुनो, ये कविता भी लिखती हैं"। १९३२ में जब उन्होंने इलाहाबाद विश्वविद्यालय से संस्कृत में एम॰ए॰ पास किया तब तक उनके दो कविता संग्रह *नीहार* तथा *रश्मि*प्रकाशित हो चुके थे।

वैवाहिक जीवन

सन् १९१६ में उनके बाबा श्री बाँके विहारी ने इनका विवाह बरेली के पास नबाव गंज कस्बे के निवासी श्री स्वरूप नारायण वर्मा से कर दिया, जो उस समय दसवीं कक्षा के विद्यार्थी थे। श्री वर्मा इण्टर करके लखनऊ मेडिकल कॉलेजमें बोर्डिंग हाउस में रहने लगे। महादेवी जी उस समय क्रास्थवेट कॉलेज इलाहाबाद के छात्रावास में थीं। श्रीमती महादेवी वर्मा को विवाहित जीवन से विरक्ति थी। कारण कुछ भी रहा हो पर श्री स्वरूप नारायण वर्मा से कोई वैमनस्य नहीं था। सामान्य स्त्री-पुरुष के रूप में उनके सम्बंध मधुर ही रहे। दोनों में कभी-कभी पत्राचार भी होता था। यदा-कदा श्री वर्मा इलाहाबाद में उनसे मिलने भी आते थे। श्री वर्मा ने महादेवी जी के कहने पर भी दूसरा विवाह नहीं किया। महादेवी जी का जीवन तो एक संन्यासिनी का जीवन था ही। उन्होंने जीवन भर श्वेत वस्त्र पहना, तख्त पर सोईं और कभी शीशा नहीं देखा। सन् १९६६ में पति की मृत्यु के बाद वे स्थाई रूप से इलाहाबाद में रहने लगीं।

कार्यक्षेत्र

महादेवी का कार्यक्षेत्र लेखन, संपादन और अध्यापन रहा। उन्होंने इलाहाबाद में प्रयाग महिला विद्यापीठ के विकास में महत्वपूर्ण योगदान किया। यह कार्य अपने समय में महिला-शिक्षा के क्षेत्र में क्रांतिकारी कदम था। इसकी वे प्रधानाचार्य एवं कुलपति भी रहीं। १९३२ में उन्होंने महिलाओं की प्रमुख पत्रिका 'चाँद' का कार्यभार संभाला। १९३० में नीहार, १९३२ में रश्मि, १९३४ में नीरजा, तथा १९३६ में सांध्यगीत नामक उनके चार कविता संग्रह प्रकाशित हुए। १९३९ में इन चारों काव्य संग्रहों को उनकी कलाकृतियों के साथ वृहदाकार में *यामा* शीर्षक से प्रकाशित किया गया। उन्होंने गद्य, काव्य, शिक्षा और चित्रकला सभी क्षेत्रों में नए आयाम स्थापित किये। इसके अतिरिक्त उनकी 18 काव्य और गद्य कृतियां हैं जिनमें *मेरा परिवार, स्मृति की रेखाएं, पथ के साथी, शृंखला की कड़ियाँ* और अतीत के चलचित्र प्रमुख हैं। सन १९५५ में महादेवी जी ने इलाहाबाद में साहित्यकार संसद की स्थापना की और पं इलाचंद्र जोशी के सहयोग से साहित्यकार का संपादन संभाला। यह इस संस्था का मुखपत्र था। उन्होंने भारत में महिला कवि सम्मेलनों की नीव रखी। इस प्रकार का पहला अखिल भारतवर्षीय कवि सम्मेलन १५ अप्रैल १९३३ को सुभद्रा कुमारी चौहान की अध्यक्षता में प्रयाग महिला विद्यापीठ में संपन्न हुआ। वे हिंदी साहित्य में रहस्यवाद की प्रवर्तिका भी मानी जाती हैं।महादेवी बौद्ध धर्म से बहुत प्रभावित थीं। महात्मा गांधी के प्रभाव से उन्होंने जनसेवा का व्रत लेकर झूसी में कार्य किया और भारतीय स्वतंत्रता संग्राम में भी हिस्सा लिया। १९३६ में नैनीताल से २५ किलोमीटर दूर रामगढ़ कसबे के उमागढ़ नामक गाँव में महादेवी वर्मा ने एक बँगला बनवाया था। जिसका नाम उन्होंने मीरा मंदिर रखा था। जितने दिन वे यहाँ रहीं इस छोटे से गाँव की शिक्षा और विकास के लिए काम करती रहीं। विशेष रूप से महिलाओं की शिक्षा और उनकी आर्थिक आत्मनिर्भरता के लिए उन्होंने बहुत काम किया। आजकल इस बंगले को महादेवी साहित्य संग्रहालय के नाम से जाना जाता है। शृंखला की कड़ियाँ में स्त्रियों की मुक्ति और विकास के लिए उन्होंने जिस साहस व दृढ़ता से आवाज़ उठाई हैं और जिस प्रकार सामाजिक रूढ़ियों की निंदा की है उससे उन्हें महिला मुक्तिवादी भी कहा गया। महिलाओं व शिक्षा के विकास के कार्यों और जनसेवा के कारण उन्हें समाज-सुधारक भी कहा गया है। उनके संपूर्ण गद्य साहित्य में पीड़ा या वेदना के कहीं दर्शन नहीं होते बल्कि अदम्य रचनात्मक रोष समाज में बदलाव की अदम्य आकांक्षा और विकास के प्रति सहज लगाव परिलक्षित होता है।

उन्होंने अपने जीवन का अधिकांश समय उत्तर प्रदेश के इलाहाबाद नगर में बिताया। ११ सितंबर १९८७ को इलाहाबाद में रात ९ बजकर ३० मिनट पर उनका देहांत हो गया।

समालोचनाठ

आधुनिक गीत काव्य में महादेवी जी का स्थान सर्वोपरि है। उनकी कविता में प्रेम की पीर और भावों की तीव्रता वर्तमान होने के कारण भाव, भाषा और संगीत की जैसी त्रिवेणी उनके गीतों में प्रवाहित होती है वैसी अन्यत्र दुर्लभ है। महादेवी के गीतों की वेदना, प्रणयानुभूति, करुणा और रहस्यवाद काव्यानुरागियों को आकर्षित करते हैं। पर इन रचनाओं की विरोधी आलोचनाएँ सामान्य पाठक को दिग्भ्रमित करती हैं। आलोचकों का एक वर्ग वह है, जो यह मानकर चलते हैं कि महादेवी का काव्य नितान्त वैयक्तिक है। उनकी पीड़ा, वेदना, करुणा, कृत्रिम और बनावटी है।

- आचार्य रामचंद्र शुक्ल जैसे मूर्धन्य आलोचकों ने उनकी वेदना और अनुभूतियों की सच्चाई पर प्रश्न चिह्न लगाया है –[घ] दूसरी ओर
- आचार्य हज़ारी प्रसाद द्विवेदी जैसे समीक्षक उनके काव्य को समष्टि परक मानते हैं।
- शोमेर ने 'दीप' (नीहार), मधुर मधुर मेरे दीपक जल (नीरजा) और मोम सा तन गल चुका है कविताओं को उद्धृत करते हुए निष्कर्ष निकाला है कि ये कविताएं महादेवी के 'आत्मभक्षी दीप' अभिप्राय को ही व्याख्यायित नहीं करतीं बल्कि उनकी कविता की सामान्य मुद्रा और बुनावट का प्रतिनिधि रूप भी मानी जा सकती हैं।
- सत्यप्रकाश मिश्र छायावाद से संबंधित उनकी शास्त्र मीमांसा के विषय में कहते हैं — "महादेवी ने वैदुष्य युक्त तार्किकता और उदाहरणों के द्वारा छायावाद और रहस्यवाद के वस्तु शिल्प की पूर्ववर्ती काव्य से भिन्नता तथा विशिष्टता ही नहीं बतायी, यह भी बताया कि वह किन अर्थों में मानवीय संवेदन के बदलाव और अभिव्यक्ति के नयेपन का काव्य है। उन्होंने किसी पर भाव साम्य, भावोपहरण आदि का आरोप नहीं लगाया केवल छायावाद के स्वभाव, चरित्र, स्वरूप और विशिष्टता का वर्णन किया।
- प्रभाकर श्रोत्रिय जैसे मनीषी का मानना है कि जो लोग उन्हें पीड़ा और निराशा की कवयित्री मानते हैं वे यह नहीं जानते कि उस पीड़ा में कितनी आग है जो जीवन के सत्य को उजागर करती है।

यह सच है कि महादेवी का काव्य संसार छायावाद की परिधि में आता है, पर उनके काव्य को उनके युग से एकदम असम्पृक्त करके देखना, उनके साथ अन्याय करना होगा। महादेवी एक सजग रचनाकार हैं। बंगाल के अकाल के समय १९४३ में इन्होंने एक काव्य संकलन प्रकाशित किया था और बंगाल से सम्बंधित "बंग भू शत वंदना" नामक कविता भी लिखी थी। इसी प्रकार चीन के आक्रमण के प्रतिवाद में हिमालय नामक काव्य संग्रह का संपादन किया था। यह संकलन उनके युगबोध का प्रमाण है।

गद्य साहित्य के क्षेत्र में भी उन्होंने कम काम नहीं किया। उनका आलोचना साहित्य उनके काव्य की भांति ही महत्वपूर्ण है। उनके संस्मरण भारतीय जीवन के संस्मरण चित्र हैं।

उन्होंने चित्रकला का काम अधिक नहीं किया फिर भी जलरंगों में 'वॉश' शैली से बनाए गए उनके चित्र धुंधले रंगों और लयपूर्ण रेखाओं का कारण कला के सुंदर नमूने समझे जाते हैं। उन्होंने रेखाचित्र भी बनाए हैं। दाहिनी ओर करीन शोमर की क़िताब के मुखपृष्ठ पर महादेवी द्वारा बनाया गया रेखाचित्र ही रखा गया है। उनके अपने कविता संग्रहों यामा और दीपशिखा में उनके रंगीन चित्रों और रेखांकनों को देखा जा सकता है।

पुरस्कार व सम्मान

उन्हें प्रशासनिक, अर्धप्रशासनिक और व्यक्तिगत सभी संस्थाओं से पुरस्कार व सम्मान मिले।

- १९४३ में उन्हें 'मंगलाप्रसाद पारितोषिक' एवं 'भारत भारती' पुरस्कार से सम्मानित किया गया। स्वाधीनता प्राप्ति के बाद १९५२ में वे उत्तर प्रदेश विधान परिषद की सदस्या मनोनीत की गयीं। १९५६ में भारत सरकार ने उनकी साहित्यिक सेवा के लिये 'पद्म भूषण' की उपाधि दी। १९७९ में साहित्य अकादमी की सदस्यता ग्रहण करने वाली वे पहली महिला थीं। 1988 में उन्हें मरणोपरांत भारत सरकार की पद्म विभूषण उपाधि से सम्मानित किया गया।
- सन १९६९ में विक्रम विश्वविद्यालय, १९७७ में कुमाऊं विश्वविद्यालय, नैनीताल, १९८० में दिल्ली विश्वविद्यालयतथा १९८४ में बनारस हिंदू विश्वविद्यालय, वाराणसी ने उन्हें डी.लिट की उपाधि से सम्मानित किया।
- इससे पूर्व महादेवी वर्मा को 'नीरजा' के लिये १९३४ में 'सक्सेरिया पुरस्कार', १९४२ में 'स्मृति की रेखाएँ' के लिये 'द्विवेदी पदक' प्राप्त हुए। 'यामा' नामक काव्य संकलन के लिये उन्हें भारत का सर्वोच्च साहित्यिक सम्मान 'ज्ञानपीठ पुरस्कार' प्राप्त हुआ। वे भारत की ५० सबसे यशस्वी महिलाओं में भी शामिल हैं।
- १९६८ में सुप्रसिद्ध भारतीय फ़िल्मकार मृणाल सेन ने उनके संस्मरण 'वह चीनी भाई' पर एक बांग्ला फ़िल्म का निर्माण किया था जिसका नाम था नील आकाशेर नीचे।
- १६ सितंबर १९९१ को भारत सरकार के डाकतार विभाग ने जयशंकर प्रसाद के साथ उनके सम्मान में २ रुपए का एक युगल टिकट भी जारी किया है।

महादेवी वर्मा का योगदान

साहित्य में महादेवी वर्मा का आविर्भाव उस समय हुआ जब खड़ीबोली का आकार परिष्कृत हो रहा था। उन्होंने हिन्दी कविता को बृजभाषा की कोमलता दी, छंदों के नये दौर को गीतों का भंडार दिया और भारतीय दर्शन को वेदना की हार्दिक स्वीकृति दी। इस प्रकार उन्होंने भाषा साहित्य और दर्शन तीनों क्षेत्रों में ऐसा महत्त्वपूर्ण काम किया जिसने आनेवाली एक पूरी पीढ़ी को प्रभावित किया। शचीरानी गुर्टू ने भी उनकी कविता को सुसज्जित भाषा का अनुपम उदाहरण माना है। उन्होंने अपने गीतों की रचना शैली और भाषा में अनोखी लय और सरलता भरी है, साथ ही प्रतीकों और बिंबों का ऐसा सुंदर और स्वाभाविक प्रयोग किया है जो पाठक के मन में चित्र सा खींच देता है।[ज] छायावादी काव्य की समृद्धि में उनका योगदान अत्यंत महत्त्वपूर्ण है। छायावादी काव्य को जहाँ प्रसाद ने प्रकृतितत्त्व दिया, निराला ने उसमें मुक्तछंद की अवतारणा की और पंत ने उसे सुकोमल कला प्रदान की वहाँ छायावाद के कलेवर में प्राण-प्रतिष्ठा करने का गौरव महादेवी जी को ही प्राप्त है। भावात्मकता एवं अनुभूति की गहनता उनके काव्य की सर्वाधिक प्रमुख विशेषता है। हृदय की सूक्ष्मातिसूक्ष्म भाव-हिलोरों का ऐसा सजीव और मूर्त अभिव्यंजन ही छायावादी कवियों में उन्हें 'महादेवी' बनाता है। वे हिन्दी बोलने वालों में अपने भाषणों के लिए सम्मान के साथ याद की जाती हैं। उनके भाषण जन सामान्य के प्रति संवेदना और सच्चाई के प्रति दृढ़ता से परिपूर्ण होते थे। वे दिल्ली

में १९८३ में आयोजित तीसरे विश्व हिन्दी सम्मेलन के समापन समारोह की मुख्य अतिथि थीं। इस अवसर पर दिये गये उनके भाषण में उनके इस गुण को देखा जा सकता है।

यद्यपि महादेवी ने कोई उपन्यास, कहानी या नाटक नहीं लिखा तो भी उनके लेख, निबंध, रेखाचित्र, संस्मरण, भूमिकाओं और ललित निबंधों में जो गद्य लिखा है वह श्रेष्ठतम गद्य का उत्कृष्ट उदाहरण है।[झ] उसमें जीवन का संपूर्ण वैविध्य समाया है। बिना कल्पना और काव्यरूपों का सहारा लिए कोई रचनाकार गद्य में कितना कुछ अर्जित कर सकता है, यह महादेवी को पढ़कर ही जाना जा सकता है। उनके गद्य में वैचारिक परिपक्वता इतनी है कि वह आज भी प्रासंगिक है।[ञ] समाज सुधार और नारी स्वतंत्रता से संबंधित उनके विचारों में दृढ़ता और विकास का अनुपम सामंजस्य मिलता है। सामाजिक जीवन की गहरी परतों को छूने वाली इतनी तीव्र दृष्टि, नारी जीवन के वैषम्य और शोषण को तीखेपन से आंकने वाली इतनी जागरूक प्रतिभा और निम्न वर्ग के निरीह, साधनहीन प्राणियों के अनूठे चित्र उन्होंने ही पहली बार हिंदी साहित्य को दिये।

मौलिक रचनाकार के अलावा उनका एक रूप सृजनात्मक अनुवादक का भी है जिसके दर्शन उनकी अनुवाद-कृत 'सप्तपर्णा' (१९६०) में होते हैं। अपनी सांस्कृतिक चेतना के सहारे उन्होंने वेद, रामायण, थेरगाथा तथा अश्वघोष, कालिदास, भवभूति एवं जयदेव की कृतियों से तादात्म्य स्थापित करके ३९ चयनित महत्वपूर्ण अंशों का हिन्दी काव्यानुवाद इस कृति में प्रस्तुत किया है। आरंभ में ६१ पृष्ठीय 'अपनी बात' में उन्होंने भारतीय मनीषा और साहित्य की इस अमूल्य धरोहर के सम्बंध में गहन शोधपूर्ण विमर्ष किया है जो केवल स्त्री-लेखन को ही नहीं हिंदी के समग्र चिंतनपरक और ललित लेखन को समृद्ध करता है।

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भारत के सुप्रसिद्ध हिन्दी - लेखक श्रीलाल शुक्ल : व्यक्तिव एंव भाषा शैली में व्यग्य रूप ।

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1. जीवन परिचय :

श्रीलाल शुक्ल जी हिन्दी साहित्य के प्रमुख साहित्यका है । श्रीलाल शुक्ल जी का जन्म लखनऊ की मोहनलाल गंज बस्ती के पास अतरैली ग्राम में 31-12-1921 में हुआ था। इनके पिता का नाम ब्रजकिशोर शुक्ल है, उनके परिवार गरीबों का था, पर उनके संस्कार गरीबी के न थे। उनके पिता को संस्कृत, हिन्दी, उर्दू का कामचलाऊ ज्ञान था । उनके जीवन कुछ समय तक खती पर, बाद में श्रीलाल शुक्ल के बडे भाई पर निर्भर रहे। श्रीलाल शुक्लजी की माँ उदारमन और उत्साही थी। प्रसाद और प्रेमचंद की कई पुस्तकें श्रीलाल शुक्ल(8वी) आठवी कक्षआ में ही पढी थी । वे इतिहास साहित्य और शास्त्रीय संगीत के प्रेमी है। उनकी मिडिल स्कूल की शिक्षा मोहनलाल गंज मे, हाईस्कूल की शिक्षा लखनऊ, और इन्टरमीडियट की शिक्षा कान्याकुब्ज कॉलेज, कानुपूर में हुई। और इस के बाद सन् 1945 में इलाहबाद के विश्वविद्यालय में बी.ए में प्रवेश लिय और उस के बाद लखनऊ विश्वविद्यालय में एम.ए. और कानून की शिक्षाओं में प्रवेश लिया था । सन् 1948 में उनकी शादी हुई थी, उनके पत्नी का नाम गिरिजा था। गिरिजा जी बहुत अच्छी श्रोता थी, साहित्य और संगीत की अच्छी जानीकारी थी। उनके तीन पुत्रियाँ और एक पुत्र है। सन् 1949 में उत्तरप्रदेश सिविल में नियुक्ति पाया, बाद में आई.ए.एस में प्रोन्नत, सन् 1983 में.भारतीय सेवानिवृत्ता सरकारी नौकरी में आ जाने के बाद लेखन से वास्ता रहा ।

2. व्यक्तित्व :

श्रीलाल शुक्ल का व्यक्तिव अपनी मिसाल आप था । अनुशासनप्रिय लेकिन अराजक, सहज लेकिन सतर्क, विनोदी लेकिन विदवान, और वे बहुत परिश्रमी, ईमानदार शीघ्र निर्णय लेने वाले आदमी है । वे अंग्रजी, उर्दू, संस्कृत और हिन्दी भाषा के विद्वान थे, औरकाम में उनका प्रैक्टिकल एप्रोच है। वे सबके प्रती समान आदरभाव रखते है। शुक्लजी संगीत के शास्त्रीय और सुगम दोनों पक्षों के रसिक - मर्मज थे । वे कई साहित्य सम्मेलन और कई 'कथाक्रम' समारोह समिति के वह अध्यक्ष रहे।

श्रीलाल शुक्ल जी ने गरीबी झेली, संघर्ष किया,पर उसके विलाप से लेखन को नहीं भरा। उन्हें नई पीढ़ी को भी सबसे अधिक समझने और पढ़नेवाले विरिष्ठ रचनाकारों में से एक रहै। न पढ़ने और लिखने के लिए लोग सैद्दांतिकी बनाते हैं। उनके रचनाओं के आधार पर समझते हैं कि शुक्लजी व्यावहारिक, साथ ही सहानुभूति भी है। श्रीलाल शुक्ल जी का व्यक्तित्व बड़ा ही सहज था, वह हमेशा मुस्कुराकर सबका स्वागत करते थे। लेकिन अपनी बात बिन लाग - लपेट कहते थे। इन के बारे में लीलाधर जगूडी समझाते है कि ''श्रीलाल शुक्ल के लिए अपना समय, अपना समाज और अपने लोग ही महत्वपूर्ण है। एक उपन्यासार के रूप में वे आजादी के बाद के समाजशास्त्री और इतिहासकार है। वे तरह-तरह से आजाती के बाद के समाज की मूल्यहीनता और आधुनिकता के संकट के साथ - साथ राजनीति के हाथों पराजित होते समाज के इतिहास में प्रवेश दिलाते हैं''। व्यक्तित्व की इसी खूबी के चलते उन्होंने सरकारी सेवा में रहते हुए भी व्यवस्थ पर करारी चोट करने वाली ''राग दरबारी'' (1968) जैसी रचना हिन्दी साहित्य को दी ।

3. कार्यक्षेत्र :

श्रीलाल शुक्ल जी उपन्यासकार व व्यंग्यकार के रूप में प्रतिष्ठित, 130 से अधिक पुस्कतों का लेखन किया था। उनका विधिवत लेखन 1954 से शुरू होता है और इसी के साथ गद्य का एक गौरवशाली अध्याय आकार लेने लगता है। उनका पहला प्रकाशित उपन्यास ।। ''सूनी घाटी का सूरज'' 1957 तथा पहला प्राकशित व्यंग ''अंगद का पाँव 1958 है। स्वतंत्रता के बाद के भारत के ग्रमीण जीवन की मूल्यहीनता को परत दर परत उघाड़ने वाले उपन्यास

''राग दरबारी'' 1968 के लिये उन्हें साहित्य अकादमी पुरस्कार से सम्मानित किया गया था। उनके इस उपन्यास पर एक दूरदर्शन - धारावाहिक का निर्माण भी हुआ । श्री लाल शुक्ल का भारत सरकार ने 2008 में. पद्मभूषण पुरस्कार से सम्मानत किया है। ऐसे महान साहित्याक भारत के सुप्रसिद्द हिन्दी लेखक श्रीलाल शुक्ल और उनके उपन्यास, रचनाएँ, निबंध, व्यंग्य निबंध, जीवनी आलोचना आदि के बारे में शोध प्रबंध प्रस्तुतकर और पी.एच.डी उपाधि पाकर एक शोधार्थी के रूप में मेरा जन्म भी सार्थक हुई।

4. भाषा शैलीः

साहित्य के प्रति श्रीलाल शुक्ल जी की रूचि अनेक रचनाएँ लिखने में सफल हुऐ । हास्य, व्यंग्य उपन्यास, अपराध कथाएँ, उनके कहानियाँ, व्यंग्य निबंध, टिप्पणियाँ, जीवनी, आलोचना, अनुवाद में सफलता पाई। इसके साथ ''हिन्दी हास्य-व्यंग्य संकलन'' का सम्पादन भी किये । उनके प्रसिद्द व्यंग्य उपन्यास ''राग दरबारी'' का सभी प्रमुख भारतीय भाषाओं में अनुवाद किया था।

श्रीलाल शुक्ल जी आधुनिक काल के कई भाषाएँ जैसे अवधी, उर्दू, अंग्रजी, हिन्दी, संस्कृत भाषा के विदवान थे। उनका भाषा शैली साहित्य अपनी मिसाल आप था। उन्होंने शिवपालगंज के रूप में अपनी अद्‌भुत भाषा शैली, मिथकीय शिल्प और देशज मुहावरों से गढ़ा था। त्रासदियों और विडंबनाओं के इसी साम्य ने 'राग दरबारी' को महान कृति बनाया, तो इस कृति ने श्रीलाल शुक्ल को महान लेखक । राग दरबारी व्यंग्य है या उपन्यास, यह एक श्रेष्ठ रचना है, जिसकी तसदीक करोड़ो पाठकों ने की है और कर रहे हैं। 'विश्रामपुर का संत', 'सूनी घाटी का सूरज' और 'यह मेरा घर नहीं जैसी कृतियाँ साहित्यिक कसौटियों में खरी साबित हुई हैं। बल्कि 'विश्रामपुर का संत' को स्वतंत्र भारत में सत्ता के खेल की सशक्त अभिव्यक्ति तक कहा गया था। राग दरबारी को इतने वर्षों बाद भी पढ़ते हुए उसके पात्र हमारे आसपास नजर आते हैं। शुक्लजी ने जब इसे लिखा था, तब एक तरह की हताशा चारों तरफ़ नजर आ रही थी। यह मोहभंग का दौर था। ऐसे निराशा भरे महौल में उन्होंने समाज की विसंगतियों को चुटीली शैली में सामने लाया था। वह श्रेष्ठ रचनाकार के साथ ही एक संवेदनशील और विनम्र इंसान भी थे।

अ) सामाजिक व्यंग्य :

श्रीलाल शुक्ल की सूक्ष्म और पैनी दृष्टि व्यवस्था की छोटी-से-छोटी विकृति को भी सहज ही देख लेती है, परख लेती है। उन्होंने अपने लेखन को सिर्फ राजनीति पर ही केंद्रित नहीं होने दिया । शिक्षा के क्षेत्र की दुर्दशा पर भी उन्होंने व्यंग्य कसा। **1963 में प्रकाशित उनकी पहली रचना 'धर्मयुग' शिक्षा के श्रेत्र में** व्याप्त विसंगतियों पर आधारित है। व्यंगय संग्रह 'अंगद का पाँव' और उपन्यास 'राग दरबारी' में श्रीलाल शुक्ल ने इसे विस्तार दिया है।

आ) ग्रामीण परिवेश :

श्रीलाल शुक्ल की रचनाओं का एक बड़ा हिस्सा गाँव के जीवन से संबंध रखता है । ग्रामीण जीवन के व्यापक अनुभव और निरंतर परिवर्तित होते परिदृश्य को उन्होंने बहुत गहराई से विश्लेषित किया है। यह भी कहा जा सकता है कि श्रीलाल शुक्ल ने जड़ों तक जाकर व्यापक रूप से समाज की छान बीन कर, उसकी नब्ज को पकड़ा है। इसीलिए यह ग्रामीण संसार उनके साहित्य में देखने को मिला है। उनके साहित्य की मूल पृष्ठभूमि ग्राम समाज है परंतु नगरीय जीवन की भी सभी छवियाँ उसमें देखने को मिलती हैं। श्रीलाल शुक्ल ने साहित्य और जीवन के प्रति अपनी एक सहज धारणा का उल्लेख करते हुए कहा है कि - कथालेखन में मैं जीवन के कुछ मूलभूत नैतिक मूल्यों से प्रतिबद्ध होते हुए भी यथार्थ के प्रति बहुत आकृष्ट हूँ। पर यथार्थ की यह धारणा इकहरी नहीं है, वह बहुस्तरीय है और उसके सभी स्तर - आध्यात्मिक, आभ्यंतरिक, भौतिक आदि जटिल रूप से अंतर्गुम्फित हैं। उनकी समग्र रूप में पहचान और अनुभूति कहीं-कहीं रचना को जटिल भले ही बनाए, पर उस समग्रता की पकड़ ही रचना को श्रेष्ठता देती है। जैसे मनुष्य एक साथ कई स्तरों पर जीता है, वैसे ही इस समग्रता की पहचान रचना को भी बहुस्तरीयता देती है।

इ) समसामयिक स्थितियों पर व्यंग्य :

श्रीलाल शुक्ल ने अपने साहित्य के माध्यम से समसामयिक स्थितियों पर करारी चोट की है। वे कहते हैं कि - 'आज मानव समाज अपने पतन के लिए खुद जिम्मेदार है। आज वह खुलकर हँस नहीं सकता ।हँसने केलिए भी 'लाफिंग क्लब' का सहारा लेना पड़ता है। शुद्ध हवा के लिए ऑक्सीजन पार्लर जाना पड़ता है। बंद बोतल का पानी पीना पड़ता है। इंस्टेंट फूड़ खाना पड़ता है। खेलने के लिए, एक-दूसने से बात करने केलिए भी वक्त की कमी है।'

कुल मिलाकर यह कह सकते हैं कि श्रीलाल शुक्ल के साहित्य में जीवन संघर्ष है और उनका साहित्य सामाजिक सरोकारों से जुड़ा है।

ई) शिक्षा व्यवस्था पर व्यंग्य :

देश के अनेक साकरी स्कूलों की स्थिति दयनीय है। उनके पास अच्छा बवन तक नहीं है । कहीं तो तंबुओं में कक्षाओं में कक्षाएँ चलाती हैं, अर्थात न्युतम सुविधाओं का भी अभाव है। श्रीलाल शुक्ल इस पर टिप्पणी करते हुए कहते हैं कि -

''यही हाल 'राग तरबारी' के छंगामल विद्यालय इंटरमीडियेट कॉलेज का भी है, जहाँ से इंटरमीडियेट पास करने वाले लड़के सिर्फ इमारत के आधार पर कह सकते है, सैनिटरी फिटिंग किस चिड़िया का नाम है। हमने विलायती तालीम तक देशी परंपरा में पाई में पाई है और इसीलिए हमें देखो, हम आज भी उतने ही प्राकृत हैं, हमारे इतना पढ़ लेने पर भी हमारा पेशाब पेड़ के तने पर ही उतरता है।''

विडंबना यह है कि आज कल अध्यापक भी अध्यपन के अलावा सब कुछ करते हैं।'राग दरबारी' के मास्टर मोतीराम की तरह वे कक्षा में पढ़ाते कम हैं और ज्यादा समय अपनी आटे की चक्की को समर्पित करते हैं। ज्यादातर शिक्षक मोतीराम ही हैं, नाम भले ही कुछ भी हो। ट्यूशन लेते हैं, दुकान चलाते हैं और तरह तरह के निजी धंधे करते हैं। छात्रों को देने केलिए उनके पास समय कहाँ बचता है?

5.उपसंहार :

भारतीय लोकतांत्रिक व्यवस्था में हज़ारों किस्म की विसंगतियों और विडम्बनाओं को क़रीब से देखते रची गई अनेक दुर्लभ कृतिय इन के समकालीन जीवन में सर्वोत्तम उपन्यास, रचनाऐं, व्यंग्य - निबंध और आलोचना, निबंध आदि के हिन्दी सुप्रसिद्द लेखक श्रीलाल शुक्ल जी के बारे में शोध प्रबंध प्रस्तुतकर एम.फिल - पि.एचडि उपाधि पाकर एक शोधायी के रूप में मेरा जन्म भी सार्थक हुई । ज्ञानपीठ पुरस्कार और पद्मभूषण से सम्मानित तथा ''राग दरबारी'' जैसे कालजयी व्यंग्य उपन्यास और बिसामपुर का संत'' समकालीन जीवन की ऐसी महागाथा लिखने वाले मशहूर व्यंग्यकार श्रीलाल शुक्ल को 16, अक्टूबर को पार्किसन बीमारी के कारण उन्हें अस्पताल में भर्ती कराया गया था। 28 अक्टूबर 2011 को शुक्रवार सुबह 11.30 बजे सहारा अस्पताल में श्रीलाल शुक्ल का निधन हो गया। वह 86 वर्ष के थे। उनके निधन पर लखनऊ के साहित्य जगत में शोक की लहर दौड़ गई ।

पुस्तक का नाम और लेखक के नाम :

1. उत्तरप्रदेश साहित्य और संस्कृति का मासिक - सम्पादक-लीलाधार जगूडी
2. कुछ साहित्य चर्चा भी - श्रीलाल शुक्ल
3.व्यंग्य समय - श्रीलाल शुक्ल

समाप्त

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Magneto Hydro Dynamics Convective Flow Past a Vertical Porous Surface in Slip-Flow Regime

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Abstract

This paper deals with the influences of heat and mass transfer on Two dimentional MHD free convection flow, laminar and boundary layer of viscous fluid along a semi vertical permeable moving plate, a uniform transverse magnetic field, thermal and concentration bouyancy effects.The governing nonlinear partial difference equations have been decreased to the coupled nonlinear ordinary differential equations by small perturbation technique. Numerical evalution of the analytical results is performed and some graphical results for the velocity, temperature and concentration profiles with in the boundary layer.

Keywords: MHD, Heat Transfer, Mass Transfer, Slip-flow, vertical plate.

1. INTRODUCTION

Combined heat and mass transfer from different processes with porous media has a wide range Engineering and Industrial applications such as enhanced oil recovery, underground energy transport, geothermal reservoirs, cooling of nuclear reactors, drying of porous solids, packed-bed catalytic reactors and thermal insulation. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the

fluid is the working medium. The applications are often found in situation the such as fiber and granules insulation, geothermal systems in the heating and cooling chamber, fossil fuel combustion, energy processes and Astro-physical flows. Further, the magneto convection place an important role in the control of mountain iron flow in the steady industrial liquid metal cooling in nuclear reactors and magnetic separation of molecular semi conducting materials. In certain porous media applications such as those involing heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and /or endothermic chemical reactions and dissociationg fluid in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important.

Convection in porous media can be applied to underground ground water hydrology, iron blast furnaces, cooling of nuclear reactors, solar power collectors, energy efficient drying processes, cooling of nuclear fuel in shipping flasks, cooling of electronic equipment's, coal gasification, and wall cooled catalytic reactors, and natural convection in earth's crust. More examinations of the applications related to convective flows in porous media in Nield and Bejan [1] can be found. The fundamental problem of flow through porous media has been investigated extensively [2, 3]. A survey of Magneto Hydro Dynamics revises in the technological spheres in Moreau [6] can be found. When heat and mass transfer occur instantaneously between the fluxes, the driving potentials are of more intricate nature. The uses of magnetic field to control the flow and heat transfer processes in fluids near different types of boundaries are familiar now. It has led to significant interest in the study of boundary layer. Hossain [5] analysed on MHD free convective heat transfer for a Newtonian fluid. The natural convection flows adjacent to both vertical and horizontal surface, which result from the combined buoyancy effects of thermal and mass diffusion, was first investigated by Gebhart et al. [7] and Pera et al. [8]. While Soundalgekar [9] investigated the situation of unsteady free convective flows wherein the effects of viscous dissipation on the flow past an infinite vertical porous plate was highlighted. In the course of analysis, it was assumed that the plate temperature oscillates in such a way that its amplitude is small. Later, Chen et al [10] studied the combined effect of buoyancy forces from thermal and mass diffusion on forced convection.

The present analysis discussed here in, is based on the study as referred and suggested by Soundalgekar [11]. Under the assumptions made by Sharma et al. [12] have also discussed the free convection flow past a vertical plate in slip-flow regime. They had quoted several applications that occur in several engineering applications wherein heat and mass transfer occurs at high degree of temperature differences.

In all above presentations, the plate was assumed to be maintained at a constant temperature, which is also the temperature of the surrounding stationary fluid. However, in many applications that occur in industrial situations are not those simple and at high temperatures, quite often the plate temperature starts oscillating about a non-zero mean temperature. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particles at the surface always possess a finite tangential velocity and it "slips" almost along the surface.

Therefore, the flow regime is called the slip - flow regime and such an effect cannot be neglected. Hence, in the fitness of the industrial and scientific applications and to be more realistic the effect of periodic heat and mass transfer on unsteady free convection flow past a vertical flat porous plate under the influence of applied transverse magnetic field has been examined. The slip flow regime it is assumed that the suction velocity oscillates in time about a non-zero constant mean because in actual practice temperature, species concentration and suction velocity may not always be uniform.

2. FORMULATION OF THE PROBLEM

An unsteady free MHD convective flow of a viscous incompressible fluid past an infinite vertical porous flat plate in slip-flow regime, with periodic temperature and concentration when variable suction velocity distribution $\left[V^* = -V_0^*\left(1+\varepsilon A e^{i\omega^* t^*}\right)\right]$ is fluctuating with respect to time is considered. A co-ordinate system is employed with wall lying vertically in $x^* y^*$ – plane. The x^* – axis is taken in vertically upward direction along the vertical porous plate and y^* – axis is taken normal to the plate. Since the plate is considered infinite in the x^* – direction, hence all physical quantities will be independent of x^*. Under these assumption, the physical variables are purely the functions of y^* and t^* only. In the fairness of the realistic situation by neglecting viscous dissipation and then assuming variation of density in the body force term (Boussinesq's approximation) the problem can be governed by the following set of equations:

$$\frac{\partial u^*}{\partial t^*} - V_0^*\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial u^*}{\partial y^*} = g\beta\left(T^* - T_\infty^*\right) + g\beta^0\left(C^* - C_\infty^*\right) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\beta_0^2}{\rho}u^* - \frac{\nu}{K^*}u^* \tag{1}$$

$$\rho C_P\left[\frac{\partial T^*}{\partial t^*} - V_0^*\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial T^*}{\partial y^*}\right] = k\frac{\partial^2 T^*}{\partial y^{*2}} \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} - V_0^*\left(1+\varepsilon A e^{i\omega^* t^*}\right)\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{3}$$

The boundary conditions of the problem are:

$$\left.\begin{aligned} &u^* = B^* e^{i\psi^* t^*},\quad T^* = T_w^* + \varepsilon\left(T_w^* - T_\infty^*\right)e^{i\omega^* t^*},\quad C^* = C_w^* + \varepsilon\left(C_w^* - C_\infty^*\right)e^{i\omega^* t^*} \quad \text{at } y^* = 0 \\ &u^* \to 0,\quad T^* \to T_\infty^*,\quad C^* \to C_\infty^* \quad \text{as } y^* \to \infty \end{aligned}\right\} \tag{4}$$

We now introduce the following non-dimensional quantities into Eqs. (1) to (4)

$$y=\frac{y^*V_0^*}{\nu},\quad t=\frac{t^*V_0^{*2}}{4\nu},\quad u=\frac{u^*}{V_0^*},\quad \omega=\frac{4\nu\omega^*}{V_0^{*2}},\quad \theta=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*},$$

$$C=\frac{C^*-C_\infty^*}{C_w^*-C_\infty^*}.Gr=\frac{g\beta\nu\left(T_w^*-T_\infty^*\right)}{V_0^{*3}},Gc=\frac{g\beta^\circ\nu\left(C_w^*-C_\infty^*\right)}{V_0^{*3}},$$

$$Pr=\frac{\mu C_p}{k}=\frac{\nu\rho C_p}{k},Sc=\frac{\nu}{D},M=\frac{\sigma\beta_0^2\nu}{\rho V_0^{*2}},K=\frac{K^*V_0^{*2}}{\nu^2},B=\frac{B^*}{V_0^*}.$$

The subscript (∞) denotes the free stream condition. Then equations (1) to (3) reduce to the following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial u}{\partial y}=Gr\,\theta+Gc\,C+\frac{\partial^2 u}{\partial y^2}-Mu-\frac{u}{K} \tag{5}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{7}$$

The boundary conditions to the problem in the dimensionless form are:

$$\left.\begin{array}{llll} u=Be^{i\psi t}, & \theta=1+\varepsilon e^{i\omega t}, & C=1+\varepsilon e^{i\omega t} & \text{at } y=0 \\ u\to 0, & \theta\to 0, & C\to 0 & \text{as } y\to\infty \end{array}\right\} \tag{8}$$

3. SOLUTION OF THE PROBLEM

Assuming the small amplitude oscillations (ε << 1), we can represent the velocity u, temperature θ and concentration C near the plate as follows:

$$u(y,t)=u_0(y)+\varepsilon u_1(y)e^{i\omega t} \tag{9}$$

$$\theta(y,t)=\theta_0(y)+\varepsilon\theta_1(y)e^{i\omega t} \tag{10}$$

$$C(y,t)=C_0(y)+\varepsilon C_1(y)e^{i\omega t} \tag{11}$$

Substituting (9) to (11) in (5) to (7), equating the coefficients of harmonic and non harmonic terms, neglecting the coefficients of ε^2, the solutions are given by

$$\theta_0(y)=e^{-\Pr y} \tag{12}$$

$$C_0(y)=e^{-Scy} \tag{13}$$

$$u_0(y) = m_1 e^{-m_2 y} + m_3 e^{-\Pr y} + m_4 e^{-Scy} \tag{14}$$

$$\theta_1(y) = m_5 e^{-m_6 y} + m_7 e^{-\Pr y} \tag{15}$$

$$C_1(y) = m_8 e^{-m_9 y} + m_{10} e^{-Scy} \tag{16}$$

$$u_1(y) = m_{11} e^{-m_{12} y} - m_{13} e^{-m_6 y} - m_{14} e^{-m_9 y} + m_{15} e^{-m_2 y} + m_{16} e^{-\Pr y} + m_{17} e^{-Scy} \tag{17}$$

Using equations (12)-(17), the velocity, temperature and concentration can be obtained as follows:

$$u(y,t) = m_1 e^{-m_2 y} + m_3 e^{-\Pr y} + m_4 e^{-Scy} + \begin{pmatrix} m_{11} e^{-m_{12} y} - m_{13} e^{-m_6 y} - m_{14} e^{-m_9 y} \\ + m_{15} e^{-m_2 y} + m_{16} e^{-\Pr y} + m_{17} e^{-Scy} \end{pmatrix} e^{i\omega t}$$

$$\theta(y,t) = e^{-\Pr y} + \left(m_5 e^{-m_6 y} + m_7 e^{-\Pr y} \right) e^{i\omega t}$$

$$C(y,t) = e^{-Scy} + \left(m_8 e^{-m_9 y} + m_{10} e^{-Scy} \right) e^{i\omega t}$$

4. RESULTS AND CONCLUSION

The effect of Grashof number on velocity profiles is illustrated in Fig. 1. It is noticed that increase in Grashof number contributes to the increase in velocity of the fluid. Further, it is noticed in the boundary layer region the velocity increases and thereafter, it decreases. Also, far away from the plate, not much of significant effect of Grashof number is noticed.

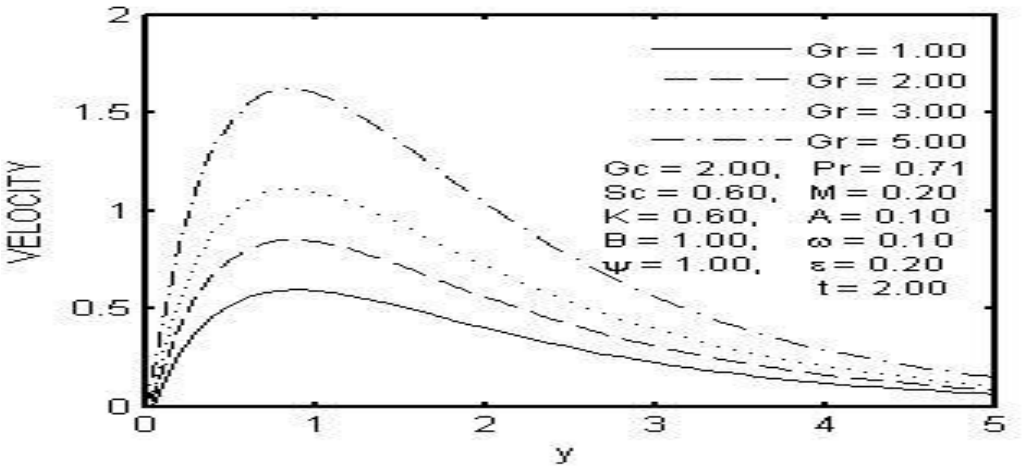

Fig.1. Effect of Gr on velocity profiles

The variation of velocity with respect to Gc is noticed in Fig. 2. It is observed that as Gc increases, in general the velocity also increases. As was seen in earlier case, the

rise in velocity is noticed in the boundary layer region and thereafter, it decreases. The contribution of Schmidt number on the velocity profiles is noticed in Fig. 3. It is seen that as Schmidt number increases, the velocity decreases. It is seen that the contribution by Sc is not that significant at the boundary but as we move far away from the plate, the dispersion due to Sc is found to be more distinct and the effect of Sc could be noticed. The contribution of the magnetic intensity over the velocity field is observed in Fig. 4. It is observed that, as the magnetic intensity is increased, the fluid velocity decreases. Such an observation is in tune with the realistic situation that, as the magnetic intensity suppresses the fluid velocity. The influence of the porosity of the fluid bed on the velocity profiles is shown in Fig. 5. It is noted that, as the pore size of the fluid bed increases, the velocity decreases. This is in agreement with the real life situation. As the pore size of the fluid bed increases, the fluid over the bed gets trapped into the pores resulting in the decrease of the fluid velocity. Variation in the velocity of the fluid medium with respect to suction parameter is illustrated in Fig. 6. It is noticed that as the suction parameter is increased, the velocity of the fluid medium is found to be decreasing. At the boundary, the suction parameter does not show any influence. However, as we move far away from the plate the influence appears to be more predominant. The influence of the amplitude on the velocity profiles is shown in Fig. 7. It is noticed that, as the amplitude increases, the fluid velocity decreases and also at times a backward flow is noticed. Due to the percolation of the fluid into the boundary such a backward flow is noticed. However, as we move far away from the bounding surface, the effect of such backward flow is found to be negligible and the influence of amplitude is not seen.

Fig. 8 illustrates the influence of Prandtl number on the temperature field. It is observed that the prandtl number has significant contribution over the temperature field. From the illustrations it is seen that as the Prandtl No increases, the temperature decreases. Further, not much of significant contribution by the Prandtl number is noticed at the boundary and also as we move far away from the plate. The influence of suction parameter on the temperature field is illustrated in Fig. 9. It is seen that as the suction parameter increases, the temperature field decreases. It is observed that the increase in the suction parameter contributes to the parabolic nature of the temperature profiles. Further, far away from the plate, it is noticed that the suction parameter tends to loose its significance. Fig. 10 illustrates the effect of Sc on concentration profiles. It is seen that as Sc increases, the concentration is found to be decreasing. Further, increase in Sc contributes to the parabolic nature of the concentration profiles. Also, the dispersion in concentration is found to be more distinctive as we move far away from the bounding surface.

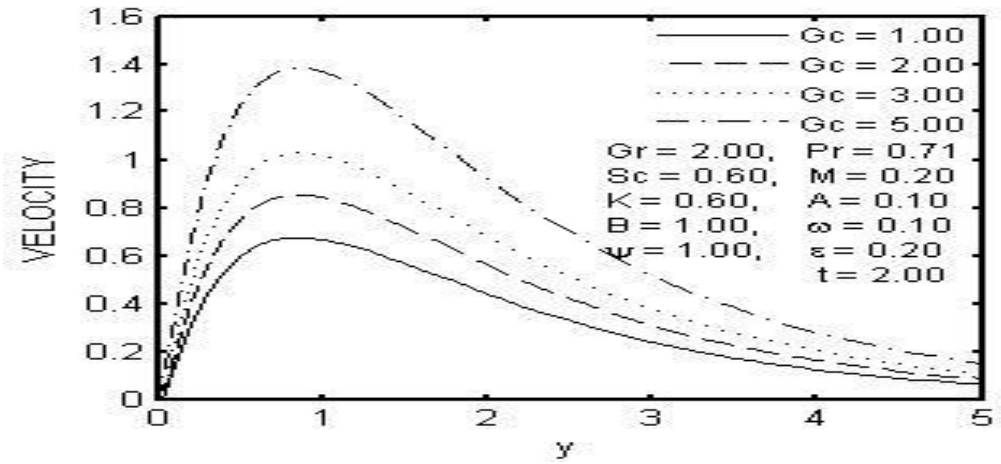

Fig. 2. Effect of Gc on velocity profiles

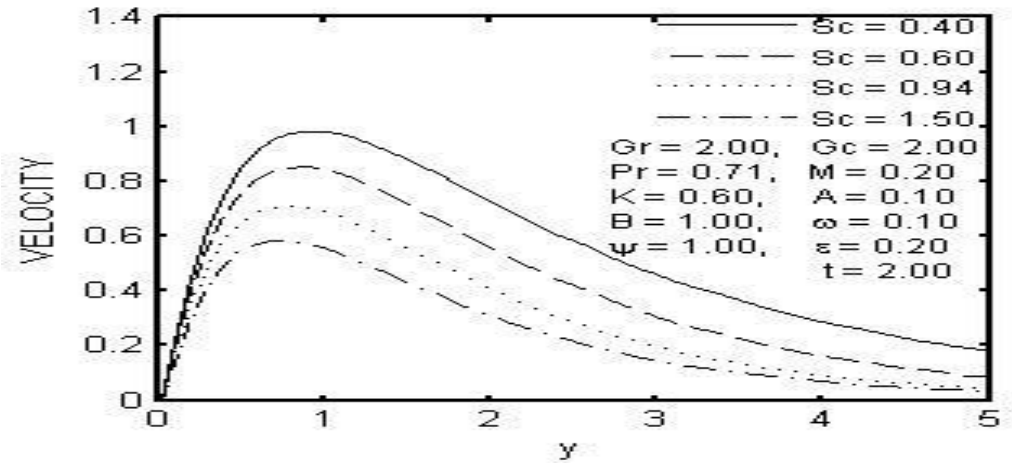

Fig. 3. Effect of Sc on velocity profiles

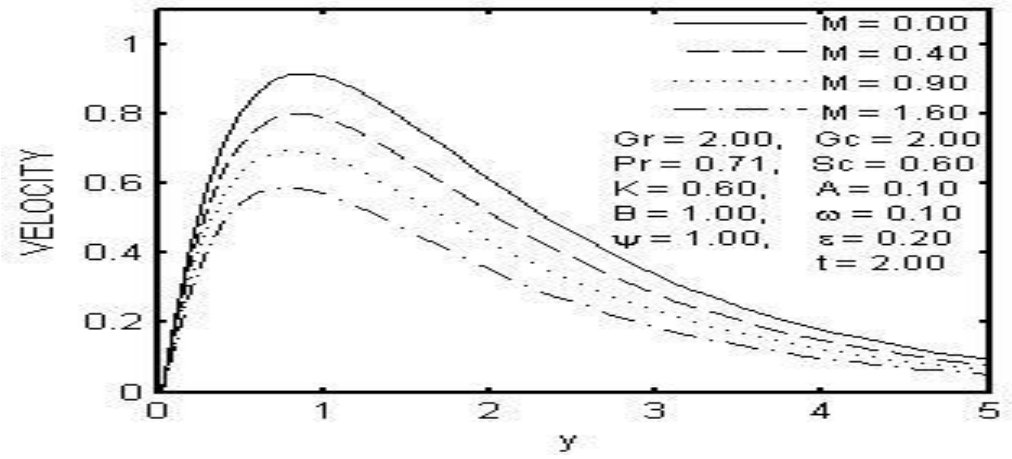

Fig. 4. Effect of Magnetic field on velocity profiles.

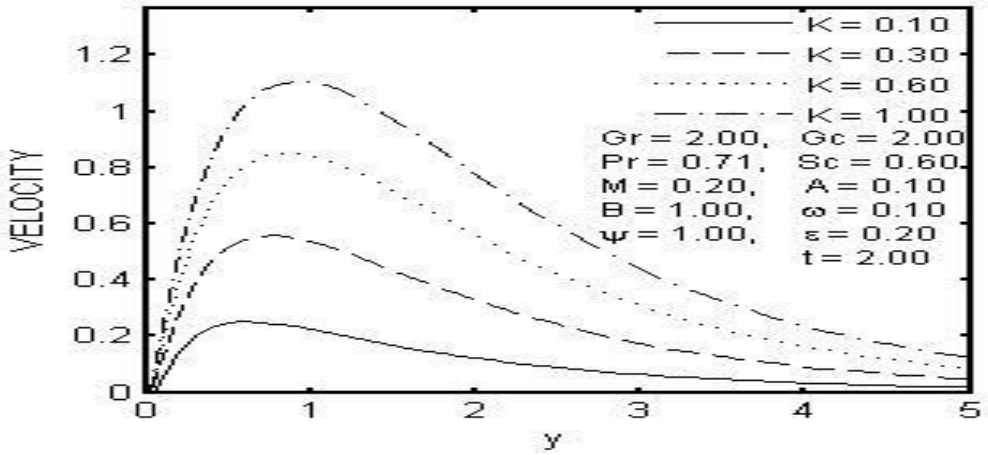

Fig. 5. Effect of Permeability parameter on velocity

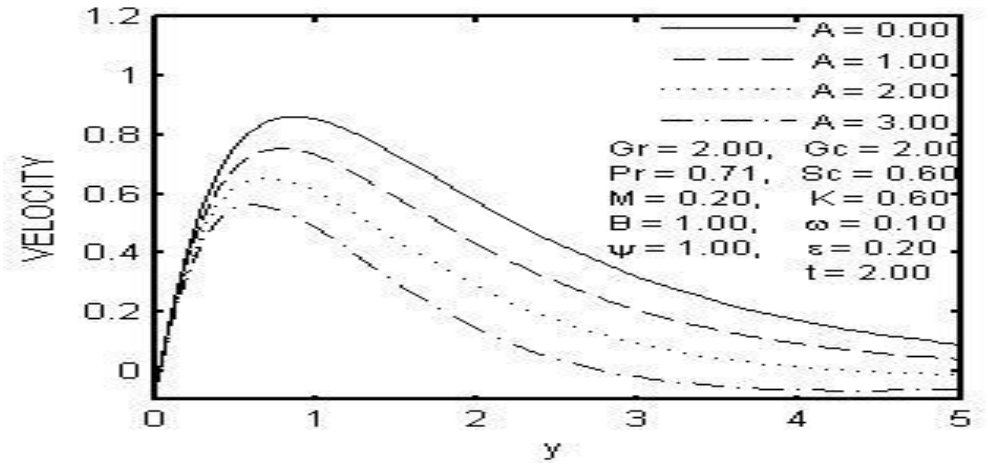

Fig. 6. Effect of Suction parameter on velocity

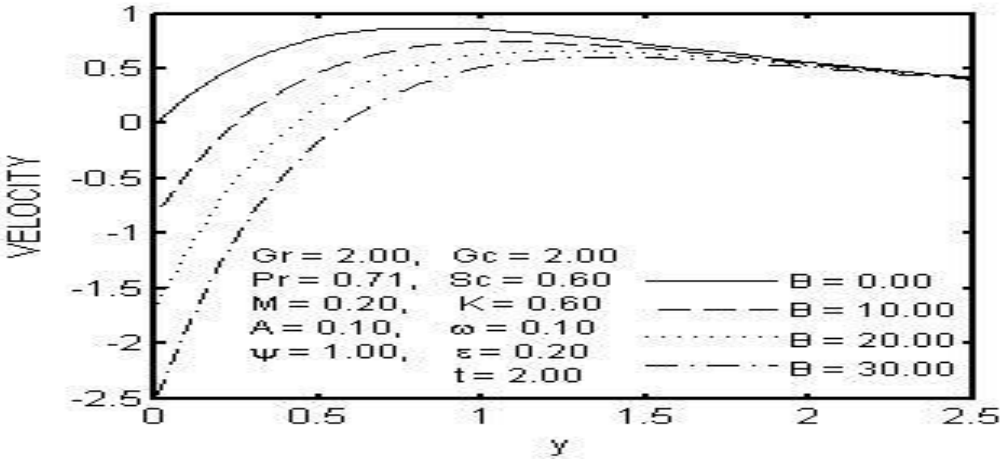

Fig. 7. Effect of B on velocity profiles.

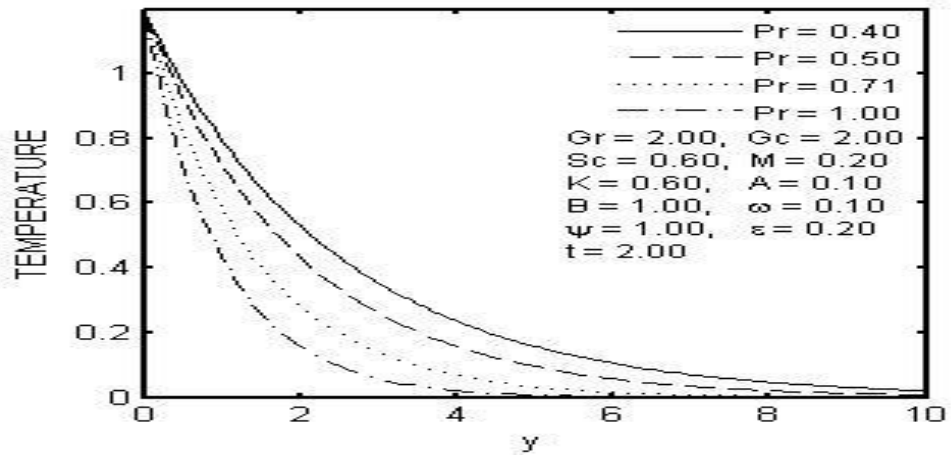

Fig. 8. Effect of Pr on temperature profiles.

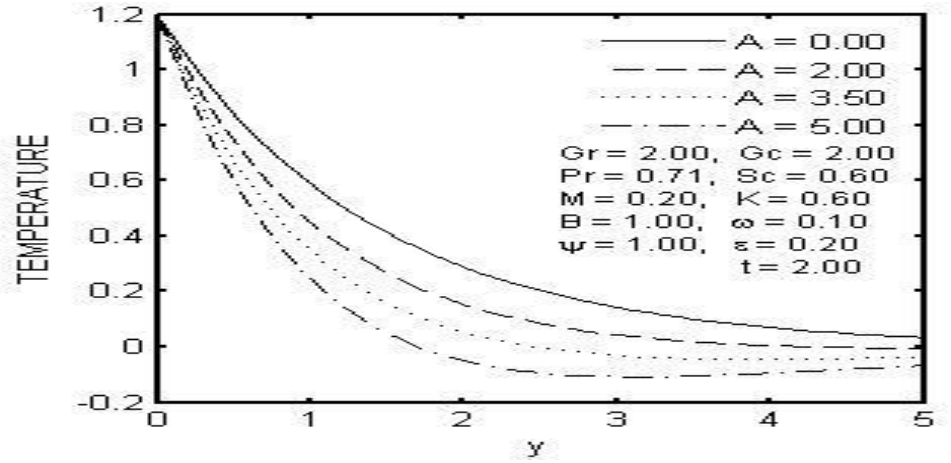

Fig. 9. Effect of Suction parameter on temperature.

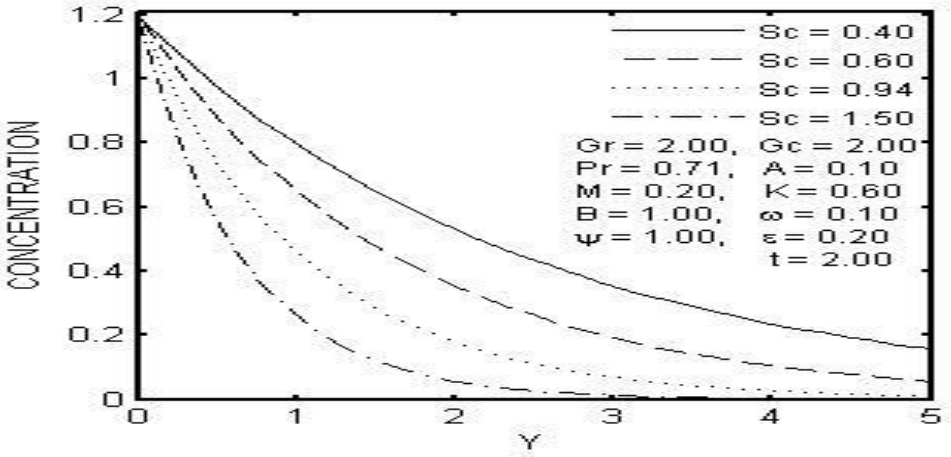

Fig.10. Effect of Sc on concentration

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REFERENCES:

[1] D .Nield, A. Bejan “Convection in porous media”,Newyork, Springer, 1999.

[2] P.Cheng, “Heat transfer in geothermal system”, Adv Heat Transfer 978,14:1-105.

[3] N. Rudraiah, “Flow through and past porous media”,Encyclopedia of Fluid mechanics, Gulf Publ,1986,5:567-647.

[4] Y.J. Kim, “Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction”, Int J Eng Sci, 2000, 38:833–45.

[5] M. A. Hossain, “Viscous and Joule heating effects on MHD free convection flow with variable plate temperature”, Int. J. Heat Mass Transfer, 35, 3485, 1992.

[6] R. Moreau, Magnetohydrodynamics, Kluwar Academic, Dordrecht, The Netherlands, 1990.

[7] Gebhart, B. and L. Pera, "The nature of vertical natural convection flow from the combined buoyancy effects on thermal and mass diffusion", Int. J. Heat Mass Transfer, 14, 2024-2050 (1971).

[8] Pera, L. and B. Gebhart, "Natural convection flows adjacent to horizontal surface resulting from the combined buoyancy effects of thermal and mass diffusion", Int. J. Heat Mass Transfer, 15, 269-278 (1972).

[9] Soundalgekar, V.M., "Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction", Int. J. Heat Mass Transfer, 15, 1253-1261 (1972).

[10] Chen, T.S., C.F. Yuh and A. Moutsoglou, "Combined heat and mass transfer in mixed convection along a vertical and inclined plates", Int. J. Heat Mass Transfer, 23, 527-537 (1980).

[11] Soundalgekar, V.M. and P.D. Wavre, "Unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer", Int. J. Heat Mass Transfer, 20, 1363-1373, (1977a).

[12] Sharma, P.K. and R.C. Chaudhary, "Effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime", Emirates Journal for Engineering Research, 8, 33-38 (2003).

APPENDIX:

$$M_1 = M + \frac{1}{K}, \qquad m_1 = -m_3 - m_4, \qquad m_2 = \frac{1+\sqrt{1+4M_1}}{2},$$

$$m_3 = -\frac{Gr}{\mathrm{Pr}^2 - \mathrm{Pr} - M_1}, \quad m_4 = -\frac{Gc}{Sc^2 - Sc - M_1}, \quad m_5 = 1 - \frac{4iA\mathrm{Pr}}{\omega},$$

$$m_6 = \frac{\mathrm{Pr}+\sqrt{\mathrm{Pr}^2 + i\omega\,\mathrm{Pr}}}{2}, \qquad m_7 = \frac{4iA\mathrm{Pr}}{\omega},$$

$$m_8 = 1 - \frac{4iASc}{\omega}, \qquad m_9 = \frac{Sc+\sqrt{Sc^2 + i\omega Sc}}{2}, \qquad m_{10} = \frac{4iASc}{\omega},$$

$$m_{11} = \frac{B}{\varepsilon}e^{i(\psi-\omega)t} + m_{13} + m_{14} - m_{15} - m_{16} - m_{17}, \qquad m_{12} = \frac{1+\sqrt{1+4M_1+i\omega}}{2},$$

$$m_{13} = \frac{m_5 Gr}{m_6{}^2 - m_6 - \left(M_1 + i\omega/4\right)}, \qquad m_{14} = \frac{m_8 Gc}{m_9{}^2 - m_9 - \left(M_1 + i\omega/4\right)},$$

$$m_{15} = \frac{m_1 m_2 A}{m_2{}^2 - m_2 - \left(M_1 + i\omega/4\right)}, \qquad m_{16} = \frac{m_3 A\mathrm{Pr} - m_7 Gr}{\mathrm{Pr}^2 - \mathrm{Pr} - \left(M_1 + i\omega/4\right)},$$

$$m_{17} = \frac{m_4 ASc - m_{10} Gc}{Sc^2 - Sc - \left(M_1 + i\omega/4\right)}.$$

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On Discrete Wrapped Exponential Distribution– Characteristics

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***Abstract*:** Here an attempt is made to propose new discrete circular model by employing the method of discretization of existing continuous circular model. Here considering the Wrapped Exponential model we discretized it to make it discrete Wrapped Exponential distribution. The probability mass function, distribution function and characteristic function of the new discrete circular model are obtained and population characteristics are studied .

***Keywords*:** Characteristic function, Distribution function, Trigonometric moments.

I. INTRODUCTION

Circular models play vital role in data analysis of angular or periodic data. A good number of continuous circular models are available in the literature. Wrapped circular models [13], stereographic circular and semicircular models [17] and Offset circular and semicircular models [1] and *l* – axial models [6] provide a rich and very useful class of models for circular as well as *l*-axial data. Scant attention was paid in investigation of discrete circular models. There is a need of developing discrete circular models which are invariant of zero direction and sense of rotation. Wrapped Discrete Binomial model derived in [14] and characteristics of Wrapped Poisson model were studied in [15]. In this paper, a new discrete circular model called Discrete Wrapped Exponential model is constructed by employing discretization on Wrapped Exponential model.

II. CIRCULAR DISTRIBUTIONS

A circular distribution is a probability distribution whose total probability is defined on the unit circle. Since each point on the unit circle represents a direction, it is a way of assigning probabilities to different directions. The range of a circular random variable θ measured in radians, can be taken as $[0, 2\pi)$ or $[-\pi, \pi)$.

Circular distributions are of two types, they may be discrete, assigning probabilities to a finite number of directions or continuous assigning probabilities to a infinite number of directions. One of the construction methods of circular models is wrapping a linear distribution. Here wrapped discrete circular random variables are discussed.

Wrapped Discrete Circular Random Variables

If *X* is a discrete random variable on the set of integers, then reduction modulo $2\pi m\left(m \in \mathbf{Z}^{+}\right)$ wraps the integers on to the group of mth roots of unity which is a sub group of unit circle.

$$\theta = 2\pi x \left(\operatorname{mod} 2\pi m\right)$$

More precisely θ is a mapping from a set of integers *G* which is a group with respect to '+' to the set of mth roots of unity G' which is a group with respect to '.' is defined as

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$\theta(x) = e^{\frac{2\pi ix}{m}}$ where $x \in G$, $e^{\frac{2\pi ix}{m}} \in G'$

Then θ is called wrapped discrete circular random variable.

Clearly θ is a homomorphism

1) $\theta(x+y) = e^{\frac{2\pi i(x+y)}{m}} = e^{\frac{2\pi ix}{m}} e^{\frac{2\pi iy}{m}} = \theta(x)\theta(y)$

2) $\theta(0) = e^{\frac{2\pi i(0)}{m}} = e^0 = 1$ where $0 \in G$, $1 \in G'$

Since θ contains a finite number of elements they are denoted by $\theta = \left\{ \frac{2\pi r}{m} \middle/ r = 0,1,2,...m-1 \right\}$ which is lattice on the unit circle.

Probability Mass Function

Suppose if θ is a wrapped discrete circular random variable then probability mass function of θ is denoted by $Pr\left(\theta = \frac{2\pi r}{m}\right)$ which satisfies the following properties

1. $Pr\left(\theta = \frac{2\pi r}{m}\right) \geq 0$
2. $\sum_{r=0}^{m-1} Pr\left(\theta = \frac{2\pi r}{m}\right) = 1$
3. $Pr(\theta) = Pr(\theta + 2\pi l)$ For any integer l (i.e Pr is periodic)

Distribution Function

Suppose if θ is a wrapped discrete circular random variable then distribution function of θ is denoted by $F_w(\theta)$ and it is defined as

$$F_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right) \quad \text{where } k = 0,1,...m-1$$

In mathematics, discretization is the process of transforming continuous functions, models, variables, and equations into its discrete counterparts. This process is usually carried out to model a given discrete phenomenon depending on the practical situation.

III. DISCRETIZATION OF A CONTINUOUS DISTRIBUTION

In this paper, we adopt discretization from [9] to construct discrete circular models by transforming existing continuous circular models. The process is as follows

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Consider a continuous circular distribution with probability density function (pdf) $f(\theta)$ on the circle to construct a probability mass function $Pr\left(\theta=\frac{2\pi r}{m}\right)$ on a set of the equally spaced points for $r=0,1,2...m-1$ with a fixed integer m equal or greater than 2.

$Pr\left(\theta=\frac{2\pi r}{m}\right)=f\left(\frac{2\pi r}{m}\right)\Big/\sum_{r=0}^{m-1}f\left(\frac{2\pi r}{m}\right)$ is probability mass function(pmf) of respective discrete circular model. $m=1$ is excluded as it is degenerate. Here we derive new discrete circular model by employing the discretization of a continuous circular model.

IV. DISCRETE WRAPPED EXPONENTIAL DISTRIBUTION

The probability density function of a Wrapped Exponential distribution [12] is $f(\theta)=\frac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}$ where λ >0 is a parameter.

Now this model is converted into Discrete Wrapped Exponential distribution by normalizing the probability density function with $\frac{1-e^{\frac{-2\pi\lambda}{m}}}{\lambda}$

Then probability mass function of the Discrete Wrapped Exponential Distribution is defined as $Pr(\theta=\frac{2\pi r}{m})=\frac{e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)$ where $m\in\mathbf{Z}^{+}$ and $r=0,1,2,....m-1$

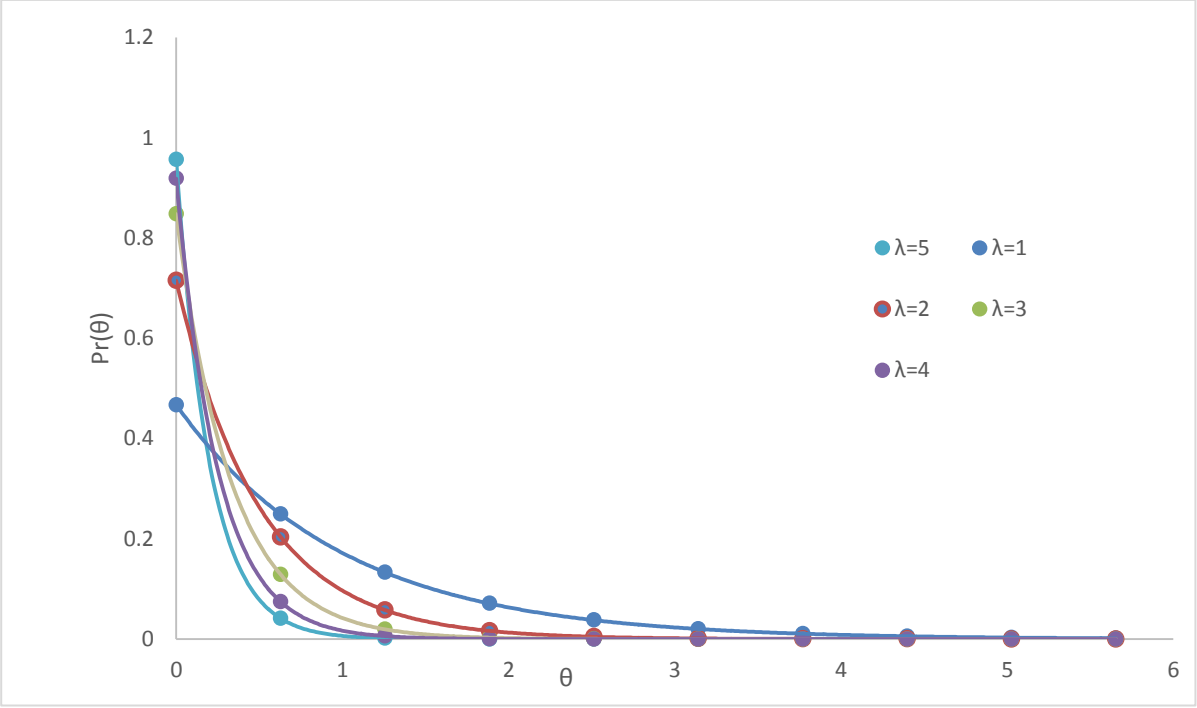

Fig.1: Graph for Probability mass function of Discrete Wrapped Exponential Distribution

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V. DISTRIBUTION FUNCTION OF DISCRETE WRAPPED EXPONENTIAL DISTRIBUTION

The Distribution function of Discrete Wrapped Exponential Distribution is defined as $F_w(\theta)=\sum_{r=0}^{k} Pr\left(\theta=\frac{2\pi r}{m}\right)$

$$=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\sum_{r=0}^{k}e^{\frac{-2\pi r\lambda}{m}}$$

$$=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\left[1+e^{\frac{-2\pi\lambda}{m}}+e^{\frac{-4\pi\lambda}{m}}+---+e^{\frac{-2\pi\lambda k}{m}}\right]$$

$$=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\left[1+e^{\frac{-2\pi\lambda}{m}}+\left(e^{\frac{-2\pi\lambda}{m}}\right)^2+---+\left(e^{\frac{-2\pi\lambda}{m}}\right)^k\right]$$

Since $\left|e^{\frac{-2\pi\lambda}{m}}\right|<1$ then by the sum of n terms of Geometric progression

$$F_w(\theta)=\frac{1-e^{\frac{-2\pi\lambda(k+1)}{m}}}{1-e^{-2\pi\lambda}}$$ Where $k=0,1,2,...m-1$

VI. CHARACTERISTIC FUNCTION OF THE DISCRETE WRAPPED EXPONENTIAL DISTRIBUTION

The Characteristic function of Discrete Wrapped Exponential distribution is defined as

$\varphi_\theta(p)=E(e^{ip\theta})$ where $p\in\mathbf{Z}$

$$=\sum_{r=0}^{m-1}e^{ip\theta}\frac{e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}(1-e^{\frac{-2\pi\lambda}{m}})=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\sum_{r=0}^{m-1}e^{ip\theta}e^{\frac{-2\pi r\lambda}{m}}$$

$$=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\sum_{r=0}^{m-1}e^{\frac{-2\pi r}{m}(\lambda-ip)}=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\left[1+e^{\frac{-2\pi}{m}(\lambda-ip)}+e^{\frac{-4\pi}{m}(\lambda-ip)}+------+e^{\frac{-2\pi(m-1)}{m}(\lambda-ip)}\right]$$

Since $\left|e^{\frac{-2\pi}{m}(\lambda-ip)}\right|<1$ then by sum of n terms in Geometric progression

$$\varphi_\theta(p)=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\left[\frac{1-e^{-2\pi(\lambda-ip)}}{1-e^{\frac{-2\pi}{m}(\lambda-ip)}}\right]=\frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}}\left[\frac{1-e^{-2\pi\lambda}e^{2\pi ip}}{1-e^{\frac{-2\pi\lambda}{m}}e^{\frac{2\pi ip}{m}}}\right]=\rho_p e^{i\mu_p}$$

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$$= \frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{-2\pi\lambda}} \left[\frac{1-e^{-2\pi\lambda}\left(\cos 2\pi p + i \sin 2\pi p\right)}{1-e^{\frac{-2\pi\lambda}{m}}\left(\cos\frac{2\pi p}{m} + i\sin\frac{2\pi p}{m}\right)} \right]$$

$$= \frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{\frac{-2\pi\lambda}{m}}\left(\cos\frac{2\pi p}{m} + i\sin\frac{2\pi p}{m}\right)}$$

$$= \frac{1-e^{\frac{-2\pi\lambda}{m}}}{1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m} - ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}}$$

$$= \frac{1-e^{\frac{-2\pi\lambda}{m}}}{\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right) - ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}} \frac{\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right) + ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}}{\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right) + ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}}$$

$$= \left(1-e^{\frac{-2\pi\lambda}{m}}\right)\left(\frac{1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m} + ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}}{1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}} \right)$$

$$= \frac{1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m} + ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}}\cos\frac{2\pi p}{m} - ie^{\frac{-4\pi\lambda}{m}}\sin\frac{2\pi p}{m}}{1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}}$$

$$= \alpha_p + i\beta_p$$

where $\alpha_p = \dfrac{1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}}\cos\frac{2\pi p}{m}}{1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}}$

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and $\beta_p = \dfrac{e^{\frac{-2\pi\lambda}{m}} \sin\dfrac{2\pi p}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\dfrac{2\pi p}{m}}{1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\dfrac{2\pi p}{m}}$ Here α_p, β_p are called p^{th} trigonometric moments

Clearly $\rho_p = \sqrt{\alpha^2{}_p + \beta_p{}^2}$ and $\mu_p = \tan^{-1}\left(\dfrac{\beta_p}{\alpha_p}\right)$

The circular mean direction is denoted by μ_1 and it is defined as

$$\mu_1 = \tan^{-1}\left[\frac{e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}}{1-e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}}\right]$$

Now ρ_1 represents the concentration towards mean direction which is defined as

$$\rho_1 = \sqrt{\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}\right)^2}{\left(1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2}\right]}$$

In general $\mu_1 = \mu$ and $\rho_1 = \rho$

The circular variance is denoted by V_o and it is defined as $V_o = 1-\rho$

$$V_o = 1 - \sqrt{\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}\right)^2}{\left(1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2}\right]}$$

And the circular standard deviation is defined as

$$\sigma_o = \sqrt{-2\log(1-V_o)}$$

$$= \sqrt{-2\log\left[\sqrt{\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}\right)^2}{\left(1+e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2}\right]}\right]}$$

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VII. CENTRAL TRIGONOMETRIC MOMENTS

The pth central trigonometric moment of θ is defined as $\varphi_\theta^*(p) = E\left(e^{ip(\theta-\mu)}\right)$

$$= E\left(e^{ip\theta}e^{-ip\mu}\right) = e^{-ip\mu}E\left(e^{ip\theta}\right)$$

$$= \left(\cos p\mu - i\sin p\mu\right)\left(\alpha_p + i\beta_p\right)$$

$$= \left(\alpha_p \cos p\mu + i\beta_p \cos p\mu - i\alpha_p \sin p\mu + \beta_p \sin p\mu\right)$$

$$= \alpha_p \cos p\mu + \beta_p \sin p\mu + i(\beta_p \cos p\mu - \alpha_p \sin p\mu)$$

$$= \alpha_p^* + i\beta_p^*$$

where $\alpha_p^* = \alpha_p \cos p\mu + \beta_p \sin p\mu$

$$\beta_p^* = \beta_p \cos p\mu - \alpha_p \sin p\mu$$

Now the Skewness for the Discrete Wrapped Exponential Distribution is defined as

$$\gamma_1 = \frac{\beta_2^*}{(V_o)^{\frac{3}{2}}}$$

$$= \frac{\beta_2 \cos 2\mu - \alpha_2 \sin 2\mu}{\left[1 - \sqrt{\left[\frac{\left(1 - e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}\right)^2}{\left(1 + e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2}\right]}\right]^{\frac{3}{2}}}$$

And the kurtosis for the Discrete Wrapped Exponential Distribution is defined as

$$\gamma_2 = \frac{\alpha_2^* - (1 - V_o)^4}{(V_o)^2}$$

$$= \frac{(\alpha_2 \cos 2\mu + \beta_2 \sin 2\mu) - \left[1 - \sqrt{\left[\frac{\left(1 - e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}\right)^2}{\left(1 + e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2}\right]}\right]^4}{\left[1 - \sqrt{\left[\frac{\left(1 - e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m} - e^{\frac{-2\pi\lambda}{m}} + e^{\frac{-4\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}} \sin\frac{2\pi}{m} - e^{\frac{-4\pi\lambda}{m}} \sin\frac{2\pi}{m}\right)^2}{\left(1 + e^{\frac{-4\pi\lambda}{m}} - 2e^{\frac{-2\pi\lambda}{m}} \cos\frac{2\pi}{m}\right)^2}\right]}\right]^2}$$

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VIII. CONCLUSION

On the lines of discretization of continuous circular model, we constructed Discrete Wrapped Exponential Distribution.Trigonometric moments are obtained from the characteristic function and we studied population characteristics using these trigonometric moments.

REFERENCES

[1]. A.J.V.Radhika,"Mathematical Tools in the Construction of New Circular Models", unpublished thesis submitted to AcharyaNagarjuna University for the award of Ph.D,2014.

[2]. A.J.V Radhika, S.V.S.Girija, and A.V DattatreyaRao,"*On Univariate Offset Pearson Type II Model – Application To Live Data",*International Journal of Mathematics and Statistics Studies, Vol.1, No. 1, pp.1-9,2013.

[3]. A.Pewsey, "A Wrapped Skew-Normal Distribution on the circle," Communication in Statistics: Theory and Methods, Vol.29, No.11, pp.2459-2472,2000.

[4]. A.V. DattatreyaRao, I. RamabhadraSarma and S.V.S Girija, "On Wrapped Version of Some Life Testing Models,"Communication in Statistics: Theory and Methods, 36, issue # 11, pp.2027-2035,2007.

[5]. A.W. Kemp, "Characterization of a Discrete Normal Distribution," Journal of Statistical Planning and Inference, Vol.63, No.2, pp.223-229,1997.

[6]. Ch.V.Sastry, "*On l – Axial Models",* unpublished thesis submitted to Acharya Nagarjuna University for the award of Ph.D,2016**.**

[7]. K.V. Mardia, and P.E.Jupp, "Directional Statistics" 2nd Edition , *Wiley, New York 2001.*

[8]. M.K. Abramowitz, I.A Stegun, "Handbook of Mathematical Functions**"** Dover, New York, 1965

[9]. Min – Zhen Wang and Kunio Shimizu, "Discrete Cardioid Distribution", presented in Conference on Advances and Applications in Distribution Theory, The Institute of Statistical Mathematics, Tokyo, Japan,2014.

[10]. S. Inusah and T.J.Kozubowski, "A Discrete Analogue of the Laplace Distribution," Journal of Statistical Planning and Inference, Vol.136, No.3,pp.1090-1102,2006.

[11]. S.RaoJammalamadaka and A.Sen Gupta, "Topics in Circular Statistics, *World Scientific Press", Singapore,2014.*

[12]. S.RaoJammalamadaka and J.Tomasz Kozubowski , "New Families of Wrapped Distributions for Modeling Skew Circular Data", Communications in Statistics - Theory and Methods, Vol .33, No. 9, pp.2059-2074,2004.

[13]. S.V.S Girija,"Construction of New Circular Models",VDM VERLAG, Germany. ISBN 978-3-639-27939-9, 2010.

[14]. S.V.S Girija, A.V.DattatreyaRao and G.V.L.N.Srihari ,"On Wrapped Binomial Model Characteristics", Horizon Research PublicationsVolume2(7),pp.231-234,2014(a).

[15]. S.V.S Girija, A.V.DattatreyaRao and G.V.L.N.Srihari, "On Characteristic Function of Wrapped Poisson Distribution", *International Journal of Mathamatical archive* Volume5(5),pp.168-173,2014(b).

[16]. S.V.S Girija, A.V.DattatreyaRao and Y.Phani, "*On Stereographic Lognormal Distribution",* International Journal of Advances in Applied Sciences (IJAAS),Vol. 2 No.3, pp.125-132,2013(a).

[17]. Y.Phani, "On Stereographic Circular and Semicircular Models", unpublished thesis submitted to AcharyaNagarjuna University for the award of Ph.D,2013.

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ON WRAPPED NEGATIVE BINOMIAL MODEL

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ABSTRACT

I*n this paper, a new discrete circular model called Wrapped Negative Binomial distribution is constructed by applying the method of wrapping for Negative Binomial distribution. The characteristic function of the Wrapped Negative Binomial Distribution is derived and the population characteristics such as Mean, Variance, Standard Deviation, Skewness and Kurtosis are studied. The graph of probability mass function for various values of parameters are plotted using MATLAB.*

Keywords: *Circular model, Characteristic function, Trigonometric moments, Mean direction.*

1. INTRODUCTION

Circular data arise in many diverse scientific contexts. Many examples of this kind of data are found in earth science, Ecology (as wind direction analysis), Biology (study of animal movement direction), Physics and more. in general any context where the study of data recorded in radians or degrees on the unit circle.

Many good number of continuous circular models are available in the literature. Wrapped circular models (Girija (2010)), stereographic circular and semicircular models (Phani (2013)) and Offset circular and semicircular models (Radhika (2014)) and *l* – axial models (Sastry (2016)) provide a rich and very useful class of models for circular as well as *l*-axial data. Scant attention was paid in construction of discrete circular models. There is a need to develop discrete circular models which are invariant of zero direction and sense of rotation. (Girija *et al.* (2014)) derived Wrapped Discrete Binomial model and studied its characteristics.

2. CIRCULAR DISTRIBUTIONS

A circular distribution is a probability distribution whose total probability is defined on the unit circle. Probabilities to different directions are assigned by representing each direction as point on the unit circle. The range of circular random variable θ measured in radians, can be taken as $[0, 2\pi)$ or $[-\pi, \pi)$.

Circular distributions are of two types, they may be discrete or continuous versions.

2.1. Wrapped Discrete Circular Random Variables

If *X* is a discrete random variable on the set of integers, then reduction modulo $2\pi m\left(m \in \mathbb{Z}^{+}\right)$ wraps the integers on to the group of mth roots of unity which is a sub group of unit circle.

i.e $\theta = 2\pi x \left(\mathrm{mod}\, 2\pi m\right)$

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More precisely θ is a mapping from a set of integers *G* which is a group with respect to '+' to the set of mth roots of unity G' which is a group with respect to '.' is defined as

$\theta(x) = e^{\frac{2\pi ix}{m}}$ where $x \in G,\ e^{\frac{2\pi ix}{m}} \in G'$

Then θ is called wrapped discrete circular random variable.

Clearly θ is a homomorphism

1) $\theta(x+y) = e^{\frac{2\pi i(x+y)}{m}} = e^{\frac{2\pi ix}{m}} e^{\frac{2\pi iy}{m}} = \theta(x)\theta(y)$

2) $\theta(0) = e^{\frac{2\pi i(0)}{m}} = e^0 = 1$ where $0 \in G, 1 \in G'$

Since θ contains a finite number of elements they are denoted by $\theta = \left\{ \frac{2\pi r}{m} \Big/ r = 0,1,2.....m-1 \right\}$ which is lattice on the unit Circle.

3. PROBABILITY MASS FUNCTION

Suppose if θ is a wrapped discrete circular random variable then probability mass function of θ is denoted by $Pr\left(\theta = \frac{2\pi r}{m}\right)$ which is defined as

$$Pr\left(\theta = \frac{2\pi r}{m}\right) = \sum_{k=-\infty}^{\infty} P(r+km) \quad \text{where } r = 0,1,...m-1 \text{ Such that } m \in \mathbb{Z}^+ \tag{3.1}$$

Further if it is to be a circular probability mass function it should satisfy the following properties.

1. $Pr\left(\theta = \frac{2\pi r}{m}\right) \geq 0$
2. $\sum_{r=0}^{m-1} Pr\left(\theta = \frac{2\pi r}{m}\right) = 1$
3. $Pr(\theta) = Pr(\theta + 2\pi l)$ For any integer l (i.e Pr is periodic)

4. DISTRIBUTION FUNCTION

Suppose if θ is a wrapped discrete circular random variable then distribution function of θ is

Denoted by $F_w(\theta)$ which is defined as

$$F_w(\theta) = \sum_{r=0}^{y} Pr\left(\theta = \frac{2\pi r}{m}\right) \quad \text{where } y = 0,1,...m-1$$

5. WRAPPED NEGATIVE BINOMIAL DISTRIBUTION

Suppose if *X* follows Negative Binomial Distribution with the parameters n, p_1 where $n \in \mathbb{Z}^+$ and $0 < p_1 < 1$ then the probability mass function of the wrapped Negative Binomial distribution defined from (3.1) as

$$Pr(\theta = \frac{2\pi r}{m}) = \sum_{k=0}^{\infty} (r+km+n-1)_{c_{n-1}} p_1^{\,n} q_1^{\,r+km}$$

Where p_1 =probability of success and q_1 =probability of failure ∋ $p_1 + q_1 = 1$

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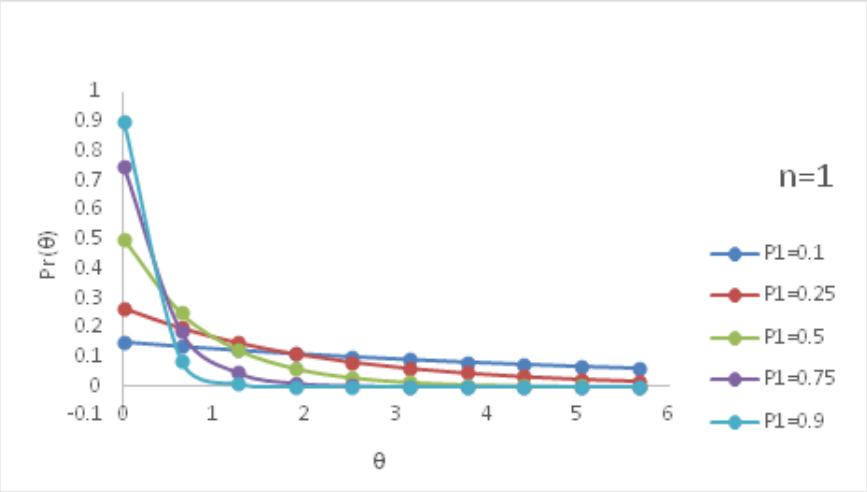

Graph for Probability Mass Function of Wrapped Negative Binomial Distribution

6. DISTRIBUTION FUNCTION OF WRAPPED NEGATIVE BINOMIAL DISTRIBUTION

The distribution function of the Wrapped Negative Binomial Distribution is defined as

$$F_w(\theta) = \sum_{r=0}^{y} Pr\left(\theta = \frac{2\pi r}{m}\right)$$

$$= \sum_{r=0}^{y} (r+km+n-1)_{c_{n-1}}\, p_1^{\,n} q_1^{\,r+km} \quad \text{where} \quad y = 0,1,\ldots m-1$$

7. CHARACTERISTIC FUNCTION OF THE WRAPPED NEGATIVE BINOMIAL DISTRIBUTION

The Characteristic function of the Wrapped Negative Binomial Distribution is defined as

$$\varphi_\theta(p) = \sum_{r=0}^{m-1} e^{ip\theta}\, Pr(\theta = \frac{2\pi r}{m}) \text{ where } \quad p \in \mathbb{Z}$$

$$= \sum_{r=0}^{m-1} e^{ip\theta} \left[\sum_{k=0}^{\infty} (r+km+n-1)_{c_{n-1}}\, p_1^{\,n} q_1^{\,r+km} \right]$$

$$= \left[\frac{p_1}{1 - e^{\frac{2\pi ip}{m}} q_1} \right]^n = \rho_p e^{i\mu_p}$$

$$= \left[\frac{p_1}{\left(\left(1 - q_1 \cos\frac{2\pi p}{m}\right) - iq_1 \sin\frac{2\pi p}{m}\right)} \frac{\left(\left(1 - q_1 \cos\frac{2\pi p}{m}\right) + iq_1 \sin\frac{2\pi p}{m}\right)}{\left(\left(1 - q_1 \cos\frac{2\pi p}{m}\right) + iq_1 \sin\frac{2\pi p}{m}\right)} \right]^n$$

$$= \left[\frac{p_1\left(1 - q_1 \cos\frac{2\pi p}{m}\right) + ip_1 q_1 \sin\frac{2\pi p}{m}}{1 + q_1^2 - 2q_1 \cos\frac{2\pi p}{m}} \right]^n$$

$$= (x+iy)^n \quad \text{Where} \quad x = \frac{p_1\left(1 - q_1 \cos\frac{2\pi p}{m}\right)}{1 + q_1^2 - 2q_1 \cos\frac{2\pi p}{m}} \quad \text{and} \quad y = \frac{p_1 q_1 \sin\frac{2\pi p}{m}}{1 + q_1^2 - 2q_1 \cos\frac{2\pi p}{m}}$$

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By taking $x = R\cos\theta$ and $y = R\sin\theta$ we get $x^2 + y^2 = R^2$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$(x+iy)^n = (R\cos\theta + iR\sin\theta)^n$$
$$= (R^n\cos n\theta + iR^n\sin n\theta)$$
$$= \alpha_p + i\beta_p$$

where $\alpha_p = R^n\cos n\theta$ and $\beta_p = R^n\sin n\theta$

Clearly

$$R = \sqrt{\left[\frac{p_1^2\left(1-q_1\cos\frac{2\pi p}{m}\right)^2 + p_1^2q_1^2\sin^2\frac{2\pi p}{m}}{\left(1+q_1^2-2q_1\cos\frac{2\pi p}{m}\right)^2}\right]}$$

$$R = \sqrt{\left[\frac{p_1^2\left(1+q_1^2-2q_1\cos\frac{2\pi p}{m}\right)}{\left(1+q_1^2-2q_1\cos\frac{2\pi p}{m}\right)^2}\right]}$$

$$R = \frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi p}{m}}}$$

Now $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{p_1q_1\sin\frac{2\pi p}{m}}{\left(p_1\left(1-q_1\cos\frac{2\pi p}{m}\right)\right)}\right]$

Then

$$\alpha_p = \left[\frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi p}{m}}}\right]^n \cos n\left[\tan^{-1}\left[\frac{p_1q_1\sin\frac{2\pi p}{m}}{\left(p_1\left(1-q_1\cos\frac{2\pi p}{m}\right)\right)}\right]\right]$$

$$\beta_p = \left[\frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi p}{m}}}\right]^n \sin n\left[\tan^{-1}\left[\frac{p_1q_1\sin\frac{2\pi p}{m}}{\left(p_1\left(1-q_1\cos\frac{2\pi p}{m}\right)\right)}\right]\right]$$

Now $\rho_p = \sqrt{\alpha_p^2 + \beta_p^2} = \left[\frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi p}{m}}}\right]^n$

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and $$\mu_p = \tan^{-1}\left[\frac{\beta_p}{\alpha_p}\right] = \left[\tan^{-1}\left[\tan n\left[\tan^{-1}\left[\left(\frac{p_1 q_1 \sin\frac{2\pi p}{m}}{p_1\left(1-q_1\cos\frac{2\pi p}{m}\right)}\right)\right]\right]\right]\right]$$

$$= \left[n\tan^{-1}\left[\left(\frac{p_1 q_1 \sin\frac{2\pi p}{m}}{p_1\left(1-q_1\cos\frac{2\pi p}{m}\right)}\right)\right]\right]$$

Now the circular mean direction is denoted by μ_1 and it is defined as $\mu_1 = n\tan^{-1}\left[\frac{p_1 q_1 \sin\frac{2\pi}{m}}{p_1\left(1-q_1\cos\frac{2\pi}{m}\right)}\right]$

Now ρ_1 represents the concentration towards mean direction which is defined as $\rho_1 = \left[\frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi}{m}}}\right]^n$

In general $\mu_1 = \mu$ and $\rho_1 = \rho$

Now the circular variance is denoted by V_o which is defined as

$$V_o = 1-\rho$$

$$= 1-\left[\frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi}{m}}}\right]^n$$

The circular standard deviation is denoted by σ_o and it is defined as

$$\sigma_o = \sqrt{-2\log(1-V_o)}$$

$$= \sqrt{-2\log\left[\frac{p_1}{\sqrt{1+q_1^2-2q_1\cos\frac{2\pi}{m}}}\right]^n}$$

$$= \sqrt{-2\left[\log {p_1}^n - \log(1+q_1^2-2q_1\cos\frac{2\pi}{m})^{\frac{n}{2}}\right]}$$

8. CENTRAL TRIGONOMETRIC MOMENTS

The pth Central trigonometric moment of θ is defined as

$$\varphi_\theta^*(p) = E\left[e^{ip(\theta-\mu)}\right] = E\left[e^{ip\theta}e^{-ip\mu}\right] = e^{-ip\mu}E\left[e^{ip\theta}\right]$$

$$= e^{-ip\mu}[\alpha_p + i\beta_p]$$

$$= (\cos p\mu - i\sin p\mu)[\alpha_p + i\beta_p]$$

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$$= \alpha_p \cos p\mu + i\beta_p \cos p\mu - i\alpha_p \sin p\mu + \beta_p \sin p\mu$$
$$= \left(\alpha_p \cos p\mu + \beta_p \sin p\mu\right) + i\left(\beta_p \cos p\mu - \alpha_p \sin p\mu\right)$$
$$= \alpha_p^* + i\beta_p^*$$

where $\alpha_p^* = \alpha_p \cos p\mu + \beta_p \sin p\mu$ and $\beta_p^* = \beta_p \cos p\mu - \alpha_p \sin p\mu$

For Wrapped Negative Binomial distribution Skewness is defined as

$$\gamma_1 = \frac{\beta_2^*}{(V_o)^{\frac{3}{2}}}$$
$$= \frac{\beta_2 \cos 2\mu - \alpha_2 \sin 2\mu}{\left(1 - \left[\frac{p_1}{\sqrt{1+q_1^2 - 2q_1 \cos\frac{2\pi}{m}}}\right]^n\right)^{\frac{3}{2}}}$$

And the kurtosis is defined as

$$\gamma_2 = \frac{\alpha_2^* - (1-V_o)^4}{(V_o)^2}$$
$$= \frac{\left(\alpha_2 \cos 2\mu + \beta_2 \sin 2\mu\right) - \left(\left[\frac{p_1}{\sqrt{1+q_1^2 - 2q_1 \cos\frac{2\pi}{m}}}\right]^n\right)^4}{\left(1 - \left[\frac{p_1}{\sqrt{1+q_1^2 - 2q_1 \cos\frac{2\pi}{m}}}\right]^n\right)^2}$$

REFERENCES

1. Abramowitz, M. and Stegun, I.A. (1965), Handbook of Mathematical Functions, Dover, New York.
2. Dattatreya Rao, A.V., RamabhadraSarma, I. and Girija, S.V.S. (2007), On Wrapped Version of Some Life Testing Models, Comm Statist,-Theor.Meth., 36, issue # 11, pp.2027-2035.
3. Girija, S.V.S. (2010). Construction of New Circular Models, VDM VERLAG, Germany. ISBN 978-3-639-27939-9.
4. Girija.S.V.S, A.V.DattatreyaRao and G.V.L.N.Srihari (2014) On Wrapped Binomial Model - Characteristics, Horizon Research PublicationsVolume2(7),pp.231-234(2014).
5. Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, John Wiley, Chichester.
6. Phani Y, (2013) "On Stereographic Circular and Semicircular Models", unpublished thesis submitted to AcharyaNagarjuna University for the award of Ph.D.
7. Phani, Y.,Girija, S.V.S., DattatreyaRao and G.V.L.N.Srihari (2015). Symmetric Circular Model Induced by Inverse Stereographic Projection On Double weibull Distribution, International Journal of soft computing, Mathematics and Control (IJSCMC), Vol 4 No.1,.pp.67-74
8. Radhika A.J.V, (2014) "Mathematical Tools in the Construction of New Circular Models", unpublished thesis submitted to AcharyaNagarjuna University for the award of Ph.D.
9. Rao Jammalamadaka S. and Sen Gupta, A. (2001), Topics in Circular Statistics, World Scientific Press, Singapore.
10. Sastry Ch.V. (2016). On *l* – Axial Models, unpublished thesis submitted to AcharyaNagarjuna University for the award of Ph.D.

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Stereographic *l* – Axial Half Logistic Distribution

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Abstract

Following Phani et al (2017) an attempt is made to develop the Stereographic-l-axial Half Logistic distribution by extending the Stereographic Semicircular Half Logistic distribution for modeling axial data. Stereographic *l* - axial Half Logistic distribution over any arc of length *l* has possible applications in Quality Control, weather studies and pollution control experiments. We consider the asymptotic behavior of the Stereographic Semicircular Half Logistic distribution, derive the first two trigonometric moments for proposed model, we state and prove some theorems that characterize the Stereographic Semicircular Half Logistic model. MATLAB is used to plot graphs and compute population characteristics.

Keywords: Circular model, Characteristic function, l-axial data, Stereographic projection, Trigonometric moments.

INTRODUCTION

Kantam and Rosaiah (1998) applied Half Logistic distribution in acceptance sampling based on life tests. Several recurrence relations satisfied by the single and product moments of order statistics from a generalized Half Logistic distribution enable one to compute the single and product moments of all order statistics. For all sample sizes in a simple recursive manner, this may be done for any choice of the shape parameter. These moments can then be used to determine the best linear unbiased estimators of location and scale parameters from complete as well as Type-II censored samples.

In some of the cases the directional / angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004) and Byoung et al (2008). Phani et al (2013) constructed some semicircular distributions by applying Inverse Stereographic projection.

Taking this as a cue, on the lines of Phani et al (2017) an attempt is made to develop the Stereographic-l-axial Half Logistic distribution by extending the Stereographic Semicircular Half Logistic distribution for modeling axial data. Stereographic *l* - axial Half Logistic distribution over *l* – arc has possible applications in Quality Control, weather studies and pollution control experiments. We consider the asymptotic behaviour of the Stereographic Semicircular Half Logistic distribution, derive the first two trigonometric moments for proposed model, we state and prove some theorems that characterize the Stereographic Semicircular Half Logistic model. MATLAB is used to plot graphs and compute population characteristics.

METHODOLOGY OF MODIFIED INVERSE STEREOGRAPHIC PROJECTION FOR

Semicircular models

Methodology of modified inverse stereographic projection for construction of circular models is well explained by Phani et al (2013).

If a linear random variable X has a support on R, then θ has a support on $(-\pi, \pi)$ and if X has a support on R^+, then θ has a support on $(0, \pi)$. These means that, after the Inverse stereographic projection is applied, we can deal circular data if the support of X is on R and we can handle semicircular data if the support of X is on R^+.

STEREOGRAPHIC SEMICIRCULAR HALF LOGISTIC DISTRIBUTION [Phani et al (2017)]

A random variable X on the real line is said to have an Half Logistic distribution with location parameter α and scale parameter $\beta > 0$, if the probability density function and probability distribution function of X are given respectively by

$$f(x) = \frac{2}{\beta}\left[1+\exp\left(\frac{-(x-\alpha)}{\beta}\right)\right]^{-2}\exp\left(\frac{-(x-\alpha)}{\beta}\right),$$
$$0 < x < \infty,\ \beta > 0. \qquad (3.1)$$

$$F(x) = \frac{1-\exp\left(\frac{-(x-\alpha)}{\beta}\right)}{1+\exp\left(\frac{-(x-\alpha)}{\beta}\right)},\ 0 < x < \infty \qquad (3.2)$$

Then by applying modified inverse stereographic projection defined by a one to one mapping $x = v\tan\left(\frac{\theta}{2}\right), v \in \mathbf{R}^+,$ this leads to a semicircular model on unit semicircle. We call this model as Stereographic Semicircular Half Logistic Distribution.

Definition:

A random variable X_{SC} on the semicircle is said to have the Stereographic Semicircular Half Logistic distribution with location parameter μ scale parameter $\sigma > 0$ denoted by **SSCHLD** (σ, μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta) = \frac{1}{\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right), \quad (3.3)$$

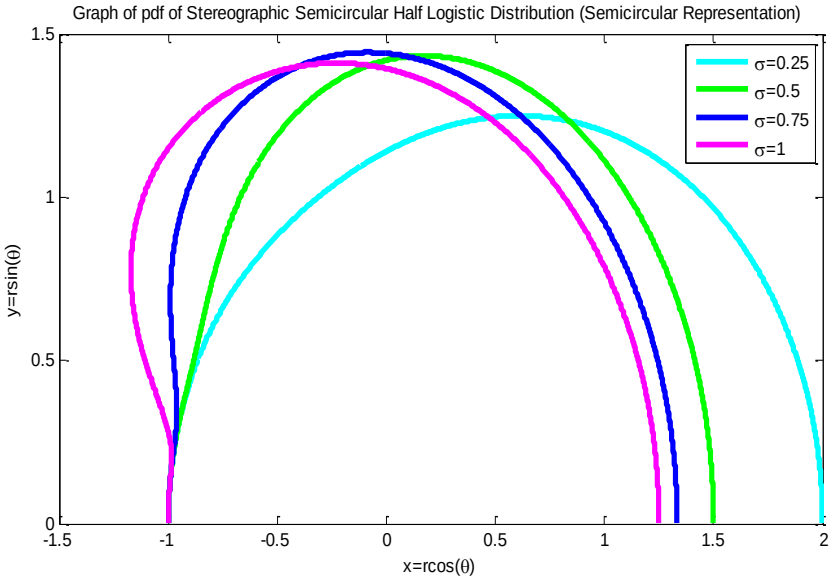
$$0<\theta<\pi,\ \sigma=\frac{\beta}{\nu}>0,\ \mu=\frac{\alpha}{\nu}$$

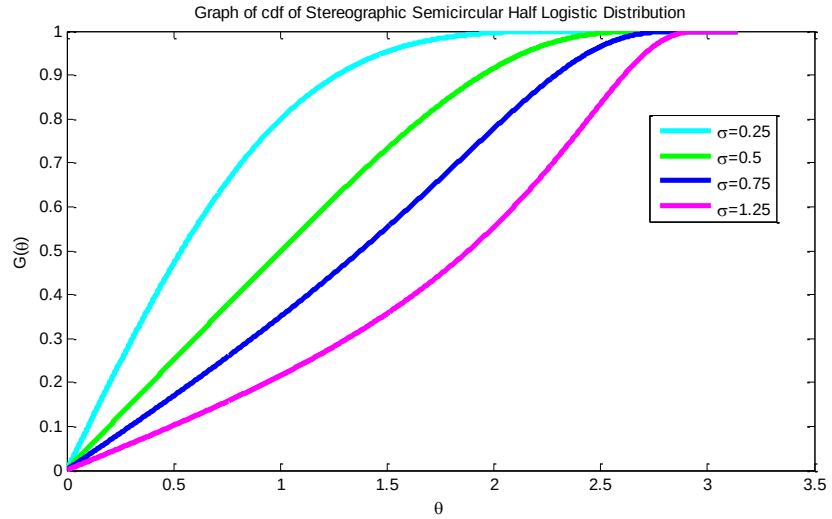
$$G(\theta) = \left[1-\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1},$$

$$0<\theta<\pi,\ \sigma=\frac{\beta}{\nu}>0,\ \mu=\frac{\alpha}{\nu} \quad (3.4)$$

Hence the proposed new model **SSCHLD** (σ, μ) is a semicircular model.

The graphs of probability density function, cumulative distribution function are plotted here.





CHARACTERISTIC FUNCTION OF STEREOGRAPHIC SEMICIRCULAR HALF

Logistic Distribution

The characteristic function of a Semicircular model with probability density function $g(\theta)$ is defined as

$$\varphi_p(\theta)=\int_0^{\pi} e^{ip\theta} g(\theta)\,d\theta\,,\ p\in\square\ .$$

If $G(\theta)$ and $g(\theta)$ are the cdf and the pdf of the Stereographic Semicircular model and $F(x)$ and $f(x)$ are the cdf and the pdf of the respective linear model, then the characteristic function of Stereographic Semicircular model is

$$\varphi_{X_{SC}}(p)=\varphi_{2\tan^{-1}\left(\frac{x}{\nu}\right)}(p),\ p\in\square$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function. Numerical integration of Weddle's rule is used for the computation of the values of the characteristic functions of both the Stereographic Semicircular models.

The characteristic function of Stereographic Semicircular Half Logistic Model is

$$\varphi_{X_{SC}}(p)=\int_0^{\pi} e^{ip\theta} g(\theta)\,d\theta$$

$$=\int_0^{\pi} e^{ip\theta}\frac{1}{\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right)d\theta$$

$$0<\theta<\pi,\ \sigma>0,\ \mu<\tan\left(\frac{\theta}{2}\right) \qquad (4.1)$$

The graph of the characteristic function for various values of parameter are plotted here

The expressions for mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions are available in Mardia and Jupp (2000). These characteristics for the Stereographic Semicircular Half Logistic distribution are also based on their respective trigonometric moments and can be expressed in terms of trigonometric moments α_p and β_p which are presented here.

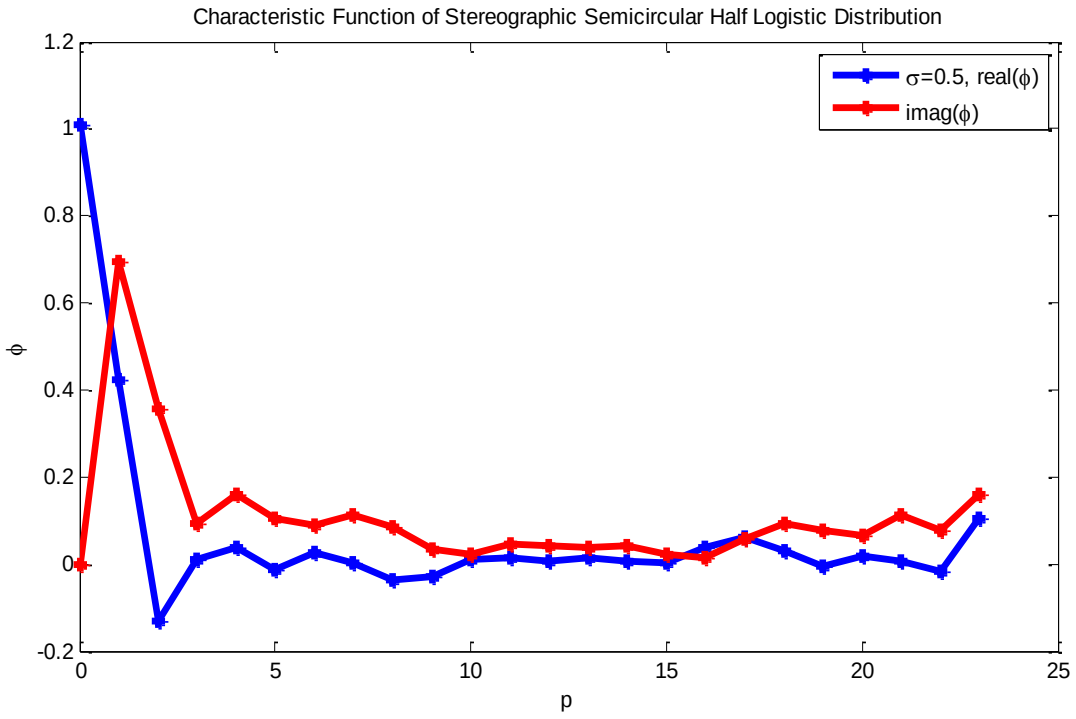


Table 4.1. Population Characteristics of SSCHLD

	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$
Trigonometric Moments				
α_1	0.7962	0.2571	0.1003	-0.1801
α_2	0.4310	-0.4301	-0.3403	-0.1897
β_1	0.3774	0.8297	0.7931	0.7390
β_2	0.4235	0.3395	0.0737	-0.2489
Resultant Length				
ρ_1	0.8811	0.8686	0.7994	0.7607
ρ_2	0.6042	0.5479	0.3482	0.3129
Mean Direction				
μ_0	0.4426	1.2703	1.4450	1.8099
Circular Variance				
v_0	0.1189	0.1314	0.2006	0.2393
Circular Standard Deviation				
σ_0	0.5032	0.5308	0.6692	0.7397
	1.0038	1.0969	1.4526	1.5243
Central Trigonometric Moments				
α_1^*	0.8811	0.8686	0.7994	0.7607
α_2^*	0.6007	0.5467	0.3479	0.2830
β_1^*	0	0	0	0
β_2^*	-0.0655	-0.0367	0.0134	0.1337
Skewness γ_1^0	-1.5967	-0.7713	0.1491	1.1416
Kurtosis γ_2^0	-0.1408	-1.3058	-1.5006	-0.9052

SOME CHARACTERIZATION THEOREMS OF STEREOGRAPHIC SEMICIRCULAR

Half Logistic Distribution

Here we recall a probability density and distribution functions of Stereographic Semicircular Exponential (Phani et al (2013)) and Circular uniform distributions that will involve in the characterization theorems of Stereographic Semicircular Half Logistic Distribution.

a) Stereographic Semicircular Exponential Distribution(Phani et al(2013))

A random variable X_{SC} on unit semicircle is said to have Stereographic Semicircular Exponential distribution with scale parameter $\sigma > 0$, denoted by SSEXP(σ), if the probability density and cumulative distribution functions are given by

$$g(\theta) = \frac{\sigma}{2}\sec^2\left(\frac{\theta}{2}\right)\exp\left(-\sigma\tan\left(\frac{\theta}{2}\right)\right),$$

for $0 \le \theta < \pi$ and $\sigma > 0$ (5.1)

$$G(\theta) = 1 - e^{-\sigma\tan\left(\frac{\theta}{2}\right)} \quad (5.2)$$

b) Semicircular Uniform Distribution

A random variable X_C on unit semicircle is said to have Semicircular Uniform distribution, if the probability density function is given by

$$g(\theta) = \frac{1}{\pi}, \quad 0 \le \theta < \pi \quad (5.3)$$

Theorem 5.1 Let θ be a continuous circular random variable with probability density function $g(\theta)$. Then the random

variable $\phi = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta}{2}\right)}}{1-e^{-\tan\left(\frac{\theta}{2}\right)}}\right)\right)$ follows Stereographic Semicircular Half Logistic distribution if and only if θ follows Stereographic Semicircular Exponential distribution by Phani et al (2013) with scale parameter $\sigma = 1$.

Proof:

Suppose θ follows Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$,

$$g(\theta) = \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\exp\left(-\tan\left(\frac{\theta}{2}\right)\right), \quad 0 \le \theta < \pi$$

Let $\phi = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta}{2}\right)}}{1-e^{-\tan\left(\frac{\theta}{2}\right)}}\right)\right)$, this implies that $\theta = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\phi}{2}\right)}}{1-e^{-\tan\left(\frac{\phi}{2}\right)}}\right)\right)$ and

$$h(\phi) = |J|\, g_\theta(\phi)$$

$$h(\phi) = \frac{\sec^2\left(\frac{\phi}{2}\right)e^{-\tan\left(\frac{\phi}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\phi}{2}\right)}\right)^2}, \quad 0 \le \phi < \pi$$, which is the density function of Stereographic Semicircular Half Logistic distribution.

Conversely, suppose that the random variable $\phi = 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta}{2}\right)}}{1-e^{-\tan\left(\frac{\theta}{2}\right)}}\right)\right)$ follows Stereographic Semicircular Half Logistic distribution, then the distribution function of θ is

$$G_\theta(\theta_0) = \Pr(\theta \le \theta_0)$$

$$= \Pr\left(2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\phi}{2}\right)}}{1-e^{-\tan\left(\frac{\phi}{2}\right)}}\right)\right) \le \theta_0\right)$$

$$= \Pr\left(\left(\frac{1+e^{-\tan\left(\frac{\phi}{2}\right)}}{1-e^{-\tan\left(\frac{\phi}{2}\right)}}\right) \le e^{\tan\left(\frac{\theta_0}{2}\right)}\right)$$

$$= 1 - \Pr\left(\phi \le 2\tan^{-1}\left(\ln\left(\frac{1+e^{-\tan\left(\frac{\theta_0}{2}\right)}}{1-e^{-\tan\left(\frac{\theta_0}{2}\right)}}\right)\right)\right)$$

$G_\theta(\theta_0) = 1 - e^{-\tan\left(\frac{\theta_0}{2}\right)}$, which is the distribution function of the Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$.

Hence the theorem.

Theorem 5.2 Let θ be a continuous circular random variable with probability density function $g(\theta)$. Then the random variable $\phi = 2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta}{2}\right)} - 1\right)\right)$ follows Stereographic Semicircular Half Logistic distribution if and only if θ follows Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$.

Proof:

Suppose θ follows Stereographic Semicircular Exponential distribution with scale parameter $\sigma = 1$,

$$g(\theta) = \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\exp\left(-\tan\left(\frac{\theta}{2}\right)\right), \quad 0 \le \theta < \pi$$

Let $\phi = 2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta}{2}\right)} - 1\right)\right)$, this implies that $\theta = 2\tan^{-1}\left(\ln\left(\frac{1+e^{\tan\left(\frac{\phi}{2}\right)}}{2}\right)\right)$ and the Jacobian of this transformation is $|J| = \frac{\sec^2\left(\frac{\phi}{2}\right)e^{\tan\left(\frac{\phi}{2}\right)}}{2\left(1+e^{\tan\left(\frac{\phi}{2}\right)}\right)}$.

Therefore, $h(\phi) = |J|\, g_\theta(\phi)$

$h(\phi)=\dfrac{\sec^2\left(\frac{\phi}{2}\right)e^{-\tan\left(\frac{\phi}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\phi}{2}\right)}\right)^2}$, $0\le\phi<\pi$, which is the density function of Stereographic Semicircular Half Logistic distribution.

Conversely, suppose that the random variable $\phi=2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta}{2}\right)}-1\right)\right)$ follows Stereographic Semicircular Half Logistic distribution, then the distribution function of θ is

$$G_\theta(\theta_0)=\Pr(\theta\le\theta_0)$$

$$=\Pr\left(2\tan^{-1}\left(\ln\left(\frac{1+e^{\tan\left(\frac{\phi}{2}\right)}}{2}\right)\right)\le\theta_0\right)$$

$$=\Pr\left(\phi\le 2\tan^{-1}\left(\ln\left(2e^{\tan\left(\frac{\theta_0}{2}\right)}-1\right)\right)\right)$$

$=1-e^{\tan\left(\frac{\theta_0}{2}\right)}$, which is the distribution function of the Stereographic Semicircular Exponential distribution with scale parameter $\sigma=1$.

Hence the theorem.

Theorem 5.3 Let θ be a circular random variable which follows Semicircular Uniform distribution, then the random variable $\phi=2\tan^{-1}\left(\ln\left(\dfrac{p-\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}\right)\right)$ has Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$ if and only if $p=\pi$.

Proof:

Suppose θ follows Semicircular Uniform distribution with probability density function

$$g(\theta)=\frac{1}{\pi},\ 0\le\theta<\pi.$$

Let $\phi=2\tan^{-1}\left(\ln\left(\dfrac{p-\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}\right)\right)$, this implies $\theta=2\tan^{-1}\left(\dfrac{p}{1+e^{\tan\left(\frac{\theta}{2}\right)}}\right)$ and the Jacobian of the transformation is $|J|=\dfrac{p\sec^2\left(\frac{\theta}{2}\right)e^{\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)^2}$.

Thus the density function of ϕ is $h(\phi)=|J|\,g_\theta(\phi)$

$h(\phi)=\dfrac{p\sec^2\left(\frac{\theta}{2}\right)e^{\tan\left(\frac{\theta}{2}\right)}}{\pi\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)^2}$, which is the density function of Stereographic Semicircular Half Logistic distribution if $p=\pi$.

Conversely, suppose ϕ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$, then

$$G_\theta(\theta_0)=Pr(\theta\le\theta_0)$$

$$=Pr\left(\theta\le 2\tan^{-1}\left(\frac{p}{1+e^{\tan\left(\frac{\theta}{2}\right)}}\right)\right)$$

$$=1-Pr\left(\phi\le 2\tan^{-1}\left(\ln\left(\frac{p-\tan\left(\frac{\theta}{2}\right)}{\tan\left(\frac{\theta}{2}\right)}\right)\right)\right)=\frac{1}{\pi}\text{ for}$$

$p=\pi$.

Hence the theorem.

Theorem 5.4 The circular random variable θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$ with density function $g(\theta)$ in (3.3) if and only if $g(\theta)$ satisfies the initial value problem $\dfrac{dg}{d\theta}-\left(\tan\left(\frac{\theta}{2}\right)+\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\right)g=\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)g^2$, with $g(0)=\frac{1}{4}$.

Proof:

Consider the differential equation

$$\frac{dg}{d\theta}-\left(\tan\left(\frac{\theta}{2}\right)+\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\right)g=\left(1+e^{\tan\left(\frac{\theta}{2}\right)}\right)g^2,\ \text{with}\ g(0)=\frac{1}{4}. \tag{5.4}$$

Suppose θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$.

Its density function is

$$g(\theta)=\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(-\left(\tan\left(\frac{\theta}{2}\right)\right)\right)\right]^{-2}\exp\left(-\left(\tan\left(\frac{\theta}{2}\right)\right)\right), \tag{5.5}$$

It is easily shown that $g(\theta)$ satisfies the initial value problem (5.4).

Conversely, assume that $g(\theta)$ satisfies the initial value problem (4.6.4), which is a non linear differential equation (in particular Bernoulli's equation), by solving this equation, we have

$$g(\theta)=\sec^2\left(\frac{\theta}{2}\right)\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)^{-2}e^{-\tan\left(\frac{\theta}{2}\right)}$$, which is the probability density function of Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$.

Theorem 5.5 The circular random variable θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$ with distribution function $G(\theta)$ in (3.4)if and only if $G(\theta)$ satisfies the initial value problem

$$\frac{dG}{d\theta}-\left(\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)}\right)G=-\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)},\ \text{with}\ G(\pi)=1.$$

Proof:

Consider the differential equation

$$\frac{dG}{d\theta}-\left(\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)}\right)G=-\frac{\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)e^{-\tan\left(\frac{\theta}{2}\right)}}{\left(1+e^{-\tan\left(\frac{\theta}{2}\right)}\right)},\ \text{with}\ G(\pi)=1. \tag{5.6}$$

Suppose θ follows Stereographic Semicircular Half Logistic distribution with scale parameter $\sigma=1$.

Its distribution function is

$$G(\theta)=\left[1-\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]\left[1+\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]^{-1}, \tag{5.7}$$

It is easily shown that $G(\theta)$ satisfies the initial value problem (4.6.6).

Conversely, assume that $G(\theta)$ satisfies the initial value problem (4.6.6), which is a non linear differential equation, by solving this equation, we have

$$G(\theta)=\left[1-\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]\left[1+\exp\left(-\tan\left(\frac{\theta}{2}\right)\right)\right]^{-1},$$

which is the distribution function of Stereographic Circular Half Logistic distribution with scale parameter $\sigma=1$.

Hence the theorem.

STEREOGRAPHIC - l -AXIAL HALF LOGISTIC DISTRIBUTION

We extend the above Stereographic Semicircular Half Logistic model to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$ so it is desirable to extend the Stereographic Semicircular Half logistic distribution to construct the Stereographic- l -axial Half Logistic distribution, we consider the density function of Stereographic Semicircular Half Logistic distribution and use the transformation $\phi=\frac{2\theta}{l}$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{l}{2\sigma}\sec^2\left(\frac{l\phi}{4}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{l\phi}{4}\right)\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{l\phi}{4}\right)}{\sigma}\right)\right),$$

$$0<\phi<\frac{2\pi}{l},\ \sigma>0\ \text{and}\ l=1,2,.. \tag{6.1}$$

We call it as **Stereographic-l-axial Half Logistic distribution**

Case (1) When $l=1$, in the probability density function (5.1), we get the density function

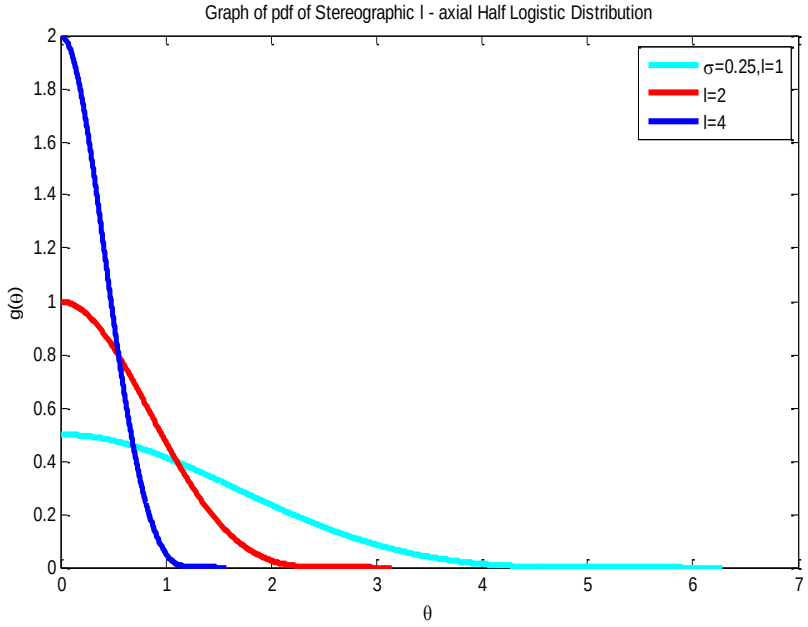
$$g(\phi)=\frac{1}{2\sigma}\sec^2\left(\frac{\phi}{4}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\phi}{4}\right)\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\phi}{4}\right)}{\sigma}\right)\right),$$

$$0<\theta<2\pi\ \text{and}\ \sigma>0 \tag{6.2}$$

We call it as **Stereographic Circular Half Logistic distribution**.

Case (2) When $l = 2$, the probability density function (6.1) is the same as that of **Stereographic Semicircular Half Logistic Distribution**

The graph of the pdf of **Stereographic *l* - axial Half Logistic Distribution** for various values of l is plotted.



REFERENCES

[1] Balakrishna, N.(1985), Order Statistics from the Half Logistic Distribution, Journal of Statistics and Computer Simulation, Vol.20,pp. 287-309.

[2] Byoung, J.A and Hyoung M.K.(2008). A New Family of Semicircular Models: The Semicircular Laplace Distributions, Communications of the Korean Statistical SocietyVol.15, pp. 775-781.

[3] Gradshteyn and Ryzhik (2007). Table of Integrals, series and products, 7th edition, Academic Press.

[4] Guardiola, J.H. (2004). The Semicircular Normal Distribution, Ph.D Dissertation, Baylor University, Institute of Statistics.

[5] JammalamadakaS. Rao and Sen Gupta, A. (2001). Topics in Circular Statistics, *World Scientific Press, Singapore.*

[6] Kantam, R.R.L. & Rosaiah, K. (1998), Half logistic distribution in acceptance sampling based on life tests , *Indian Association for Productivity Quality & Reliability (IAPQR) Transactions*, **23(2),** 117-125.

[7] Mardia, K.V. andJupp, P.E. (2000), Directional Statistics, *John Wiley, Chichester.*

[8] Olapade, A. K. (2003), On Characterizations of the Half Logistic Distribution. InterStat, February Issue,**2**, http://interstat.stat.vt.edu/InterStat/ARTICLES/2003articles/F06002.pdf.

[9] Phani.Y, GirijaS.V.S., and Dattatreya RaoA.V., (2013). On Construction of Stereographic Semi Circular Models, Journal of Applied Probability and Statistics. Vol 8, No. 1, pp. 75-90.

[10] Phani Yedlapalli, P Srinivasa Subrahmanyam, S.V.S.Girija and A.V.Dattatreya Rao, (2017). Stereographic Semicircular Half Logistic Distribution, International Journal of Pure and Applied Mathematics Volume 113 No. 11 2017, 142 – 150.

[11] Ugai. S.K., Nishijima, M. and Kan,T.(1977), Characteristics of raindrop size and raindrop shape, *Open symposium* URSI *Commission F*, 225-230.

[12] Toshihiro Abe, Kunio Shimizu and Arthur Pewsey, (2010), Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection, *J. Japan Statist. Soc.*, Vol. 40 (No. 1), pp. 45-61.

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Semicircular Logistic Distribution induced by Simple Projection Method

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Abstract

For modeling Semicircular data, here we introduce Semicircular Logistic distribution by applying a Simple projection method on Logistic model. We extend it to the l-axial Logistic distribution by a Simple projection for modeling angular data on any arc of arbitrary length. Also derive the characteristic function and the first four trigonometric moments for the evaluation of population characteristics of proposed model. A bivariate version of *l*-axial Logistic distribution is also developed.

Keywords: Circular model, Characteristic function, l-axial data, Projection, Trigonometric moments.

1 Introduction

Phani (2013) proposed new method of constructing circular and semicircular models by applying modified inverse stereographic projection on linear models. By inducing inverse stereographic projection on Weibull and Exponential distributions, Phani et al (2013) constructed Stereographic Semicircular versions of Weibull and Exponential models respectively, whose range spans in $(0,p)$ as the corresponding linear models are in $(0,¥)$ and Girija et al (2014) have constructed Offset Arc Beta model and Offset Arc Exponential type models and this aspect of existence of semicircular/arc models can be viewed as "**natural occurrence**".

The fact that in some practical situations full circular models are not required and it is noted in Guardiola (2004), Jones (1968) and Byoung et al(2008).

Taking this as cue, on the lines of Byoung et al (2008) the **Semicircular Logistic distribution** (**SCLD**) is constructed by projecting Logistic distribution over a semicircular segment. This paper is devoted in constructing Semicircular Logistic distribution, to derive results on the relationship between modality and parameter. We plot the graphs of the density function and distribution function for various values of parameters. We consider the asymptotic behaviour of the Semicircular Logistic distribution, derive the characteristic function and first four trigonometric moments for proposed model. We extend this model for l-axial also. We concentrate on developing Bivariate Semicircular Logistic model.

2 Semicircular Logistic distribution

A random variable X on the real line is said to have Logistic Distribution with location parameter γ and scale parameter $\lambda > 0$, if the probability density function and cumulative distribution function of X for $x, \gamma \in ¡$ and $\lambda > 0$ are given by

$$f(x) = \frac{1}{\lambda}\left[1+\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right]^{-2} \exp\left(\frac{-(x-\gamma)}{\lambda}\right)$$
$$= \frac{1}{4\lambda}\sec h^2\left(\frac{x-\gamma}{2\lambda}\right) \qquad (2.1)$$

$$F(x) = \left[1+\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right]^{-1}$$
$$= \frac{1}{2}\left[1+\tanh\left(\frac{1}{2}\left(\frac{x-\gamma}{\lambda}\right)\right)\right]$$

respectively.

Then by applying Simple projection defined by a one to one mapping

$x = v\tan(\theta)$ or $\theta = \tan^{-1}\left(\frac{x}{v}\right)$ $v > 0 \in ¡$, which leads to a Semicircular Logistic distribution.

Definition:

A random variable X_{SC} on the Semicircle is said to have the Semicircular Logistic distribution with location parameter μ scale parameter $\sigma > 0$ denoted by **SCLD**(σ, μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta) = \frac{1}{\sigma}\sec^2(\theta)\left[1+\exp\left(\frac{-(\tan(\theta)-\mu)}{\sigma}\right)\right]^{-2} \exp\left(-\left(\frac{\tan(\theta)-\mu}{\sigma}\right)\right),$$

$$\text{Where } \sigma = \frac{\lambda}{\nu} > 0, \mu = \frac{\gamma}{\nu} \text{ and } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \qquad (2.2)$$

or $$g(\theta) = \frac{1}{4\sigma}\sec^2(\theta)\sec h^2\left[\frac{\tan(\theta)}{2\sigma}\right]$$

$$G(\theta) = \left[1+\exp\left(\frac{-(\tan(\theta)-\mu)}{\sigma}\right)\right]^{-1}, \text{where } \sigma > 0, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Hence the proposed new model **SCLD**(σ, μ) is a Semicircular model.

We consider the asymptotic behavior of the Semicircular Logistic distribution when $\sigma \to 0$. Suppose θ follows **SCLD** (σ, μ) **.** Let $y = \frac{\theta}{\sigma}$ and then use the change of variable technique. For sufficiently small σ , we have $\tan(\sigma y)$; σy and $\sec(\sigma y)$; 1 by first order approximation of the Taylor series expansion. Hence, the distribution of θ becomes Logistic distribution (linear). So, for sufficiently small σ , the Semicircular Logistic distribution can be approximated by a linear Logistic distribution.

Theorem 5.2.1: Semicircular logistic distribution is **Symmetric** about $\mu = 0$ and is **unimodal** if $\sigma < 0.5$ and **bimodal** if $\sigma > 0.5$

Proof: The probability density function of Semicircular logistic distribution is

$$g(\theta) = \frac{1}{\sigma}\sec^2(\theta)\left[1+\exp\left(\frac{-(\tan(\theta))}{\sigma}\right)\right]^{-2} \exp\left(-\left(\frac{\tan(\theta)}{\sigma}\right)\right), \text{where } \sigma > 0, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

equivalently $$g(\theta)=\frac{1}{4\sigma}\sec^2(\theta)\sec h^2\left[\frac{\tan(\theta)}{2\sigma}\right]$$

Differentiating $g(\theta)$ with respect to θ, we get

$$g'(\theta)=\frac{1}{4\sigma}\left[\sec^2(\theta)\tan(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)-\frac{1}{\sigma}\sec^4(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)\tanh\left(\frac{\tan(\theta)}{2\sigma}\right)\right]$$

$$=\frac{1}{4\sigma}\sec^2(\theta)\sec h^2\left(\frac{v\tan(\theta)}{2\sigma}\right)\left[\tan(\theta)-\frac{1}{2\sigma}\sec^2(\theta)\tanh\left(\frac{\tan(\theta)}{2\sigma}\right)\right]$$

For Stationary points $g'(\theta)=0$

$$\Rightarrow \tan(\theta)-\frac{1}{2\sigma}\sec^2(\theta)\tanh\left(\frac{v\tan(\theta)}{2\sigma}\right)=0$$

$$\Rightarrow \tanh\left(\frac{\tan(\theta)}{2\sigma}\right)-\sigma\sin\theta=0$$

$\Rightarrow \theta=0,\ 2\pi,\ 4\pi,...,$ are the stationary points.

$\theta=0$ is the only stationary point which lies in the domain of $g(\theta)$

Differentiating $g'(\theta)$ with respect to θ, we get

$$g''(\theta)=\frac{1}{4\sigma}\left[\begin{array}{l}\sec^2(\theta)\tan^2(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)+\frac{1}{2}\sec^4(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)\\ -\frac{1}{2\sigma}\sec^4(\theta)\tan(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)\tanh\left(\frac{\tan(\theta)}{2\sigma}\right)\end{array}\right]$$

$$-\frac{1}{8\sigma^2}\left[\begin{array}{l}2\sec^2(\theta)\tan(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)\tanh\left(\frac{\tan(\theta)}{2\sigma}\right)\\ -\frac{1}{2\sigma}\sec^6(\theta)\sec h^2\left(\frac{\tan(\theta)}{2\sigma}\right)\tanh^2\left(\frac{\tan(\theta)}{2\sigma}\right)\\ +\frac{1}{4\sigma}\sec^2(\theta)\sec h^4\left(\frac{\tan(\theta)}{2\sigma}\right)\end{array}\right]$$

At $\theta = 0$

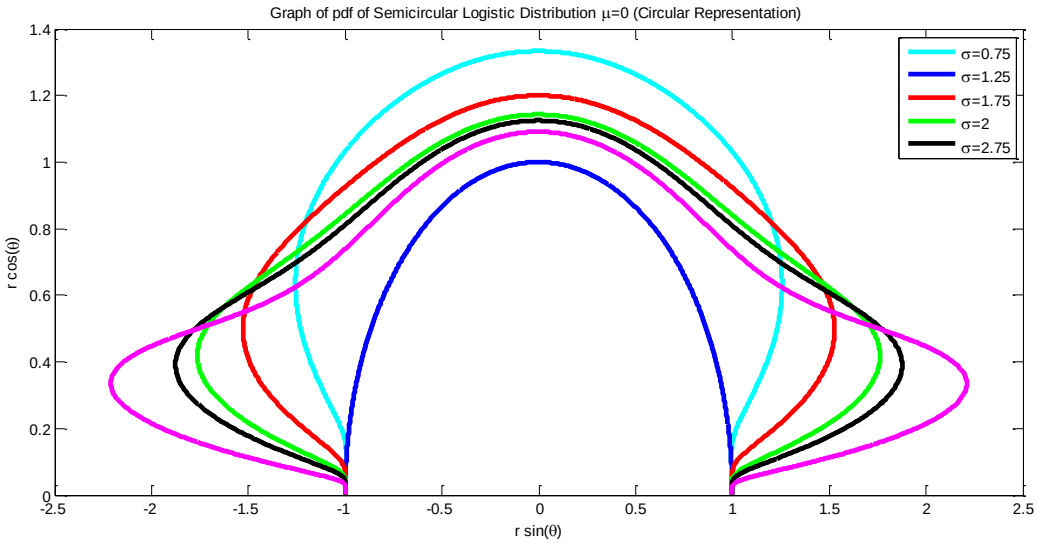
$$g''(0) = \frac{1}{16\sigma} - \frac{1}{32\sigma^3} = \frac{2\sigma^2 - 1}{32\sigma^3} = \left[\frac{2\sigma^2 - 1}{32\sigma^3}\right]$$

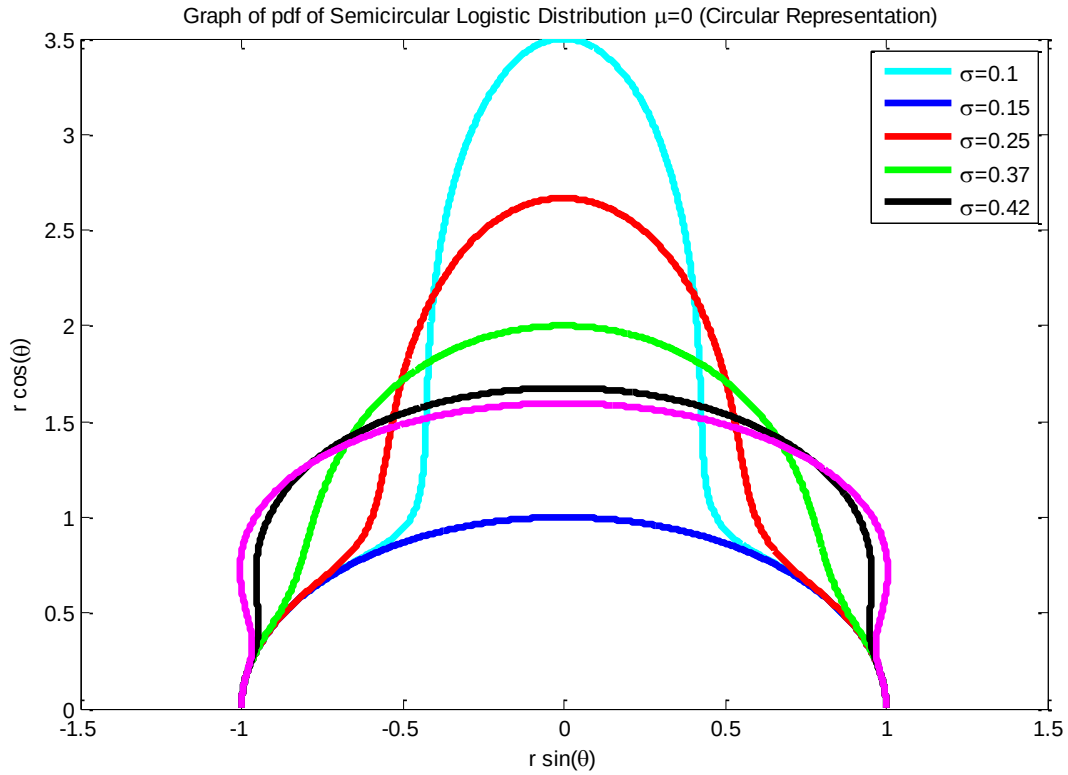
$g(\theta)$ has maximum value at $\theta = 0$ if and only if $g''(0) < 0 \Leftrightarrow \sigma < 0.5$

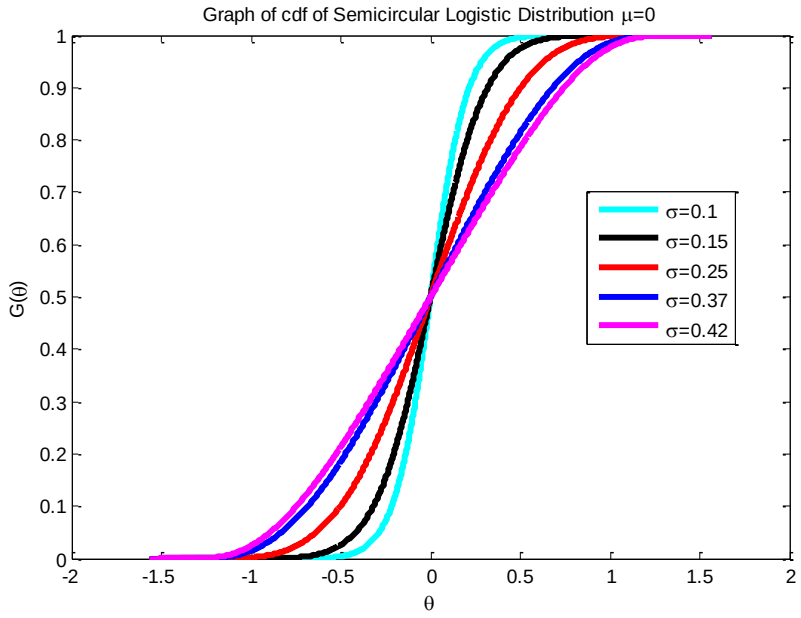
$g(\theta)$ has minimum value at $\theta = 0$ if and only if $g''(0) > 0 \Leftrightarrow \sigma > 0.5$

Hence Stereographic logistic distribution is unimodal if $\sigma < 0.5$ and bimodal if $\sigma > 0.5$

Graphs of the probability density and cumulative distribution function of the Semicircular Logistic distribution for various values of σ and μ




Graph of pdf of Semicircular Logistic Distribution μ=0 (Circular Representation)
σ=0.1
σ=0.15
σ=0.25
σ=0.37
σ=0.42
r cos(θ)
r sin(θ)
3.5
3
2.5
2
1.5
1
0.5
0
-1.5
-1
-0.5
0
0.5
1
1.5


Graph of cdf of Semicircular Logistic Distribution μ=0
σ=0.1
σ=0.15
σ=0.25
σ=0.37
σ=0.42
G(θ)
θ
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

3. The Characteristic function of Semicircular Logistic distribution:

$$\Phi_{X_{SC}}(p)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta} g(\theta)\,d\theta$$

$$=\frac{1}{\sigma}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta}\sec^2(\theta)\left[1-\exp\left(-\frac{1}{\sigma}\tan(\theta)\right)\right]^{-2}\exp\left(-\frac{1}{\sigma}\tan(\theta)\right)d\theta$$

$$=\frac{2}{\sigma}\int_{0}^{\frac{\pi}{2}} \cos p\theta\sec^2(\theta)\left[1-\exp\left(-\frac{1}{\sigma}\tan(\theta)\right)\right]^{-2}\exp\left(-\frac{1}{\sigma}\tan(\theta)\right)d\theta, \quad \text{Since}$$

$\sin p\theta$ is an odd function

$$\Phi_{X_{SC}}(p)=\frac{2}{\sigma}\int_{0}^{\infty}\cos p\left(\tan^{-1}(x)\right)\left[1+\exp\left(-\frac{x}{\sigma}\right)\right]^{-2}\exp\left(-\frac{x}{\sigma}\right)dx, \text{ putting } x=\tan(\theta)$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function.

Trigonometric moments of the Semicircular Logistic distribution

The trigonometric moments of the distribution are given by $\{\varphi_p : p=\pm1,\pm2,\pm3,\ldots\}$, where $\varphi_p=\alpha_p+i\beta_p$, with $\alpha_p=E(\cos p\theta)$ and $\beta_p=E(\sin p\theta)$ being the pth order Cosine and sine moments of the random angle θ, respectively. Because the Semicircular logistic distribution is symmetric about $\mu=0$, it follows that the sine moments are zero. Thus $\varphi_p=\alpha_p$.

Theorem 3.1 Under the pdf of Semicircular logistic distribution is symmetric with $\mu=0$, the first four $\alpha_p=E(\cos p\theta)$, $p=1,2,3,4$ are given as follows

$$\alpha_1=\frac{\pi}{\sigma}\sum_{n=1}^{\infty}(-1)^{n-1}\left[H_0\left(\frac{n}{\sigma}\right)-Y_0\left(\frac{n}{\sigma}\right)\right],$$

$$\alpha_2 = \frac{\sqrt{2\pi}}{\sigma^{\frac{3}{2}}}\sum_{n=1}^{\infty}(-1)^{n-1}\sqrt{n}\left[H_{-1/2}\left(\frac{n}{\sigma}\right)-Y_{-1/2}\left(\frac{n}{\sigma}\right)\right]-1,$$

$$\alpha_3 = -\frac{\pi}{\sigma^2}\sum_{i=1}^{\infty}(-1)^{n-1}n\left\{H_{-1}\left(\frac{n}{\sigma}\right)-Y_{-1}\left(\frac{n}{\sigma}\right)\right\}-\frac{6}{\pi\sigma}\sum_{i=1}^{\infty}(-1)^{n-1}G_{13}^{31}\left(\frac{n^2}{4\sigma^2}\middle|\begin{matrix}-\frac{1}{2}\\ 0,\ 0,\frac{1}{2}\end{matrix}\right)$$

$$\alpha_4 = 16\left[1-\frac{1}{2\sqrt{\pi}\sigma}\sum_{n=1}^{\infty}(-1)^{n-1}G_{13}^{31}\left(\frac{n^2}{4\sigma^2}\middle|\begin{matrix}-\frac{1}{2}\\ \frac{1}{2},\ 0\ \frac{1}{2}\end{matrix}\right)\right]$$

Where $\int_0^{\infty}\left(u^2+x^2\right)^{v-1}e^{-\mu x}dx = \frac{\sqrt{\pi}}{2}\left(\frac{2u}{\mu}\right)^{v-\frac{1}{2}}\Gamma(v)\left[H_{v-\frac{1}{2}}(uv)-Y_{v-\frac{1}{2}}(uv)\right]$

for $\left|\arg u\pi\right|<\frac{\pi}{2}, \operatorname{Re}\mu>0$ and $\operatorname{Re}v>0$ and $G_{13}^{31}\left(\frac{\mu^2u^2}{4}\middle|\begin{matrix}1-v\\ 1-Q-v,0,\frac{1}{2}\end{matrix}\right)$ is called as Meijer's **G-**function (Gradshteyn and Ryzhik, 2007, formula no. 3.389.2).

4. Extension to l-axial distribution

We extend the proposed model to the l-axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$ so it is desirable to extend the Semicircular Logistic distribution. To construct the l-axial Logistic distribution, we consider the density function of Semicircular Logistic distribution and use the transformation $\phi = 2\theta/l$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{l}{2\sigma}\sec^2\left(\frac{l\phi}{2}\right)\left[1+e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}\right]^{-2}e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}$$

$$\text{where}\quad -\frac{\pi}{l}<\phi<\frac{\pi}{l} \qquad (4.1)$$

Case (1) When $l=2$, the probability density function (5.1) is the same as the probability density

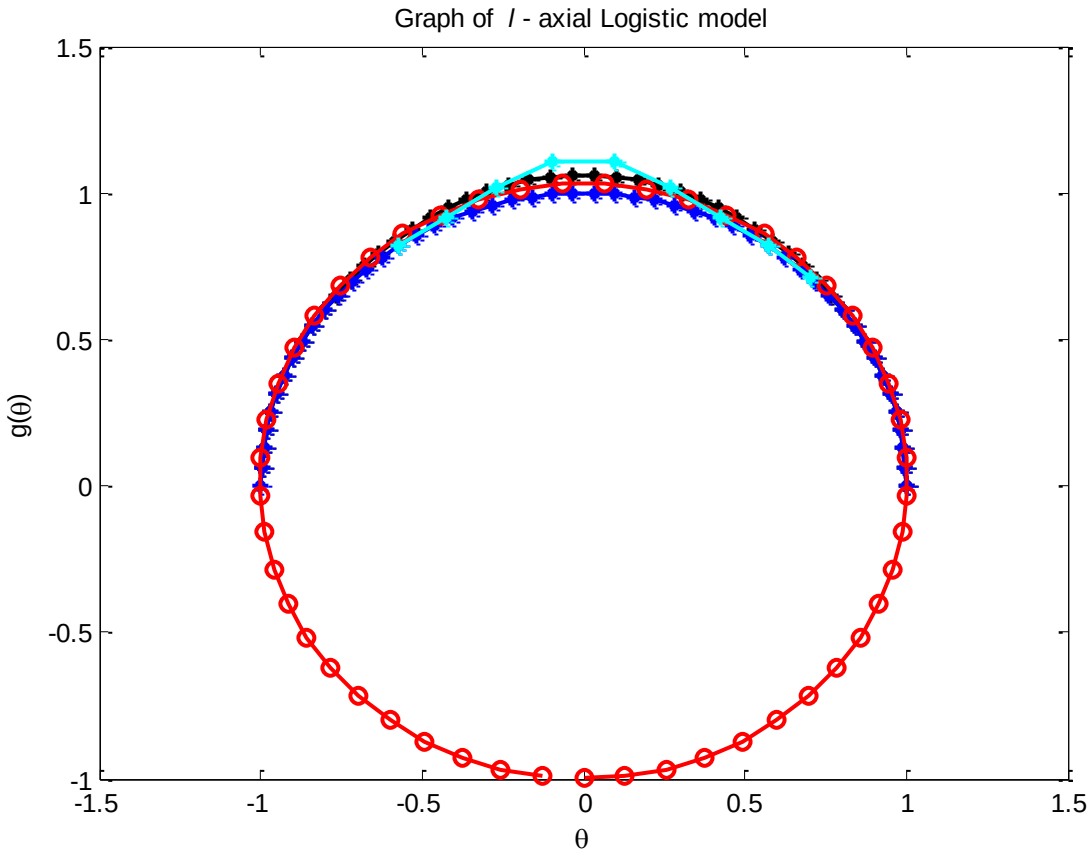
function of **Semicircular Logistic distribution**.

Case (2) When $l=1$, the probability density function (5.1) is the same as that of **Stereographic**

Logistic Distribution[Dattatreyarao et al (2016)] which is circular distribution.

$$g(\phi)=\frac{1}{2\sigma}\sec^2\left(\frac{\phi}{2}\right)\left[1+e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}\right]^{-2}e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}$$

$$\text{where}\quad -\pi<\phi<\pi \qquad (4.2)$$



5. Bivariate Semicircular Logistic Distribution

We can construct a bivariate Semicircular Logistic distribution in a manner similar to the construction of a univariate Semicircular Logistic distribution. We shall use the same semicircular transformation applied in a bivariate case. The probability density function of the bivariate Semicircular Logistic distribution is defined as

$$g(\theta_1,\theta_2)=\frac{2\sec^2(\theta_1)\sec^2(\theta_2)}{\sigma}e^{-\frac{1}{\sigma}(\tan(\theta_1)+\tan(\theta_2))}\left[1+e^{-\frac{1}{\sigma}(\tan(\theta_1))}+e^{-\frac{1}{\sigma}(\tan(\theta_2))}\right]^{-2}$$

$$-\frac{\pi}{2}<\theta_i<\frac{\pi}{2},i=1,2 \qquad (5.1)$$

To construct this density, we begin with the bivariate Logistic density

$$f(x_1,x_2)=\frac{2}{\lambda}e^{-\frac{1}{\lambda}(x_1+x_2)}\left[1+e^{-\frac{1}{\lambda}x_1}+e^{-\frac{1}{\lambda}x_2}\right]^{-2},\qquad -\infty<x_i<\infty,\quad i=1,2 \qquad (5.2)$$

Considering the transformation $x_i = v\tan(\theta_i),\ i = 1,2.$ The Jacobian is $J = v^2 \sec^2(\theta_1)\sec^2(\theta_2)$. Consequently, the probability density function of a bivariate Semicircular distribution is obtained using simple algebra.

Similar to the method of constructing univariate l **- axial Logistic distribution,** it is simple to construct **bivariate** l **- axial Logistic distribution.** Let $\theta_i^* = \frac{2\theta_i}{l},\ i = 1,2$ and $l = 1,2,3,...,$ then the probability function is given by

$$g(\theta_1^*, \theta_2^*) = \frac{2l^2 \sec^2\left(\frac{l\theta_1^*}{2}\right)\sec^2\left(\frac{l\theta_2^*}{2}\right)}{\sigma} e^{-\frac{1}{\sigma}\left(\tan\left(\frac{l\theta_1^*}{2}\right)+\tan\left(\frac{l\theta_1^*}{2}\right)\right)}\left[1+e^{-\frac{1}{\sigma}\left(\tan\left(\frac{l\theta_1^*}{2}\right)\right)}+e^{-\frac{1}{\sigma}\left(\tan\left(\frac{l\theta_1^*}{2}\right)\right)}\right]^{-2}$$

$$-\frac{\pi}{l} < \theta_i^* < \frac{\pi}{l}, i = 1,2 \qquad (5.3)$$

Case(1) When $l = 1$ in (5.3) we get

$$g(\theta_1^*, \theta_2^*) = \frac{2\sec^2\left(\frac{\theta_1^*}{2}\right)\sec^2\left(\frac{\theta_2^*}{2}\right)}{\sigma} e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta_1^*}{2}\right)+\tan\left(\frac{\theta_1^*}{2}\right)\right)}\left[1+e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta_1^*}{2}\right)\right)}+e^{-\frac{1}{\sigma}\left(\tan\left(\frac{\theta_1^*}{2}\right)\right)}\right]^{-2}$$

$$-\pi < \theta_i^* < \pi, i = 1,2 \qquad (5.4)$$

We call it as bivariate circular Logistic distribution.

Case(2) When $l = 2$ in (5.3) give the **bivariate semicircular Logistic distribution.**

References

1. Byoung, J.A and Hyoung M.K.(2008). A New Family of Semicircular Models: The Semicircular Laplace Distributions,Communications of the Korean Statistical SocietyVol.15, pp. 775-781.
2. Dattatreya RaoA.V., PhaniY., GirijaS.V.S. Stereographic Logistic Model – Application to Noisy Scrub Birds Data, (2016), Chilean Journal of Statistics, , Vol. 7, No. 2, 69-79.
3. Gradshteyn and Ryzhik (2007). Table of Integrals, series and products, 7th edition, Academic Press.

4. Guardiola, J.H. (2004). The Semicircular Normal Distribution, Ph.D Dissertation, Baylor University, Institute of Statistics.
5. Jammalamadaka S. Rao and Sen Gupta, A. (2001). Topics in Circular Statistics, *World Scientific Press, Singapore.*
6. Jones, T.A. (1968). Statistical Analysis of orientation data, Journal of Sedimentary Petrology, 38, 61-67.
7. Girija, S.V.S., Radhika, A.J.V. & Dattatreya Rao, A.V. (2014f). *On Offset l-Arc Models*,Mathematics and Statistics, 2(3): pp. 127-136.
8. Phani, Y. (2013). On Stereographic Circular and Semicircular Models,Unpublished Ph.D. thesis from Acharya Nagarjuna University.
9. Phani, Y., Girija, S.V.S. & Dattatreya Rao, A.V. (2013a). *On Construction of Stereographic Semicircular Models*, Journal of Applied Probability and Statistics, Vol. 8, no. 1, May 2013, pp. 75-90.
10. Toshihiro Abe, Kunio Shimizu & Arthur Pewsey (2010). *Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection*, J Japan Statistical Society, 40 (1), 45-61.

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STEREOGRAPHIC *l* – AXIAL REFLECTED LOG - LOGISTIC DISTRIBUTION

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ABSTRACT

I*n this paper, we introduce a new symmetric circular distribution by using a technique of modified inverse stereographic projection on a reflected log-logistic distribution and its characteristic function.Population characteristics are also evaluated. And it is further extended to Stereographic l – axial Reflected Log – logistic distribution.*

Keywords: *Circular data, Stereographic projection, Symmetric distribution, Trigonometric moments.*

1. INTRODUCTION

The Log-logistic distribution is proposed by Shoukri *et al.* (1988) and has been used in hydrology for modeling stream flow rates and precipitation. Extreme values like maximum rainfall in oneday and river discharge per month or per year often follow the Log-logistic distribution.The Log-logistic distribution has been used as a simple model of the distribution of wealth or income in Economics where it is known as the Fisk distribution. The log-logistic has been used by Gago-Benítez *et al.* in 2013 as a model for the period of time beginning when some data leaves a software user application in a computer and the response is received by the same application after travelling through and being processed by other computers, applications and network segments, most or all of them without hard real time guarantees (for example, when an application is displaying data coming from a remote sensor connected to the Internet).

The Reflected Log-logistic distribution has received considerable attention as an appropriate model in various fields. The Reflected Log-logistic distribution can be introduced by reflecting Log – logistic distribution symmetrically to $(-\infty, 0]$. Glancing the literature, it appears that little attention was paid to construct circular models induced by inverse stereographic projection. Minh and Farnum (2003) proposed a new method of generating probability distributions by applying Stereographic Projection, which maps every point on the unit circle onto the point on the real line. Basing on this, Toshihiro Abe *et al.* (2010) constructed symmetric unimodal distributions by inverse stereographic projection. On the lines of Minh and Farnum and Toshihiro Abe *et al.* (2003; 2010), here an attempt is made to derive circular model by inducing inverse stereographic projection on Reflected Log – logistic distribution.

This paper is organized into five sections. In section 2 methodology of modified inverse stereographic projection is explained. Section 3 is devoted to present the proposed model i.e., Stereographic Reflected Log-logistic distribution. In section 4 its characteristic function and population characteristics are also evaluated and Program listings developed in MATLAB for graphs and computations of population characteristics are presented. Stereographic *l* – axial Reflected Log – logistic distribution is derived in section 5.

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2. METHODOLOGY OF MODIFIED INVERSE STEREOGRAPHIC PROJECTION [Phani et al., 2012]

Probability distributions (both circular and linear) can be generated by applying Stereographic Projection, which yields one to one correspondence between the points on the unit circle and those on the real line. Inverse Stereographic Projection is defined by a one to one mapping given by $T(\theta) = x = u + v\tan\left(\frac{\theta}{2}\right)$, where $x \in (-\infty, \infty)$, $\theta \in [-\pi, \pi)$, $u \in \mathbb{R}$, *and* $v > 0$. Suppose *x* is randomly chosen on the interval $(-\infty, \infty)$. Let $F(x)$ and $f(x)$ denote the cumulative distribution and the probability density functions of the random variable *X* respectively. Then $T^{-1}(x) = \theta = 2\tan^{-1}\left\{\frac{(x-u)}{v}\right\}$ is a random point on the unit circle. Let $G(\theta)$ and $g(\theta)$ denote the cumulative distribution and the probability density functions of this random point θ respectively. Then $G(\theta)$ and $g(\theta)$ can be written in terms of $F(x)$ and $f(x)$ using the following Theorem.

Theorem: For $v > 0$,

i) $$G(\theta) = F\left(u + v\tan\left(\frac{\theta}{2}\right)\right) = F\left(x(\theta)\right) \tag{2.1}$$

ii) $$g(\theta) = v\left(\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{2}\right) f\left(u + v\tan\left(\frac{\theta}{2}\right)\right) \tag{2.2}$$

3. STEREOGRAPHIC REFLECTED LOG – LOGISTIC DISTRIBUTION

Definition: A continuous random variable X is said to have a **Reflected Log-logistic model** with scale parameter $\alpha > 0$ and shape parameter $\beta > 0$, if its probability density function and distribution function are respectively given by

$$f(x) = \frac{\alpha\left(\frac{|x|}{\beta}\right)^{\alpha-1}}{2\beta\left[1+\left(\frac{|x|}{\beta}\right)^{\alpha}\right]^2},\ \alpha, \beta > 0,\ -\infty < x < \infty \tag{3.1}$$

$$F(x) = \frac{1}{2}\left[\frac{2+\left(\left|\frac{x}{\beta}\right|\right)^{-\alpha}}{1+\left(\left|\frac{x}{\beta}\right|\right)^{-\alpha}}\right], \alpha, \beta > 0,\ -\infty < x < \infty \tag{3.2}$$

Then applying inverse Stereographic projection defined by a one-one mapping $x = v\tan\left(\frac{\theta}{2}\right), -\pi < \theta < \pi,\ v > 0$, we get a two parametric Symmetric circular distribution on a unit circle ,whose pdf and cdf are respectively given by

$$g(\theta) = \frac{\alpha\sigma\sec^2\left(\frac{\theta}{2}\right)\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2},\ \text{where}\ \ \alpha, > 0, \sigma = \frac{v}{\beta} > 0, -\pi < \theta < \pi \tag{3.3}$$

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and

$$G(\theta)=\begin{cases}1-\frac{1}{2}\left[\frac{2+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}{1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}\right], \alpha,\sigma>0, -\pi<\theta<0 \\ \frac{1}{2}\left[\frac{2+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}{1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}\right], \alpha,\sigma>0, 0<\theta<\pi\end{cases} \tag{3.4}$$

Definition: A random variable X_S on unit circle is said to have Stereographic Reflected Log-logistic Distribution with shape parameter $\alpha>0$, scale parameter $\sigma>0$ and concentration parameter $\nu>0$, denoted by SRLLG(α,σ), then the probability density and cumulative distribution functions are given by

$$g(\theta)=\frac{\alpha\sigma\sec^2\left(\frac{\theta}{2}\right)\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2}, \text{where } \alpha,\sigma>0, -\pi<\theta<\pi \tag{3.5}$$

and

$$G(\theta)=\begin{cases}1-\frac{1}{2}\left[\frac{2+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}{1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}\right], \alpha,\sigma>0, -\pi<\theta<0 \\ \frac{1}{2}\left[\frac{2+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}{1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{-\alpha}}\right], \alpha,\sigma>0, 0<\theta<\pi\end{cases} \tag{3.6}$$

Hence the proposed new model SRLLG(α,σ) is a circular model named as "**STEREOGRAPHIC REFLECTED LOG - LOGISTIC MODEL".** It can be seen that the Stereographic Reflected Log - logistic Model is symmetric model.

Graphs of probability density function and cumulative distribution function of Stereographic Reflected Log-logistic Distribution for various values of α and σ are presented here.

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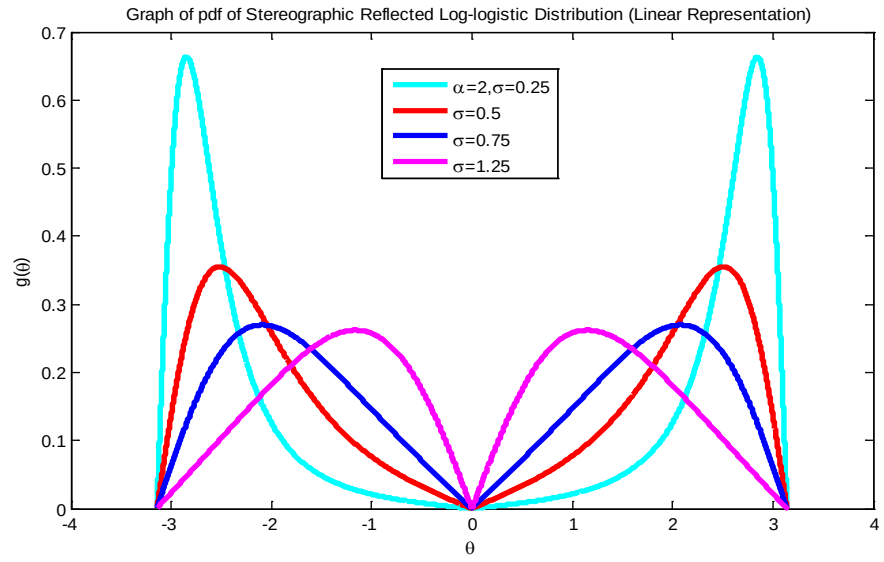

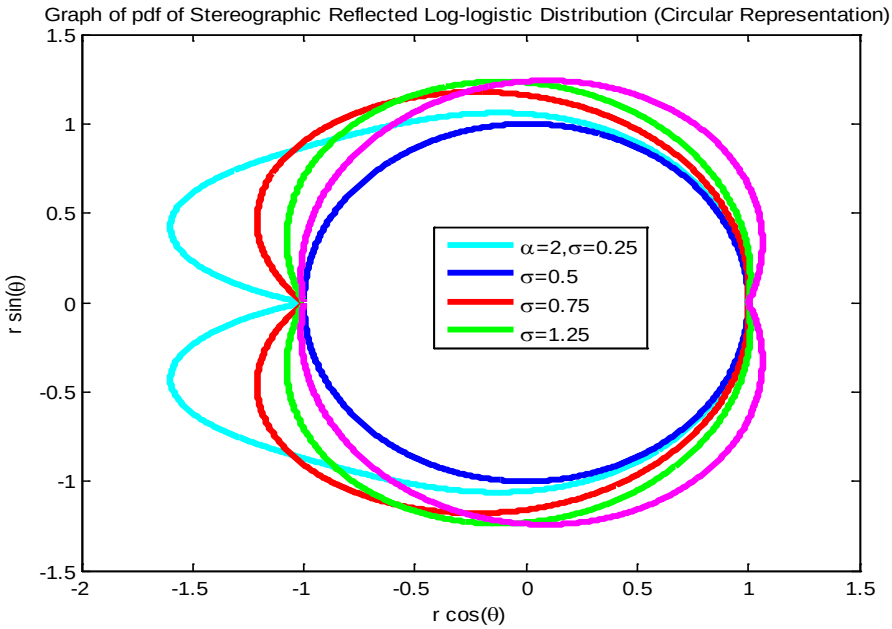

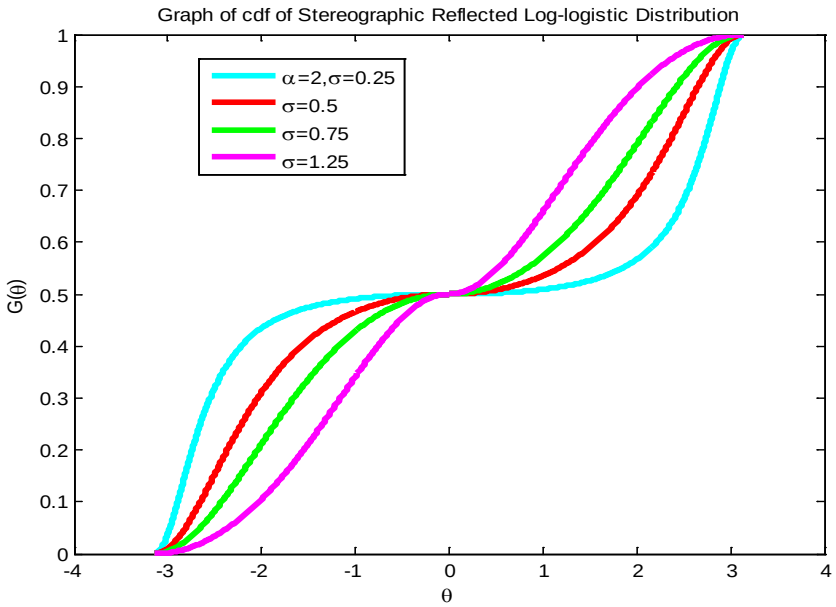

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4. THE CHARACTERISTIC FUNCTION OF STEREOGRAPHIC REFLECTED LOG-LOGISTIC MODEL

$$\Phi_{X_S}(p)=\int_{-\pi}^{\pi} e^{ip\theta} g(\theta)\,d\theta$$

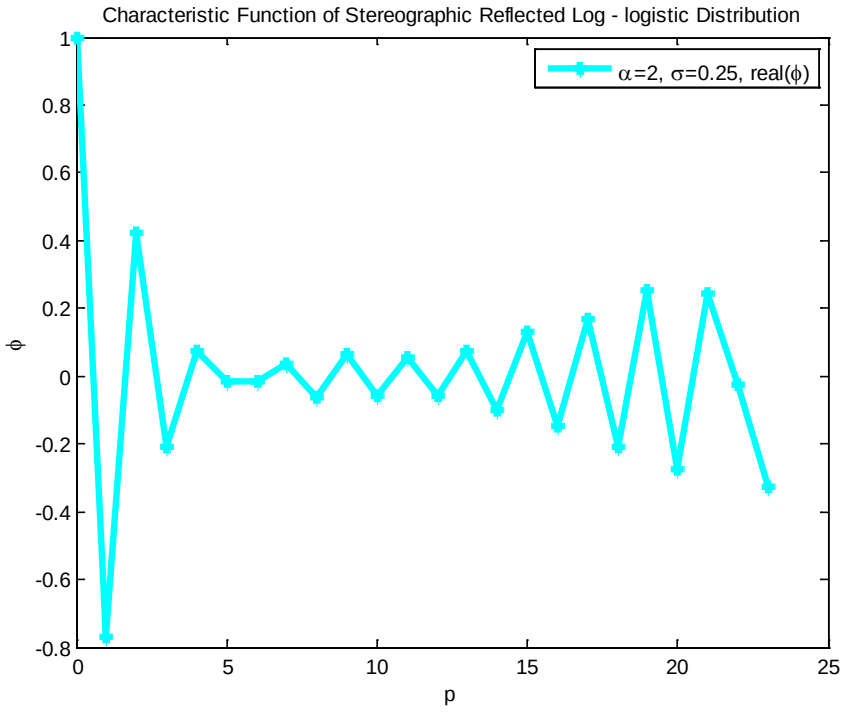
$$=\int_{-\pi}^{\pi}\cos p\theta\,\frac{\alpha\sigma\sec^2\left(\frac{\theta}{2}\right)\left[\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right]^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2}\,d\theta+i\int_{-\pi}^{\pi}\sin p\theta\,\frac{\alpha\sigma\sec^2\left(\frac{\theta}{2}\right)\left[\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right]^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2}\,d\theta$$

$$=\int_{0}^{\pi}\cos p\theta\,\frac{\alpha\sigma\sec^2\left(\frac{\theta}{2}\right)\left[\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right]^{\alpha-1}}{2\left(1+\left(\sigma\left|\tan\left(\frac{\theta}{2}\right)\right|\right)^{\alpha}\right)^2}\,d\theta\text{ , since }\sin p\theta\text{ is odd}$$

Trigonometric moments of the Stereographic Reflected Log-Logistic Model

The trigonometric moments of the distribution are given by $\{\varphi_p : p=\pm1,\pm2,\pm3,\ldots\}$, where $\varphi_p=\alpha_p+i\beta_p$, with $\alpha_p=E(\cos p\theta)$ and $\beta_p=E(\sin p\theta)$ being the pth order cosine and sine moments of the random angle θ, respectively. Because the **Stereographic Reflected Log-Logistic** distribution is symmetric, it follows that the sine moments are zero. Thus $\varphi_p=\alpha_p$.

The graphs of real part of the characteristic function are plotted here.



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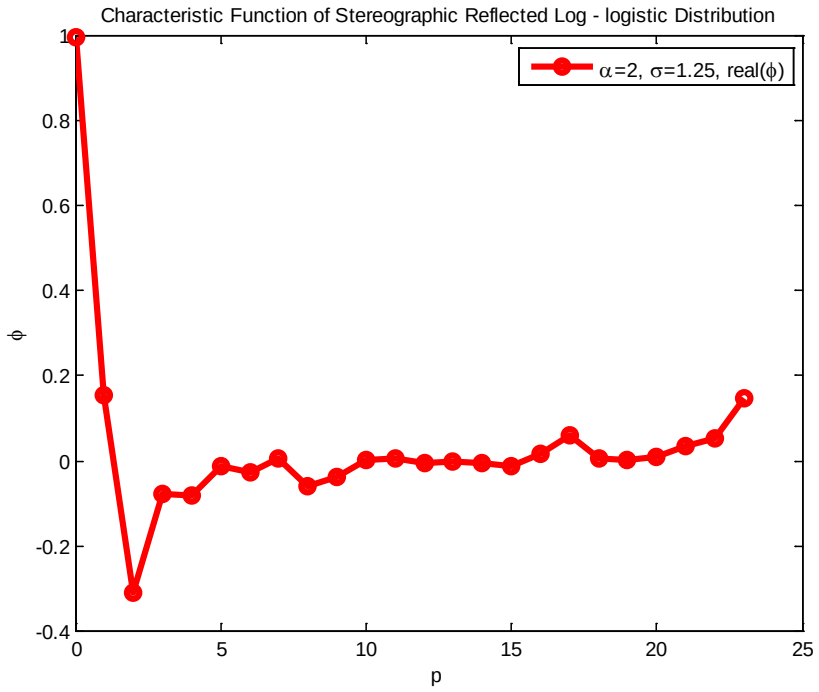

Mardia and Jupp (2000) gave expressions of mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions in terms of first four moments. These characteristics for the Stereographic Reflected Log - logistic model are also based on their respective trigonometric moments. These can be expressed in terms of trigonometric moments α_p and β_p and are presented here. If one has to evaluate the characteristic function for all integral values of p, numerical integration of Weddle's rule is to be used.

Table-4.1: Population Characteristics of SRLGLGD for $\alpha = 2$

	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$
Trigonometric Moments				
α_1	-0.7457	-0.4382	-0.1685	0.1548
α_2	0.3666	-0.0979	-0.3210	-0.3102
Resultant Length				
ρ_1	0.7457	0.4382	0.1685	0.1548
ρ_2	0.3666	0.0979	0.3210	0.3102
Mean Direction				
μ_0	0	0	0	0
Circular Variance				
v_0	0.2543	0.5618	0.8315	0.8452
Circular Standard Deviation				
σ_0	0.7661	1.2846	1.8874	1.9317
	1.4167	2.1558	1.5076	1.5302
Central Trigonometric Moments				
α_1^*	0.7457	0.4382	0.1685	0.1548
α_2^*	0.3666	0.3479	0.3210	0.3102
Kurtosis				
γ_2^0	0.8872	0.5934	0.4630	0.4333

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5. STEREOGRAPHIC *l* - AXIAL REFLECTED LOG – LOGISTIC DISTRIBUTION

We extend the above **Stereographic Reflected Log – logistic distribution** to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$ so it is desirable to extend the **StereographicReflected Log – logistic** distribution to construct the **Stereographic** l **-axial Reflected Log – logistic** distribution, we consider the density function of **Stereographic Reflected Log – logistic** distribution and use the transformation $\phi=\frac{2\theta}{l}$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{l\alpha\sigma\sec^2\left(\frac{l\phi}{2}\right)\left(\sigma\left|\tan\left(\frac{l\phi}{2}\right)\right|\right)^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{l\phi}{2}\right)\right|\right)^{\alpha}\right)^2},\text{where } \alpha,\sigma>0\ ,\frac{-\pi}{l}<\phi<\frac{\pi}{l} \tag{5.1}$$

We call it as **Stereographic -** l **-axial Reflected Log – logistic distribution**

Case-(1): When $l=1$, in the probability density function (5.1), we get the density function

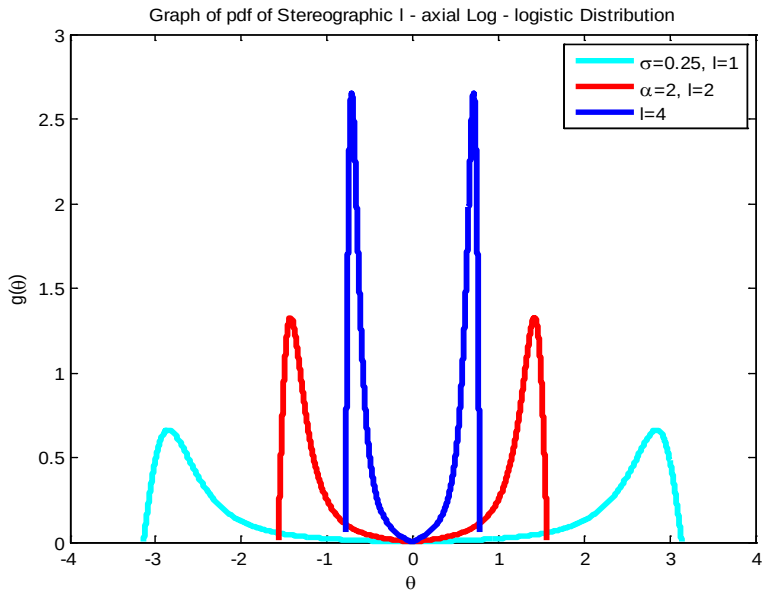
$$g(\phi)=\frac{\alpha\sigma\sec^2\left(\frac{\phi}{2}\right)\left(\sigma\left|\tan\left(\frac{\phi}{2}\right)\right|\right)^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan\left(\frac{\phi}{2}\right)\right|\right)^{\alpha}\right)^2},\text{where } \alpha,\sigma>0\ ,-\pi<\phi<\pi \tag{5.2}$$

Which is the density function of **Stereographic Circular Reflected Log – logistic distribution**.

Case-(2): When $l=2$, the probability density function (5.1) is the same as that of **Stereographic Semicircular Reflected Log – logistic Distribution**

$$g(\phi)=\frac{2\alpha\sigma\sec^2(\phi)\left(\sigma\left|\tan(\phi)\right|\right)^{\alpha-1}}{4\left(1+\left(\sigma\left|\tan(\phi)\right|\right)^{\alpha}\right)^2},\text{where } \alpha,\sigma>0\ ,\frac{-\pi}{2}<\phi<\frac{\pi}{2} \tag{5.3}$$

The graph of the pdf of **Stereographic** ***l*** **- axial Reflected Log – logistic Distribution** for various values of l is plotted.



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REFERENCES

1. DattatreyaRao, A.V., Ramabhadra Sarma, I. and Girija, S.V.S. (2007), On Wrapped Version of Some Life Testing Models, *Comm Statist, - Theor.Meth*. , 36, issue # 11, pp.2027-2035.
2. Dattatreya Rao,A.V., Girija,S.V.S., Phani.Y. (2011), Differential Approach to Cardioid Distribution, Computer Engineering and Intelligent Systems, Vol. 2, No.8, pp. 1-6.
3. Dattatreya Rao, A.V., Girija, S.V.S., and Phani, Y.(2016), Stereographic Logistic Model - Application to Noisy Scrub Birds Data, Chilean Journal of Statistics, Vol. 7, No. 2, pp.69-79.
4. Girija, S.V.S., (2010), Construction of New Circular Models, VDM – VERLAG, Germany.
5. Jammalamadaka S. Rao and Sen Gupta, A. (2001), Topics in Circular Statistics, *World Scientific Press, Singapore.*
6. Lukacs, E. and Laha, R.G. (1970), Applications of Characteristic Functions, *Charles Griffin and Company Limited, London, second edition.*
7. Mardia, K.V. andJupp, P.E. (2000), Directional Statistics, *John Wiley, Chichester.*
8. Minh, Do Le and Farnum, Nicholas R. (2003), Using Bilinear Transformations to Induce Probability Distributions, *Communication in Statistics – Theory and Methods*, 32, 1, pp. 1 – 9.
9. Phani, Y., Girija, S.V.S. and Rao, A.V.D., Circular model induced by inverse stereographic projection on extreme-value distribution. *Engineering Science and Technology*, 2 (5), pp. 881-888, (2012).
10. Phani, Y., Girija, S.V.S. & Dattatreya Rao, A.V. (2013). On *Construction of Stereographic Semicircular Models*, Journal of Applied Probability and Statistics, Vol. 8, no. 1, pp. 75-90.
11. RamabhadraSarma, I. A.V.Dattatreya Rao and S.V.S.Girija (2009), On Characteristic Functions of Wrapped Half Logistic and Binormal Distributions, *International Journal of Statistics and Systems*, Volume 4 Number 1, pp. 33–45.
12. Ramabhadra Sarma, I. A.V.Dattatreya Rao and S.V.S.Girija (2011), On Characteristic Functions of Wrapped Lognormal and Weibull Distributions, *Journal of Statistical Computation and Simulation*, Vol. 81, No. 5, 579–589 .
13. Toshihiro Abe, Kunio Shimizu and Arthur Pewsey, (2010), Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection, *J. Japan Statist. Soc.*, Vol. 40 (No. 1), pp. 45-61.
14. Shoukri, M.M.; Mian, I.U.M.; Tracy, D.S. (1988), "Sampling Properties of Estimators of the Log-Logistic Distribution with Application to Canadian Precipitation Data", The Canadian Journal of Statistics, 16 (3): 223–236, doi:10.2307/3314729, JSTOR 3314729.
15. Jump up Ashkar, Fahim; Mahdi, Smail (2006), "Fitting the log-logistic distribution by generalized moments", Journal of Hydrology, 328 (3–4): 694–703, doi:10.1016/j.jhydrol.2006.01.014.
16. Gago-Benítez, A.; Fernández-Madrigal J.-A., Cruz-Martín, A. (2013), Log-Logistic Modeling of Sensory Flow Delays in Networked Telerobots, IEEE Sensors 13(8), pp. 2944–2953.

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ON CONSTRUCTION OF DISCRETE WRAPPED LINDLEY DISTRIBUTION

By

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ABSTRACT

In this paper, a new discrete circular distribution is constructed by applying the method of discretization for existing continuous circular distribution. The Wrapped Lindley Distribution is discretized and new circular model is Discrete Wrapped Lindley distribution. The probability mass function, cumulative distribution function, and characteristic function of the Discrete Wrapped Lindley Distribution are derived and population characteristics are studied using first two trigonometric moments. The graphs of the probability mass function and cumulative distribution function are plotted for different values of parameter by using MATLAB.

Keywords: Probability Mass Function, Distribution Function, Characteristic Function, Trigonometric Moments.

INTRODUCTION

Directional data are collected in many scientific fields, including Biology, Geology, Geography, Meteorology, Physics, Political Science, Image Analysis, etc., and statistically analyzed. Examples of such applications of directional data are given in the much acclaimed book titled Directional Statistics (Mardia & Jupp, 2000). A set of observations on directions is referred to as directional data. In particular, directional data of two dimensions is called circular data. Directional data arise in the form of circular / semicircular / axial, symmetric / asymmetric, uni / bimodal data, in practical situations of varied fields listed above.

Circular models play a vital role in analyzing the angular data. Many number of continuous circular models are available in the literature. Circular distributions are of two types, they may be discrete, assigning probabilities to a finite number of directions or continuous assigning probabilities to an infinite number of directions. Angular models viz, circular, semicircular and I - axial models furnished in the previous chapters are continuous distributions. A cursory glance at the literature reveals that work on construction of discrete angular models is not paid much attention.

Variables, continuous functions, models, and equations can be converted to their respective discrete counterparts. In Mathematics, this transformation is known as Discretization and is applied in this paper to derive discrete circular model.

1. Review of Literature

All the required knowledge, concepts and expressions for the development of this thesis are considered from the illustrious book titled Directional Statistics, authored by (Mardia & Jupp, 2000) and a Textbook on "Topics in Circular Statistics", authored by (Jammalamadaka & Sengupta, 2001). Wrapped circular models (Girija, 2010), stereographic circular and semicircular models (Phani, 2013), Offset circular and semicircular models (Radhika, 2014), and I-axial models (Sastry, 2016) provide a rich and very useful class of models for circular as well as I-axial data. Yedlapalli, Girija, Rao, and Srihari (2015) discussed on Stereographic Double Weibull model. Yedlapalli, Girija, and Rao (2017) worked on I-axial Chi-Square Distribution and Devaraaj, Girija, and Rao (2017) proposed Cut-off points for various tests for Circular Uniformity. Scant attention was paid in construction of discrete circular models. There is a need to develop discrete circular models which are invariant of zero direction and sense of rotation.

Jacob and Jayakumar (2013) presented Wrapped Geometric Distribution. Girija, Rao, and Srihari (2014a) derived Wrapped Binomial model and the characteristics of Wrapped Poisson model were studied by Girija, Rao, and Srihari (2014b). Srihari, Girija, and Rao (2018a) derived the Discrete Wrapped Exponential Distribution. The Wrapped Negative Binomial model and the Wrapped Logarithmic Distribution are derived by (Srihari, Girija, & Rao, 2018b; 2018c), respectively.

2. Circular Distributions

A circular distribution is a probability distribution of assigning probabilities to the different directions on the circumference of the unit circle. The range of a circular random variable θ measured in radians, can be taken as $[0, 2\pi)$ or $[-\pi, \pi)$.

Circular distributions are of two types, they may be discrete, assigning probabilities to a finite number of directions or continuous assigning probabilities to an infinite number of directions. One of the construction methods of circular models is wrapping a linear distribution around a unit circle. Here the construction of discrete circular model by applying the methodology of wrapping is discussed.

2.1 Wrapped Discrete Circular Random Variables

If X is a discrete random variable on the set of integers, then reduction modulo $2\pi m (m \in Z^+)$ wraps the integers on to the group of mth roots of unity, which is a sub group of unit circle.

$\theta = 2\pi x \pmod{2\pi m}$

Since θ contains a finite number of elements they are denoted by,

$$\theta = \left\{ \frac{2\pi r}{m} \Big/ r = 0,1,2,...m-1 \right\}$$

2.2 Probability Mass Function

Suppose if θ is a wrapped discrete circular random variable then probability mass function of θ is denoted by $Pr\left(\theta = \frac{2\pi r}{m}\right)$, which satisfies the following properties.

1. $g_W(\theta) = Pr\left(\theta = \frac{2\pi r}{m}\right) \geq 0$

2. $g_W(\theta) = \sum_{r=0}^{m-1} Pr\left(\theta = \frac{2\pi r}{m}\right) = 1$

3. $Pr(\theta) = Pr(\theta + 2\pi l)$

   $\Rightarrow g_W(\theta) = g_W(\theta + 2\pi l)$ for any integer l (i.e Pr is periodic)

2.3 Distribution Function

Suppose if θ is a Wrapped discrete circular random variable then distribution function of θ is denoted by $G_w(\theta)$ and it is defined as,

$$G_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right) \quad \text{where } k = 0,1,...m-1$$

2.4 Characteristic Function

The Characteristic function of the Wrapped discrete circular random variable θ and these are related as,

$$\varphi_\theta(p) = \sum_{r=0}^{m-1} e^{ip\theta} Pr(\theta = \frac{2\pi r}{m}) = \rho_p e^{i\mu_p} \qquad \text{where } p \in Z$$

$$= \alpha_p + i\beta_p$$

One can also see these trigonometric moments α_p, β_p in terms of $\alpha_p = E(\cos p\theta)$, $\beta_p = E(\sin p\theta)$ and these are related as,

$$\rho_p = \sqrt{\alpha_p^2 + \beta_p^2} \text{ and } \mu_p = \tan^{-1}\left(\frac{\beta_p}{\alpha_p}\right)$$

2.5 Central Trigonometric Moments

The pth Central trigonometric moment of θ is defined as,

$$\varphi_\theta^*(p) = E\left[e^{ip(\theta-\mu)}\right] = \alpha_p^* + i\beta_p^*$$

Now the Skewness for the circular distribution is defined as $\gamma_1 = \frac{\beta_2^*}{(V_o)^{\frac{3}{2}}}$ and kurtosis for the Circular distribution is defined as,

$$\gamma_2 = \frac{\alpha_2^* - (1-V_o)^4}{(V_o)^2}$$

3. Discretization of a Continuous Distribution

In mathematics, discretization is the process of converting continuous functions, models, variables, and equations into its discrete counterparts. This process is usually carried out to model a given discrete phenomenon depending on the practical situation.

Wang and Shimizu (2014) initiated a method of transforming existing continuous circular models to construct discrete circular models. The process is as follows,

Consider a continuous circular distribution with Probability Density Function (PDF) f(θ) on the circle to construct a probability mass function $Pr\left(\theta = \frac{2\pi r}{m}\right)$ on a set of the equally spaced points for r=0, 1, 2, ...m-1 with a fixed integer m equal or greater than 2.

Then $Pr\left(\theta = \frac{2\pi r}{m}\right) = f\left(\frac{2\pi r}{m}\right) \Big/ \sum_{r=0}^{m-1} f\left(\frac{2\pi r}{m}\right)$ is Probability Mass Function (PMF) of respective discrete circular model. m=1 is excluded as it is degenerated.

4. Discrete Wrapped Lindley Distribution

The probability density function of wrapped Lindley distribution (Joshi & Jose, 2018) is defined as,

$$f(\theta) = \frac{\lambda^2}{1+\lambda} e^{-\lambda\theta}\left[\frac{(1+\theta)(1-e^{-2\pi\lambda}) + 2\pi e^{-2\pi\lambda}}{(1-e^{-2\pi\lambda})^2}\right] \text{ where } \lambda > 0 \text{ is a parameter}$$

Now the probability mass function of the Discrete Wrapped Lindley Distribution is

$$g_W(\theta) = Pr\left(\theta = \frac{2\pi r}{m}\right) = \left[\frac{\frac{\lambda^2}{1+\lambda} e^{-\lambda\theta}\left(\frac{(1+\theta)(1-e^{-2\pi\lambda}) + 2\pi e^{-2\pi\lambda}}{(1-e^{-2\pi\lambda})^2}\right)}{\sum_{r=0}^{m-1} \frac{\lambda^2}{1+\lambda} e^{-\lambda\theta}\left(\frac{(1+\theta)(1-e^{-2\pi\lambda}) + 2\pi e^{-2\pi\lambda}}{(1-e^{-2\pi\lambda})^2}\right)}\right]$$

$$= \frac{e^{-\lambda\theta}\left((1+\theta)(1-e^{-2\pi\lambda}) + 2\pi e^{-2\pi\lambda}\right)}{\sum_{r=0}^{m-1} e^{-\lambda\theta}\left((1+\theta)(1-e^{-2\pi\lambda}) + 2\pi e^{-2\pi\lambda}\right)}$$

$$\text{Now } \sum_{r=0}^{m-1}(1-e^{-2\pi\lambda})e^{-\lambda\theta} = (1-e^{-2\pi\lambda})\left[1 + e^{\frac{-2\pi\lambda}{m}} + \left(e^{\frac{-2\pi\lambda}{m}}\right)^2 + \left(e^{\frac{-2\pi\lambda}{m}}\right)^3 + \ldots\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\right] = \frac{\left(1-e^{-2\pi\lambda}\right)^2}{1-e^{\frac{-2\pi\lambda}{m}}}$$

$$\text{and } \sum_{r=0}^{m-1}(1-e^{-2\pi\lambda})\theta e^{-\lambda\theta} = (1-e^{-2\pi\lambda})\frac{2\pi}{m} e^{\frac{-2\pi\lambda}{m}}\left[1 + 2e^{\frac{-2\pi\lambda}{m}} + 3\left(e^{\frac{-2\pi\lambda}{m}}\right)^2 + \ldots(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-2}\right]$$

Then from Arithmetico-Geometric progression,

$$\sum_{r=0}^{m-1}(1-e^{-2\pi\lambda})\theta e^{-\lambda\theta}=\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[\frac{1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}-(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)}{\left(1-e^{\frac{-2\pi\lambda}{m}}\right)^{2}}\right]$$

Clearly $\sum_{r=0}^{m-1}2\pi e^{-2\pi\lambda}e^{-\lambda\theta}=2\pi e^{-2\pi\lambda}\frac{\left(1-e^{-2\pi\lambda}\right)}{1-e^{\frac{-2\pi\lambda}{m}}}$

$$Pr\left(\theta=\frac{2\pi r}{m}\right)=\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}}\right)^{2}e^{-\lambda\theta}\left((1+\theta)(1-e^{-2\pi\lambda})+2\pi e^{-2\pi\lambda}\right)}{s}\right]$$

with $s=\left(1-e^{-2\pi\lambda}\right)^{2}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)+2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right)$

$$+\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}-(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)\right]$$

The graph of Probability Mass function of Discrete Wrapped Lindley Distribution is drawn in MATLAB and presented in Figure 1.

5. Cumulative Distribution Function of Discrete Wrapped Lindley Distribution

The cumulative distribution function of Discrete Wrapped Lindley Distribution is defined as,

$$G_w(\theta)=\sum_{r=0}^{k}Pr\left(\theta=\frac{2\pi r}{m}\right)\quad\text{where}\quad k=0,1,2..m-1$$

$$=\sum_{r=0}^{k}\frac{e^{-\lambda\theta}\left((1+\theta)(1-e^{-2\pi\lambda})+2\pi e^{-2\pi\lambda}\right)}{\sum_{r=0}^{m-1}e^{-\lambda\theta}\left((1+\theta)(1-e^{-2\pi\lambda})+2\pi e^{-2\pi\lambda}\right)}\quad\text{where}\quad k=0,1,2..m-1$$

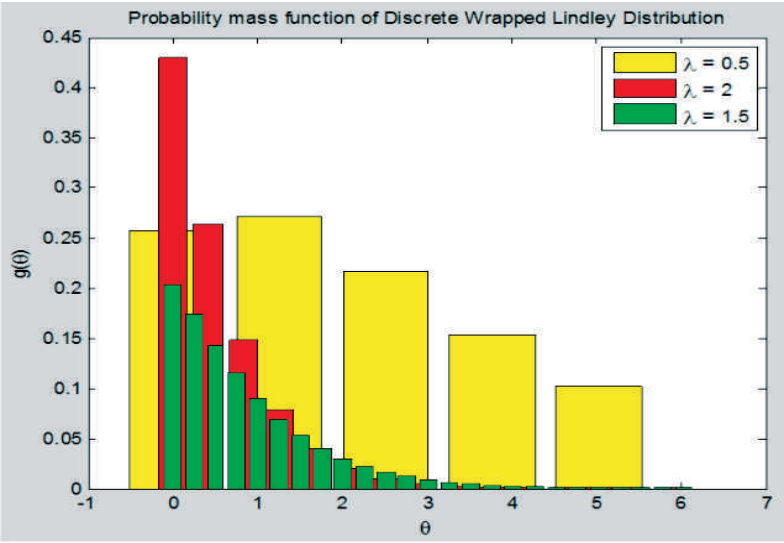

Figure 1. Probability Mass Function of Discrete Wrapped Lindley Distribution

$$= \sum_{r=0}^{k} \left[\begin{array}{l} \dfrac{\left(1-e^{\frac{-2\pi\lambda}{m}}\right)^2 e^{-\lambda\theta}\left[(1+\theta)(1-e^{-2\pi\lambda})+2\pi e^{-2\pi\lambda}\right]}{\left(1-e^{-2\pi\lambda}\right)^2\left(1-e^{\frac{-2\pi\lambda}{m}}\right)+2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right)} \\ +\dfrac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}-(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)\right] \end{array} \right]$$

Now $\sum_{r=0}^{k}(1-e^{-2\pi\lambda})e^{-\lambda\theta} = (1-e^{-2\pi\lambda})\left[1+e^{\frac{-2\pi\lambda}{m}}+\left(e^{\frac{-2\pi\lambda}{m}}\right)^2+\left(e^{\frac{-2\pi\lambda}{m}}\right)^3+\ldots\left(e^{\frac{-2\pi\lambda}{m}}\right)^k\right]$

$$= (1-e^{-2\pi\lambda})\left[\frac{1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{k+1}}{1-e^{\frac{-2\pi\lambda}{m}}}\right]$$

And $\sum_{r=0}^{k}(1-e^{-2\pi\lambda})\theta e^{-\lambda\theta} = (1-e^{-2\pi\lambda})\frac{2\pi}{m}e^{\frac{-2\pi\lambda}{m}}\left[1+2e^{\frac{-2\pi\lambda}{m}}+3\left(e^{\frac{-2\pi\lambda}{m}}\right)^2+\ldots k\left(e^{\frac{-2\pi\lambda}{m}}\right)^{k-1}\right]$

Then from Arithmetico-Geometric Progression,

$$\sum_{r=0}^{k}(1-e^{-2\pi\lambda})\theta e^{-\lambda\theta} = \frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[\frac{1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^k-k\left(e^{\frac{-2\pi\lambda}{m}}\right)^k\left(1-e^{\frac{-2\pi\lambda}{m}}\right)}{\left(1-e^{\frac{-2\pi\lambda}{m}}\right)^2}\right]$$

Clearly, $\sum_{r=0}^{k}2\pi e^{-2\pi\lambda}e^{-\lambda\theta} = 2\pi e^{-2\pi\lambda}\frac{1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{k+1}}{1-e^{\frac{-2\pi\lambda}{m}}}$

$$G_w(\theta) = \left[\begin{array}{l} \left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda(k+1)}{m}}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right)++2\pi e^{2\pi\lambda}\left(1-e^{\frac{-2\pi\lambda(k+1)}{m}}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right) \\ +\dfrac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^k-k\left(e^{\frac{-2\pi\lambda}{m}}\right)^k\left(1-e^{\frac{-2\pi\lambda}{m}}\right)\right] \\ \hline \left(1-e^{-2\pi\lambda}\right)^2\left(1-e^{\frac{-2\pi\lambda}{m}}\right)+2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right) \\ +\dfrac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}-(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)\right] \end{array} \right]$$

The graph of cumulative distribution function of Discrete Wrapped Lindley distribution is plotted in Figure 2.

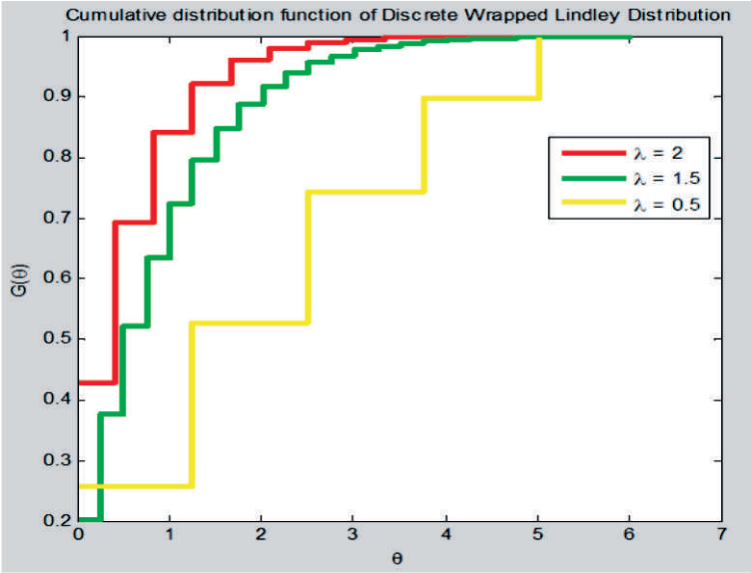

Figure 2. Cumulative Distribution Function of Discrete Wrapped Lindley Distribution

6. Characteristic Function of the Discrete Wrapped Lindley Distribution

The characteristic function of the Discrete Wrapped Lindley Distribution is defined as $\varphi_\theta(p) = E(e^{ip\theta})$, where p∈Z

$$= \sum_{r=0}^{m-1} e^{ip\theta} Pr\left(\theta = \frac{2\pi r}{m}\right)$$

For p∈Z,

$$\varphi_\theta(p) = \sum_{r=0}^{m-1} \left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}}\right)^2 e^{-\theta(\lambda-ip)}\left[(1+\theta)(1-e^{-2\pi\lambda})+2\pi e^{-2\pi\lambda}\right]}{\left(1-e^{-2\pi\lambda}\right)^2\left(1-e^{\frac{-2\pi\lambda}{m}}\right)+2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right)} + \frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}-(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)\right] \right]$$

Now $$\sum_{r=0}^{m-1}(1-e^{-2\pi\lambda})e^{-\theta(\lambda-ip)} = (1-e^{-2\pi\lambda})\left[1+e^{\frac{-2\pi(\lambda-ip)}{m}}+\left(e^{\frac{-2\pi(\lambda-ip)}{m}}\right)^2+\left(e^{\frac{-2\pi(\lambda-ip)}{m}}\right)^3 +\ldots\left(e^{\frac{-2\pi(\lambda-ip)}{m}}\right)^{m-1}\right]$$

$$= \left(1-e^{-2\pi\lambda}\right)\frac{1-e^{-2\pi(\lambda-ip)}}{1-e^{\frac{-2\pi(\lambda-ip)}{m}}} = (1-e^{-2\pi\lambda})^2\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)+ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}}{1-e^{\frac{-4\pi\lambda}{m}}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}}\right]$$

and $\sum_{r=0}^{m-1}(1-e^{-2\pi\lambda})\theta e^{-\theta(\lambda-ip)}=(1-e^{-2\pi\lambda})\frac{2\pi}{m}e^{\frac{-2\pi(\lambda-ip)}{m}}\left[1+2e^{\frac{-2\pi(\lambda-ip)}{m}}+3\left(e^{\frac{-2\pi(\lambda-ip)}{m}}\right)^2+\ldots(m-1)\left(e^{\frac{-2\pi(\lambda-ip)}{m}}\right)^{m-2}\right]$

Then from Arithmetico-Geometric Progression,

$$=\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)\left[\frac{e^{\frac{-2\pi(\lambda-ip)}{m}}-e^{-2\pi(\lambda-ip)}-(m-1)e^{-2\pi(\lambda-ip)}\left(1-e^{\frac{-2\pi(\lambda-ip)}{m}}\right)}{\left(1-e^{\frac{-2\pi(\lambda-ip)}{m}}\right)^2}\right]$$

$$=\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)\left[\frac{\begin{array}{l}\left(e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}-e^{-2\pi\lambda}-(m-1)e^{-2\pi\lambda}+(m-1)e^{\frac{-2\pi\lambda}{m}}e^{-2\pi\lambda}\cos\frac{2\pi p}{m}\right)\left(1+e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)\\+\left(e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}+(m-1)e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)\left(e^{\frac{-4\pi\lambda}{m}}\sin\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)\\+i\left[\left(e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}+(m-1)e^{\frac{-2\pi\lambda}{m}}e^{-2\pi\lambda}\sin\frac{2\pi p}{m}\right)\left(1+e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)\right.\\\left.-\left(e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}-e^{-2\pi\lambda}-(m-1)e^{-2\pi\lambda}+(m-1)e^{-2\pi\lambda}e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)\left(e^{\frac{-4\pi\lambda}{m}}\sin\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)\right]\end{array}}{\left(1+e^{\frac{-8\pi\lambda}{m}}+4e^{\frac{-4\pi\lambda}{m}}-4e^{\frac{-6\pi\lambda}{m}}\cos\frac{2\pi p}{m}+2e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-4e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}\right]$$

Clearly, $\sum_{r=0}^{m-1}2\pi e^{-2\pi\lambda}e^{-\theta(\lambda-ip)}=2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)+ie^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}}{1+e^{\frac{-4\pi\lambda}{m}}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}}\right]$

$\therefore \varphi_\theta(p)=\alpha_p+i\beta_p$

$$\alpha_p=d\left[\frac{\left(1-e^{-2\pi\lambda}\right)^2\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}{\left(1+e^{\frac{-4\pi\lambda}{m}}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}+\frac{2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}{\left(1+e^{\frac{-4\pi\lambda}{m}}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}\right.$$

$$\left.\frac{\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)\left[\begin{array}{l}\left(e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}-e^{-2\pi\lambda}-(m-1)e^{-2\pi\lambda}\right.\\\left.+(m-1)e^{\frac{-2\pi\lambda}{m}}e^{-2\pi\lambda}\cos\frac{2\pi p}{m}\right)\left(1+e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)\\+\left(e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}+(m-1)e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)\left(e^{\frac{-4\pi\lambda}{m}}\sin\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)\end{array}\right]}{\left(1+e^{\frac{-8\pi\lambda}{m}}+4e^{\frac{-4\pi\lambda}{m}}-4e^{\frac{-6\pi\lambda}{m}}\cos\frac{2\pi p}{m}+2e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-4e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}\right]$$

and

$$\beta_p = d\left[\frac{\left(1-e^{-2\pi\lambda}\right)^2\left(e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)}{\left(1+e^{\frac{-4\pi\lambda}{m}}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}+\frac{2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)}{\left(1+e^{\frac{-4\pi\lambda}{m}}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}\right.$$
$$\left.\frac{\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)\left[\begin{array}{l}\left(e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}+(m-1)e^{\frac{-2\pi\lambda}{m}}e^{-2\pi\lambda}\sin\frac{2\pi p}{m}\right)\left(1+e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)\\ -\left(\begin{array}{l}e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}-e^{-2\pi\lambda}-(m-1)e^{-2\pi\lambda}+\\ (m-1)e^{-2\pi\lambda}e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\end{array}\right)\left(e^{\frac{-4\pi\lambda}{m}}\sin\frac{4\pi p}{m}-2e^{\frac{-2\pi\lambda}{m}}\sin\frac{2\pi p}{m}\right)\end{array}\right]}{\left(1+e^{\frac{-8\pi\lambda}{m}}+4e^{\frac{-4\pi\lambda}{m}}-4e^{\frac{-6\pi\lambda}{m}}\cos\frac{2\pi p}{m}+2e^{\frac{-4\pi\lambda}{m}}\cos\frac{4\pi p}{m}-4e^{\frac{-2\pi\lambda}{m}}\cos\frac{2\pi p}{m}\right)}\right]$$

where, $$d=\left[\frac{\left(1-e^{\frac{-2\pi\lambda}{m}}\right)^2}{\left(1-e^{-2\pi\lambda}\right)^2\left(1-e^{\frac{-2\pi\lambda}{m}}\right)+2\pi e^{-2\pi\lambda}\left(1-e^{-2\pi\lambda}\right)\left(1-e^{\frac{-2\pi\lambda}{m}}\right)}\right.$$
$$\left.+\frac{2\pi}{m}\left(1-e^{-2\pi\lambda}\right)e^{\frac{-2\pi\lambda}{m}}\left[1-\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}-(m-1)\left(e^{\frac{-2\pi\lambda}{m}}\right)^{m-1}\left(1-e^{\frac{-2\pi\lambda}{m}}\right)\right]\right]$$

α_p and β_p are p^{th} order trigonometric moments,

Clearly $\rho_p=\sqrt{\alpha^2{}_p+\beta^2{}_p}$ and $\mu_p=\tan^{-1}(\beta_p/\alpha_p)$

Now μ_1 represents mean direction and $\rho_1(0\le\rho_1\le1)$ represents concentration towards the mean direction. In general $\mu_1=\mu$ and $\rho_1=\rho$.

With first two trigonometric moments, population characteristics are evaluated using MATLAB.

Conclusion

Discretization of continuous linear distributions to their discrete counterparts is available in the literature. Modifying this concept formula for discretization of continuous circular models to discrete circular models is developed and applied on continuous circular model, Wrapped Lindley distribution is used to construct Discrete Wrapped Lindley distribution. The characteristic function of newly proposed discrete circular model is derived and population characteristics are evaluated with first two trigonometric moments using MATLAB.

References

[1]. Devaraaj, V. J., Girija, S. V. S., & Rao, A. D. (2017). Cut-off points for various tests for circular uniformity. *i-manager's Journal on Mathematics,* 5(4), 29-38. https://doi.org/10.26634/jmat.6.4.13863.

[2]. Girija, S. V. S., Rao, A. D., & Srihari, G. V. L. N. (2014a). On wrapped binomial model characteristics. *Mathematics and Statistics,* 2(7), 231-234.

[3]. Girija, S. V. S., Rao, A. D., & Srihari, G. V. L. N. (2014b). On characteristic function of wrapped poisson distribution. *International Journal of Mathematical Archive,* 5(5), 168-173.

[4]. Girija, S. V. S. (2010). *Construction of New Circular Models,* VDM Verlag Dr. Müller GmbH & Co.

[5]. Jacob, S. & Jayakumar, K. (2013). Wrapped geometric distribution: A new probability model for circular data. *Journal of Statistical Theory and Applications,* 12(4), 348-355.

[6]. Jammalamadaka, S. R., & Sengupta, A. (2001). *Topics in Circular Statistics.* World Scientific Press, Singapore.

[7]. Joshi, S., & Jose, K. K. (2018). Wrapped lindley distribution. *Communications in Statistics-Theory and Methods,* 47(5), 1013-1021.

[8]. Mardia, K. V., & Jupp, P. E. (2000). *Directional Statistics.* John Wiley, Chichester.

[9]. Phani, Y. (2013). *On Stereographic Circular and Semicircular Models* (Unpublished Ph.D. Thesis, Acharya Nagarjuna University)..

[10]. Radhika, A. J. V. (2014). *Mathematical Tools in the Construction of New Circular Models* (Unpublished thesis submitted to Acharya Nagarjuna University).

[11]. Rao, A. V. D., Sarma, I. R., & Girija, S. V. S. (2007). On wrapped version of some life testing models. *Communications in Statistics-Theory and Methods,* 36(11), 2027-2035.

[12]. Sastry, Ch. V. (2016). *On I-Axial Models* (Unpublished PhD. Thesis, Acharya Nagarjuna University).

[13]. Srihari, G. V. L. N., Girija, S. V. S., & Rao, A. V. D. (2018a). On discrete wrapped exponential distribution-characteristics. *International Journal of Scientific Research in Mathematical and Statistical Sciences,* 5(2), 57-64.

[14]. Srihari, G. V. L. N., Girija, S. V. S., & Rao A.V. D. (2018b). On wrapped negative binomial model. *International Journal of Mathematical Archive,* 9(5), 1-6.

[15]. Srihari, G. V. L. N., Girija, S. V. S., & Rao, A.V. D. (2018c). On characteristics of wrapped logarithmic model. *International Journal of Current Trends in Science and Technology,* 8(4), 20413-20419.

[16]. Wang, M., & Shimizu, K. (2014). Discrete Cardioid Distribution. *Conference on Advances and Applications in Distribution Theory.* The Institute of Statistical Mathematics, Tokyo, Japan.

[17]. Yedlapalli, P., Girija, S. V. S., Rao, A. D., & Srihari, G. V. L. N. (2015). Symmetric circular model induced by inverse stereographic projection on double Weibull distribution with application. *International Journal of Soft Computing, Mathematics and Control,* 4(1), 69-76.

[18]. Yedlapalli, P., Girija, S. V. S., & Rao, A. V. D. (2017). On I-Axial Chi-Square Distribution. *i-manager's Journal on Mathematics,* 6(4), 51-58.

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Correlation between Bond Energy and Electronegativity Differences in a Few Semiconductors

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Abstract: ***Studies of bond energies of diatomic molecules are of great importance in thermochemistry, astrophysics and combustion physics. The bond energies of diatomic molecules and various semiconductors are evaluated. The bond energies are experimentally verified by thermal mass and spectroscopic techniques. The present method is an improvement over Pauling's relation. The bond energies evaluated by this present method are in excellent agreement with that evaluated by other methods. For e.g., in the case of BN (Boron Nitride) the bond energy, D_{AB} obtained by present study is 86.88 and that obtained by Pauling equation is 86.62. In the case of GaSb, it is 43.66 by the present equation is 43.66 by the Pauling equation it is 43.66. Also the average percentage deviation by the present study is 15.00, by the Pauling equation it is 14.20 and by Match et al it is 14.66. the studies of bond energies of various semiconductors taken up by the author are in good agreement with the evaluated equations proposed by literature, Pauling and Matcha.***

Keywords: ***Electronegativity, ionicity, energy gap, bond energy***

I. INTRODUCTION

In the chemical approaches taken for the approximate prediction of band gaps of semiconductors, correlations have been proposed between band gaps and various other quantities like single bond energies[1] , heats of formation per mole[2], heats of formation per equivalent[3], atomic numbers[4], electronegativities[5] etc. Some of these approaches especially those seeking relationships between bond energy and band gaps have also been presented in the form of reviews either detailed[6] or synoptic[1] .

According to Welker's postulate[7] the energy gap depends on binding energy and the effective charges of atoms, the energy gap being related to a great number of parameters[8-10] that are directly or indirectly connected with the binding energy or the lattice energy - of these the bond length taken by Goodman[11] leads to a linear relation between 'E_g' and 'd', where 'E_g' is the energy gap and 'd' is the bond length in diamond structures. According to Pauling[12], the covalent bond energy between two atoms sharing equal number of electrons at a minimum bond distance, is proportional to the reciprocal of bond length. So, the energy gap of a pure covalent semiconductor of diamond type structure will exhibit a similar relationship as a function of bond strength or an inverse function of bond length. Suchet[13] after a study of the importance of the covalent coordination and of the structure type in the prediction of the semiconductivity of inorganic compounds, relates the atomic and structural results to the value of the energy gap by means of empirical formulae allowing the computation of homopolar and heteropolar contributions to this value. The energy gap has a tendency to be progressively increasing in going from element of II-IV compound semiconductor. This tendency is a result of increasing degree of ionicity in the chemical bond. The difference between covalent and ionic bonding is the charge transfer that is involved in forming an ionic bond. Ge and Si are considered to be covalent semiconductors while lead salts are considered as polar semiconductors. Information concerning charge transfer and hence the degree of ionicity can be obtained by computing the static and optical dielectric constants. Pauling's[12] procedure gives fairly accurate estimate of bond energy between two uni-valent atoms held together by a single bond. However, this approach emphasizes the correlation between ionicity and type of bonding through electronegativities. Degree of ionicity in a chemical bond can be understood through bond energy. It was well emphasized by Pauling[14] in a graph between electronegativity difference and amount of ionic character. Recently Manca[1,15] has attempted to calculate the energy of a single bond for different semiconductors crystalizing in the diamond and blende structures by means of Pauling relation[16]. In this equation he utilised the work on electronegativities of elements by Gordy and Thomas[17].

In the absence of reasonably complete quantum mechanical calculations, empirical and semi-empirical approaches are employed in the construction of potential energy terms and to understand the nature of molecular binding and bond energies. Bond energies of diatomic molecules have a great importance in thermochemistry, combustion physics and astrophysics. In quantitative problems of

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valency, statistical calculations of equilibria at high temperatures, chemical bonding and in other problems, bond energies play a fundamental role. Astrophysicists, chemists and spectroscopists are therefore concerned with the determination of reliable values for bond energies of the diatomic molecules. Thermal and mass spectroscopies are the most important experimental methods used to evaluate bond energy values[18-20].

In view of the above importance a study of investigation of bond energies of various semiconductors is taken up.

II. METHODOLOGY

For calculations of bond energies some of the methods available are

A. Thermochemical Process

In this process the bond energy of NaCl is obtained by the equation

$$D(Na\text{-}Cl) = \Delta H_s + \Delta H_{diss} + \Delta H_f \quad \text{---------- (1)}$$

Where D(Na-Cl) is the required bond energy, ΔH_s is the heat of sublimation of Na metal per mole, ΔH_{diss} is the heat of dissociation of Cl_2 molecule into Cl atoms per mole, ΔH_f is the heat of formation of NaCl per mole in its standard state. The bond energy in eq.(1) is the heat of atomization of NaCl. In the case of NaCl, only one shared electron pair is involved in the bonding. In the case of complex compounds like Ta_2O_5 more than one bond (i.e., shared electron pair) is involved and hence a suitable normalizing factor must be used to obtain from eq.(1). The heat of atomization per mole is normalized with respect to the participating valencies to obtain heat of atomization per equivalent, which is the bond energy. For example heat of atomization per equivalent (i.e., the bond energy) for Ta_2O_5 is of the heat of atomization per mole as calculated from eq.(1).

The bond energies thus obtained after second order corrections (e.g., spin correlation stabilization energies and coordinate valence) as discussed by Howald[21] are the actual values of the average bond energies as obtained from experimental and theoretical data. The accuracy of these energies is limited only by the accuracy of the thermodynamic data used in computing them. Vijh[22] concluded that this procedure is strictly valid for alkali halides only, because alkali halides do not involve in any coordinate bonds.

B. Pauling Process

Pauling[14] stated that the energy of an actual bond between unlike atoms is greater than the energy of a normal covalent bond between the same atoms. This additional energy is due to ionic resonance or ionic character i.e.,

$$D(A\text{-}B) \geq [D(A\text{-}A) + D(B\text{-}B)] \quad \text{---------- (2)}$$

If the additional ionic resonance energy is indicated by Δ then eq.(2) can be written as

$$\Delta = D(A\text{-}B) - [D(A\text{-}A) + D(B\text{-}B)] \quad \text{----------(3)}$$

This postulate of additive law is not valid for alkali hydrides. When quantum mechanics is applied to study electron bonds, arithmetic mean can be replaced by geometric mean. The values of energies for normal covalent bonds obtained using geometric mean is involved is more satisfactory than that obtained using arithmetic mean.

$$\Delta(A-B) = D(A-B) - [D(A-A) + D(B-B)]^{1/2} \quad \text{------- (4)}$$

$\Delta(A-B)$ is obtained directly from heats of reaction using the relation

$$\Delta(A-B) = 30\,(XA - XB)^2 \quad \text{----------(5)}$$

Where X_A and X_B are the Pauling electronegativities of the atoms. The bond energy (K cal/mole) is evaluated through Pauling's[14] relation.

$$D(A-B) = [(D(A-A).D(B-B)]^{1/2} + 30\,[\Delta X]^2 \quad \text{---------(6)}$$

C. Matcha Process

Matcha[23] used simple quantum mechanical theory to study the interaction of potential between two atoms and obtained an appropriate form of bond energy, electronegativity relationship which is valid for both ionic as well as covalent bond. The final equation obtained is

$$D_{AB} = (1\text{-}f)\,D_{AB} + \frac{f}{R_e}\left[R\,D_{ion} - \frac{R_e E_{ct}}{f}\right] \quad \text{---------- (7)}$$

E_{ct} is the energy associated with partial charge transfer, D_{AB} is the average covalent bond energy, f is the effective charge transferred in the curve crossing region during bond formation and R_e is the inter nuclear distance of the molecule.

Substituting the constants E_{ct}, R_e and D_{ion}

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$$D_{AB} = (1\text{-}f)\, D_{AB} + 252 \frac{f}{R_e} \text{ ---------- (8)}$$

A comparison between $\frac{f}{R_e}$ and i (ionic character defined by Pauling) suggests a strong correlation between these quantities. This suggests such that D_{AB} can be related to ΔX by replacing $\frac{f}{R_e}$ by $\frac{2i}{3}$

Where I = $1 - e^{-i\frac{\Delta X^2}{4}}$

Matcha gave a relation for the construction electronegativity scale as

$$D_{AB} = D_{AB} + \frac{f}{R_e}[\,252 - R_e D_{AB}] \text{ -------(9)}$$

The term in the square brackets is approximately constant (169) for a series of bonds. Thus the above equation can be written as

$$D_{AB} \approx D_{AB} + 169 \left(\frac{f}{R_e}\right) \text{ ---------- (10)}$$

Replacing $\frac{f}{R_e}$ by $\frac{2i}{3}$

$$D_{AB} \approx D_{AB} + 113\, i \text{ ----------(11)}$$

To increase the accuracy of this equation, Matcha added two adjustable parameters k & i,

Thus $D_{AB} = D_{AB}$ ki ---------- (12)

For smaller values of ΔX, ki becomes

$$K\left[1 - e^{-\frac{\alpha X^2}{1}}\right]$$

Matcha found that k= 103 and $\alpha = 0.29$

Finally $D_{AB} = D_{AB} + K\left[1 - e^{-\frac{\alpha X^2}{1}}\right]$ ---------- (13)

D. Reddy et al., process:

The Pauling's empirical relation[16] fares badly when applied to ionic bonds. The error in estimated ionic bond energies is as much as 190%. So Reddy et al.,[24-26] worked out a relation, using which they calculated bond energies of both ionic and covalent molecules. The relation developed by them[24-26] is simple and straight forward, which is deduced as follows.

The empirical relation for bond energies proposed by Pauling is

$$D_{AB} = D_{AB} + 30\,[\Delta X]^2 \text{ ----------(14)}$$

Where $D_{AB} = [(D_{AA}.D_{BB})]^{1/2}$ and $\Delta X = X_A - X_B$ (Pauling electro negativity difference).

Equation (14) gives a reasonable accurate estimates of covalent bond energies. Matcha[23] derived an equation

$$D_{AB} = D_{AB}(1\text{-}f) + 252 \left(\frac{f}{R_e}\right) \text{ ---------- (15)}$$

It was later transformed to the following form with adjustable parameters k and i as,

$$D_{AB} = D_{AB} + k\, i \text{ ---------- (16)} \qquad \text{(where k=103)}$$

For ionic cases Reddy et.al., derived an empirical relation

$$D_{AB} = D_{AB}(1 - 0.2075\, \Delta X\, r_e) + 52.29\, \Delta X \text{ ---------- (17)}$$

On comparison of equations (15) and (17) we get

$$\frac{f}{R_e} = 0.2075\, \Delta X \text{ ---------- (18)}$$

But Matcha[23] gave $\frac{f}{R_e} = \frac{2i}{3}$ ----------(19)

From equations (18) and (19)

$$i = 0.31125\, \Delta X \text{ ---------- (20)}$$

substituting the value of I from equation (20) and k= 103 in Matcha's relation (16), Reddy et al., obtained

$$\boxed{D_{AB} = D_{AB} + 32.058\, \Delta X} \text{ --------- (21)}$$

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Table 1.
Single Bond Energies (K CAL / MOLE)

S. No.	Molecule	Single bond energy(K cal / mole)
1	B-B	70.00
2	Ai-Al	45.00
3	Ga-Ga	27.00
4	In-In	24.00
5	Si-Si	53.00
6	Zn-Zn	164.30*
7	Cd-Cd	90.13*
8	N-N	40.00
9	P-P	48.00
10	As-As	35.00
11	Sb-Sb	70.60
12	S-S	64.00
13	Se-Se	41.00
14	Te-Te	30.00
15	C-C	83.00

Table 2.
BOND ENERGY, D_{AB} OF SEMICONDUCTORS

S.No	Sem-iconductor	ΔX	Bond energy, D_{AB} (K cal / mole)			
			Literature [Ref.1,22&27]	Pauling Equ. (6)	Matcha Equ.(13)	Present study Equ.(21)
1	BN	1.06	95.69	86.62	81.55	86.88
2	BP	0.05	84.76	58.04	58.04	59.56
3	BAs	0.19	69.41	50.58	50.57	55.58
4	BSb	0.19	64.56	71.38	71.37	76.38
5	AlN	1.60	106.76	119.20	96.40	93.70
6	AlP	0.59	71.71	56.91	56.36	65.38
7	AlAs	0.73	58.11	55.67	54.43	63.08
8	AlSb	0.35	50.73	60.03	59.95	67.58
9	GaN	1.25	96.85	79.73	70.39	72.92
10	GaP	0.24	60.41	37.72	37.70	43.69
11	GaAs	0.38	48.88	35.07	34.96	42.91
12	GaSb	0.00	42.19	43.66	43.66	43.66
13	InN	1.58	85.78	105.87	84.04	81.62
14	InP	0.57	47.27	43.68	43.20	52.20
15	SiC	0.76	71.94	83.69	82.25	90.72
16	ZnS	0.78	127.54	120.70	119.20	101.23
17	ZnSe	0.18	87.84	83.04	83.03	88.31
18	ZnTe	0.35	81.42	73.81	73.80	81.63
19	CdS	0.98	107.35	104.76	100.98	84.86
20	CdSe	1.02	93.48	92.00	87.61	73.79
21	CdTe	0.55	69.62	61.07	60.64	67.56
Average percentage deviation				14.20	14.66	15.00

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III. RESULTS AND DISCUSSION

In order to evaluate the bond energies of different semiconductors, we use the experimental homo nuclear bond energies of constituent atoms of the semiconductor and Pauling electronegativity reported in Lide[27,] Pauling[14] Manca[1] and Sandersen[28]. The estimated bond energies from equations (6), (13) and (21) are presented in table 2. Bond energies obtained with the above formulae are in excellent agreement with the literature values[1,22,27]. The average percentage deviation in table 2 indicates that the calculated values obtained with the above formulae are in good agreement with the literature values. The magnitude of the electronegativity difference between two atoms indicates the degree of ionicity. It has been observed from the tables that the magnitude of ΔX of the semiconductor is below 1 in most of the cases. It indicates the nature of the bonding is covalent. It is noted that when different species of atoms are involved in covalent bonding, their electronegativities may influence their bonding. Electronegativity describes the pre-disposition of an atom to absorb electrons, its units are the square root of bond strength. Thus an atom with higher electronegativity will be more reactive chemically than one with a lower electronegativity. When the electronegativities with an atom engaged with covalent bonding are similar, this factor has little influence on bonding. However, when the difference in electronegativities of the two compounds is equal to or more than about 0.2 units, small amounts of ionic bonding may take place along with the covalent bonding. Larger electronegativity differences involve higher degree of ionicity in the bonding. It is probable that perfectly pre covalent bonding normally does not exists in compounds because no two atoms have identical electronegativities, small degrees of ionicity are present . BP, BAs, BSb, GaSb and ZnSe are some semiconductors will fall in this category. A careful observation of the table 2 reveals that their bond energies are below 90 K cal/mole.

According to Paulling[14], the percentage of ionic bonding in a compound increases from ionicity 0.2 to 0.63 to 0.89 as difference between the electronegativity values of its elements increases from 1 to 2 to 3, where the values of electronegativity is not sufficiently accurate in comparing the relative degree of ionicity in different compounds. The percentage of ionic bonding based on Pauling's criterion, is estimated to be 22 % in CdS and ZnS. The value of ionicity quoted by Phillips[29] are accurate in comparing the pualing's[14] values. The above study is extended to understand the relation between ionization potentials, energy gaps and optical electronegativity of few semiconductors.

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REFERENCES

[1] P. Manca, J. Phys. Chem. Solids, 20, 268(1961).
[2] W. Ruppel, A. Rose and H.J Gerritsen, Helv. Phy. Acta., 30.238 (1957).
[3] A. K. Vijh, J. Phys. Chem. Solids, 29, 2233 (1968).
[4] R. H. Bube, 'Photoconductivity of Solids', Wiley, New York (1960).
[5] N.B. Hannay, 'Solid State Chemistry', Prentice Hall, New York(1967).
[6] H. Gaatos and A. Rosenberg, 'The Physics and Chemistry of Ceramics', Gorden and Breach.
[7] H. Welker, Z. Naturf, 7a,774 (1952); 8a, 248 (1953).
[8] C.H.L Goodman, Proc. Phys. Lond., B67, 258 (1954).
[9] T. Miyaucht, J. Phys. Soc. Japan, 12, 308 (1957).
[10] F.N.Z Hooge, Phys. Chem., 24, 275 (1960).
[11] C.H.L. Goodman, J. Electronics, 1,115 (1955).
[12] L. Pauling, J. Phys. Chem. Solids, 58, 662 (1954).
[13] J.P. Suchet, J. Phys. Chem. Solids, 12, 74 (1959); 16,265 (1960).
[14] L. Pauling, 'The Nature of the chemical bond', Oxford & IBH Pub. C0., Delhi (1969).
[15] R.R Reddy, R. Viswanath and T.V.R. Rao, Proc. of Ind. Natn. Sci. Acad., 56, 421 (1990).
[16] L. Pauling, 'The Nature of Chemical bond', Cornell University Press, Ithaca, New York, 58-75 (1942).
[17] W. Gordy and D.E. Thomas, J. Chem. Phys., 24, 439 (1956).
[18] G. Herzberg, 'Molecular Spectra and Molecular Structure', Van Nostrand, New York (1955).
[19] D.L Hildenbrand and E. Murad, J. Chem. Phys., 51,807 (1969).
[20] M.R. Litzow and T.R. Splading,' 'Mass Spectrometry,' American Elsevier, New York (1973).
[21] R. Howald, J. Chem. Educ., 45, 163 (1968).
[22] Ashok K. Vijh, J.Phys. Chem. Solids, 30, 1999 (1969).
[23] R.L. Matcha, J.Am. Chem. Soc., 105, 4859 (1983).
[24] R.R. Reddy, T.V.R. Rao and R. Viswanath, J. Am. Chem. Soc., 111, 2914 (1989).
[25] R.R. Reddy and R. Viswanath, J. Astro phys. Astr., 10, 157 (1989).
[26] R.R. Reddy, T.V.R. Rao and R. Viswanath, Ind. J. Phys., 63B, 376 (1989).
[27] David R. Lide, 'CRC Hand book of Chemistry and Physics', 74 ed., CRC press, New York (1993-94).
[28] R.T. Sandersen, J. Am. Chem. Soc. 105, 2259 (1983).
[29] J.C. Phillips, Rev. Mod. Phys., 42,317 (1970).

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A Correlation between Hardness and Bond Ionicity in Crystals

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Abstract: ***Hardness is a degree of measurement of resistance in materials against permanent deformation. Materials of high hardness are made in use of designing and preparing tools, engine components and dyes. Bond ionicity and hardness are also related to ceramics and glasses. The prominent scale of Mohs is used to design scratch less glass and glasses which are resistant to heat, temperature are designed to shield the modern photo voltaic modules. Not much work is done in determining hardness of a material in terms of chemical properties like bond ionicity. The author attempted successfully in evaluating an empirical formula which relates bond ionicity to the hardness of a material.***
Keywords: Bond ionicity, hardness, compressibility, lattice energy, indentation, hardness scale

I. INTRODUCTION

The hardness of a material has been defined as "its resistance scratching'' or "to plastic deformation". The hardness is a measure of resistance to surface penetration by indenter with a force applied to it. The indentation process takes place by plastic deformation in metals and alloys. The hardness is inherently related to plastic flow resistance in these materials. However, brittle solids, such as glass and polycrystalline ceramics at room temperature are subjected to indentation, their hardness tested. The indentation is accomplished by applying a load to a suitable indenter positioned on the surface of the material. To test of the hardness, the indenter material must be harder than the material to be indented. Typical indenters are hardened steel and diamond. The hardness is tested by various methods of combinations of load and indenter on the material to be indented.

The hardness studies on some semiconducting compounds have been attempted by many authors[1]-[4]. Correlation lattice between the micro hardness and other characteristic material parameters like the coordination of atoms in the crystal lattice, the melting or Debye temperature, the interatomic distance, the average atomic mass of the constituent atoms of compounds, the bond ionicity and the elastic properties have been identified and discussed in the literature[5]-[7]. Several attempts have been made in the literature to correlate the bulk modulus and the microhardness. Plendtl et al [8],[9] assumed that with in a given class of non-metallic compounds the bulk modulus B directly proportional to the microhardness H. In the case of the tetrahedrally coordinated elemental and binary semiconductors the most successful attempt to quantify the relation between micro hardness and a few well known material parameters have been reported by Garbo et al. and Rucci [10]. In the case of the diamond and zinc blende semiconductors, it has been shown by various authors that the elastic properties depend only on a few, usually well known material parameters, namely the bond volume and the spectroscopically defined bond ionicity. Micro mechanical hardness testing provides useful information concerning the mechanical behaviour of metals and minerals. Buckle has pointed out the possibility of investigating various material properties by means of microhardness measurements. Plendl and Gieline[8],[9] have given a correlation between hardness, the lattice energy per unit volume and compressibility. Goble and Scott[11] have established a relation between second derivative of lattice energy and interatomic distance.

There are many scales for the measurement of hardness of materials[12]. Scales like Moh's hardness, Brinell hardness and Rockwell hardness are used to measure hardness. Other hardness tests are also common. The Vicker's hardness test, Microhardness and the knoop test are other types of tests available in the literature. Many attempts[13] have been made to relate the hardness of a crystal with many physical properties such as vibrational frequencies, atomic concentrations and surface energy. Oxides, sulphides and silicate compounds are hard materials and have high hardness. Sulphates, carbonates and phosphates are relatively soft. these common features of crystals suggest that stable chemical compositions consisting of different types of bond nature have strong correlation with the degrees of hardness. The bond ionicity is an important physical property which distinguishes a covalent crystal from an ionic crystal Yang et al[14] have established a relation between compressibility and hardness of the crystal. The present work is an attempt to find a definite correlation between hardness and ionicity of the crystals with different bonding.

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II. METHODOLOGY

For calculations of hardness the following different methods are available here with.

1) *Experimental details of hardness:* One of the most common hardness tests is in Brinell test[12]. In it, the indenter is a 10 mm diameter hardened steel ball, the applied mass is usually 300 kg, the test procedure is shown in fig.1. since a hard material will resist indentation more than a 'soft' material. The average indentation depth, surface area, and projected area are small if the material is hard and relatively large. The Brinell hardness number (BHN) is defined "As the load, 'F' divided by the surface area of indentation". BHN can be evaluated as follows:

$$BHN = \frac{2F}{\pi D\left[D-(D^2-d^2)^{\frac{1}{2}}\right]} = \frac{F}{\pi Dt} \quad \text{-------- (1)}$$

Where 'D' is the ball diameter, 't' is the indentation depth and 'd ' is the indentation diameter. In performing the test, 'F' and 'D' are known as priori. So measurements of 'd' allows calculation of the BHN. The BHN has dimensions of stress (N/m^2), but hardness numbers are often quoted in terms of mass, rather than force, per unit area eg. (Kg/m^2).

Vikers[15] micro hardness measures have been performed by Vickers indentor attached to the Universal Research Microscope. All the indentations were done at room temperature on freshly cleaned samples. The indentations were performed on a load of 100 P and the time of indentation being 10 seconds. The microhardness value is calculated using the expression.

$$BHN = \frac{1.8544\,P}{d^2} \quad \text{---------- (2)}$$

where 'P' is the load applied in grams and 'd' is the length of the diagonal of the indentor impression in mm. the length of the diagonal was measured using a micrometer. At least ten indentations were performed on each sample and a number of specimens were taken from each crystal . Microhardness value is an average of all such measurements.

2) *Julg Method*: Julg[16] proposed an empirical relation between the ionicity (I) and the Moh's hardness (H) of a crystal. The relation is as follows.

$$H = k-\frac{2}{3}\ K\ I^4 \quad \text{---------- (3)}$$

Here the value of 'K' is purely determined by the rows to which atoms in the chemical bond belongs in the periodic table. This means 'K' must be either related to atomic number, atomic weight or atomic radius. From the above equation it is inconceivable that which of these atomic properties come with the ionicity term to give the hardness value. Although the validity of the above equation has been claimed and verified by Julg. But it is admitted that a great uncertainty arises due to the presence of the fourth power of the ionicity term and therefore for weak bond ionicity values of electron charges and hardness estimated may not be accurate.

The procedure has been followed by Sreedharan Pillai[13] suggested a straightforward relationship between hardness (H) and ionicity (I) without incorporating additional parameters. The proposed empirical relation is in the form of second degree polynomial of the type:-

$H = aI^2 + bI + C$ ---------- (4) where a, b and c are the constants.

Yang et al[14], have been correlated the hardness of a solid and compressibility and proposed a relation as

$H = \frac{1}{n\beta}$ ---------- (5) of the solid this formula becomes

Where 'n' is the number of densities. In terms of weight density ρ (gram/cc)

$$H=\frac{M}{q\rho\beta} = \frac{M_a}{\rho\beta} \quad \text{----------(6)}$$

Where 'M' is the molecular weight, 'q' is the number of atoms in the molecule and $M_a = \frac{M}{q}$ is the average atomic weight.

3) Present method: Sreedharan Pillai has applied a second degree polynomial relation for some crystals only. In view of Moh's hardness data for different crystals listed by Yang et al., the quadratic relation of the following type is proposed.

$H = 14.455 – 27.511 + 16.002\ I^2$ -------- (7)

Where 'H' and 'I' are Moh's hardness and ionicity of the compounds respectively.

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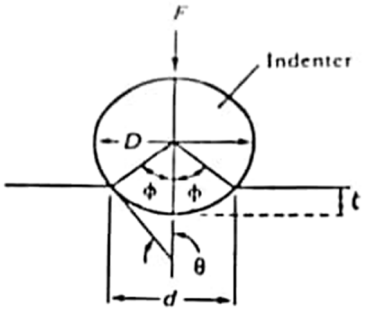

Figure 1

Table 1. Bond energy, D_{AB} of semiconductors

S.No	Compound	Ionicity(I)	Hardness, H (Mohs)			
			Experimental Equ.	Yang et al Equ. (6)		Present study Equ.(21)
1	AgBr	0,850	2.50	5.90		2.63
2	CuBr	0.785	2.40	4.96		2.72
3	CuCl	0.746	2.25	4.76		2.24
4	Cul	0.692	2.50	5.64		3.08
5	CaO	0.913	4.50	9.59		2.68
6	PbS	0.633	2.65	9.71		3.45
7	HgS	0.640	2.50	3.46		3.40
8	PbTe	0.551	3.00	8.36		4.16
9	SrO	0.926	3.50	9.08		2.70
10	SrS	0.914	2.61	---		2.68
11	CdTe	0.675	2.80	8.69		3.18
12	ZnTe	0.546	2.80	7.76		4.20
13	ZnS	0.623	3.75	9.06		3.53
14	SiC	0.177	9.50	15.35		10.09
15	ZnSe	0.676	3.75	7.92		3.17
16	CdS	0.685	3.20	9.18		3.12
17	CdSe	0.699	3.00	8.84		3.04
18	KBr	0.952	2.00	3.26		2.77
19	KCl	0.953	2.00	3.38		2.77
20	KF	0.955	2.80	3.78		2.78
21	KI	0.950	2.20	2.22		2.70
22	LlBr	0.899	2.50	3.22		2.66
23	LlCl	0.935	3.00	3.22		2.72
24	LiF	0.915	3.75	3.26		2.68
25	Lll	0.890	2.80	3.08		2.65
26	NaBr	0.934	2.45	3.02		2.72
27	NaCl	0.935	2.00	3.29		2.72
28	NaF	0.946	2.25	4.02		2.75
29	Nal	0.92	2.30	3.29		84.86

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30	RbBr	0.957	2.70	3.40		2.78
31	RbCl	0.955	2.60	3.45		2.78
32	RbF	0.960	2.30	4.01		2.79
33	Rbl	0.951	2.68	3.32		2.76
34	BaS	0.935	3.00	6.87		2.72
35	BaF_2	0.890	3.00	6.77		2.65
36	CaF_2	0.980	4.00	7.04		2.86
37	CdF2	0.790	4.00	6.74		2.71
38	SrF_2	0.900	3.50	6.90		2.66
39	$CaCl_2$	0.890	3.00	3.94		2.65
40	Al_2O_3	0.450	9.00	12.45		5.32

III. CONCLUSIONS

The bond ionicity data are available for many crystals and minerals[13],[18]. Recently Yang et al[14]., have given an extensive hardness data for different crystals. This prompted the author to correlate the hardness and the ionicity of the crystal. The quadratic form of the relation yielded two values of l, of which one was always very small or negative. Higher values were considered for covalent nature of chemical bond eq.(7), is found suitable in estimating the hardness of the crystal, the evaluated values are in good agreement with the hardness values quoted by Yang et al[14].

The bond between the positive and negative ions in an ionic compound results exclusively due to electrostatic attraction. The electron cloud repels the positive, negative ions to come too close to each other. As the two ions approach each other, the spherical electron cloud of the negative ion become much distorted by the influence of the electron cloud of positive ion which results in gradual change of ionic picture to covalent nature of crystal. There are the two extreme types of bond character. In mixed bond nature, as the anions are larger than cations, a portion of electron cloud of the anions becomes pulled over towards the region of the cation. As the distortion is increased, the bond between anion and cation takes more and more covalent character[13]. The bond energies of different semiconductors, were given by Sanderson[17].

This work is a part of M.Phil., titled, 'Bonding Characteristics of Materials' submitted by the author at S K University during 1997.

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REFERENCES

[1] A S Borshchevskii, N A Goryvnova and N K Takhtareva, Soviet Phys. Tech. Phys., **2.** 1301 (1957)
[2] B Stone and D Hill, Phys. Rev. Letters, 4, 282 (1960)
[3] M S Albova, A R Regal,Soviet Phys. – Solid State, 4, 282 (1962)
[4] J W Allen, Phil. Mag., 2, 1475 (1957)
[5] G Beckmann Kristall und Technik, 6, 109 (1971)
[6] A S Borsh Chevskil, N A Goryunova and N K Takhtareva, Z Tekh., F_{1z} 27,1408 (1957)
[7] VM Gold Shmidt, USP. F_{1z} Nauk, 9, 811 (1929)
[8] J N Plendl,S S Mittra P J Gieline, Phys Rev., 125, 828 (1961).
[9] J N Plendl,S S Mittra P J Gieline, Phys Stat. Solid, 12, 367 (1965).
[10] L Garbato and A Rucci, Phil. Mag., 35, 1681 (1977)
[11] R Y Goble and S D Scott, Can. J. Mineral, 23, 273 (1985)
[12] T H Courtney, 'Mechanical behavior of Materials', Mc Graw-Hill Pub. Co., New York (1990)
[13] K Sreedharan Pillai, Ind J Pure and Appl. Phys., 20, 46 (1982)
[14] Weitao Yang, Robert G. Parr and Lieve Uytterhoeven, Phys., Chem. Minerals, 15, 191 (1987)
[15] C Hays E G Kendal, Metallography, 6, 275 (1973)
[16] R T Sandersen, J. Am. Chem.Soc, 105, 2259 (1983)
[17] J C Phillips, Rev. Mod. Phys., 42, 317 (1970)

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Employees' attitude towards information technology

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Abstract: *In this paper an attempt has been made to find out whether the employees have a faourable attitude towards Information Technology and difference of attitudes by gender wise, age wise, job nature wise by using statistical tools like Tables, Diagrams, Tests of hypothesis etc., to analyze and present the data. It is found that the nature of jobs (Government or Private) and age influence the attitude towards Information Technology. But the gender differences may not have significant influence on the attitude towards Information Technology.*
Key words: *Information Technology, capabilities, educated illiterate*

INTRODUCTION

Now a days Computers and communications are becoming integral parts of our lives. The Information Technology or IT is revolutionizing the way, in which we live and work. It is challenging all aspects of our life and life style. The digital revolution has given mankind the ability to treat information with mathematical precision, to transmit it at very high accuracy and to manipulate it at will. These capabilities are bringing into being a whole world within and around the physical world. The amount of calculational power that is available to mankind is increasing at an exponential rate. A few decades back communications used to be between people, one person to another. But now inanimate objects are getting into the act, books can tell the cash registers how much they cost, identity cards can tell the door lock whether to open or not, automated guided vehicles can tell the post computer where they are in the shop floor and what they are carrying and when they will be free, missiles can compare the landscape with their own map and hit the target with pin point precision. On the internet people engage in lively chats and discussions, shopping , play games even if they are physically into different continents.

It is known fact that no field is untouched by computers. Unless one has the ability to make use of computers in the respective fields, he/she is considered to be an illiterate even though he/she is educated. Otherwise, he/she is known as an educated illiterate in the modern era. Even though, having a computer is considered to be a status symbol and many people are posing, as if they are using the computers effectively. They do not possess adequate knowledge about computers and their operating procedures. It is because of the fact, that many people are very much afraid of operating the computers, as the operations involve many technical terms or jargons. Therefore, on many occasions they keep themselves a little away from the computer circle, even though the

computers have a lot of applications and user friendly in nature.

At present people feel that the world has become a small village because one can talk to anyone, enter into library, seek any information, discuss on any academic problem, share any idea, search the information which is not commonly available, etc.,. This shows that one has an access to every nook and corner of the world. All this has been made possible due to invention of internet, Voice mail, E-mail, Cyber café etc. Recently virtual university has come to existence; people are studying in virtual classrooms, one need not to travel to other countries and can complete education staying at his own place of residence. This shows that accessibility has increased very rapidly.

INDIA-INTERNET USAGE STATS AND TELECOMMUNICATIONS MARKET REPORT

TABLE-1

1998	1,400,000	1,094,870,677	0.1 %	ITU
1999	2,800,000	1,094,870,677	0.3 %	ITU
2000	5,500,000	1,094,870,677	0.5 %	ITU
2001	7,000,000	1,094,870,677	0.7 %	ITU
2002	16,500,000	1,094,870,677	1.6 %	ITU
2003	22,500,000	1,094,870,677	2.1 %	ITU
2004	39,200,000	1,094,870,677	3.6 %	**C.I. Almanac**
2005	50,600,000	1,112,225,812	4.5 %	**C.I. Almanac**
2006	40,000,000	1,112,225,812	3.6 %	**IAMAI**
2007	42,000,000	1,129,667,528	3.7 %	**IWS**
2009	81,000,000	1,156,897,766	7.0 %	**ITU**
2010	100,000,000	1,173,108,018	8.5 %	**IWS**
2012	137,000,000	1,205,073,612	11.4 %	**IAMAI**
2015	375,000,000	1,251,695,584	30.0 %	**IAMAI**
2016	462,124,989	1,266,883,598	36.5 %	**IAMAI**

The world is changing at an incredible pace. The agricultural revolution and the industrial revolution are followed by information revolution and knowledge revolution. Today the disparity is not so much between those who 'have more' and those who 'have less' but between those who 'know more' and those who 'know less'. It was Peter Drucker, the great management expert

who coined the term 'knowledge workers' several years ago. Knowledge is doubling in every two years in almost all occupations. That means we have to keep pace with it by doubling our knowledge every two years even to stay where we are. Otherwise instead of forging ahead we fall behind. Instant communication across the seven continents has become a common day-to-day experience. Distance is abridged and the world is shrinking. The global society which was envisaged by our ancient seers and visionaries in the concepts of 'Vasudhaika kutumbam' and 'Sarvejana sukhinobhavanthu' can become a reality today on account of TV and Internet, Web and other tools of Information Technology.

Here an attempt has been made to find out whether the employees have a favourable attitude towards computers. So the study entitled **"EMPLOYEES' ATTITUDE TOWARDS INFORMATION TECHNOLOGY"** has been undertaken.

OBJECTIVES OF THE PRESENT STUDY

- To study the employees' attitude towards Information Technology
- To study the difference of attitude towards Information Technology between Government & Private employees
- To study the difference of attitude towards Information Technology between Male & Female employees
- To study the difference of attitude towards Information Technology between the employees below 40years of age and above 40years of age

METHODOLOGY

The present research study mainly focus on the employees' general attitude towards computer. It also deals with various other issues of comparing the attitudes of Government and Private employees, Male and Female employees, the employees below 40 years and above 40 years of age.

The field work was undertaken in Kavali mandal in Nellore district of Andhra Pradesh. For the purpose of survey , I met different employees in different departments. I have selected a sample of employees of size 200 using Stratified Random Sampling technique. I have selected the sample randomly from different departments, which covers Government and Private employees, Male and Female employees of different age groups. Here all the employees of private institutions, corporations like RTC, LIC , Banks etc., are treated as Private employees and all the Government departments including Government Aided institutions are treated as Government employees.

TOOL

For the present study I prepared a questionnaire comprising of 21 questions by which the awareness of Information Technology can be assessed. I have used two options as well as three option questions. For two option questions I have given the weightage as 1 and 0 for yes or no respectively. For three option questions I have given the weightage as 2,1 and 0 respectively. An individual can get a score range from 0 to 26. So if anyone gets a score of 13 and above I considered him as favourable towards Information Technology.

I used statistical tools like tables, diagrams, tests of hypothesis etc., to analyze and present the data.

ANALYSIS

After considering the awareness scores I found that 125 employees out of 200 have favourable attitude towards Information Technology, which says that 62.5% of the respondents have favourable attitude towards Information technology and 37.5% of the respondents have unfavourable attitude towards Information Technology.

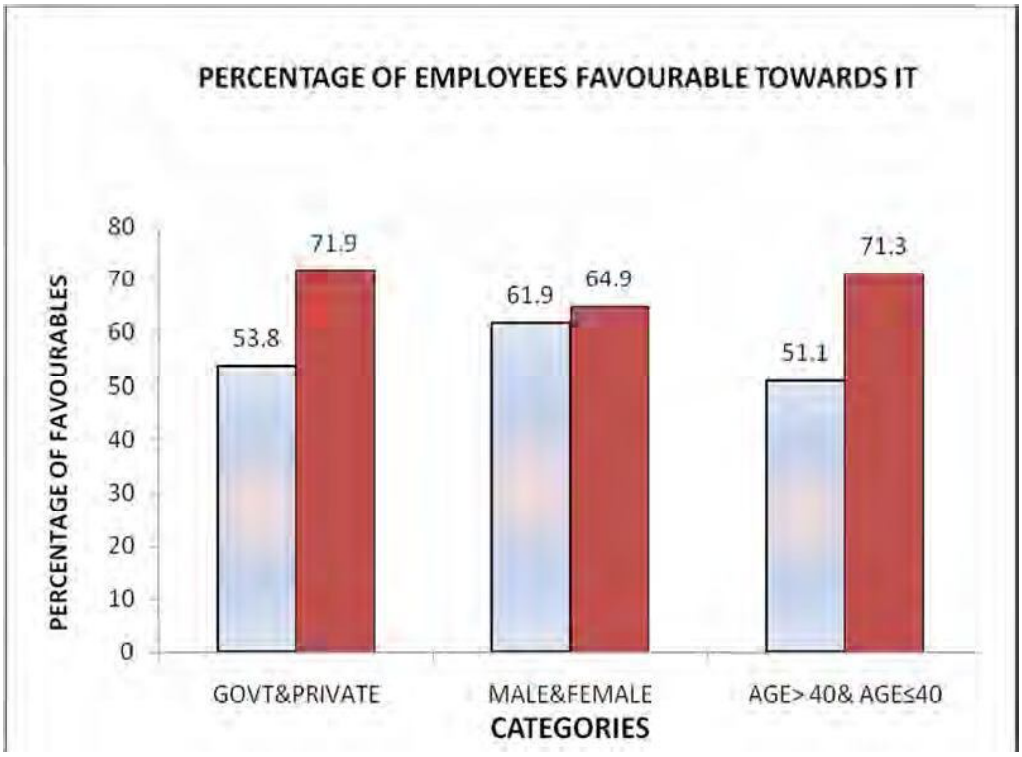
If we consider the category wise responses, 56 Government employees out of 104 have favourable attitude towards Information Technology, which says that 53.5% of the respondents have favourable attitude towards Information technology and 46.5% of the respondents have unfavourable attitude towards Information Technology. Similarly 69 Private employees out of 96 have favourable attitude towards Information Technology, which says that 71.9% of the respondents have favourable attitude towards Information technology and 28.1% of the respondents have unfavourable attitude towards Information Technology. 78 Male employees out of 126 have favourable attitude towards Information Technology, which says that 61.9% of the respondents have favourable attitude towards Information technology and 38.1% of the respondents have unfavourable attitude towards Information Technology. 48 Female employees out of 74 have favourable attitude towards Information Technology, which says that 64.9% of the respondents have favourable attitude towards Information technology and 35.1% of the respondents have unfavourable attitude towards Information Technology. 47 members of Age above 40 employees out of 92 have favourable attitude towards Information Technology, which says that 51.1% of the respondents have favourable attitude towards Information technology and 48.9% of the respondents have unfavourable attitude towards Information Technology. 77 members of Age below 40 employees out of 108 have favourable attitude towards Information Technology, which says that 71.3% of the respondents have favourable attitude towards Information technology and 28.7% of the respondents have unfavourable attitude towards Information Technology.

The following table gives category wise responses of survey and Category wise percentage of employees favourable towards Information Technology.

TABLE-2: CONSOLIDATION OF SURVEY

CATEGORY	*Total*	*Favourable*	*Not Favourable*	*Percentage of Favourables*
Total number of Employees considered for the survey	200	125	75	62.5
Government Employees	104	56	48	53.8
Private Employees	96	69	27	71.9
Male Employees	126	78	48	61.9
Female Employees	74	48	26	64.9
Age above 40	92	47	45	51.1
Age <=40	108	77	31	71.3

We can display the above particulars using multiple bar diagram by which an ordinary man can also understand easily.



Now for the purpose of testing the following null hypotheses were formulated:

- There is no significant difference between **Government & Private** employees in respect of their attitude towards IT
- There is no significant difference between **Male &Female** employees in respect of their attitude towards IT
- There is no significant difference between employees of **age below 40** and **age above 40** in respect of their attitude towards IT

As we are having the proportions of favourables for different categories , to test the above hypotheses we have to adopt **" *Test for Proportions"***. Being the sample size is 200, these are large sample tests. The test statistic for this type of test is

$$t_c = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

The details of calculations for this test and results obtained are shown in the following table

TABLE-2: TESTS OF HYPOTHESES FOR PROPORTIONS OF DIFFERENT CATEGORIES

Category	*Sample size*	*Favourable*	*Proportions of FavourableS*	*Critical value*	*Table value*	*Significance at 5% LOS*
Government	104	56	0.538	2.6413	1.96	Significant
Private	96	69	0.719			
Male	126	78	0.619	0.4242	1.96	Not significant
Female	74	48	0.649			
Age above 40	92	47	0.511	2.9334	1.96	Significant
Age below 40	108	77	0.713			

From the above table we can say that the first and third hypotheses are significant at 5% level of significance and the second hypothesis is not significant at 5% level of significance.

CONCLUSION

Thus the present study has shown that

- The majority of the employees are favourable attitude towards Information Technology
- The Private employees are favourable towards Information Technology than Government employees
- The employees of age below 40 years are favourable towards Information Technology than above 40 years
- There is no significant difference between the attitudes of Male and Female employees towards Information Technology

Totally we can conclude that the nature of jobs (Government or Private) and age influence the attitude towards Information Technology. But the gender differences may not have significant influence on the attitude towards Information Technology.

References

Milliron, M.D. (2000). Internet Vision: Unleashing the power of the internet in the higher education enterprise. In M.D. Milliron, and C. Miles, (Eds.), Taking the Big Picture Look at Technology Learning and the Community College, (pp.317-331).

Mission Viejo: League for Innovation in the Community College. Networks and Development Foundation (2002, February). Languages and Cultures. [Online] http://www.funredes.org/LC/L5/ultimas.html [June 10, 2003].

Brown, M M. (April 2001). Can ICTs address the needs of the poor? UNDP News, 2-5. Byron, I. and

Gagliardi, R. (August 2000). Communities and the information society: The role of Information and Communication Technologies in education. [Online] http://www.anais.org/ARTICLES/DOC22.HTML [June 23, 2001]

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Tobacco product users in Kavali Town- a Sample Survey

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Abstract: *Prevalence of all kinds of tobacco consumption is higher among poor income groups in India. Roughly 85% of the tobacco consumption in India is non-cigarette type i.e.. In the form of Bidi (It is an indigenous tobaccoprepartion in India made by rolling a dried piece of Temburini leaf) (Diospyros melnoxylon) with 0.15 to 0.25 g of sun-dried flaked tobacco into a conical shape and securing the roll with a thread. As per the interpretations in Kavali the percentage of men and women consuming more compared with estimated National data. Hence, it is suggested that the Govt., and Non-Govt., Social Service Organizations are requested to concentrate on this issue and enlighten the public through canvass and other media to eradicate the tobacco product usage by the householders to reduce their financial burden and also medical expenditure to cure various tobacco related diseases.*

Key words: *neo-plastic diseases, smokers, drinking alcohol*

Introduction:

Tobacco has become the predominant cause of preventable deaths in the world (Reddy & Gupta 2004). In India estimated 60 % of men and 33% of women consume tobacco products. India is home to nearly 17 % of the smokers in the world. Probability of various respiratory, vascular and neo-plastic diseases and mortality rate are found to be higher among tobacco users compared to non-users. Estimates of household expenditure from National Sample Survey Organist ion (NSSO) in India on consumption of addictive goods account for 4.4% of total budget of a household who choose to consume any of the addictive goods viz., Smoking, Chewing tobacco, use of snuff, drinking alcohol etc. in both rural and urban India.

High prevalence of tobacco use coupled with a suitable proportion of the family budget on its consumption will have far reaching implications on the welfare of public, in general, and the tobacco using households in particular. In addition to the financial burden that directly affects the householder and there is an indirectly financial burden in the form of medical expenditure to cure numerous tobacco related diseases such as Cancer, tuberculosis, heart diseased and various acute respiratory diseases.

Prevalence of all kinds of tobacco consumption is higher among poor income groups in India. Roughly 85% of the tobacco consumption in India is non-cigarette type i.e.. In the form of Bidi (It is an indigenous tobaccoprepartion in India made by rolling a dried piece of Temburini leaf) (Diospyros melnoxylon) with 0.15 to 0.25 g of sun-dried flaked tobacco into a conical shape and securing the roll with a thread (Gupta et al 1992)). As high as cigarettes contribute 86% of the excise revenue collected from the

tobacco products where as bidi contribute only 5% and chewing tobacco consumption also attracts only a very small tax. To regulate the use of all tobacco products, taxation is an important price instrument. In India Taxes on cigarettes in India are levied on the basis of its length, and it ranged from Rs. 135 to Rs. 450 per thousand units for non-filtered cigarettes and Rs. 670 to Rs. 1450 for filter cigarettes in the year 2002-03as given in the table. Where as tax on thousand units of bidis is only a megre Rs.7., Chewing tobacco attracts an ad valorem tax in India. A study sponsored by Tobacco Institute of India (Sen et al 1998) found that frequent hikes in excise duties help in showing down the consumption of tobacco. The fact is that bidis, which are consumed by the majority of tobacco users in India, are more harmful than cigarettes. Though bidis contain only a small amount of tobacco compared to cigarette, they deliver as much as 45 mg. – 50 mg. of Tar and 1.74mg – 2.5 mg. of Nicotine compared to 18 mg. – 28 mg. and 1.55 mg. – 1.92 mg. of Tar and Nicotine respectively in Indian cigarettes (Gupta et al 1992).

As per the latest (6 March 2006, the Hindu) information Director, Udayan Lall TII (Tobacco Institute of India) the tobacco country `s share is 35% in the global trade. The flue-cured Virginia tobacco had tremendous potential to enhance Agri-income, to increase rural employment, to boost exports and to maximize Govt., revenue. Currently, a cigarette, with their merger 14% share of total tobacco consumption, contributes 85% of the revenue. Taxes on a per Kg., basis were 30 times higher on cigarettes as compared to bidi and chewing tobacco. In my present paper I want to study how the tobacco products are being purchased in Kavali Town through a sample survey conducted in different areas of the town and estimate the men and women using tobacco products.

DATA ON CONSUPTION OF TOBACCO PRODUCTS:

National Sample Survey Organization (NSSO) conducted National wide sample surveys on the consumption of addictive goods habits of households in India during it's Quinquennial surveys (a large sample survey carried out every five years). The last such survey that was carried out during July 1999 – June 2000 collected information on consumption of addictive goods from 1,20,309 households spread over 10,140 villages in India. The goods of consumption of on which it collected the information included a wide variety of addictive goods such as tobacco, pan and alcohol products along with 500 food and non-food items. Various household characteristics were also surveyed with it. The quantity purchased as well as the expenditure incurred for consumption of various products over the last 30 days prior to the date of interview were recorded (table: 3). Addictive goods that are surveyed included eight tobacco products, 6 pan products (pan consists of betel leaf, areca nut, slaked lime, catechu and tobacco- tobacco forms only a small portion of pan and the amount of tobacco varies in different pan products). Pan includes pan leaf and finished pan. The data indicates that 65% of the rural households and 49% of the urban households report consumption at least one of those addictive goods in the last 30 days prior to the date of interview. Tobacco users constitute the major chunk of households consuming addictive goods. Approximately 95% of the reporting tobacco users consume either bide,

cigarette, leaf tobacco or a combination of these. Except cigarette and foreign liquor, consumption of every other items higher among rural households than their urban counterparts. Table 3 gives average unit value over those households who bought the addictive good and the average over all households of the share of the total expenditure that is devoted to each goods with the fraction of households consuming each of them. Portion of households budget spent on addictive goods, average by the number of all households, include those who do not consume any of it. Shows that 3% of the rural and 2.2%of the urban household budget is spent on consuming addictive goods. Table 5 gives the information of tobacco product users in Kavali Town as per the conducted Sample Survey.

INTERPRETATIONS:

Total percentage of tobacco product users in Kavali Town is 76.11% among them 84.7% of Men and 62.8% of women were estimated as per the sample survey report. Area wise reported percentages are 1) Vengal Rao nagar: - 71.85% 2) Vykuntapuram: 27.64% 3) Jantapet: 34.1% 4) Trunk road: 85.88% 5) Old Town: 27.64% are accounted. As per the test for proportions for the total town Z_c = 10.91 in the case of Men and Zc = 13.9 in women both are rejected and interpretated as when compared with National value of tobacco product consumers are high in Kavali . It is also estimated that totally 60896 of 79862 populations were consuming tobacco products, among which 34583 Men and 26313 women are consuming tobacco products. As per the area wise estimations inVengalrao Nagar, Trunk Road and Old Town the percentage of men users are more than the Indian Men consumers i.e. 65%. And in the case of women the consumption is very high only in the Trunk Road on the basis of the data collected from the floating population that come from rural area. It is also little more in Old Town as it has rural background.

CONCLUSIONS :-

As per the interpretations in Kavali the percentage of men and women consuming more compared with estimated National data. Hence, it is suggested that the Govt., and Non-Govt., Social Service Organizations are requested to concentrate on this issue and enlighten the public through canvass and other media to eradicate the tobacco product usage by the householders to reduce their financial burden and also medical expenditure to cure various tobacco related diseases.

References:-

Rijo M John, 'Price Elasticity Estimates for Tobacco and Other Addictive Goods in India', Aug 2005.

Chaloupka, Frank and Kenneth E Warner, 2000, The economics of smoking, In A. J.Culyer and J. P.Newhouse, eds, 'Handbook of Health Economics', Vol. 1B, Elsevier Science, Amsterdam, pp. 1539–1627.

Gajalakshmi, V, Richard Peto, T.S Kanaka and Prabhat Jha, 2003, 'Smoking and mortality from tuberculosis and other diseases in India: Retrospective study of 43000 adult male deaths and 35000 controls', *The Lancet* **362**: 507–515.

Gupta, Prakash C, James E Hamner and P.R Murti, eds, 1992, *Control of Tobacco-Related Cancers and Other Diseases*, Oxford University Press, Bombay.

John, Rijo M, 2004, 'An analysis of household tobacco consumption decisions: Evidence from india', Working Paper No. 002-2004, Indira Gandhi Institute of Development Research, Mumbai.

John, Rijo M, 2005, 'Tobacco consumption patterns and its health implications in India', *Health Policy* **71**[2]: 213–222.

NSSO, 2000, 'Socio-economic survey, 55th round: July 1999 - june 2000, household schedule 1.0: Consumer expenditure', Government of India, New Delhi.

NSSO Expert Group, 2003, 'Suitability of different reference periods for measuring household consumption', *Economic and Political Weekly* **38**[4]: 307–321.

Rani, Manju, S Bonu, P Jha, S.N. Nguyen and L Jamjoum, 2003, 'Tobacco use in India: Prevalence and predictors of smoking and chewing in a national cross sectional household survey', *Tobacco Control* **12**[4].

Reddy, K Srinath and Prakash C Gupta, eds, 2004, *Report on Tobacco Control in India*, Ministry of Health and Family Welfare, Government of India.

Sarma, Atul, 2000, 'Regulating tobacco use: Role of taxes', *Economic and Political Weekly* **35**[52]: 4613–4616.

Sen, Anindya, Subrata Sarkar and Ajit Ranade, 1998, *Opportunities and Challenges in Tobacco*, Indira Gandhi Institute of Development Research, Report of Study Sponsored by Tobacco Institute of India, New Delhi.

Shafey, Omar, Suzanne Dolwick and G Emmanuel Guindon, eds, 2003, *Tobacco Control Country Profiles*, 2 edn, World Health Organization.

Shimkhada, R and J.W Peabody, 2003, 'Tobacco control in India', *Bulletin of The World Health Organisation* **81**[1]: 48–52.

World Health Organization, 2002, Country profiles on tobacco or health, WHO, Regional Office for South-East Asia, New Delhi, India, pp. 27–44.

Figure 1: Projected consumption of and revenue from bidis and cigarettes

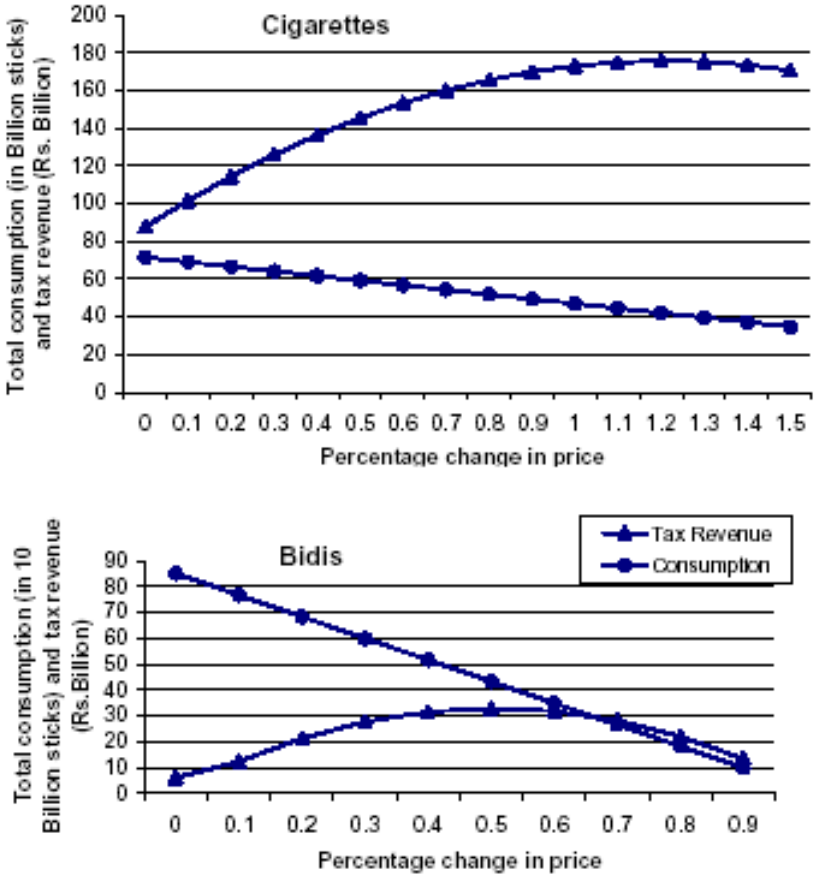

Table 1: Share of different tobacco products in total Union excise duty (basic and additional) collected from tobacco.

Year	Cigarettes	Bidi	Chewing	Others	Total Revenue (Rs.Billion)
1994-95	86.71	6.90	4.69	1.62	31.577
1995-96	85.86	4.54	5.29	4.21	39.877
1996-97	85.60	5.18	5.59	3.57	46.529
1997-98	86.30	5.66	5.37	2.61	51.376
1998-99	86.29	5.70	5.99	1.98	55.954
1999-00	86.06	5.77	6.25	1.87	55.660
2000-01	85.10	5.40	7.14	2.33	60.389

Source: Calculated from P.240, Reddy and Gupta (2004).

Table 2: Excise duty rate of different tobacco products in India as on 2003-04.

Tobacco Products	Duty per 1000 units in Rs.
Non Filter Cigarettes	
not exceeding 60 mm in length	135
Between 60 mm and 70 mm	450
Filter Cigarettes	
not exceeding 70 mm in length	670
between 70 mm and 75 mm	1090
between 75 mm and 85 mm	1450
Bidis	7
Rates of duty (*ad valorem*)	
Cigars and cheroots	16%
Other Tobacco products	60%
Pan masala	55%
Pan masala containing Tobacco	60%

Notes: The *ad valorem* taxes for cigars and cheroots are only from basic excise duty (BED) whereas that of other tobacco products and pan masala containing tobacco comprise of 16% BED, 16% Special duty, 18% Additional duty, and 10% National Calamity Contingency Duty (NCCD). The duty on pan masala comprises of BED and special duty 16% each and NCCD 23%.

Source: Central Board of Excise and Customs, Government of India.

Table 3: Unit values and budget shares of different addictive goods

	Rural India					
	Unweighted			Weighted		
Commodty	% Consuming	Unit value	Share	% consuming	Unit value	Share
Bidis	35.33	0.22	0.97	36.50	0.17	1.08
Cigarettes	5.46	1.59	0.18	3.69	1.40	0.14
Leaf-Tob	18.68	70.00	0.23	19.42	69.86	0.24
Pan	22.54	0.70	0.36	18.47	0.65	0.30
Alcohol	15.65	48.02	0.82	16.40	47.02	0.72
Total	64.77	NA	3.16	64.23	NA	2.92
	Urban India					
Bidis	18.93	0.21	0.50	19.84	0.21	0.54
Cigarettes	11.32	1.47	0.43	9.61	1.32	0.39
Leaf-Tob	7.39	74.98	0.09	7.25	74.73	0.08
Pan	18.79	1.13	0.39	15.46	1.13	0.32
Alcohol	9.65	99.10	0.59	10.20	95.07	0.56
Total	48.95	NA	2.27	44.16	NA	2.14

Notes: Total includes all the addictive goods including the ones that are not listed here but are there in the NSS data. Unit of measurement for bidi, cigarette and pan is number, leaf tobacco is Kg., and alcohol is litre. Unit values are all in Rupees. Budget shares (averaged for consuming and non-consuming households alike) are in percentages.

Source: Author's calculation from NSSO (2000) data.

Table 4 : Own price and Expenditure elasticities for 50th round

	Rural India		Urban India	
Items	own elsty	η_x	own elsty	η_x
Bidi	-0.769	0.65	-0.707	0.589
Cigarette	0.601	2.24	0.251	1.527
Leaf Tobacco	-0.003	0.44	-0.141	0.669
Pan	-0.665	1.03	-0.592	1.042
Alcohol	-1.162	1.34	-1.050	1.690

Table: 5.Area wise tobacco products purchasing consumers in KAVALI Town.

	No. Of consumers surveyed			No. Purchasing tobacco products		
Area	**Men**	**Women**	**Total**	**Men**	**Women**	**Total**
Vengal rao nagar	43	26	69	35	7	52
Vykuntapuram	103	20	123	32	2	34
Janatapet	31	13	44	13	2	15
Trunk Road	525	410	935	516	287	803
Old Town	44	15	64	40	6	46
GRAND TOTAL	**751**	**484**	**1235**	**636**	**304**	**940**

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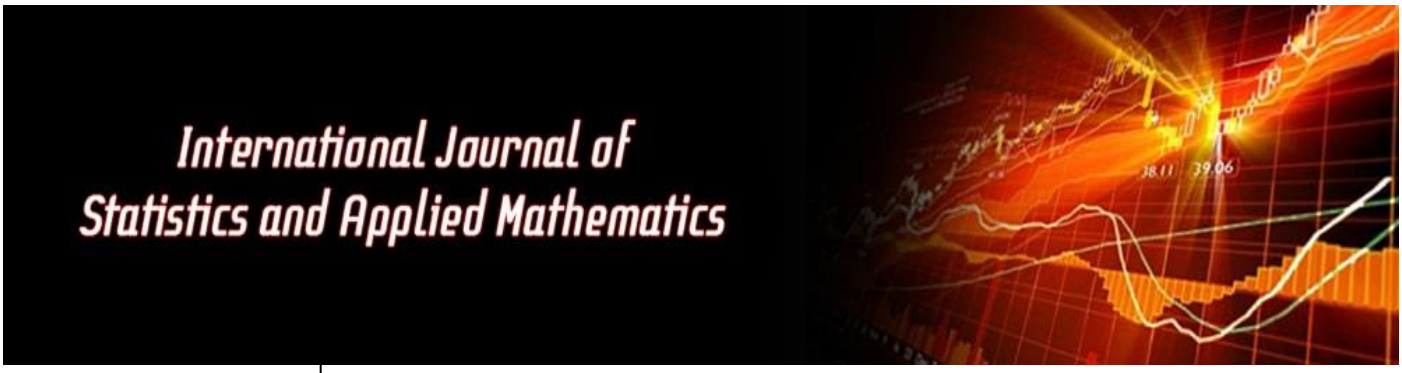

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Stock market prediction using time series analysis

N Viswam and G Satyanarayana Reddy

Abstract
Stock market is a market that enables seamless exchange of buying and selling of company stocks. Every Stock Exchange has their own Stock Index value. Index is the average value that is calculated by combining several stocks. This helps in representing the entire stock market and predicting the market's movement over time. The Equity market can have a profound impact on people and the country's economy as a whole. Therefore, predicting the stock trends in an effective manner can minimize the risk of investing and maximize profit. In our paper, we are using the Time Series Forecasting methodology for predicting and visualizing the predictions. Our focus for prediction will be based on the technical analysis using historic data and ARIMA Model. Autoregressive Integrated Moving Average (ARIMA) model has been used extensively in the field of finance and economics as it is known to be robust, efficient and has a strong potential for short-term share market prediction.

Keywords: Stock market, prediction using, time series analysis

1. Introduction
The stock market is a general term which refers to the collection of markets where the issuing and trading of equities, bonds and other sorts of securities takes place through various physical and electronic exchanges and over the counter market. The stock market is one of the most important components of a market economy, because it provides companies with access to capital by allowing investors to buy shares of ownership in a company. The arena of stock market is constantly developing under the process of refinements. Considering the variations it brings every day, investors need to plan intensively to make profit.

Forecasting the stock exchange data includes an assumption that the information publicly available at present has some predictive relationships to the future stock returns. Stock trend forecasting is one of the most difficult tasks to achieve in finance market because of the difficulty in the highly intricate world of stock market. The investors in the stock market always find a technique that can guarantee easy profiting by forecasting the stock trends and minimize the risk of investing. This motivates the researchers in the domain field to develop new forecasting models.

Stock prices are not randomly generated values rather they can be treated as a discrete time series model which is based on a set of well-defined numerical data items collected at successive points at regular intervals of time. Since, it is essential to identify a model to analyze trends of stock prices with adequate information for decision making, it recommends that transforming the time series using ARIMA is a better algorithmic approach than forecasting directly, as it gives more authentic results. Autoregressive Integrated Moving Average (ARIMA) Model converts a non-stationary data to a stationary data before working on it. It is one of the most popular model to predict linear time series data.

R is a programming language and environment for statistical processing and graphics. The R dialect is generally utilized by data analysts for statistical programming and data analysis. R language was created by Ross Ihaka and Robert Gentleman at the University of Auckland, New Zealand and is presently maintained by the R development core team. R-Studio is a free and powerful integrated development environment for R language which allows the user to implement ARIMA model.

In this paper, we have mainly focused on the amount of accuracy of forecasting stock values for various sectors which will aid new investors understand the market and make a wise decision to invest in the stock market.

2. Literature Survey

The author [Banerjee D] [1] has tried to develop a model that helps forecast the future Indian stock market values, based on the past information collected on the monthly closing stock indices. Using ARIMA model he has predicted the future stock indices which have strong performance on the economy of India.

The authors [Ayodele A] [Adebiyi] [2] have used the ARIMA model to develop a process of building stock price predictive model by taking data from NYSE and NSE. Artificial Neural Networks (ANNs) model is popular for its ability to make patterns from data and deduce solution from unknown data. The obtained results from real-life data reveals the potential strength of ARIMA models in short-term prediction that can aid investor's decsion making process.

The authors [Devi, B. Uma] [D. Sundar] [P. Alli.] [3] has applied ARIMA to solve the real world problems of the stock market by forecasting the stock prices with the top four companies in Nifty Midcap-50 using MATLAB along with performance measure.

The authors [Jarrett] [E. Jeffrey] [Eric Kyper] [4] have used ARIMA as both analytical and forecasting models in the PACAP-CCER China Database, developed by the Pacific.

The authors [J. Contreras][R. Espinola][F. J. Nogales][A.J. Conejo] [5] have provided a method of using ARIMA model for predicting or forecasting electricity price more specifically of the next day.

The authors [Prapanna Mondal] [Labani Shit] [Saptarsi Goswami] [6] have conducted a study to predict amount of accuracy for various sectors of Indian stocks using ARIMA model. As a result it has been found that accuracy of in predicting stock prices using ARIMA model is above 85%.

The authors [M. Angadi] [A. Kulkarni] [7] have attempted to develop a model for predicting the trends of stock market based on the technical analysis using historical time series stock market data and ARIMA model. The experimental results reveal that the potential of ARIMA model to predict future indices of stock price is strong. This provide guidance for the investors in the to make a decision of profitable investment in stock market.

3. Implementation

For in-depth understanding and visualization of results, we have used two different techniques for stock prediction.

3.1 QuantMod package

In this approach we have collected data from Google Finance and Yahoo Finance using R script. The appropriate ticker symbol needs to be specified to load data from the above sources. The OHLC data cannot be downloaded directly from Google Finance, we need to call the getSymbols method. By default it uses Yahoo for data retrieval.

3.1.1 First step is to install the Quant Mod package

install. packages (quantmod') Next, we need to select a secure CRAN Mirror CRAN Mirror allows us to select a download site close to us for better bandwidth/latency.

Add the QuantMod library

library ("quantmod")

3.1.2 To retrieve data set from Quant Mod

get Symbols ("AAPL") AAPL For retrieving data of a particular period Get Symbols ("AAPL", from=2016-01-01", to="2017-01-01") We can obtain the first several rows of a data set using the head function head (AAPL)

3.1.4 Plot Visualization using charts

chart Series (MSFT) chart Series (MSFT, subset="2017-01: 2017-06") By default it shows the OHLC data and the volume data For detailed analysis we can use add MACD and add B Bands Moving Average Convergence Divergence (MACD) is a centred oscillator that fluctuates above and below the zero line. It helps us analyse the strength, weakness or direction of momentum of a security's move. BBands allows us to compare the volatility and price levels of a stock over a period of time.

3.1.5 View and analyse results

To get the short-term predictions, correlations can be found out once we plot the results.

The default MACD formula used in almost all charting packages is - MACD 12,26,9 The MACD line is plotted by taking the difference between the value of 12 day and 26 day Exponential Moving Average (EMA). The difference between the 26 day EMA and 12 day EMA gives the MACD line. Then a 9 day EMA of the MACD line is superimposed over the MACD as a 'signal line'.

MACD produces signals from 3 sources for prediction

1. MACD/signal line crossovers - They are the most common signals sent out but are not very reliable as most of the times it sends a false signal. When the MACD line crosses above the signal line it is considered as bullish moving average and when the MACD is below the signal line it is considered as bearish moving average crossover.
2. MACD central line crossover - MACD line below the centre line crossing over the centre line indicates bullish crossover and vice versa.
3. Divergence with price - It is the least frequent signal that MACD generates but it the most powerful and reliable signal. In an uptrend, when price makes a higher high, but the MACD line makes a lower high diverging with the actual price. In a downtrend, divergence occurs when price makes a lower low but the MACD line makes a higher low. Whenever a divergence is spotted, there is a higher chance of price retracement.

	AAPL.Open	AAPL.High	AAPL.Low	AAPL.Close	AAPL.Volume	AAPL.Adjusted
2015-01-02	111.39	111.44	107.35	109.33	53204600	103.86647
2015-01-05	108.29	108.65	105.41	106.25	64285500	100.94039
2015-01-06	106.54	107.43	104.63	106.26	65797100	100.94989
2015-01-07	107.20	108.20	106.70	107.75	40105900	102.36544
2015-01-08	109.23	112.15	108.70	111.89	59364500	106.29853
2015-01-09	112.67	113.25	110.21	112.01	53645500	106.41255
2015-01-12	112.60	112.63	108.80	109.25	49650800	103.79047
2015-01-13	111.43	112.80	108.91	110.22	67091900	104.71200
2015-01-14	109.04	110.49	108.50	109.80	48956600	104.31299
2015-01-15	110.00	110.06	106.66	106.82	60014000	101.48190
2015-01-16	107.03	107.58	105.20	105.99	78513300	100.69338
2015-01-20	107.84	108.97	106.50	108.72	49012100	103.28696
2015-01-21	108.95	111.06	108.27	109.55	48575900	104.07549
2015-01-22	110.26	112.47	109.72	112.40	53796400	106.78306
2015-01-23	112.30	113.75	111.53	112.98	46051200	107.33408
2015-01-26	113.74	114.36	112.80	113.10	55615000	107.44807
2015-01-27	112.42	112.48	109.03	109.14	95568700	103.68597
2015-01-28	117.63	118.12	115.31	115.31	146477100	109.54764
2015-01-29	116.32	119.19	115.56	118.90	84436400	112.95824
2015-01-30	118.40	120.00	116.85	117.16	83745500	111.30518
2015-02-02	118.05	119.17	116.08	118.63	62739100	112.70172
2015-02-03	118.50	119.09	117.61	118.65	51640300	112.72072
2015-02-04	118.50	120.51	118.31	119.56	70149700	113.58526
2015-02-05	120.02	120.23	119.25	119.94	42246200	114.39597
2015-02-06	120.02	120.25	118.45	118.93	43372000	113.43264
2015-02-09	118.55	119.84	118.43	119.72	38889800	114.18613
2015-02-10	120.17	122.15	120.16	122.02	62008500	116.37981
2015-02-11	122.77	124.92	122.50	124.88	73561800	119.10761
2015-02-12	126.06	127.48	125.57	126.46	74474500	120.61459
2015-02-13	127.28	127.28	125.65	127.08	54272200	121.20593
2015-02-17	127.49	128.88	126.92	127.83	63152400	121.92125
2015-02-18	127.63	128.78	127.45	128.72	44082300	122.77013
2015-02-19	128.48	129.03	128.33	128.45	37362400	122.51260
2015-02-20	128.62	129.50	128.05	129.50	48948400	123.51408
2015-02-23	130.02	133.00	129.66	133.00	70974100	126.85228





3.2 By importing data set in R

We have downloaded the historical data of APPLE from Yahoo Finance. The historical data of any company can be downloaded. A csv file is generated which we will import in R studio for our analysis.

3.2.1 Importing the data

```
aapl=read.csv("C:\\Users\\Laptop\\Downloads\\AAPL.csv",header=T)
head (aapl)
```

3.2.2 Creating a time series function

In our data the closing prices are listed in the 5th column, we use the ts() function to create time series. We have taken data from October 2015 to October 2017.

aapl.ts=ts(rev(aapl) [5], start=c (2015, 10), end=c (2017, 10), freq=12)

aapl. ts

In addition, we also take the log of values and place them both in an accessible data frame. We have saved the closing values and the log values in a separate file.

>aapl=data. frame (closing=aapl. ts, lclosing=log(aapl. ts))

> save (aapl, file="aapl. df. RData")

> load ("aapl. df. RData")

3.2.3 Plotting the stock values

Plot the data frame x-axis year and y-axis closing price

plot (aapl$closing, main="AAPLE Stock Prices(APPLE)", lwd=2,sub="October 2015 to October 2017",ylab="Closing Price")

3.3 Decomposition

The function stl() removes the seasonal component from the data set and the trend is found after smoothening the remainder.

aapl. stl=stl (aapl $closing, s. window="periodic")

plot (aapl. stl, main="APPLE Stock decomposition")

3.4 Forecasting using stl()

We use the forecast package to predict the stock price over the next 2 years.

Prediction method=ARIMA Model

h=24 (Predict 2 years into the future)

level =95 (95% confidence level)

install. packages ('forecast')

library (forecast)

aapl. f=forecast (aapl. stl, method="arima", h=24, level=95)

plot (aapl. f, ylab="Stock Price", xlab="Year", sub="Forecast from October 2017 to October 2019")

	AAPL.Open	AAPL.High	AAPL.Low	AAPL.Close	AAPL.Volume	AAPL.Adjusted
2015-01-02	111.39	111.44	107.35	109.33	53204600	103.86647
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2015-01-09	112.67	113.25	110.21	112.01	53645500	106.41255
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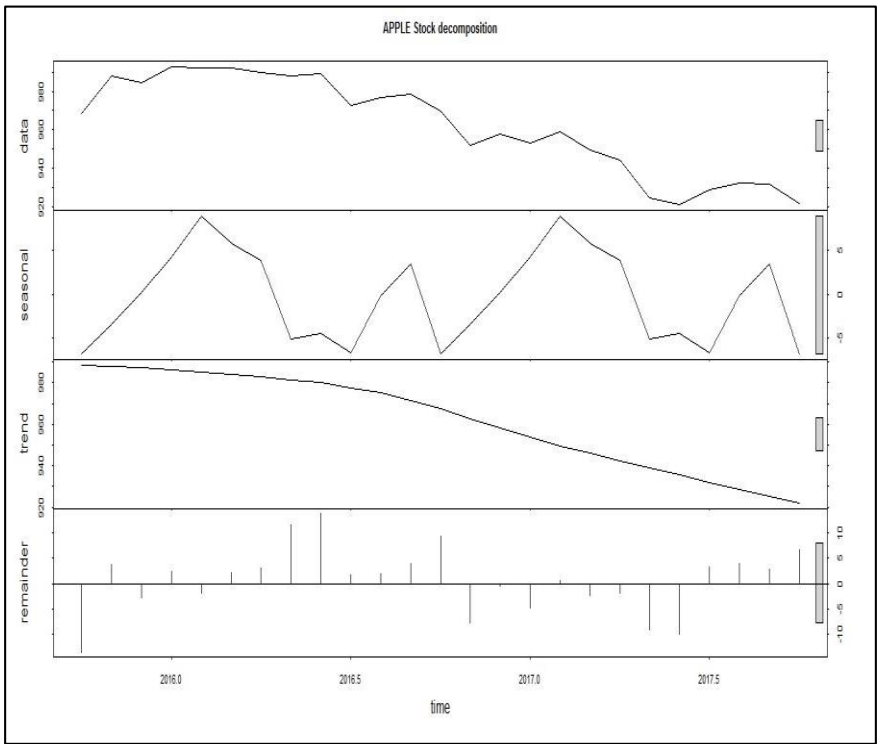
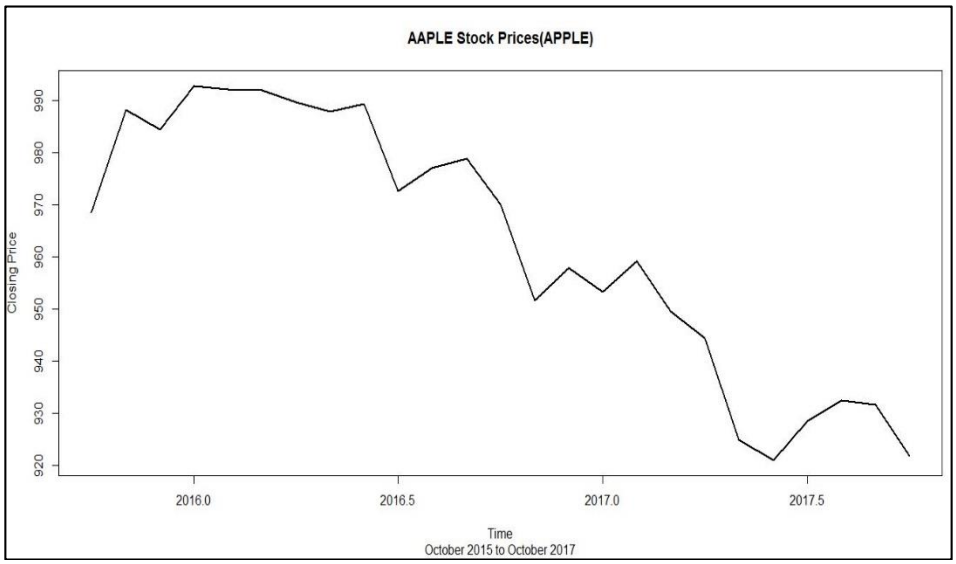
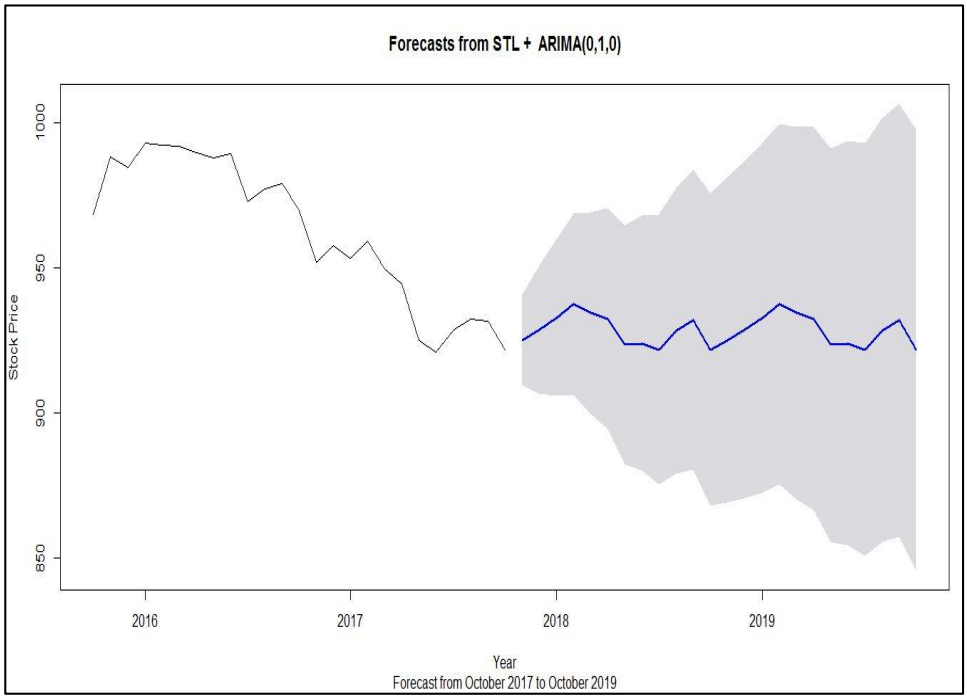
2015-02-23	130.02	133.00	129.66	133.00	70974100	126.85228







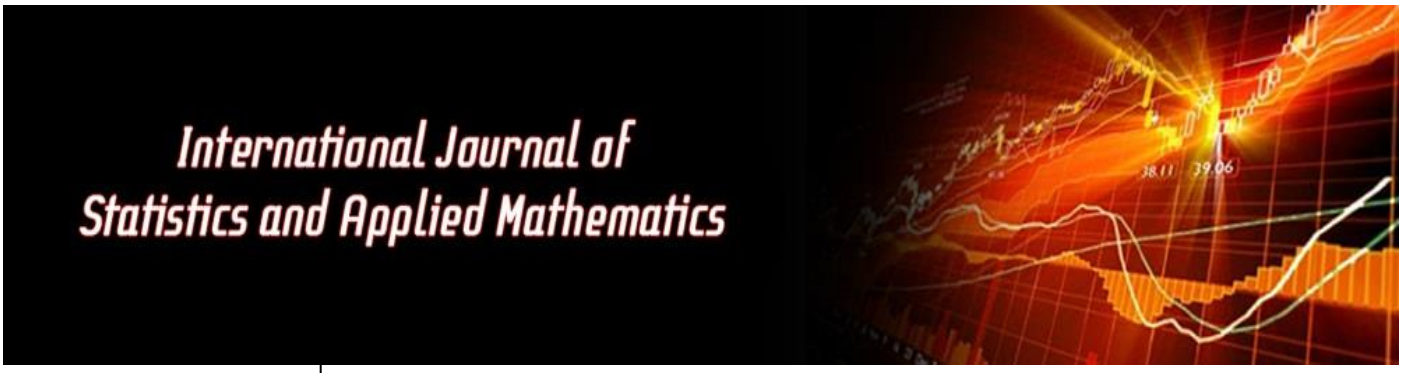
Conclusion

In this paper we have predicted the share market trends by considering a company's historical time series data. The ARIMA Model played a key role in predicting the short-term trends of a stock market. The investors can do a thorough analysis of the share market by visualizing the graphical plots. There are various technical ways in which the plots can be visualized. For accurate prediction the MACD line should be compared with the signal line indicator. The divergence with price signal is generated very rarely but is the most effective technique for precise analysis. This can help the investors make a profitable decision on whether to buy/sell/hold a stock.

5. References

1. Banerjee D. Forecasting of Indian stock market using time-series ARIMA model, 2nd IEEE International Conference on Business and Information Management (ICBIM), 2014, 131-135.
2. Ayodele A Adebiyi, Aderemi O Adewumi, Charles K Ayo. Stock price prediction using the ARIMA model, 16th IEEE International Conference on Computer Modelling and Simulation (UKSim), 2014, 106 -112.
3. Uma Devi B, Sundar D, Alli P. An Effective Time Series Analysis for Stock Trend Prediction Using ARIMA Model for Nifty Midcap-50., International Journal of Data Mining & Knowledge Management Process (IJDKP) 2013, 3.
4. Jarrett E Jeffrey, Kyper E. ARIMA modeling with intervention to forecast and analyze chinese stock prices, International Journal of Engineering Business Management, 2011; 3(3)
5. Contreras JR, Espinola FJ, Nogales AJ. Conejo, ARIMA models to predict next-day electricity prices, IEEE Transactions on Power Systems, 2003.
6. Mondal P, Shit L, Goswami S. Study of effectiveness of time series modeling (ARIMA) in for casting stock prices, International Journal of Computer Science, Engineering and Applications, 2014.
7. Angadi M, Kulkarni A. Time Series Data Analysis for Stock Market Prediction using Data Mining Techniques with R International Journal of Advanced Research in Computer Science, 2015.

International Journal of Statistics and Applied Mathematics 2018; 3(1): 476-480



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Logistic regression model applications in retail industry

G Satyanarayana Reddy and N Viswam

Abstract
The Indian retail industry has emerged as one of the most dynamic and fast-paced industries due to the entry of several new players. It accounts for over 10 per cent of the country's Gross Domestic Product (GDP) and around 8 per cent of the employment. India is the world's fifth-largest global destination in the retail space. The Boston Consulting Group and Retailers Association of India published a report titled, 'Retail 2020: Retrospect, Reinvent, Rewrite', highlighting that India's retail market is expected to nearly double to US$ 1 trillion by 2020 from US$ 600 billion in 2015, driven by income growth, urbanization and attitudinal shifts. Retail spending in the top seven Indian cities amounted to Rs.3.58 trillion (US$ 53.7 billion), with organized retail penetration at 19 per cent as of 2014.

Keywords: Logistic regression, model applications, retail industry

Introduction
Promotions can be categorized based on the intended coverage of a single promotional message. For example, a single television advertisement for a major event, an in store promotion or a promotion on a newspaper could be seen by hundreds of customers at the same time. Such mass promotion, intended to reach as many people as possible, has been a mainstay of Retailers' promotional efforts for a long time. Unfortunately, while mass promotions are delivered to a large number of people, the actual number that fall within the marketer's target market may be small. Because of this, many who use mass promotion techniques find it to be an inefficient way to reach desired customers. Instead, today's marketers are turning to newer techniques designed to focus promotional delivery to only those with a high probability of being in the marketer's target market. In so doing, the marketing focus shifts away from the breadth of customer base to the depth of each customer's needs. The performance metric changes from market share to so-called "wallet share".
Data mining is used to construct six types of techniques aimed at solving business problems: classification, regression, time series, clustering, association analysis, and sequence discovery. The first two, classification and regression, are used to make predictions, while association and sequence discovery are used to describe behavior. Clustering can be used for either forecasting or description.
This article examines the performance of the mass promotions Vs target promotion here we use logistic regression technique with qualitative and quantitative variable to predict the response of a customer to a particular promotion and we also propose how can we use the customer segmentations to know a customer at a 360^0 view and how to use these segmentations to select correct populations for any target promotions. Finally, we draw conclusions based on the discussion.

Logistic Distribution
A random variable X is said t have Logistic distribution with mean μ and variance σ^2, if it has a cumulative distribution function,

$$F(x,\mu,\sigma) = \left\{1+\exp\left[\frac{-\pi(x-\mu)}{\sigma\sqrt{3}}\right]\right\}^{-1} \quad -\infty < x < \infty, \quad -\infty<\mu<\infty \quad \sigma>0 \tag{2.1}$$

and probability density function related to its distribution function,

$$f(x,\ \mu,\ \sigma) = \frac{\pi}{\sigma\sqrt{3}} F(x,\mu,\sigma)\,[1-F(x,\mu,\sigma)] \tag{2.2}$$

Alternatively, these functions may be expressed as

$$F(x,\mu,\sigma) = \frac{1}{2}\left\{1+\tanh\left[\frac{\pi(x-\mu)}{2(\sigma\sqrt{3})}\right]\right\} \tag{2.3}$$

and $$F(x,\mu,\sigma) = \frac{\pi}{4\sigma\sqrt{3}}\operatorname{sech}^2\left[\frac{\pi}{2}\frac{(x-\mu)}{(\sigma\sqrt{3})}\right] \tag{2.4}$$

Properties of Logistic Distribution

1. The density f (x, μ, σ) is bell shaped and symmetrical with heavier tails than a normal density with the same mean and variance.
2. The canonical form of the logistic distribution, which corresponds to the random variable z, with mean μ = 0 and variance $\sigma^2 = \frac{\pi^2}{3}$ and has cumulative distribution function and probability density functions,

$$G(z) = \frac{1}{1+e^{-z}} \tag{2.5}$$

$$\text{and } g(z) = G(z)\,(1-G(z)) \tag{2.6}$$

Equation (2.6) and therefore equation (2.2) characterizes the logistic distribution and it is equivalent to the linearity of the transformation (known as logit)

$$\text{Log}\left[\frac{G(z)}{1-G(z)}\right] = z \tag{2.7}$$

1. The most popular application of the logistic distributions is the logit in the content of modeling Quantal response data and performing Logistic regression.
2. The distribution function of the standardized random variable $z/(\pi\sqrt{3})$ is very close to the standard normal distribution.
3. The sum of independent logistic random variables is not a logistic random variable.
4.
5. The characteristic function of z is given by

$$\phi_z(t) = \Gamma(1-it)\,\Gamma(1+it) = \prod_{j=1}^{\infty}\left(1-\frac{t^2}{j^1}\right)^{-1} \tag{2.8}$$

6. The absolute moments are given by

$$E|z|^k = 2\,\Gamma(k+1)\left[1-\frac{1}{2^{k+1}}\ \xi(k)\right] \tag{2.9}$$

7. The Logistic distribution can be obtained from a mixture of the extreme value distributions and the exponential distribution. The Logistic distribution is infinitely divisible.

8. If a random variable Y is uniformly distributed on [0,1] then the logit transformation of Y, say $\log\left[\frac{y}{(1-y)}\right]$ has the logistic distribution function G.

Specification of Logistic Regression Model

Generally linear regression model is used to approximate the relationship between a continuous response variable and a set of predictor variables. However the response variable is often categorical rather than continuous variable, for such cases, linear regression is not appropriate, but the biostatistician can turn to an analogous method, Logistic regression, which is similar to linear regression in many ways.

Logistic regression refers to techniques for describing the relationship between a categorical response variable and a set of predictor variables. In other words the good of a logistic regression analysis is to find the best fitting and most parsimonious, yet biologically reasonable, model to describe the relationship between a response variable and a set of predictor or independent variables. Generally the response variable in the Logistic regression model is categorical and usually Binary or bichotonous variable.

There are two main reasons for choosing the Logistic regression model by Biostatistician. These are:

(i) In the mathematical point of view, it is an extremely flexible and easily used function;

(ii) It trends itself to a biologically meaningful interpretation.

Suppose that $\pi(x) = E[y/x]$ be the conditional mean of y given x. The logistic regression model is given by

$$\pi(x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} \tag{3.1}$$

The logit transformation g(x) in terms of π (x) is given by

$$g(x) = \lin\left[\frac{\pi(x)}{1-\pi(x)}\right] = \beta_0 + \beta_1 x \tag{3.2}$$

The logit g(x) has many of the desirable properties of linear regression model. The logit g(x) is linear in its parameters, may be continuous and may range from -∞ to +∞ depending on the range of x.

In the linear regression analysis, one may assume, that an observation of the outcome variable may be expressed as

$$y = E[y/x] + \in \tag{3.3}$$

Such that the error variable ∈ follows a normal distribution with mean zero and some constant variance That is constant across the levels of the independent variable. This implies that the conditional distribution of the dependent variable (y) given x is normal with mean E [y/x] and constant variance.

In the case of Logistic regression model, one may express the value of the response variable given x as

$$Y = \pi(x) + \in \tag{3.4}$$

Here, ∈ may assume one of are two possible values. If y = 1 then ∈ = 1 π (x) with probability π (x) and if y = 0 then ∈ = -π (x) with probability 1-π (x). i.e., ∈ has a distribution with mean zero and variance equal to $\pi(x)[1-\pi(x)]$.

Thus, the conditional distribution of the response variable follows a Binomial distribution with probability given by the conditional mean π(x).

A simple example of logistic regression is as follows: suppose that medical researchers are interested in exploring the relationship between patient age (x) and the presence (1) or absence (0) of a particular disease (y). For a number of patients, generally, this relationship shows logistic regression.

Estimating the Logistic Regression Model

Consider (x_i, y_i), i = 1,2,……., n be a sample of n independent pairs of observations on two variables X and Y, where y_i denotes the value of a dichotomous response variable and x_i denotes the value of the independent variable for the ith subject.

Assume that Y has been coded as 0 or 1 representing the absence or presence of the characteristic respectively.

Write the logistic regression model as

$$\pi(x) = \frac{\exp(\beta_o + \beta_1 x)}{1 + \exp(\beta_o + \beta_1 x)} \tag{4.1}$$

To estimate the logistic regression model, the maximum likelihood method of estimation may be used to obtain optimum estimators for β_o and β_1.

Suppose that Y is coded as 0 or 1, then one may have

$$\pi(x) = P(y=1/x) \text{ and } 1-\pi(x) = P(y=0/x)$$

For the pairs (x_i, y_i), where $y_i = 1$ and $y_i = 0$, the likelihood function may be expressed as

$$L(\beta_o, \beta_1) = \prod_{i=1}^{n} \pi(x_i)^{y_i} \left[1-\pi(x_i)\right]^{1-y_i} \quad (4.2)$$

$$\Rightarrow \ln L(\beta_o, \beta_1) = \sum \left\{ y_i \ln\left[\pi(x_i)\right] + (1-y_i)\ln\left[1-\pi(x_i)\right]\right\} \quad (4.3)$$

Under the method of maximum likelihood estimation, one may obtain the maximum likelihood equations as

$$\sum_{i-1}^{n}\left[y_i - \pi(x_i)\right] = 0 \quad (4.4)$$

$$\text{and } \sum_{i-1}^{n} x_i \left[y_i - \pi(x_i)\right] = 0 \quad (4.5)$$

Since, the maximum likelihood equations are nonlinear in parameters, Iterative methods may be used to obtain the maximum likelihood estimators $\hat{\beta}_o$ and $\hat{\beta}_1$ for β_o and β_1 respectively. In particular, the solutions to the equations (4.4) and (4.5) may be obtained by using a generalized weighted least squares estimation.

By substituting $\hat{\beta}_o$ and $\hat{\beta}_1$, the maximum likelihood estimate of $\pi(x_i)$ is obtained as $\hat{\pi}(x_i)$.

Hence, $\hat{\pi}(x_i)$ gives the fitted or predicted value for y_i.

Since, $P\left(y=1 \middle/ x\right) = \pi(x_i)$

From (4.4), one may obtain

$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \hat{\pi}(x_i) \quad (4.6)$$

Sum of the observed values of y_i = sum of the predicted values of y_i.

Conclusion

In the Growing retail industry promotions and customer retention has become one of the major challenge. They need to focus on using the statistical methodologies like predictive modelling, segmentation, cluster analysis, factor analysis, logistic regression and decision theory to understand their customers well and they need to plan all their promotional and customer retention policies based on the statistical results. The latest software technologies will help them in visualizing and analyzing the vast amount of data they have.

References

1. Camm J, Cochran J, Fry M, Ohlmann J, Anderson D. A Categorization of Analytical Methods and Models. In Essentials of Business Analytics, 2014, 5-7.
2. Cox DR, SNELL EJ. The Analysis of Binary Data, 2nd ed. London: Chapman and Hall, 1989.
3. Fisher D, De Line R, Czerwinski M, Drucker S. Interactions with big data analytics. Interactions. 2012; 19(3):50-59.
4. Hastie T, Tibshirani R, Friedman J. The Elements of Statistical Learning: Data Mining, Inference, and Prediction. 2nd ed. Springer, 2009.
5. Hosmer DW, Lemeshow S. 'A goodness-of-fit test for the multiple logistic regression model', Communications in Statistics. 1980; A10:1043-1069
6. Hosmer DW, Lemeshow S. Applied Logistic Regression, Wiley, New York, 1989
7. Information Discovery, Inc. A characterization of data mining technologies and processes: an Information Discovery, Inc. White Paper
8. Lal Rajiv, David E Bell. The Impact of Frequent Shopper Programs in Grocery Retailing, Quantitative Marketing and Economics. 2003; 1(2):179-202.
9. Lam, Shun Y, Mark Vandenbosch, John Hulland, Michael Pearce. Evaluating Promotions in Shopping Environments: Decomposing Sales Response into Attraction, Conversion,and Spending Effects, Marketing Science. 2001; 20(2):194-215.

10. Leenheer, Jorna, Tammo HA, Bijmolt. Adoption and Effectiveness of Loyalty Programs: The Retailer's Perspective, Research Report No. 03-124. Cambridge, MA: Marketing Science Institute, 2003,
11. Maddala GS. Limited-Dependent and Qualitative Variables in Econometrics. Cambridge University Press, 1983.
12. Peter SH, Leeflang, Dick R Wittink. The Estimation of Pre- and Postpromotion Dips with Store-Level Scanner Data, Journal of Marketing Research. 2000; 37:383-95.
13. Petrissans A. Customer relationship management: the changing economics of customer relationships. White Paper prepared by Cap Gemini and International Data Corporation, 1999.
14. Rice J. ed. Mathematical Statistics and Data Analysis. Cengage Learning, 2006.
15. Roland T Rust, Katherine N Lemon. The Customer Pyramid: Creating and Serving Profitable Customers, California Management Review. 2001; 43(4):118-42.
16. Rossi, Peter E, Greg M. Allenby A Bayesian Approach to Estimating Household Parameters, Journal of Marketing Research. 1993; 30:171-82.
17. Subrahmanyan S. Using Quantitative Models for Setting Retail Prices. Journal of Product & Brand Management. 2000; 9(5)
18. Talluri KT, Van Ryzin GJ. The Theory and Practice of Revenue Management. Springer, 2005.
19. Winer R. A Price Vector Model of Demand for Consumer Durables: Preliminary Developments Marketing Science, 1985.
20. Wolverson R. High and Low: Online Flash Sales Go Beyond Fashion to Survive. Time Magazine, 2012.

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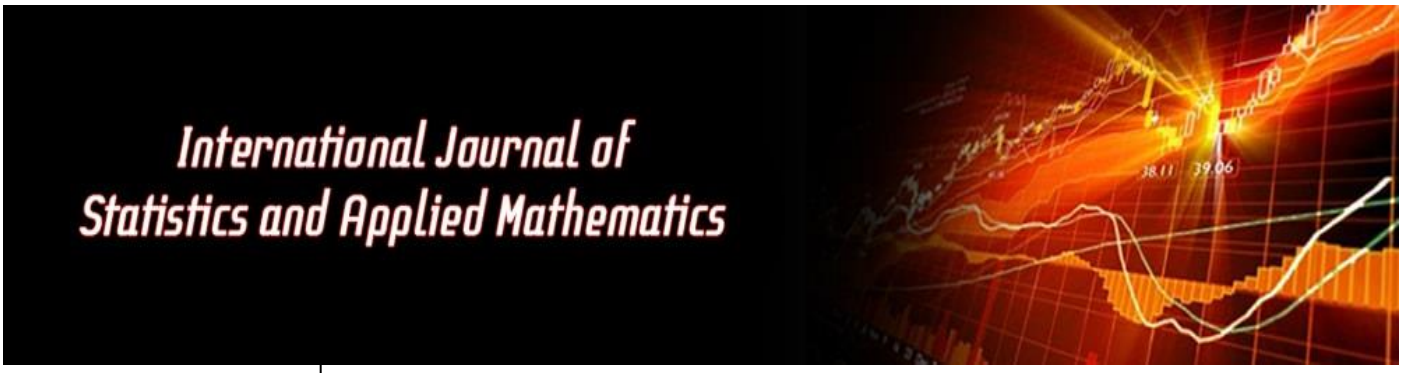

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Control charts and cusum charts under linear trend with log-logistic distribution

K Sai Swathi, CHV Aruna, N Viswam and GVSR Anjaneyulu

Abstract
This paper intends to assess the performance of Control Charts and CUSUM charts under linear trend with Log-Logistic distribution. A distinct approach, in which the operation of the scheme is regarded as forming a Markovian Chain, is set out. The Run Length properties of Control Charts and CUSUM schemes under conditions of slippage in the mean level by step-change to a new sustained level are well documented. Such control procedures are often used where a genuine out of control signal may result from gradual, rather than step changes. This paper presents, the results of evaluation of run length under linear trend with Rayleigh distribution.

Keywords: Control chart, cumulative sum technique, average run length, run length distribution, linear trend, non-homogeneous morkov chain, transition matrix

1. Introduction
Cumulative Sum Schemes Control Charts were introduced in 1954 by Page (1954) [11]. These charts may be used in several situations where a production process is expected to change at an unknown time from an" in control state" to an "out of control state". As soon as one has evidence that the out of control state has observed one would wishes to stop the production process to take remedial measures. CUSUM schemes have proven to be optimal stopping rules in the sense that to minimize expected run length under the out of control state given that the stopping rules has a fixed expected run length under the in-control state Moustakides (1986) [10].

CUSUM control charts have found an interesting variety of applications since their introduction. Several researchers namely Johnson (1966) [5], Hinkley (1970) [3], Brook and Evans (1972) [1], Ashish and Srivastava (1975) [14], Hawkins (1977) [4], Khan (1978) [8], Koning and Does (1988) [9], Rendtel (1990) [12], Rogerson (2006) [13], Cox (2009) [2] have attempted performance of CUSUM charts under various conditions. In most of the research problems the CUSUM chart performance is mainly assessed based on the Average Run Length or its distribution. In other words, the effectiveness of monitoring procedures like Shewart charts with Action limit only, control charts with Warning lines and CUSUM procedures can be demonstrated when there is a slippage in mean level from a target value. This can be done with the help of ARL or other features of run length distributions. The ARL is usually measured on the assumption of step change i.e. abrupt change from the process average. The main purpose of this paper is to assess the performance of control charts and CUSUM charts under linear trend with non-normal distribution namely, Rayleigh. This distribution is considered because of its applications in the real world. This distribution and its importance is discussed as and when the CUSUM schemes and other control charts performance are assessed. In the subsequent sections we discuss Shewart chart with action line, control chart with warning line and CUSUM procedures.

2. Shewart Control Chart with Action Lines
In the construction of control charts we are using two sets of limits such as action limits or outer limits and warning limits or inner limits. When action lines point plots outside of this limit, a search for an assignable cause is made and corrective action is taken if necessary.

Shewart control chart with only action lines, it is denoted by 'Scheme A' and specified distributional assumptions, the evaluation of run length properties follows Geometric distribution with parameter P_A that is

$$\text{ARL} = \frac{1}{P_A} \tag{2.1}$$

Where P_A is the Probability of action limit for a specified process mean.

3. Shewart Control Chart with Warning Lines

In a Shewart control chart with action and warning lines we take decisions with monitoring procedures depend on preceding observations as well as the most recent value. It is denoted by 'Scheme W'. If one or more points fall between the warning line and the central line or very close to the warning line, we should be suspicious that the process may not be operating properly. One possible action to take when this occurs is to increase the sampling frequency. The use of warning limit can increase the sensitivity of the control chart. The complete run length distribution is obtained by using successive powers of the transition matrix. In particular, the ARL is found to be

$$\frac{1+P_W-P_A}{P_A+P_W(P_W-P_A)} \tag{3.1}$$

Where P_W is the Probability of a violation of warning line which includes more extreme action line violationP_A. The action line scheme is having only two states one Transient and another one is absorbing.

4. Transition Matrices for Control Chart and Cusums with Rayleigh Distribution

4.1 The Importance of Log Logisticdistribution

In Probability and Statistics, the log-logistic distribution is a continuous probability distribution for a non-negative random variable. It is used in survival analysis as a parametric model for events whose rate increases initially and decreases later, for example mortality from cancer following diagnosis or treatment. It has also been used in hydrology to model stream flow and precipitation, and in economics as a simple model of the distribution of wealth or income. The log-logistic distribution is the probability distribution of a random variable whose logarithm has a logistic distribution. It is similar in shape to the log-normal distribution but has long tails.

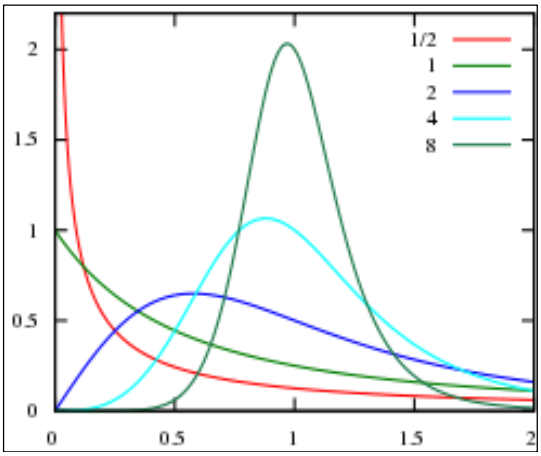
The parameter $\alpha > 0$ is a scale parameter and is also the median of the distribution. The parameter $\beta > 0$ is a shape parameter.

The cumulative distribution function is

$$\begin{aligned} F(x;\alpha,\beta) &= \frac{1}{1+(x/\alpha)^{-3}} \\ &= \frac{(x/\alpha)^3}{1+(x/\alpha)^3} \\ &= \frac{x^3}{\alpha^3+x^3} \end{aligned} \tag{4.1}$$

Where $x > 0$, $\alpha > 0$, $\beta > 0$.

Probability density function



α = 1, values of β as shown in legend

The probability density function is

$$f(x;\alpha,\beta)=\frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{[1+(x/\alpha)^{\beta}]^2} \tag{4.2}$$

Where, $x > 0$, $\alpha > 0$, $\beta > 0$.

The kth raw moment exists only when $k < \beta$, when it is given by

$$\begin{aligned}\mathrm{E}(X^k) &= \alpha^k B(1-k/\beta,\ 1+k/\beta)\\ &= \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)}\end{aligned} \tag{4.3}$$

Where B () is the beta function.

$$\mathrm{E}(X)=\alpha b/\sin b,\quad \beta>1,$$

And the variance is

$$\mathrm{Var}(X)=\alpha^2(2b/\sin 2b-b^2/\sin^2 b),\ \beta>2.$$

If X has a log-logistic distribution with scale parameter α and shape parameter β then $Y = \log(X)$ has a log-logistic distribution with location parameter log (α) and scale parameter 1/β. This situation is of special interest because a CUSUM appropriate to a Log Logistic distribution may be used.

4.2 Transition Matrix for Control Chart

The Transition matrices representation for control charts are give below. In case of row labels refer to states at sample (i-1) and column heading to states at sample i. The upper left column partition is the reduced transition matrix after deleting row and column for the absorbing states.

Table 4.1

A. Transition matrix for "Action only" (Shewart) chart		
	clear	**signal**
Clear	1-P_A	P_A
Signal	0	1

Table 4.2

W. Transition matrix for "Action and Warning" control chart			
	clear	**Warning**	**signal**
clear	1-P_A	P_W-P_A	P_A
Warning	1-P_W	0	P_W
Signal	0	0	1

4.3 Transition Matrix for Cusum Chart with Rayleigh Distribtuion

Brooks and Evans (1972) show that CUSUM procedures may be viewed as Markov chains. However for, continuous distributions, it is necessary to consider the discretization for the markov chain representation and the various states then corresponds to values of the CUSUM at any step. For an instance consider a scheme with decision interval H and reference value K are designed to detect upward shift from a target value. A set of (m + 1) states can be interpreted as the CUSUM values of

$$\leq 0,0\,\text{to}\,\frac{H}{m},\frac{H}{m}\,\text{to}\,\frac{2H}{m},\text{etc}\text{-------}(\text{m-1})\frac{H}{m}\,\text{to}<\text{H},\text{-------}\geq \text{H}. \tag{4.3.1}$$

The last of these states that is violation of the decision interval can be thought of as an absorbing barrier.

In the usual Markov chain notation with Transition matrix P and reduced matrix are one which is obtained from the deletion of the row and column representing the absorbing barrier. The well-known result for obtaining ARL from an initial zero CUSUM is the sum of the elements in the first row of $(I-R)^{-1}$. While considering the states the degree of discretization has some effect on the accuracy of ARL determination. In the present study 20 states transition matrices were used. Thus for H = 5 and K = 0.5 with μ at the target value. The transition matrix in general and for the particular study is shown in tables 4.3 and 5.1 respectively.

Table 4.3: C. Transition matrix for CUSUM scheme H, K (m + 1 states)

		0	$\frac{H}{m}$	$\frac{2H}{m}$	- ----------	$\frac{(m-1)H}{m}$	≥H
	≤0	$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	----------	$P_{0,m-1}$	$P_{0,m}$
CUSUM at	$\frac{H}{m}$	$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	----------	$P_{1,m-1}$	$P_{1,m}$
(i - 1)th sample	$\frac{2H}{m}$	$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	----------	$P_{2,m-1}$	$P_{2,m}$
	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -
	$\frac{(m-1)H}{m}$	$P_{m-1,0}$	$P_{m-1,1}$	$P_{m-1,2}$	----------	$P_{m-1,m-1}$	$P_{m-1,m}$
	≥H	0	0	0	----------	0	1

The entries in the above matrix need some explanation. In the first row, all entries corresponding moves from an initial zero CUSUM, and in the first entry, it indicates that a sample i, the CUSUM remains at or below zero. This means the sample value should not exceed the reference value k. Thus

$$P_{0,0} = P(x \le K) \tag{4.3.2}$$

For a move from state zero to H/m, the ith sample must have a value between K and (K +H/m). So that the subtraction of reference value gives a CUSUM contribution of H/m. After the discretization,

$$P_{0,1} = P(x = K + 2H/m) \tag{4.3.3}$$

5. Run Length Calculation under Linear Trend with Log Logistic Distribution

For calculating Run length distribution under linear trend with Log Logistic distribution, we make use of the discussion and procedure given in chapter-II. From this discussion we can obtain Transition Probability Matrix for the standard control charts viz., Action scheme, Warning scheme and CUSUM scheme.

For an illustration, consider W chart with action line at $3.09\sigma_e$ from a target value and warning limit at $1.96\sigma_e$. Let the process slippage to a value $1.00\sigma_e$ from the target value, then for Log-logistic variable

$$P_A = 1 - \Phi(3.09 - 1) = 0.015299$$

$$P_W = 1 - \Phi(1.96 - 1) = 0.0.146607$$

The transition matrix in this case is given by

$$P = \begin{bmatrix} 0.853393 & 0.131308 & 0.015299 \\ 0.853393 & 0 & 0.146607 \\ 0 & 0 & 1 \end{bmatrix}$$

The ARL is

$$\frac{1 + P_W - P_A}{P_A + P_W(P_W - P_A)} = 32.74439194$$

The probability of a signal at the first sample $P_{0,3}$ is 0.015299 . Squaring P, we get

$$\begin{bmatrix} 0.844923 & 0.110376 & 0.04470 \\ 0.724147 & 0.110376 & 0.155077 \\ 0 & 0 & 1 \end{bmatrix}$$

The element $P_{0.3}^{2}$ is 0.04470. Obviously the probability of a signal a sample 2 is 0.030542. Similarly $P_{0.3}^{3}$ is 0.072441, gives 0.030542 is the probability run length for three samples.

In the case of non-homogenous transition matrix, it is necessary to multiply original P by new transition matrix obtained after allowing a step change in the mean level. We denote the rate of change by Δ and we use ^{1}P, ^{2}P etc, for the first, second etc samples transition matrices are deduced. In general the Cumulative probability of signal before the ith sample is (0, m) th element of the product, i.e. $^{1}P\ ^{2}P\ ^{3}P$-------^{i}P.

Individual terms of the run length probability distribution are obtained by successive differences of cumulative probabilities.
Reconsider W scheme with A = 3.09, W = 1.96, with 0 shift we get
P_A =0.0002
P_W =0.020433

We get

$$^1P = \begin{bmatrix} 0.979567 & 0.020232 & 0.0002 \\ 0.979567 & 0 & 0.020433 \\ 0 & 0 & 1 \end{bmatrix}$$

At the second sample, the $0.5\sigma_e$ shift i.e. $\Delta = 0.5$ gives
P_A = 0.002427
P_W = 0.063859

We get

$$^2P = \begin{bmatrix} 0.936141 & 0.061431 & 0.002427 \\ 0.936141 & 0 & 0.063859 \\ 0 & 0 & 1 \end{bmatrix}$$

Also

$$^1P\ ^2P = \begin{bmatrix} 0.935954 & 0.060176 & 0.00387 \\ 0.917014 & 0.060176 & 0.022810 \\ 0 & 0 & 1 \end{bmatrix}$$

From the above calculations we get the cumulative probabilities of a signal at sample 2 is 0.008545. Subtracting the $(0, m)^{th}$ element of ^1P gives 0.00367. The above computations are presented for the sake of illustration allowing slippages at different levels similar type of calculated and re-designated as 1P 2P 3P-------
For different slippages in mean level, say Δ = 0.01, 0.02, 0.05, 0.1, 0.2 are computed and summarized below.
In case of CUSUM scheme the transition matrix can be obtained by making use of the formulae given in section 4.3.3 These are summarized in the following matri.

Table 5.1: Signaling Table for Action & Warning Charts

Shift	**P_A**	**P_W**		**0.9801589**	**0.0177707**	**0.0020704**					
0	0.0020704	0.0198411	^1P	0.9801589	0	0.0198411					
				0	0	1		0.9777296	0.0177699	0.0045005	
							$^1P * ^2P$	0.9603186	0.0177699	0.0219114	0.00243
0.01				0.9797581	0.0181297	0.0021123		0	0	1	
	0.0021123	0.0202419	^2P	0.9797581	0	0.0202419					
				0	0	1		0.9772805	0.0181215	0.0045979	
							$^2P*^3P$	0.9595253	0.0181215	0.0223532	0.002486
0.02				0.9793492	0.0184959	0.0021549		0	0	1	
	0.0021549	0.0206508	^3P	0.9793492	0	0.0206508					
				0	0	1		0.9759645	0.019234	0.0048014	
							$^3P*^4P$	0.9578742	0.019234	0.0228917	0.002646
0.05				0.9780722	0.0196396	0.0022882		0	0	1	
	0.0022882	0.0219278	^4P	0.9780722	0	0.0219278					
				0	0	1		0.9735333	0.0212292	0.0052375	
							$^4P*^5P$	0.9543696	0.0212292	0.0244012	0.002949
0.1				0.975766	0.0217051	0.0025288		0	0	1	
	0.0025288	0.024234	^5P	0.975766	0	0.024234					
				0	0	1		0.9679466	0.0258683	0.0061851	
							$^5P*^6P$	0.9468839	0.0258683	0.0272478	0.003656
0.2				0.9704006	0.0265107	0.0030887		0	0	1	
	0.0030887	0.0295994	^6P	0.9704006	0	0.0295994					
				0	0	1		0.9431442	0.0468759	0.00998	
							$^6P*^7P$	0.9180633	0.0468759	0.0350609	0.006891
0.5				0.9460663	0.0483057	0.005628		0	0	1	
	0.005628	0.0539337	^7P	0.9460663	0	0.0539337					
				0	0	1		0.8485901	0.1242265	0.0271834	
								0.8073664	0.1242265	0.0684071	0.021555
1				0.853393	0.1313085	0.0152985	$^7P*^8P$	0	0	1	
	0.0152985	0.146607	^8P	0.853393	0	0.146607					
				0	0	1					

In the above tables, if we multiply 3.2with 3.3 it gives cumulative probability of signal by sample 2 is 0.002064. Similarly one can compute the moments with expected values of appropriate functions of run length.

6. Run Length Properties of Standard Control Charts under Linear Trend with Log Logistic Distribution

The following table 6.1 gives the average run length and other properties of the run length distributions for three basic control charts namely a scheme, W scheme and CUSUM scheme. In these tables Δ values ranges from 0 to 1. Here we note that the shift will be detected rapidly when the trend is greater than 1 standard error per sample in all charts methods.

Table 6.1: Run Length Properties of Control Procedures under Linear Trend

			Scheme	
Δ		**A**	**W**	**C**
0	ARL	471.9074	328.6031	370.7045
	DEL*ARL	0	0	0
0.01	ARL	447.7653	313.0634	192.5529
	DEL*ARL	4.477653	3.130634	1.925529
0.02	ARL	425.1167	298.4103	172.5522
	DEL*ARL	8.502335	5.968205	3.451045
0.05	ARL	365.1412	259.2238	139.6568
	DEL*ARL	18.25706	12.96119	6.982838
0.1	ARL	286.8255	207.0704	129.5647
	DEL*ARL	28.68255	20.70704	12.95647
0.2	ARL	185.0777	137.0706	125.9978
	DEL*ARL	37.01555	27.41412	25.19957
0.5	ARL	69.9616	52.43937	118.6587
	DEL*ARL	34.9808	26.21969	59.32937
1	ARL	33.06159	22.27797	116.6856
	DEL*ARL	33.06159	22.27797	116.6856

The entries in the third and fourth column are obtained by using equation (2.1) and (3.1) the first column ARL are obtained by using from initial zero CUSUM which is the sum of the elements in the first row of $(I - R)^{-1}$ of respective slippage values. Here CUSUM scheme is operated with H = 5, K = 0.5 under log logistic distribution. The following table gives values of ARL and Δ*ARL (displacement ARL) for these basic control charts and the corresponding data for further two CUSUM scheme with H =8, K=0.25 and H = 2.5, K =1.

Control Charts							**Cusum schemes**							
		A=3.09		**A=3**		**A=3.09,W=1.96**	**A=3,W=2**		**H=8,K=0.25**		**H=5,K=0.5**		**H=2.5,K=1**	
Δ	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**
0	471.9074	0	328.6031	0	131.1323	0	134.1699	0	349.8561	0	370.7045	0	360.8578	0
0.005	459.6429	2.298215	320.7187	1.603594	129.5022	0.647511	132.2731	0.661365	250.2714	1.251357	242.5532	1.212766	231.3672	1.156836
0.01	447.7653	4.477653	313.0634	3.130634	127.8953	1.278953	130.4084	1.304084	188.856	1.88856	192.5529	1.925529	192.3664	1.923664
0.015	436.261	6.543915	305.6296	4.584445	126.3115	1.894673	128.5753	1.928629	149.8559	2.247839	182.5526	2.738288	181.3656	2.720484
0.02	425.1167	8.502335	298.4103	5.968205	124.7504	2.495008	126.7733	2.535467	143.868	2.877359	172.5522	3.451045	171.3648	3.427295
0.025	414.3201	10.358	291.3983	7.284958	123.2117	3.080291	125.002	3.125049	139.8557	3.496393	154.5519	3.863799	161.364	4.0341
0.03	403.8588	12.11576	284.5871	8.537613	121.695	3.650851	123.2606	3.697819	133.9155	4.017464	148.6097	4.458291	154.3022	4.629067
0.04	383.8963	15.35585	271.5413	10.86165	118.7268	4.749073	119.8661	4.794645	131.9423	5.277693	142.6338	5.705352	151.2722	6.050887
0.05	365.1412	18.25706	259.2238	12.96119	115.8435	5.792176	116.5858	5.829292	149.9662	7.498308	139.6568	6.982838	147.2451	7.362256
0.06	347.5121	20.85072	247.5891	14.85534	113.0428	6.78257	113.4159	6.804955	129.9897	7.799379	135.6797	8.140782	143.2181	8.593085
0.08	315.3352	25.22681	226.2008	18.09607	107.6805	8.614443	107.3923	8.591388	127.0356	10.16285	132.7255	10.61804	141.1641	11.29313
0.1	286.8255	28.68255	207.0704	20.70704	102.6229	10.26229	101.767	10.1767	123.8687	12.38687	129.5647	12.95647	138.3536	13.83536
0.15	228.7016	34.30524	167.4577	25.11866	91.20719	13.68108	89.27462	13.39119	122.186	18.3279	127.8849	19.18273	136.9764	20.54646
0.2	185.0777	37.01555	137.0706	27.41412	81.36279	16.27256	78.73662	15.74732	120.2839	24.05677	125.9978	25.19957	133.8433	26.76866
0.25	151.9843	37.99607	113.5382	28.38455	72.88245	18.22061	69.83811	17.45953	118.3742	29.59355	123.11	30.7775	130.711	32.67775
0.3	126.6212	37.98635	95.14802	28.54441	65.58263	19.67479	62.31365	18.6941	117.4573	35.2372	122.2214	36.66643	126.5795	37.97385
0.4	91.6571	36.66284	69.1273	27.65092	53.89679	21.55872	50.53079	20.21232	116.6038	46.64151	120.4418	48.17672	124.3188	49.72753
0.5	69.9616	34.9808	52.43937	26.21969	45.23702	22.61851	42.00476	21.00238	115.7263	57.86315	118.6587	59.32937	122.0611	61.03056
0.6	56.04004	33.62402	41.4113	24.84678	38.79635	23.27781	35.771	21.4626	115.3414	69.20482	117.8721	70.72324	119.8063	71.88376
0.8	40.56173	32.44938	28.70338	22.9627	30.3401	24.27208	27.70622	22.16498	114.981	91.98484	117.2871	93.82971	119.3048	95.44385
1	33.06159	33.06159	22.27797	22.27797	25.43894	25.43894	23.07402	23.07402	114.8988	114.8988	116.6856	116.6856	116.8139	116.8139
1.25	28.39658	35.49572	18.1321	22.66513	21.9301	27.41263	6.503641	8.129552	114.7245	143.4056	116.1587	145.1984	116.2148	145.2686
1.5	26.033	39.0495	15.96246	23.94369	19.93487	29.90231	4.608738	6.913108	114.2181	171.3272	115.6026	173.404	115.6317	173.4475
2	17.25648	34.51296	10.24578	20.49156	5.264789	10.52958	2.031458	4.062916	113.8119	227.6238	114.3994	228.7987	115.1991	230.3982
2.5	6.321457	15.80364	5.21478	13.03695	3.215478	8.038695			113.7027	284.2569	114.7966	286.9915	114.4628	286.1569
3	2.124578	6.373734	2.312457	6.937371	3.215478	9.646434			113.3756	340.1267	114.63	343.89	114.3805	343.1414

7. Conclusions

The various control schemes considered here are, in effect, continuous hypothesis tests. These hypotheses can be stated as

$$H_0 : \mu = T$$

Against the alternatives

$H_1 : \mu < T$
$H_1 : \mu < T$ } "Single-sided" schemes

$H_0 : \mu \neq T$ "Two-sided" schemes

In real world, it is frequently unknown whether the process averages μ will change suddenly or gradually. Most of the ARL calculations are based on the one standard deviation schemes under a trend alternative.
If trend is expected in the process average, this prior knowledge will be incorporated in to the design of control procedures while considering sampling intervals.
In the case of Log Logistic distribution suggest that there exists less difference in performance among schemes A, W and C under the linear trend than under step change conditions. Where as in scheme C with Log Logistic distribution the lower ARL for slippage of 0.2 to 0.1 standard errors is noted. It can be observed that the standard C scheme with Log Logistic distribution gives somewhat quicker response over the range $0.015 \leq \Delta \leq 0.6$ as compared with schemes of A and W schemes over the range $0.03 \leq \Delta \leq 0.3$.
These results are broadly compatible for those relating to step changes, in that, for example with ARL ≅ 6, at $\Delta = 0.3$ for W and C schemes, the process mean must have shifted above two standard errors by the time the trend is detected. Similarly, for A and C shift is about 3 standard error with ARL ≅ 4.9 at $\Delta = 0.6$. For step changes greater than 2. $5\sigma_e$, it is observed that lower ARL for W when compared with C scheme. The same situation is prevailed in the case of slippage greater than $2.5\sigma_e$. In table 6.2, the values of Δ ARL gives for further clarification on the point of selection A, W and C alternatives.

8. References

1. Brook D, Evans DA. An approach to the probability distribution of Cusum run length, Biomertika, 1972, 59.
2. Cox MAA. A comparison of Formulas for the design of cumulative sum control charts. Quality Reliability Engineering. 2009; 25:731-737.
3. Hinkley DV. Inference about the change point in a sequence of random variables. Biomertika. 1970; 57:1-17.
4. Hawkins DM. Testing a sequence of observations for a shift in location. Journal of the American Statistical Association. 1977; 72:180-186.
5. Johnson NL. Cumulative sum control charts and the Weibull Distribution. Technometrics. 1966; 8:481-491.
6. James Benneyan C. Performance of Number-Between g-Type Statistical Control Charts for Monitoring Adverse Events. Health Care Managemen Science. 2001; 4:319-336.
7. Johnson NL. Cumulative sum control charts and the Weibull Distribution. Technometrics. 1966; 8:481-491.
8. Khan RA. Wald's approximations to the average run length in CUSUM procedures. Statist. Plan. And Inf. 1978; 2:63-77.
9. Koning AJ, Does RJMM. Cusum charts for preliminary analysis of individual observations. Manuscript, submitted for publication. 1988; 3:133-137.
10. Moustakides GV. Optimal Stopping Times for detecting changes in distributions. Ann. Statist. 1986; 14:1379-1387.
11. Page ES. Continuous inspection schemes. Biometrika. 1954; 41:100-114.
12. Rendtel U. Cusum schemes with variable sampling intervals and sample sizes. Statistical papers, Springer Verlag. 1990; 31:103-118.
13. Rogerson PA. Formulas for the design of CUSUM quality control charts. Communications in Statistics-Theory and Methods. 2006; 35:373-383.
14. Sen A, Srivastava MS. On tests for detecting change in mean when variance is unknown. Annuals of the Institute of Statistical Mathematics. 1975; 27:479-486.

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3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal

2019-20

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link to the recognition in UGC enlistment of the Journal		
							Link to website of the Journal	Link to article / paper / abstract of the article	Is it listed in UGC Care list
1	*Influence of Soil Elements on Photosynthesis and Secondary Metabolites in Selected Medicinal Plants*	Dr. Krishna Naragani	Department of Botany	Research Journal of Life Sciences, Bioinformatics, Pharmaceutical and Chemical Sciences	2019	ISSN: 2454-6348	www.rjlbpcs.com	DOI: 10.26479/2019.0502.68	NO
2	*Phylogenetic Characterization of Potential Bioactive Metabolites Producing Actinomycetes from Mangrove Sediments*	Dr. Krishna Naragani	Department of Botany	International Journal of Pharmaceutical Sciences and Research	2020	E-ISSN: 0975-8232; P-ISSN: 2320-5148	https://ijpsr.com/	DOI: 10.13040/IJPSR.0975-8232.11(12).6124-29	NO
3	*Health Insurance in Andhra Pradesh: Dr. NTR Vaidya Seva Health Insurance Scheme*	Smt. V. SailajaVani	Department of Commerce	Journal of Emerging Technologies and Innovative Research	2020	ISSN-2349-5162	www.jetir.org	https://www.jetir.org/papers/JETIR1905A72.pdf	http://jetir.org/jetir%20ugc%20approval.pdf
4	*Songs and Music in Shakespeare's Plays*	Smt. V. Beulah Rani	Department of English	Research Journal of English Language and Literature (RJELAL)	2019	ISSN:2395-2636 (P); 2321-3108(O)	http://www.rjelal.com/	http://www.rjelal.com/7.4.19/172-176%20V.%20BEULAH%20RANI.pdf	NO
5	*Thematic Analysis of Satish Chender's Telugu Story "It Is Wrong." In English*	Sri Kumbha V Koteswara Rao	Department of English	Research Journal Of English (RJOE)	2019	ISSN: 2456-2696	https://www.rjoe.org.in/	https://www.rjoe.org.in/Files/VOL4ISS4/27.RJOE-KOTESWA%20RAO_249-252_OK_1.pdf	NO
6	*Application of Krasnoselskii Theorem for Multi valued Operators*	Dr. B. Rami Reddy	Department of Mathematics	Research J. Science and Tech	2019	ISSN 0975-4393 (Print) 2349-2988 (Online)	https://rjstonline.com/	DOI: 10.5958/2349-2988.2019.00029.9	NO

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7	*Application of Fuzzy Quasi Metric Spaces in the Domain of Words*	Dr. B. Rami Reddy	Department of Mathematics	Journal of Emerging Technologies and Innovative Research (JETIR)	2019	ISSN-2349-5162	www.jetir.org	https://www.jetir.org/view?paper=JETIR1905D04	http://jetir.org/jetir%20ugc%20approval.pdf
8	*The Sum of ith Smallest Parts of r - partitions of n*	Dr. K. Hanuma Reddy Lt. Dr. A. Majusree	Department of Mathematics	International Journal of Recent Scientific Research	2020	ISSN: 0976-303	http://www.recentscientific.com	DOI: 10.24327/IJRSR	http://recentscientific.com/approved-ugc
9	*The number of ith smallest parts of of r - partitions n*	Dr. K. Hanuma Reddy Lt. Dr. A. Majusree	Department of Mathematics	International Journal of Creative Research Thoughts (IJCRT	2020	ISSN: 2320-2882	www.ijcrt.org	https://ijcrt.org/archivelist.php	https://ijcrt.org/ugc%20approval.jpg
10	*On Semicircular Extreme-value distribution*	Dr. S.V.S. Girija Sri R. Srinivas	Department of Mathematics	International Journal of Applied Engineering Research	2019	ISSN 0973-4562	https://www.ripublication.com/ijaer.htm	https://www.ripublication.com/ijaer19/ijaerv14n9_18.pdf	UGC Approved Journal - 2017 (Journal No. - 64529)
11	*On Discrete Wrapped Cauchy Model*	Dr. S.V.S. Girija Sri R. Srinivas	Department of Mathematics	Mathematical Theory and Modeling	2019	ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)	https://iiste.org/Journals/index.php/MTM	DOI: 10.7176/MTM	NO
12	*On Stereographic Semicircular Quasi Lindley Distribution*	Dr. S.V.S. Girija	Department of Mathematics	JOURNAL OF NEW RESULTS IN SCIENCE (JNRS)	2019	ISSN: 1304-7981	https://dergipark.org.tr/en/pub/jnrs	https://dergipark.org.tr/tr/download/article-file/822866	NO
13	*On the Rising Sun offset Pearson Type II Model*	Dr. S.V.S. Girija	Department of Mathematics	International Journal of Mathematical Archive	2019	ISSN 2229 – 5046	http://www.ijma.info/index.php/ijma	http://www.ijma.info/index.php/ijma/article/view/6058/3573	NO

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S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
14	*The Discrete Rising Sun Wrapped Cauchy Model*	Dr. S.V.S. Girija Sri Y. Sreekanth	Department of Mathematics	i-manager's Journal on Mathematics	2019	ISSN, 2277-5137	https://imanagerpublications.com/journalsfulldetails/27/JournalonMathematics	https://www.researchgate.net/publication/337891456_THE_DISCRETE_RISING_SUN_WRAPPED_CAUCHY_MODEL	NO
15	*Analytical Study Of Sio $_2$-Water Based Nano Fluid Flow In A Rotating Frame With Thermo Phoresis*	Dr. Y. Uday Kumar	Department of Mathematics	International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)	2019	ISSN: 2249-6890 (print), 2249-8001 (online)	https://journals.indexcopernicus.com/journal/45150	https://oa.mg/work/10.24247/ijmperdaug201943	NO
16	*Under Water Pollution Monitoring System to Prevent Fatalities of Aqua Ecosystem*	Dr. Y. Uday Kumar	Department of Mathematics	The Patent Office Journal No. 42/2020	2020	Application No.202041044264 A	https://search.ipindia.gov.in/DynamicUtility/Journal/Patent	https://www.quickcompany.in/patents/under-water-pollution-monitoring-system-to-prevent-fatalities-of-aqua-ecosystem	NO
17	*QOS Modelling for Software Service Improvement using Adaptive Learning*	Sri Ch. Subbarao	Department of Computer Science (PG)	International Journal of Innovative Technology and Exploring Engineering (IJITEE)	2019	ISSN: 2278-3075	https://www.ijitee.org/	DOI: 10.35940/ijitee.L3553.1081219	NO
18	*Challenges of Student-Centered Learning A Note*	Dr. K. Sreelakshmamma	Department of Political Science	Research Journal of English Language and Literature (RJELAL)		ISSN:2395-2636 (P); 2321-3108(O)	http://www.rjelal.com/	http://www.rjelal.com/8.S1.2020/113-117.pdf	NO
19	*श्रीभाष्यं विजयसारथेः मन्दाकिनी काव्यस्य महौत्कृष्टत्वम्*	Dr. D.N. Deekshit	Department of Sanskrit	International Journal of Sanskrit Research	2020	ISSN: 2394-7519 IJSR 2020; 6(5): 333-336	https://www.anantaajournal.com/	https://www.anantaajournal.com/archives/?year=2020&vol=6&issue=5&part=F&ArticleId=1186	NO

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S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
20	श्रीविष्णु महापुराणे कलिधर्म निरूपणम्	Dr. D.N. Deekshit	Department of Sanskrit	International Journal of Sanskrit Research	2020	ISSN: 2394-7519 IJSR 2020; 6(5): 333-336	https://www.anantaajournal.com/	https://www.anantaajournal.com/archives/?year=2020&vol=6&issue=1&part=C	NO
21	धन्वन्तरिणा उक्तं पशुभिः प्रयोजनम्	Dr. D.N. Deekshit	Department of Sanskrit	International Journal of Sanskrit Research	2020	ISSN: 2394-7519 IJSR 2020; 6(5): 333-336	https://www.anantaajournal.com/	https://www.anantaajournal.com/archives/?year=2020&vol=6&issue=1&part=C	NO
22	*Comparison of Control with Cusum Charts – A Case Study*	Dr. N. Viswam	Department of Statistics	BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH	2019	ISSN: 2348-0580	http://www.bomsr.com/4.1.16.html	http://www.bomsr.com/7.2.19/39-47%20K.%20SRINIVASA%20RAO.pdf	NO
23	*Modern Regression with R Programming*	Dr. N. Viswam	Department of Statistics	GIS SCIENCE JOURNAL	2020	ISSN NO : 1869-939	https://gisscience.net/	https://gisscience.net/volume-7-issue-11-2020/	NO
24	*Global Epidemiology of Lung Cancer Prediction using Regression Analysis*	Dr. N. Viswam	Department of Statistics	International Journal of Creative Research Thoughts (IJCRT)	2020	ISSN: 2320-2882	www.ijcrt.org	https://ijcrt.org/papers/IJCRT2011126.pdf	NO



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INFLUENCE OF SOIL ELEMENTS ON PHOTOSYNTHESIS AND SECONDARY METABOLITES IN SELECTED MEDICINAL PLANTS

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ABSTRACT: The influence of mineral nutrients on photosynthesis and production of secondary metabolites was studied and analyzed in the plants *Alstonia scholaris*, *Tabebuia argentea* and *Jacquinia barbasco* growing in Botanical Garden of Acharya Nagarjuna University, Guntur during 2015 and 2016. Of all the plants maximum mineral nutrients uptake was observed in *T. argentea*. The rate of photosynthesis (A_{max}), rate of transpiration (E) and stomatal conductance (g_s) were found to be more in *T. argentea* i.e 7.09 µmoles $m^{-2}s^{-1}$ CO_2, 2.95 mmol $m^{-2}s^{-1}$, and 0.07 mmol CO_2 $m^{-2}s^{-1}$ respectively. Results also indicated that an optimum accumulation of mineral nutrients increased the rate of photosynthesis and secondary metabolite production in *T. argentea*.

KEYWORDS: photosynthesis, soil nutrients, phytochemicals, secondary metabolites.

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1.INTRODUCTION

Photosynthesis is a key physiological process influences all other cellular activities by providing ATP requirements. Adenosine tri phosphate produced during phosphorylation reactions not only utilized in sugars production but also involved in synthesis of so many metabolic intermediates and precursors for various secondary metabolites [1,2,3]. But the rate of photosynthesis depends on so many factors. Availability of minerals is one of such factors regulating the photosynthetic process [4]. These mineral nutrients may include both macro and micro elements/nutrients such as N, P, K, Zn, Fe, Mn, Mg, Cuand S. Of these mineral nutrients some are metal ions. These metal

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ions may bound to proteins and make them ready for the transport of molecular species [5,6]. Copper is an important constituent of plastocyanin (Pc) and Cu/Zn superoxide dis mutase. Former acts as an electron carrier between PS I and PS II in photosynthetic reactions [7] and latter acts as a protection agent during oxidative stress [8,9]. In leaves 'Fe' present in Ferretin forms and mineralized with phosphate [10]. These Ferretins may be potential 'Fe' donors in buildup of chloroplast. On the other side 'Fe' also plays an important role in formation of PS I, PS II, Cyt b 559 [11] and Ferridoxin. Previous literature reported that in order to catalyze the water oxidation process with in the OEC complex plants require 'Mn' to form the 'Mn' cluster in PS II which is located in the donor side [12,13,14]. Photophosphorylation in chloroplasts membranes may be accomplished by the reversal of ATPases activated by Ca^{+2}. Thus reduction in 'Ca' results in disturbances in energy conversions in chloroplasts [15,16]. The catalization of the rapid inter-conversion of CO_2 and H_2O to HCO^{-3} during photosynthetic 'C' fixation is promoted by Zn containing metalloenzyme β-carbonic anhydrase (β-CA) which is located in the stroma of chloroplast [17]. The coordinated action of both nitrogen and phosphorus influence Ribulose 1,5 – bisposphate carboxylase oxygenase (Rubisco) the key enzyme in photosynthetic fixation [18,19]. As the phospholipids are important constituents of membranes, decreased availability of phosphorus may disturb the membrane integrity of cellular organelles [16,20]. Potassium plays a significant role in 'CO_2' assimilation, where it is involved in stomatal regulation, ATP synthesis and enzyme activation [21]. However non stomatal regulation of 'K' deficiencies like reduced chlorophyll content, inhibition of PS II activity and '$^{-}$e' transport were also observed [22]. Magnesium (Mg) involved in grana stacking and LHC II was found to participate in the cation mediated formation of grana [23]. Magnesium also catalyzes many enzyme reactions involving phosphate transfer and ATP metabolism [24]. Chloroplasts contain protein rich in sulfur. The decrease in chlorophyll content might result in 'S' deficiency which further results in disruption of chloroplast structure [25]. Sulfur is an important component of Fe-S cluster which are co-factors of proteins that perform electrons transfer, redox and nonredox catalysis [26]. The opening and closure of stomata is mediated by the fluxes of potassium and accompanying anions such as malate and chloride [27]. Chloride is necessary for the water- splitting reaction in PS II. Besides 'Mn' chloride plays a fundamental role in the water-splitting system of PS II [28]. On the other side 'Cl' acts as a bridging ligand for stabilization of the oxidized state of manganese [29]. As discussed, the impact of nutrient availability on assimilatory process was well established in most of the medicinal plants. On the other side it is well evident that effective photosynthetic processes will enhance the production of primary and secondary metabolites in so many plants including medicinal plants [30,31,32]. The available literature confirmed that there is a strong correlation between nutrients, photosynthesis and secondary metabolite production. Plants such as *Alstonia scholaris, Tabebuia argentia* and *Jacquinia barbasco* are the traditional medicinal plants and the

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extracts obtained from these plants were used to treat many ailments such as fever, malaria [33], tumors [34], asthma [35], antidiabetic [36] and antioxidant [37]; antitumor activity, antimetastatic activity anti-microbial and antifungal activity [38,39], antiviral activity [40], anti-inflammatory [41], antiparasitic activity [42], leishmanicidal activity [43]and molluscicidal activity [44]; antifungal, antibacterial, antioxidant activity [45] respectively. But their photosynthetic efficiency in response to nutrient availability was not explored till now. By studying the photosynthetic capacity of these plants we can further understand the ability of these plants in producing secondary metabolites.

2. MATERIALS AND METHODS

The present study was conducted in the Botanical Garden of Acharya Nagarjuna University (latitude 16°22'31.16" N and longitude 80°31'042.9" E). The soil is red loamy and the average rainfall is around 831 mm with a monthly average temperature of 33.6 °C (midsummer) and 24.1 °C (midwinter). Three traditional medicinal plants *Alstonia scholaris* (Devil tree), *Tabebuia argentea* (Silver trumpet tree) *and Jacquinia barbasco* (torchwood tree) of the same age group growing in Botanical Garden of Acharya Nagarjuna University are selected for this study and the data was taken on the following parameters. Photosynthetic rate (μmoles $m^{-2}s^{-1}$ CO_2), rate of transpiration (mmol $m^{-2}s^{-1}$) and stomatal conductance (mmol CO_2 $m^{-2}s^{-1}$) were measured with a portable open-path gas exchange system with CO_2 control (LC pro portable IRGA (ADC, UK). Measurements were undertaken using the standard 20 × 30 mm chamber equipped with blue-red light emitting diodes mounted on the top of the chamber with a photosynthetic photon flux density (PPFD) of 1600 μmol m^{-2} s^{-1} and CO_2 of 370 ppm. After enclosure in the chamber, the leaves were left to equilibrate until a constant CO_2 flux was observed (up to 5 min). Measurements were made between 09:00 and 11:00 h Indian standard time (IST). Chlorophyll and carotenoid contents were determined by using dimethyl sulfoxide (DMSO) method [46]. Chlorophyll 'a' and chlorophyll 'b' were measured at 645 nm and 663 nm. Carotenoids were estimated at 480 nm and 510 nm using UV-VIS Spectrophotometer (Elico *SL 159*). Saturation extracts of soils were analyzed for electrical conductivity using an electrical conductivity meter. Soil pH was determined on air-dried samples using a 1: 2.5 soil to 1 M KCl ratio. Micro and macro nutrients such as K^+, Zn^{2+}, Cu^{2+}, Mn^{+2}, S, Fe, N_2 and Mg^{2+} were determined by atomic absorption (Varian Spectra AA-10, Mulgrave, Australia). Phosphorus was determined by the molybdenum blue procedure [47] and a spectrophotometer (Beckman DU 640, Fullerton, USA). Leaf protein content was determined according microkzeldal method [48].

Qualitative Phytochemical Screening: Shade dried plant material was extracted with methanol and preliminary phytochemical screening was done by using the standard tests.

Alkaloid Test (Dragendroff's Test): A 2 ml of plant extract was acidified with few drops of dilute hydrochloric acid. To this acidic medium, 1 ml of Dragendroff's reagent (Potassium

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bismuth iodide) was added. An orange or reddish brown precipitate produced indicates the presence of alkaloids.

Anthocyanidins: To the plant extract an equal volume of methanolic HCl was added. Appearance of red or purple color indicated the presence of anthocyanidins.

Flavonoid Test (Shinoda Test): The presence of flavonoids was confirmed by treating the alcoholic plant extract with few fragments of magnesium ribbon and hydrochloric acid. The reaction mixture develops pink colour indicating the presence of flavonoids.

Saponin Test (Foam Test): One millilitre of each extract was shaken with 10 ml of distilled water and it was agitated in a graduated cylinder for 10 min. The formation of persistent honey-comb like froth indicated the presence of saponins.

Quinone Test: A small amount of extract was treated with concentrated HCl and observed for the formation of yellow colour precipitate.

Tannin Test (Lead Acetate Test): To 2 ml of each extract few drops of 10% Lead acetate was added. The appearance of white precipitate indicated the presence of tannins.

Terpenoids and Steroids: To the test tube containing a mixture of methanolic HCl and acetic anhydride, 50% H_2SO_4 was added. Change in color, from green to blue-green indicated the presence of terpenoids and steroids.

Phenol Test: To 2 ml of extract 0.5 ml of $FeCl_3$ (w/v) solution was added, formation of an intense colour indicated the presence of phenols.

Coumarin: To the methanolic extract, a few drops of alcoholic sodium hydroxide was added. Formation of yellow color indicated the presence of coumarins.

Test for Glycosides: Methanolic extract was mixed with a little anthrone on a watch glass. Few drops of conc. H_2SO_4 was added and warmed gently over water bath. The presence of glycosides was identified by dark green color formation.

Resins: Plant extracts were treated with acetone. To this, small amount of water was added and shaken. The appearance of turbidity indicates the presence of resins.

Quantitative determination of phytochemicals

Total phenols

Total phenolic content in the plant methanolic extracts was determined using spectrophotometric method [49]. The reaction mixture was prepared by mixing 0.5 ml of plant extract solution, 2.5 ml of 10% Folin-Ciocalteu's reagent dissolved in water and 2.5 ml of 7.5% of $NaHCO_3$ aqueous solution. The samples were thereafter incubated in a thermostat at 45 ºC for 45 minutes. The absorbance was determined using spectrophotometer at wave length 765 nm. The same procedure was repeated for the standard solution of gallic acid and the calibration line was constructed. Based on the measured absorbance, the concentration of gallic acid equivalent expressed in terms of (mg of GA/g of extract).

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Crude saponins

To the 20 g of extracted methanolic sample 100 ml of 20 % aqueous ethanol was added. The solution was heated for 4 h with constant stirring at 55°. Solution was filtered and was extracted with 200 ml 20 % ethanol. After that both extracts were mixed and solvent was evaporated till 40 ml volume of extract remains. The concentrate was extracted with 20 ml of diethyl ether in separating funnel. The aqueous layer was recovered while the ether layer was discarded. The aqueous extracts were purified by adding 60 ml n-butanol. Further it was washed with twice 10 ml of 5 % aqueous NaCl. The solution was dried and the saponin content was calculated as percentage [50,51].

Tannins

Amount of tannins present in the methanolic leaf extracts were determined by Folin-Ciocalteu method [52]. About 0.1 ml of the sample extract was added to a 10 ml volumetric flask containing 7.5 ml of distilled water and 0.5 ml of Folin-Ciocalteu phenol reagent, 1 ml of 35% sodium carbonate solution and diluted to 10 ml by adding distilled water. The mixture was shaken well and kept at room temperature for 30 minutes. Absorbance was measured against the blank at 700 nm with an UV/Visible spectrophotometer. The tannin content was expressed in terms of mg of tannic acid equivalents/ g of dried sample.

Alkaloids

The plant extract (1 mg) was dissolved in dimethyl sulfoxide (DMSO), 1 ml of 2 N HCl was added and filtered. This solution was transferred to a separating funnel, 5 ml of bromocresol green solution and 5 ml of phosphate buffer were added. The mixture was shaken with 1, 2, 3 and 4 ml chloroform by vigorous shaking and collected in a 10 ml volumetric flask and diluted to the volume with chloroform. A set of reference standard solutions of atropine (20, 40, 60, 80 and 100 µg/ml) were prepared in the same manner as described earlier. The absorbance for test and standard solutions were determined against the reagent blank at 470 nm with an UV/Vis spectrophotometer. The total alkaloid content was expressed as mg of AE/g of extract [53,51].

Steroids

The steroids present in the plant sample was estimated [54]. A 1ml of Methanolic extract of steroid solution was transferred into 10 ml volumetric flasks. Sulfuric acid (4N, 2ml) and iron (III) chloride (0.5% w/v, 2 ml), were added, followed by potassium hexacyanoferrate (III) solution (0.5% w/v, 0.5 ml). The mixture was heated in a water-bath maintained at 70±20 °C for 30 minutes with occasional shaking and diluted to the mark with distilled water. The absorbance was measured at 780 nm against the reagent blank.

Coumarins

Coumarin content was estimated following the standard methods [55,56]. To 500 µl of plant extract, 2 ml distilled water and 500 µl of lead acetate (5%, w/v) solution were added in a test

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tube. After shaking thoroughly, 7 ml of distilled water was added and mixing well, 2 ml of this solution was taken in another test tube and 8 ml of 0.1 M (v/v) hydrochloric acid solution was added. The solution was kept for 30 minutes at room temperature and absorbance was recorded at 320 nm using UV-Visible spectrophotometer. The total coumarin content was expressed as mg of coumarin equivalents per gm of sample extract (mg CE/g).

Cardiac glycosides

Cardiac glycosides were determined according Muhammad and Abubakar [57]. To the 8 ml of plant extract 60 ml of H_2O and 8 ml of 12.5% lead acetate were added, mixed and filtered. From this 50 ml of the filtrate was transferred into another 100 ml flask and 8 ml of 47% Na_2HPO_4 were added to precipitate excess Pb^{2+} ion. This was mixed and completed to volume with water. The mixture was filtered twice through same filter paper to remove excess lead phosphate. Now 10 ml of purified filtrate was transferred into clean Erlyn – Meyer flask and treated with 10ml Baljet reagent. A blank titration was carriedout using 10ml distilled water and 10ml Baljet reagent. This was allowed to stand for one hour for complete colour development. The colour intensity was measured colorimetrically at 495 nm.

Calculation

$$\% \text{ Total glycosides} = \frac{A \times 100}{77}$$

Where A = absorbance

3. RESULTS AND DISCUSSION

Results

Reduced mineral nutrients may affect the photosynthesis and in turn secondary metabolite production. To identify the effect of soil elements on photosynthesis and secondary metabolite production the present study was conducted.

Gas exchange parameters

In present study the mean photosynthetic rate was ranged between 2.35 to 7.09. Of the three plants studied highest maximum photosynthetic rate (A_{max}) 7.09 µmoles $m^{-2}s^{-1}$ CO_2 was recorded by *T. argentea*, followed by *A. scholaris* and *J. barbasco* (Fig. 1). The mean stomatal conductance (g_s) and transpiration rate (E) was ranged from 0.024 to 0.076 m mol CO_2 $m^{-2}s^{-1}$ and 1.26 to 2.95 mmol $m^{-2}s^{-1}$ respectively. Coupled with maximum A_{max} high g_s and E was observed in *T. argentea* i.e 0.076 m mol CO_2 $m^{-2}s^{-1}$ and 2.95 mmol $m^{-2}s^{-1}$ (Fig. 1).

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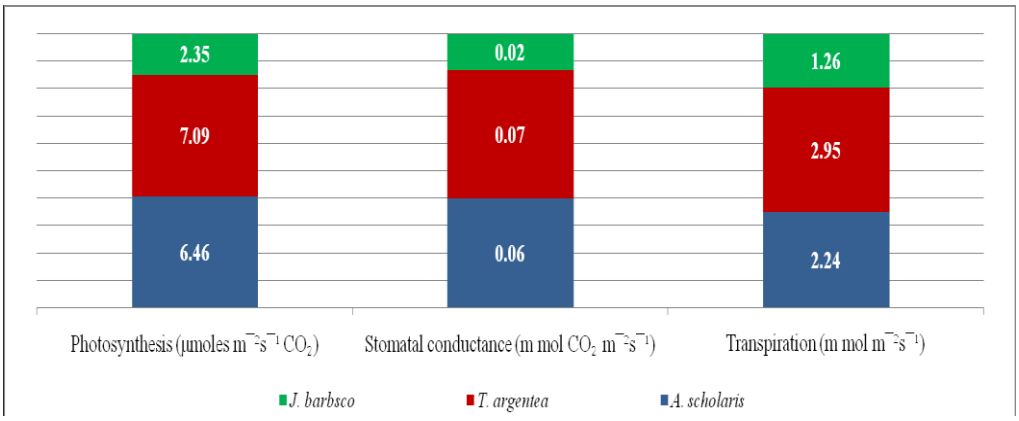

Figure 1. Variation in Gas exchange parameters

Leaf chlorophyll

Leaf chlorophyll characters such as Chl 'a', Chl 'b' and total chlrophyll content were estimated (Fig. 2). Maximum chl 'a' (1.25 mg/g) and Chl 'b' (0.94 mg/g) along with total chlorophyll content was observed in *T. argentea* (1.95 mg/g). Coupled with chlorophyll content the maximum carotenoid content (0.90 mg/g) was observed in *T. argentea*.

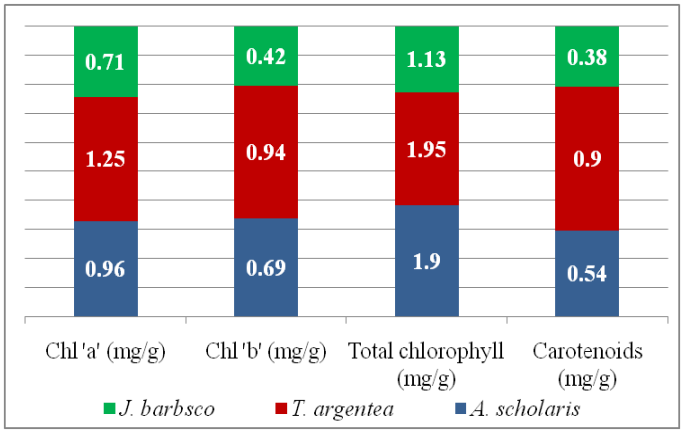

Figure 2. Leaf chlorophyll characters of the studied plants

Leaf protein

Proteins would play major role in metabolic activities such as photosynthesis and production of secondary metabolites. In present study high leaf protein content (0.019 µg/mg) (Fig. 3) was reported in *T. argentea* followed by *A. scholaris* (0.018 µg/mg) and *J. barbasco* (0.016 µg/mg).

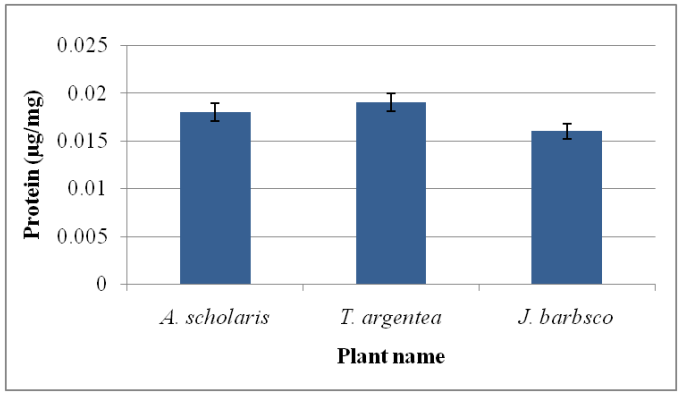

Figure 3. Total protein content of the leaves

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Qualitative phytochemical screening

The present study revealed that the methanolic leaf extracts of *T. argentea*, *A. scholaris* and *J. barbasco* confirmed the existence of different phytochemicals (Table 1). Coumarins, phenols, saponins, tannins, steroids and alkaloids are present in all the three methanolic leaf extracts. But the existence of coumarins and glycosides are present only *T. argentea* and *J. barbasco*.

Table 1. Phytochemical screening of methanolic leaf extracts

Phytochemical	***A. Scholaris***	***T. Argentea***	***J. barbasco***
Anthocyanidins	-	-	-
Phenols	+	+	+
Coumarins	-	+	+
Quinones	+	+	+
Glycosides	-	+	+
Saponins	+	+	+
Tannins	+	+	+
Flavonoids	-	-	-
Terpenoids/Steroids	+	+	+
Resins	+	+	+
Alkaloids	+	+	+

Quantitative determination of phytochemicals

Total secondary metabolites content were determined by UV spectrophotometric method. The present investigation shows that significant variation in the contents like alkaloids, saponins, phenols, tannins, cardiac glycosides, coumarins, and steroids. The alkaloid content was examined in plant extracts and expressed in terms of atropine equivalent as mg of AE/g of extract ($y=0.0002x+0.002$, $R^2=0.988$) shown in Fig. 4. The total alkaloid content ranging from 28.10±2.13 to 48.14±1.4 % w/w. The highest alkaloid concentration 48.14±1.4 % w/w was recorded in *T. argentea* (Table 2).

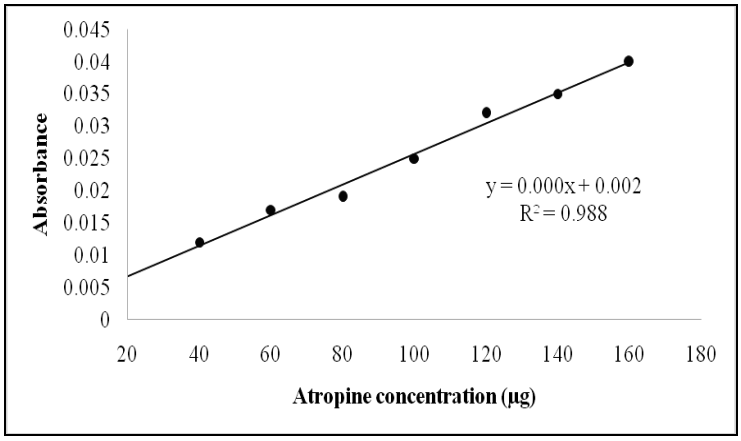

Figure 4. Standard curve of Atropine for alkaloids

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Total saponin content ranging between 1.82±1.85 to 2.56±0.49 % w/w. Of all the plants saponin content was found to be more (2.56±0.49 % w/w) in *T. argentea* (Table 2). Phenolic compounds are essential for the growth and reproduction of plants, and plays prominent role in plant defense mechanism. Using the standard plot of gallic acid (y=0.0289x+0.0311, R^2=0.9936) (Fig. 5), the phenolic contents of leaf methanolic extract of *T. argentea*, *A. scholaris* and *J. barbasco* were estimated. The phenolic content was ranging from 0.81±1.21 to 2.89±0.45 mg GA/g sample and in present study maximum phenol content was recorded in *T. argentea* (Table 2).

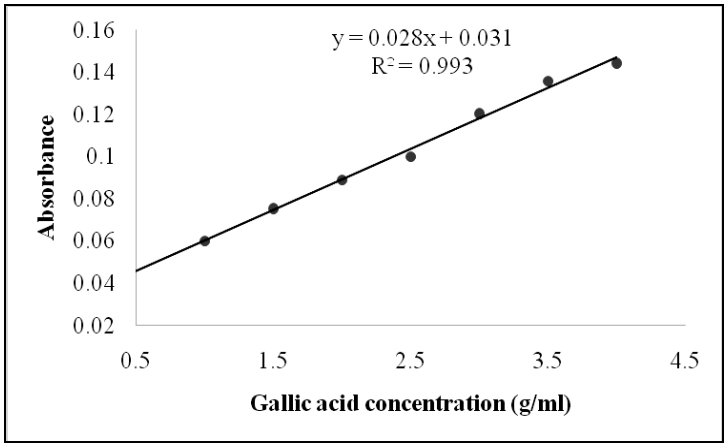

Figure 5. Standard curve of Gallic acid for phenols

To determine the amount of tannins present in the methanolic leaf extracts a standard plot was drawn by using tannic acid as standard (0.0005x+0.0487, R^2=0.9947) shown in Fig. 6. In present study total tannin content was varied between 2.14±2.15 to 7.99±0.84 %w/w. The total tannin content was found to be more in *T. argentea* (7.99± % w/w) (Table 2).

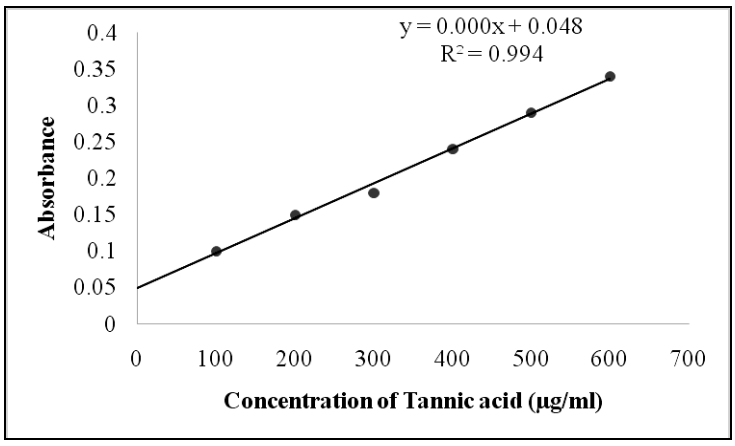

Figure 6. Standard curve of Tannic acid for tannins

Cardiac glycoside and coumarins were not found in *A. scholaris*. The maximum cardiac glycosides and coumarins content 2.10±0.48 g/100g and 17.32±0.82 mg/g was reported in *T. argentea* respectively (Table 2). Coumarin content was estimated using coumarin equivalents as standard (CE mg/g) (y=13.9x-0.392, R^2=0.966) presented in Fig. 7.

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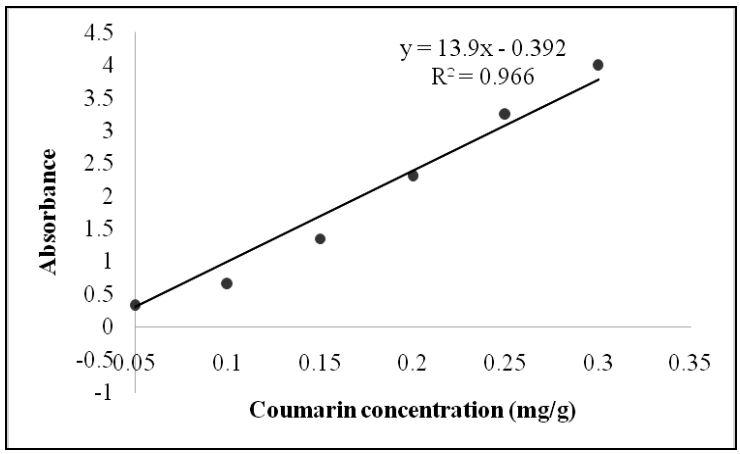

Figure 7. Standard curve of coumarin for coumarins

The steroids content was ranging from 76.36 µg to 118.12 µg and the maximum steroids content was observed in *T. argentea* (Table 2). A standard graph was plotted for the steroids by using cycloartenol as standard (y=3.74+0.048, R^2=0.996) and expressed in µg/mg (Fig. 8).

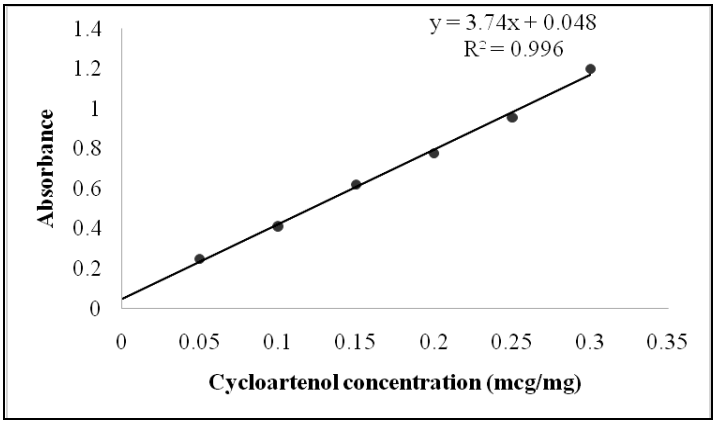

Figure 8. Standard curve of Cycloartenol for steroids

Table 2. Quantitative estimation of methanolic leaf extracts

Secondary metabolite	*A. scholaris*	*T. argentea*	*J. barbasco*
Alkaloids (% w/w)	36.28	48.14	28.10
Saponins (% w/w)	2.10	2.56	1.82
Phenols (mg/g)	1.72	2.89	0.81
Tannins (% w/w)	5.41	7.99	2.14
Cardiac glycosides (g/100g)	-	2.10	1.16
Coumarins (mg/g)	-	17.32	11.15
Steroids (µg/mg)	95.21	118.12	76.36

Elemental analysis

Availability different micro and macro nutrients of soils of Botanical Garden, Acharya Nagarjuna University and experimental plant leaves were studied and were presented in Fig. 9. Data presented in Fig. 9 represents that nitrogen content ranged from 8.10 µg/mg to 10.95 µg/mg. The maximum nitrogen content was recorded with *T. argentea* (10.95 µg/mg, followed by *A. scholaris*

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(8.86 µg/mg) and *J. barbasco* (8.10 µg/mg). The values of phosphorus accumulated in the studied plants ranged from 1.89 µg/mg to 3.20 µg/mg and it is clear that the leaf samples of *T. argentea* (3.20 µg/mg). The potassium content of the plants leaves ranged from 3.82 µg/mg to 5.25 µg/mg and the maximum potassium content was observed in *T. argentea* (3.82 µg/mg). As seen from the data presented in Fig. 9, the studied plants varied significantly in accumulating zinc and it was ranged from 0.59 µg/mg to 1.10 µg/mg. Of the three plants studied highest accumulation of zinc was reported in *T. argentea* (1.10 µg/mg). The accumulation of the iron content in the studied plants ranged from 5.81 µg/ to 8.92 µg/mg. Highest iron accumulation was recorded in *T. argentea* followed by *A. scholaris* (6.98 µg/mg) and *J. barbasco* (5.81 µg/mg). Significant amount of manganese was reported in the studied plants and it was ranged between 1.76 µg/mg to 3.92 µg/mg. Maximum accumulation of manganese was reported in *T. Argentea* (3.92 µg/mg). The copper content ranged from 1.21 µg/mg to 2.18 µg/mg (Fig.2). The values of copper content in *T. argentea* (2.18 µg/mg), *A. scholaris* (1.81 µg/mg) and *J. barbasco* (1.21 µg/mg). The sulfur contents recorded leaf samples of the studied plants was ranged from 1.80 µg/mg to 3.60 µg/mg. The highest sulfur content was observed in leaf samples of *T. argentea* (3.60 µg/mg). It is clear from the Figure 2 that the magnesium content of plants was ranged from 1.42 µg/mg to 2.74 µg/mg. And the least magnesium content was reported with leaves of *J. barbasco* (1.42 µg/mg).

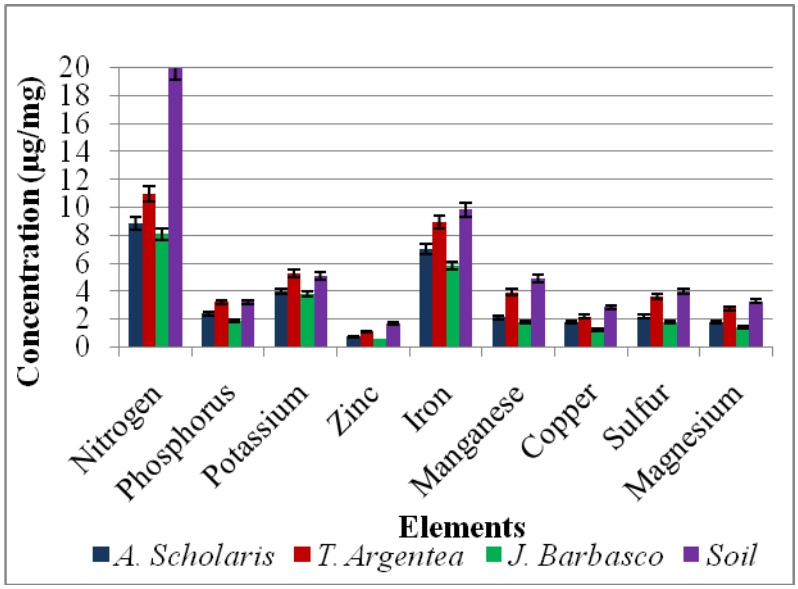

Figure 9. Nutrient availability in studied plants and corresponding soil

DISCUSSION

The present study was conducted to identify the effect of available soil mineral elements on photosynthesis coupled with secondary metabolites accumulation. Plants exhibit a remarkable ability to sense environmental factors such as soil mineral elements. Ribulose 1,5 – bisposphate carboxylase oxygenase (Rubisco) is the key enzyme in photosynthetic fixation process and its production is entirely depends upon available N_2 [23]. In present study highest A_{max} (7.09 µmoles $m^{-2}s^{-1}$ CO_2) along with g_s (0.07 mmol CO_2 $m^{-2}s^{-1}$) and E (2.95 m mol CO_2 $m^{-2}s^{-1}$) was observed in

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T. Argentea in which high leaf N_2 content was present (Fig. 9). Adequate accumulation of N_2 on the other hand tend to produce more production of alkaloids in *T. argentea* (Table 2), which is previously reported in basil [58]. Moreover limitations in Mg, P and Fe also influence the synthesis of alkaloids [59]. In present study the accumulation of these nutrients found to be low in leaves of *J. barbasco* and *A. scholaris* and this might be the reason for the decreased production of alkaloids (Table 2). Quantification of chlorophyll is considered as an important parameter to verify the concentration of pigments involved in light absorption and energy transfer during the phytochemical process of photosynthesis. Of the three trees high chlorophyll content was observed in *T. Argentea* (Fig. 2) which shows a direct relationship between photosynthetic rate and chlorophyll content. In turn chlorophyll production depends on Mg and S, where Mg deficiency has been shown to induce an impairment of the linear photosynthetic electron transport [60] later inhibit the chlorophyll synthesis by causing a shortage of the proteins required for chlorophyll formation [61]. By maintaining optimum uptake of Mg and S the plant *T. argentea* showed high A_{max}. Coupled with Mg and S, Fe and Zn also had a significant effect on *Chl*'a' and *Chl*'b' by reducing chlorophyll biosynthesis and poor development of chloroplast [52]. The carotenoid content of the leaf was very much reduced by the deficiency of Fe followed by Mg and Zn deficiency. A similar influence of Fe-deficiency on carotenoid content was previously reported [63,64]. On the other side the Zn-deficiency had caused inhibition in both PS I and PS II activation [13]. In present study *T. argentea* showed maximum *Chl* 'a' content and carotenoid content by maintaining adequate amounts of Zn and Fe. Copper plays a prominent in role structural composition of different electron carriers which involves in photosynthesis and respiration [65,66]. By maintaining the adequate amounts of 'Cu' *T. argentea*is is able to manage the electron carriers such as plastocyanin and cytochrome oxidase and thereby maximum photosynthesis. Required concentrations of Zn, Cu, Fe reported to be play a prominent role in bio-synthesis of phenols in present study (Table 2). This is previously reported in vetiver [67]. The adequate leaf potassium concentration makes the plants to have a steady net CO_2 assimilation rate (A_{max}) and in turn leaf chlorophyll concentration [68]. Coupled with 'Fe' and 'Zn', the availability of 'Mn' also plays a prominent role in photosynthesis. In present study higher leaf 'Mn' concentration (Table 2) was observed in *T. Argentea* and its concentration is directly proportional to photosynthesis. In order to catalyze the water oxidation process with in the OEC complex plants require 'Mn' to form the 'Mn' cluster in PS II which is located in the donor side [12]. On the other side the formation of Mn cluster requires 'Mn' ions along with Ca^{+2} ion [69]. Besides 'Mn' chloride plays a fundamental role in the water-splitting system of PS II [70]. Optimal availability of Mn along with Fe increased the flavanoids concentration in present study especially in *T. argentea* (Table 2). This is in agreement with previous reports [71,72]. Potassium impacts photosynthesis through both stomatal and non-stomatal aspects of photosynthesis. At the stomatal

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level, regulation of conductance is closely impacted by the leaf K^+ concentration through the reversible flux of K^+ ions into and out of the stomatal guard cells [73]. The influx of K^+ into the guard cell lowers the osmotic potential of the cell causing water to flow into the cell. In response to this inward flow of water, the cell elongates and thereby forces the stomata open. This situation makes the leaves to have a high efficiency to CO_2, and there by more CO_2 fixation in *T. Argentea*. Coupled with 'K', zinc plays a prominent role in transpiration process [74]. On the other side the accumulation of required potassium levels in plants tend to increase the bio-synthesis of phenols and flavonoids in present study especially in *T. argentea* (Fig. 2). This is may be due to the enhancement of total non-structural carbohydrate [75]. This might due to role of 'K' in stimulating and trans-location activity of both photosynthesis and carbohydrates to plant parts [76].

4. CONCLUSION

The present study confirms that the availability of mineral nutrients may influence the photosynthesis and there by secondary metabolite production, which is clearly seen in *T. argentea*. By proper accumulation of soil nutrients *T. argentea* maintained optimum photosynthesis and thus high secondary metabolite concentration when compared to *A. scholaris* and *J. barbasco* grown in same soil with similar physico-chemical properties.

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CONFLICT OF INTEREST

Authors do not have any conflict of interest.

REFERENCES

1. Brooks A and Farquhar GD. Effect of temperature on the CO_2/O_2specificity of ribulose-1,5-bisphosphate carboxylase/oxygenase and the rate of respiration in the light. Estimates from gas-exchange measurements on spinach. Planta.1985;165:397–406.
2. Amthor JS. Plant respiratory responses to elevated carbon dioxide partial pressure. Advances in Carbon Dioxide Effects Research. 1997; 35–77.
3. Xin Li, Lan Zhang, Golam JalalAhammed, Zhi-Xin Li, Ji-PengWei, ChenShen, PengYan., Li-Ping Zhang,and Wen-Yan Han. Stimulation in primary and secondary metabolism by elevated carbon dioxide alters green tea quality in *Camellia sinensis* L. Scientific Reports. 2017; 7(7937): 1-12.
4. MerleTrankner, Ershad Tavakol, and Balint Jakli. Functioning of potassium and magnesium in photosynthesis, photosynthate translocation and photoprotection. Physiologia Plantarum, 2018; 163: 414–431.
5. Chapman SK. Protagonists in chemistry: Volume dedicated to Professor AG Sykes. Inorganica Chimica Acta, 2002; 331(1): 25-27.

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6. Yruela I. Transition metals in plant photosynthesis.Metallomics. 2013;5(9):1090-109.
7. Magdolna Droppa, Norman Terry, and Gabor Horvath.Effects of Cu deficiency on photosynthetic electron transport. Procedings of Natural Academy of Sciences, US A.1984; 81(8): 2369–2373.
8. Lindsey RF and Jonathan DG. Oxidative stress induced by loss of Cu,Zn-superoxide dismutase (SOD1) or superoxide-generating herbicides causes axonal degeneration in mouse DRG cultures. Acta Neuropathology. 2010;19(2): 249–259.
9. Muhammad Shahbaz, Karl Ravet, Graham Peers and Marinus Pilon. Prioritization of copper for the use in photosynthetic electron transport in developing leaves of hybrid poplar. Frontiers in Plant Sciences.2015; 6(407):1-12.
10. Waldo GS, Wright E, Wang ZH, Briat JF, Theil EC, Sayers DE. Formation of the ferritin iron mineral occurs in plastids: an X-ray absorption spectroscopy study. Plant Physiology.1995;109:797–802.
11. Kaminskaya O, Kurreck J, Irrgang KD, Renger G, Shuvalov VA. Redox and spectral properties of cytochrome b559 in different preparations of photosystem II. Biochemistry, 1999; 38:16223-16235.
12. George MC and Iris FMartin. Site of manganese function in photosynthesis. Biochimica et Biophysica Acta (BBA) - Bioenergetics. 1968; 153(4): 819-837.
13. Schmidt SB, Powikrowska M, Krogholm KS,Naumann-Busch B, Schjoerring JK, Husted S, Jensen PE, and Pedas PR. Photosystem II functionality in barley responds dynamically to changes in leaf manganese status. Frontiers in Plant Sciences, 2016; 7:1772.
14. Nagashima H, Asada M, Mino H. Magnetic structure of manganese cluster in photosystem II investigated by electron paramagnetic resonance. Biophysics and Physicobiology. 2018;15: 45-50.
15. Hui Chen, Jinlu Hu, Yaqin Qiao, Weixian Chen, Junfeng Rong, Yunming Zhang, Chenliu He, Qiang Wang. Ca^{2+}-regulated cyclic electron flow supplies ATP for nitrogen starvation-induced lipid biosynthesis in green alga. Scientific Reports, 2015; 5 (15117):1-12.
16. Singh R, Parihar P, PrasadSM. Sulfur and Calcium simultaneously regulate photosynthetic performance and nitrogen metabolism status in As-Challenged *Brassica juncea* L. Seedlings. Frontiers in Plant Sciences. 2018; 9:772.
17. Andrejic G, Gajic G, Prica M, Dzeletovic Z, Rakic T. Zinc accumulation, photosynthetic gas exchange, and chlorophyll a fluorescence in Zn-stressed Miscanthus × giganteus plants. Photosynthetica. 2018;56(4):1249-1258.
18. Hendrik Poorter and John RE. Photosynthetic nitrogen-use efficiency of species that differ inherently in specific leaf area. Oecologia.1998; 116(1/2): 26-37.

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19. Wright IJ, Reich PB, Westoby M, Ackerly DD, Baruch Z, BongersF, Cavender-Bares J, Chapin T, Cornelissen JH, Diemer M, Flexas J, Garnier E, Groom PK, Gulias J, Hikosaka K, Lamont BB, Lee T, Lee W, Lusk C, Midgley JJ, Navas ML, Niinemets U, Oleksyn J, Osada N, Poorter H, Poot P, Prior L, Pyankov VI, Roumet C, ThomasSC, Tjoelker MG, Veneklaas EJ, Villar R.The worldwide leaf economics spectrum. Nature. 2004; 22, 428(6985): 821-827.
20. Andreas Carstensen, Andrei Herdean, Sidsel Birkelund Schmidt, Anurag Sharma, Cornelia Spetea, Mathias Pribil and Soren Husted. The impacts of phosphorus deficiency on the photosynthetic electron transport chain. Plant Physiology Preview. 2018; 1-38.
21. Lebaudy A, Hosy E, Simonneau T, Sentenac H, Thibaud JB, Dreyer.Heteromeric K^+ channels in plants. The Plant Journal.2008; 54(6): 1076-82.
22. Basile B, Reidel EJ, Weinbaum SA, DeJong TM.Leaf potassium concentration, CO_2 exchange and light interception in almond trees (*Prunus dulcis*(Mill)D.A.Webb), Scientia Horticulture-AMSTERDAM. 2003; 98(2): 185–194.
23. Jing Wang, Xuefa Wen, Xinyu Zhang, Shenggong Li, Da-Yong Zhang. Co-regulation of photosynthetic capacity by nitrogen, phosphorus and magnesium in a subtropical Karst forest in China. Scientific Reports. 2018; 8(7406):1-9.
24. Hewitt EJ and Smith TA. Plant MineralNutrition, John Wiley & Sons, New York, 1974.
25. Hall JD, Barr R, Al-Abbas AH and CraneFL. The ultrastructure of chloroplasts inmineral-deficient maize leaves. Plant Physiology.1972; 50:404-409.
26. Muhammad Ali Raza, Ling Yang Feng, Nasir Iqbal, Abdul Manaf, Muhammad Hayder Bin Khalid, Sana ur Rehman, Allah Wasaya, Muhammad Ansar, Masum Billah, Feng Yang, and Wenyu Yang. Effect of sulfur application on photosynthesis and biomass accumulation of sesame varieties under rainfed conditions. Agronomy.2018; 8(149):1-16.
27. Marschner H. Mineral Nutrition of Higher Plants. New York: Academic Press. 1995.
28. Alain Boussac, Naoko Ishida, Miwa Sugiura, Fabrice Rappaport. Probing the role of chloride in Photosystem II from *Thermosynechococcus elongatus* by exchanging chloride for iodide. Biochimica et Biophysica Acta. 2012; 1817: 802–810.
29. Kenneth Olesenand Lars-Erik Andréasson. The function of the chloride ion in photosynthetic oxygen evolution. Biochemistry. 2003; 42(7): 2025-2035.
30. Zhong JJ, Seki T, Kinoshita S and Yoshida.1991. Effect of light irradiation onanthocyaninproduction by suspended culture of *Perillafrutescens*. Biotechnology and Bioengineering.1991; 38: 653–658.
31. Kurata H, Matsumura S, Furusaki S. Light irradiation causes physiological and metabolic changes for purine alkaloid production by a *Coffea arabica* cell suspension culture. PlantScience. 1997; 123(1-3):197-203.
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32. Kawoosa T, Singh H, Kumar A, Sharma SK, Devi K, Dutt S, Kumar Vats S, Sharma M, Singh Ahuja P, Kumar S. Light and temperature regulated terpene biosynthesis: hepatoprotective monoterpene picroside accumulation in *Picrorhiza kurrooa*. Functional Integrative Genomics. 2010;10:393–404.
33. Olanlokun JO, Bolaji OM, Agbedahunsi JM, Olorunsogo OO. Therapeutic effects of various solvent fractions of*Alstonia boonei* (Apocynaceae) stem bark on *Plasmodium berghei*-induced malaria. African Journal of Medicine and Medical Sciences. 2012; 41: 27-33.
34. Wang W, Cheng MH, Wang XH. Monoterpenoid indole alkaloids fromAlstonia rupestris with cytotoxic, anti-inflammatory and antifungal activities. Molecules. 2013; 18(6): 7309-22.
35. Arai H, Zaima K, Mitsuta E, Tamamoto H, Saito A, Hirasawa Y, Rahman A, Kusumawati I, Zaini NC. Morita H. Alstiphyllanines I-O, ajmaline type alkaloids from *Alstonia macrophylla* showing vasorelaxant activity. Bio Organic and Medicinal Chemistry. 2012; 20(11): 3454-9.
36. Rahmatullah M, Azam NK, Khatun Z, Seraj S, Islam F, Rahman AS, Aziz S. 2012. Medicinal plants used for treatment of diabetes by the Marakh sect of the Garo tribe living in Mymensingh District, Bangladesh. African Journal of Traditional, Complementary and Alternative Medicine. 2012; 9(3): 380–385.
37. Ramachandra YL, Ashajyothi C, Padmalatha RS. Antioxidant activity of *Alstonia scholaris* extracts containing flavonoids and phenolic compounds. International Journal of Pharmacy and Pharmaceutical Sciences. 2012; 4(3): 424-6.
38. Da Silva MN, Da Souza MCBV, Ferreira VF, Pinto AV, Pinto MCRF, Solange MSV, Wardell SMSV, Wardell JL. Synthesis of new aldehyde derivatives from β-lapachone and nor-β-lapachone. Arkivoc. 2003; 56-168.
39. Sadananda TS, Nirupama R, Chaithra K, Govindappa M, Chandrappa CP, Vinay Raghavendra. Antimicrobial and antioxidant activities of endophytes from *Tabebuia argentea* and identification of anticancer agent (lapachol). Journal of Medicinal Plants Research. 2011; 5(16): 3643-3652.
40. Breger J, Fuchs BB, Aperis G, Moy TI, Ausubel FM, Mylonakis E. Antifungal chemical compounds identified using a *C. elegans* pathogenicity assay. Plos Pathogy.2007; 3(2):168-178.
41. Almeida ER, Filha AAS, Santos ERD, Lopes CAC. Anti inflammatory action of lapachol. Journal of Ethnopharmacology.1990; 29: 239-241.
42. Murray MT, Pizzorno JE. Encyclopedia of Natural Medicine 2nd ed. PA4 Rocklin, CA: Prima Pub. 1998
43. Teixeira MJ, Almeida YM, Viana JR, Holanda Filha JG, Rodrigues TP, Prata JRC, Jr Coelho ICB, Rao VS, Pompeu MML. In vitro and in vivo leishmanicidal activity of 2-hydroxy-3-(3-methyl-2- butenyl)-1,4-naphthoquinone (lapachol). Phytotherapy Research, 2001; 15: 44-48.
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44. Silva TMS, Camara CA, Barbosa TP, Soares AZ, Cunha LC, Pinto AC, Vargas MD. Design, synthesis and antifungal activity of a novel water soluble prodrug of antifungal triazole. Bioorganic and Medicinal Chemistry. Letters.2005; 13:193-196.
45. Padmavathi CH, Narendra K, Suman Joshi DSD, Rathnakar Reddi KVN, Sowjanya KM, Swathi J, Krishna Satya A. 2014. Active ingredient estimation and in-vitro analysis of *Jacquinia barbasco* (Loefl.) Mez plant extracts. Journal of Pharmacognosy and Phytochemistry. 2014; 3(4): 99-104.
46. Hiscox JD and Israelstam GF. A method for extraction of chlorophyll fromleaf tissue without maceration. Canadian Journal of Botany. 1979; 57:1332-1334.
47. Hunter A. New techniques and equipment for routine soil/plant analytical procedures. In: Soil Management in Tropical America. (eds E. Borremiza & A. Alvarado). N.C. State University, Raleigh, NC. 1975.
48. Jackson NE, Miller RH, Franklin RE. The influence of VAM on uptake of 90sr from soil by soybeans. Soil Biology and Biochemistry. 1973; 5(2): 205-212.
49. Alhakmani F, Kumar S, Khan SA. Estimation of total phenolic content, *in-vitro* antioxidant and anti-inflammatory activity of flowers of *Moringa oleifera.* Asian Pacific Journal of Tropical Biomedicine. 2013; 3(8):623-7.
50. Eleazu CO, Eleazu KC, Awa E, Chukwuma SC. Comparative study of the phytochemical composition of the leaves of five Nigerian medicinal plants. Journal of Biotechnology and Pharmaceutical Research.2012; 3: 42-46.
51. Soni AK, Jha J, Dwivedi P, Soni. Qualitative and quantitative determination of phytoconstituents in some antifertility herbs.Indian Journal of Pharmaceutical Sciences. 2018; 80(1): 79-84.
52. PolshettiwarSA, Ganjiwale RO, Wadher SJ, YeolePG.Spectophotometric estimation of total tannins in some Ayurvedic eye drops. Indian Journal of Pharmaceutical Sciences, 2007; 69: 574-576.
53. Tambe VD, Bhambar RS.Estimation of total Phenol, tannin, alkaloid and flavonoid in *Hibiscus tiliaceus* Linn. Wood Extracts. Journal of Pharmacognosy and Phytochemistry. 2013; 2: 41-44.
54. Narendra D, RamaLakshmi N, Satyanarayana B, Sudeepthi P, Hemachakradhar N, Pavankumar Raju. Preliminary phytochemical screening, quantitative estimation and evaluation of antimicrobial activity of *Alstonia macrophylla* stem bark. International Journal of Science Inventions Today. 2013; 2(1): 31-39.
55. Osorio OK, Martins JLS. Determinacao de cumarina emextrato fluido tintura de guaco porespectrofotometria derivada deprimeira ordem. Brazilian Journal of Pharmaceutical Sciences. 2004; 40(4): 481 486.
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56. Rajat Buragohain. Screening and quantification of phytochemicals and evaluation of antioxidant activity of *Albizia chinensis* (vang): One of the tree foliages commonly utilized for feeding to cattle and buffaloes in Mizoram. International Journal of Current Microbiology and Applied Sciences. 2015; 4(9): 305-313.
57. Shamsu Adnan Muhammad and Abubakar SM. Qualitative and quantitative determination of phytochemicals in aqueous extract of C*hrysophyllumalbidum* seed kernel. Biosciences Biotechnology Research Asia. 2016; 13(2): 1201-1206.
58. Sifola MI, Barbieri G. Growth, yield and essential oil content of three cultivars of basil grown under different levels of nitrogen in the field. Scientia Horticulturae.2006; 108: 408-413.
59. Siregar LAM, Keng CL, Lim BP. Effects of medium constituents on growth and canthinone accumulation in cell suspension cultures of *Eurycoma longifolia* Jack. Hayati Journal of Biosciences. 2009;16: 69–77.
60. Farhat N, Ivanov AG, Krol M, Rabhi M, Samoui A, Abdelly C, HunerNP. Preferential damaging effects of limited magnesium bioavailability on photosystem I in *Sulla carnosa* plants. Planta. 2015; 241: 1189–1206.
61. Kumawat RN, Rathore PS, Mahajan SS. Dry matter partitioning and yield of mungbean (*Vigna radiata* L.) with sulfur and iron nutrition. Journal of Arid Legumes. 2009; 6: 52-56.
62. Abadia J and Abadia A. Iron and plant pigments. In: Barton LL, Hemming BC. eds. Iron chelation in plants and soil microorganisms. New York, 1993.
63. Val J, Monge E, Heras L, Abadia J. Changes in photosynthetie pigment composition in higher plants as affected by iron nutrition status. Journal of Plant Nutrition.1987; 10: 995-1001.
64. Hanke GT and Mulo P. Plant type ferredoxins and ferredoxin-dependent metabolism. Plant Cell Environment. 2013; 36: 1071–1084.
65. Tapken W, Kim J, Nishimura K,Van Wijk KJ, Pilon M. The Clp protease system is required for copper ion-dependent turnover of the PAA2/HMA8 copper transporter in chloroplasts. New Phytologist. 2015a; 205: 511–517.
66. Tapken W, Ravet K, Shahbaz M, and Pilon M. Regulation of Cu delivery to chloroplast proteins. Plant Signaling and Behaviour.2015b; 10(7): 1-5.
67. Funzani AM, Thierry Regnier Rob I, McCrindleNtebogeng S, Mokgalaka S. Impact of Metals on Secondary Metabolites Production andPlant Morphology in Vetiver Grass (*Chrysopogon zizanioides*). African Journal of Chemistry. 2012; 65: 178–183.
68. Wang XG, Zhao XH, Jiang CJ, Li CH, Cong S, Wu D, Chen YQ, Yu HQ, Wang CY.2015. Effects of potassium deficiency on photosynthesis and photoprotection mechanisms in soybean (*Glycine max* (L.) Merr.).Journal of Integrative Agriculture, 2015; 14:856-863.
69. Ferreira KN, Iverson TM, Maghlaoui K, Barber J, Iwata S. Architecture of the photosynthetic oxygen-evolving center. Science.2004; 303: 1831–1838.
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70. Ball MC, Chow WS, Anderson JM. Salinity induced potassium deficiency causes a loss of functional photosystem II in leaves of grey mangroves, *Avicennia marina*, through depletion of the atrazine-binding polypeptide. Australian Journal of Plant Physiology. 1987; 14:351-361.
71. Jung S, Chung IM, Kim SH, Kim MY, Ahmad A, Praveen N. In vitro antioxidant activity, total phenolics and flavonoids from celery (*Apium graveolens*) leaves. Journal of Medicinal Plants Research.2011; 5(32): 7022-7030.
72. Nakao M, Takio S, Ono K. 1998. Alkyl peroxyl radical-scavenging activity of catechins. Phytochemistry. 1998; 49: 2379-2382.
73. Fischer RA and Hsiao TC. Stomatal opening in isolated epidermal strips of *Vicia faba*. Li. Responses to KC1 concentration and the tole of potassium absorption. Plant Physiology.1968; 43:1953-58.
74. Mattiello Edson M, Ruiz Hugo A, Neves Julio CL, Ventrella Marilia C, Araujo Wagner L. Zinc deficiency affects physiological and anatomical characteristics in maize leaves. Journal of Plant Physiology.2015; 183: 138-143.
75. Kutaiba Ibrahim Alzand and Mohamed Abdalkarim Mohamed. 2012. Flavonoids: Chemistry, Biochemistry and Antioxidant activity. Journal of Pharmacy Research. 2012; 5(8): 4013-4020.
76. Ghasemzadeh A, Jaafar HZE, Rahmat A. Antioxidant activities, total phenolics and flavonoids content in two varieties of Malaysian young ginger (*Zingiber officinale* Roscoe). Molecules.2010; 15: 4324- 4333.

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PHYLOGENETIC CHARACTERIZATION OF POTENTIAL BIOACTIVE METABOLITES PRODUCING ACTINOMYCETES FROM MANGROVE SEDIMENTS

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Mangrove ecosystem, Microbial diversity, Phylogenetic analysis and Antimicrobial activity

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ABSTRACT: An attempt has been made in the present study to isolate and identify actinomycetes, which can produce bioactive metabolites possessing antimicrobial activity. The soil samples were collected from mangrove sediments of Krishna district, Andhra Pradesh. The potent secondary metabolite producing strains were isolated and designated as VLK-15, VLK-24, and VLK-56. Identification of the strains was carried out basing on morphological, cultural, physiological and biochemical characters. Phylogenetic analysis of 16S rRNA gene sequence showed that the strains VLK-15, VLK-24, VLK-56 form distinct clade within the genus *Streptomyces*. The 16S rRNA sequences were deposited in the GenBank database of NCBI under the accession numbers MG309759 (*Streptomyces olivoverticillatus* VLK-15), MG309760 (*Strepto-myces rubrus* VLK-24), and MF952650 (*Streptomyces sampsonii*-VLK-56). The antimicrobial activity of the strains was evaluated by using agar well diffusion assay. The ethyl acetate extracts were highly effective against *Candida albicans*, *Staphylococcus aureus*, *Bacillus megaterium* followed by *Bacillus subtilis*, *Escherichia coli* and *Klebsiella pneumoniae*.

INTRODUCTION: Actinomycetes are sapro-phytic, free-living, Gram-positive bacteria widely distributed in different habitats, frequently filamentous and sporulating with DNA rich in G+C (55-75%). They are the potential source for many bioactive secondary metabolites [1]. The hunt for new antimicrobial agents is very useful to prevent infectious diseases [2]. They have been used in areas of medicine as well as in water sanitation to inhibit the growth of microorganisms in drinking water [3].

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Mangrove ecosystems are the unique woody plant communities in the tropical and subtropical coastal regions [4]. They are viewed as the ecosystems that comprise unexplored microbial diversity, including actinomycetes [5]. The mangrove ecosystems exist between terrestrial and marine environments that support a rich and diverse group of microorganisms [6]. There may be no other group of plants with such highly developed morphological, biological, ecological, and physiological adaptations to extreme conditions.

Mangrove derived actinomycetes remain as an unexploited and luxuriant source of pharmaceutical thrust. The expedition for the microbial groups with therapeutic possessions continues to receive great attention as researchers investigate mangrove

microbes for novel molecules with antimicrobial, antitumor, and biochemical activities [7].

Uncaged mangrove habitats provide more chances for isolating new species of streptomyces with unique chemical structures preventing many microbial diseases and cancers [8]. Hence, the present work was designed to study the diversity and polyphasic taxonomy of actinobacterial population of the mangrove ecosystems of the Krishna district of Andhra Pradesh, India.

MATERIALS AND METHODS:

Collection of Mangrove Sediment Sample: Sediment samples were collected at bimonthly intervals from mangrove ecosystems of the coastal region, Krishna district of Andhra Pradesh. The samples were collected from 6-10 cm depth and transported to the laboratory in sterile bags, and air-dried at room temperature.

Isolation of Actinomycetes: The air-dried and pretreated soil samples were suspended in sterile distilled water (1g in 100 ml), homogenized by vortexing, and 0.1ml of serially diluted sample (10^{-3} dilution) was spread over the surface of selective culture medium (starch-casein agar) containing 3% NaCl supplemented with tetracycline and secnidazole [9]. After incubation for one week at 30 °C, distinct strains were selected for subculturing to maintain pure culture on agar slants.

Extraction of Secondary Metabolites: Modified yeast extract, malt extract, and dextrose broth was used as a production medium for the extraction of crude secondary metabolites. All the actinobacterial isolates were inoculated separately and incubated at 30 °C in a rotatory shaker (120 rpm) for primary screening. The crude culture filtrate obtained from a five-day-old culture was extracted with equal volumes of ethyl acetate to extract the anti-microbial compounds. The solvent extracts were evaporated to dryness in a water bath at 40 °C, and the compound obtained was tested for its activity against the test microorganisms by agar well diffusion method [10].

Test Microorganisms: The antimicrobial metabolites produced by the strains were tested against test bacteria such as *Staphylococcus aureus* (MTCC 3160), *Escherichia coli* (ATCC 35218), *Bacillus subtilis* (ATCC 6633), *B. cereus* (MTCC-430), *B. megaterium* (NCIM2187) *Klebsiella pneumoniae (*ATCC10031*), Vibrio parahaemolyticus* (ATCC 43996) and fungi like *Candida albicans* (ATCC 10231) by agar well diffusion assay.

Identification: Micro-morphological, biochemical and physiological characteristics together with molecular (16S rRNA gene sequencing) analysis were carried out to identify the strains.

Molecular Identification: The Molecular identification of the potential strains was carried out by amplifying the bacterial genome. DNA samples were extracted using an Insta Genetm Matrix (BIO-RAD.). The primers 27F 5' (AGA GTT TGA TCM TGG CTC AG) 3,' and 1492R 5' (TAC GGY TAC CTT GTT ACG ACT T) 3' were used for the PCR. The PCR reaction was performed with 20 ng of genomic DNA as the template in a 30 µL reaction mixture using EF-Taq (SolGent, Korea) as follows: activation of Taq polymerase at 95 °C for 2 min, 35 cycles at 95 °C for one minute, 55 °C and 72 °C for one minute each were performed, finishing with a 10-min step at 72 °C.

The amplification products were purified with a multi-screen filter plate (Millipore Corp., Bedford, MA, USA). The sequencing reaction was performed using a PRISM Big Dye Terminator v3.1 Cycle Sequencing Kit. The DNA samples containing the extension products were added to Hi-Di formamide (Applied Biosystems, Foster City, CA). The mixture was incubated at 95 °C for 5 min, followed by 5 min on ice, and then analyzed by ABI Prism 3730XL DNA analyzer (Applied Biosystems, Foster City, CA).

Phylogenetic Analysis: The 16S rRNA sequence was compared with the known sequences in Gene Bank using the Basic Local Alignment Search Tool (BLAST) against the gene library available in the NCBI and the GenBank. The phylogenetic tree was constructed using the Maximum Parsimony method. The closely related homologous strains were identified, retrieved, and compared to the sequence of the isolated strains using CLUSTAL W available with the MEGA-X Version [11].

Gene Bank Accession Number: The 16S rRNA gene sequence of the strains are deposited in the National Center for Biotechnology Information (NCBI).

RESULTS AND DISCUSSION: The air-dried soil samples were pretreated with calcium carbonate [12]. The serial dilution plate technique was used for the isolation of actinobacteria. After incubation for a week at 30 °C, distinct colonies were selected based on smooth, granular, powdery appearance [9]. A total of 60 actinobacterial strains were isolated and designated as VLK-1 to VLK-60 from the mangrove ecosystem of coastal regions of the Krishna district. Some reports indicate the isolation of rare actinobacterial strains from Krishna mangrove ecosystems [14, 15]. The strains designated as VLK-15, VLK-24, and VLK-56 exhibited typical morphological characteristics of the genus Streptomyces [16]. The morphological, physiological, and biochemical characteristics of the strains are presented in **Table 1**.

TABLE 1: MORPHOLOGICAL, PHYSIOLOGICAL AND BIOCHEMICAL CHARACTERISTICS OF STRAINS

Character	Response		
	VLK-15	VLK-24	VLK-56
Morphological characters			
Colour of aerial mycelium	White	Brown	Pale brown
Colour of substrate mycelium	Pale yellow	Dark brown	Dark brown
Sporophore morphology	Rectiflexible	Rectiflexible	Rectiflexible
Mycelial form	Branched	Branched	Branched
Physiological characters			
Gram's reaction	+	+	+
Production of melanin pigment	-	-	-
Range of pH for growth	4-9	4-10	4-9
Optimum pH for growth	7.0	7.5	7.0
NaCl tolerance	5%	7%	3%
Biochemical characters			
Catalase production	+	+	+
Urease production	+	+	+
Hydrogen sulphide production	-	-	-
Nitrate reduction	-	-	-
Starch hydrolysis	+	+	+
Gelatin liquefaction	-	-	-
Methyl red test	-	-	-
Voges proskauer test	-	-	-
Indole production	-	-	-
Citrate utilization	+	+	+

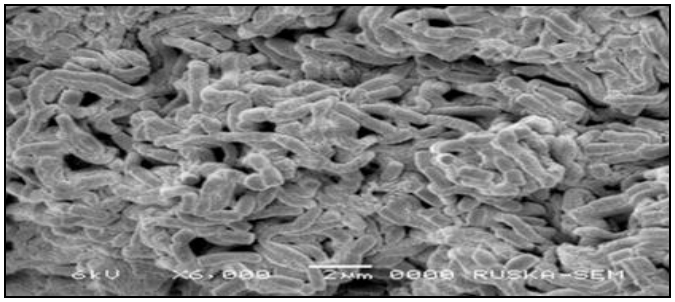

FIG. 1: SCANNING ELECTRON MICROSCOPY PHOTOGRAPH OF VLK-15

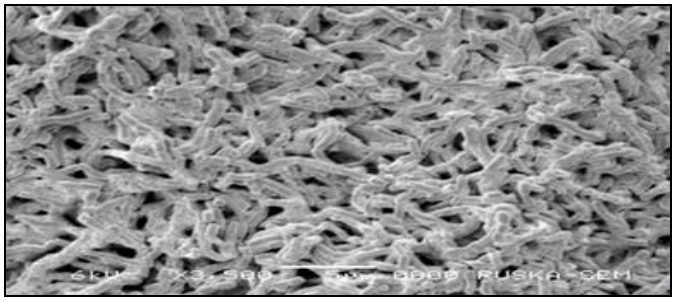

FIG. 2: SCANNING ELECTRON MICROSCOPY PHOTOGRAPH OF VLK-24

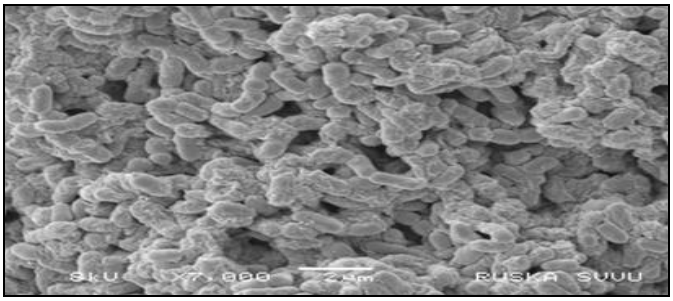

FIG. 3: SCANNING ELECTRON MICROSCOPY PHOTOGRAPH OF VLK-56

The strains exhibited good growth on ISP-1, ISP-2, ISP-7, starch casein agar, Czapek-Dox agar, and maltose tryptone agar. The growth was moderate on ISP-4 and ISP-5 and nutrient agar media, while it was poor on ISP-3 agar. The color of aerial mycelium varied from white to brown, while the substrate mycelium varied from brown to dark brown. Production of melanin pigment was not found by the strains on ISP-7 medium.

Micromorphology of the strains was observed by the coverslip culture method. The cultures showed extensively branched aerial mycelium with a long chain of spores. The strains may be placed in the rectus-flexibilis group of Streptomyces [17, 18]. Spore characters of the strains were recorded with a Scanning Electron Microscope (SEM) **Fig. 1-3**.

The utilization of different carbon sources by the strains indicated its wide pattern of carbon source consumption **Table 2**. The strains efficiently utilized the carbon sources such as glucose, maltose, sucrose, lactose, and starch.

TABLE 2: UTILIZATION OF CARBON SOURCES BY THE STRAINS

Carbon source	Response		
	VLK-15		VLK-15
Glucose	+++	Glucose	+++
Maltose	+++	Maltose	+++
Sucrose	++	Sucrose	++
Lactose	+++	Lactose	+++
Fructose	++	Fructose	++
Galactose	+	Galactose	+
Starch	++	Starch	++
Mannitol	-	Mannitol	-

(+++) = Efficient, (++) = Good, (+) = Moderate (-) = poor

The partial gene sequence of 16S rRNA of the strains was blasted against the nucleotide database of the NCBI. The library search reported matching strains, and the sequences were aligned with the set of published sequences on the basis of by nucleotide BLAST similarity search analysis.

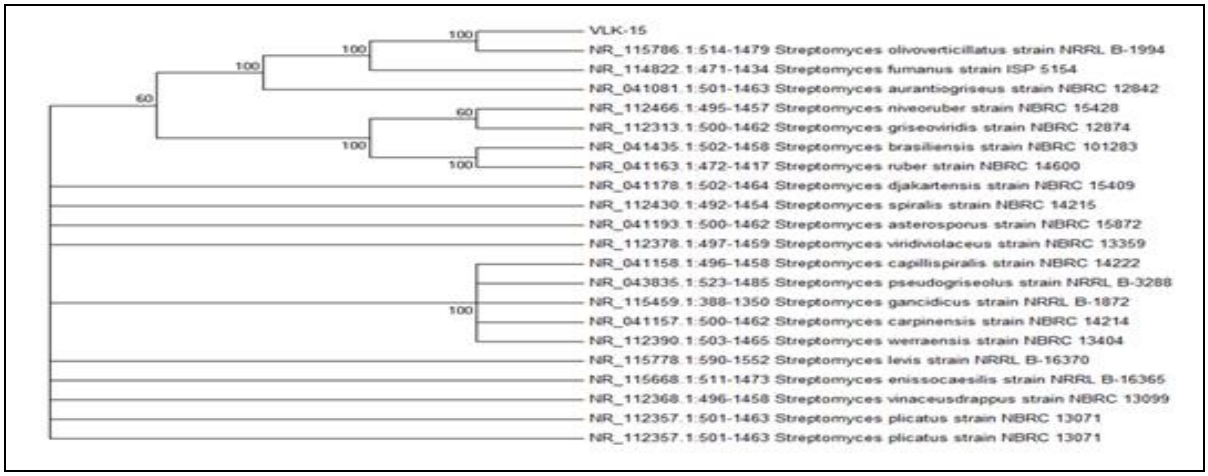

FIG. 4: MAXIMUM PARSIMONY TREE BASED ON PARTIAL 16S rRNA GENE SEQUENCE SHOWING RELATIONSHIP BETWEEN THE ISOLATE VLK-15 AND RELATED MEMBERS OF THE GENUS STREPTOMYCES

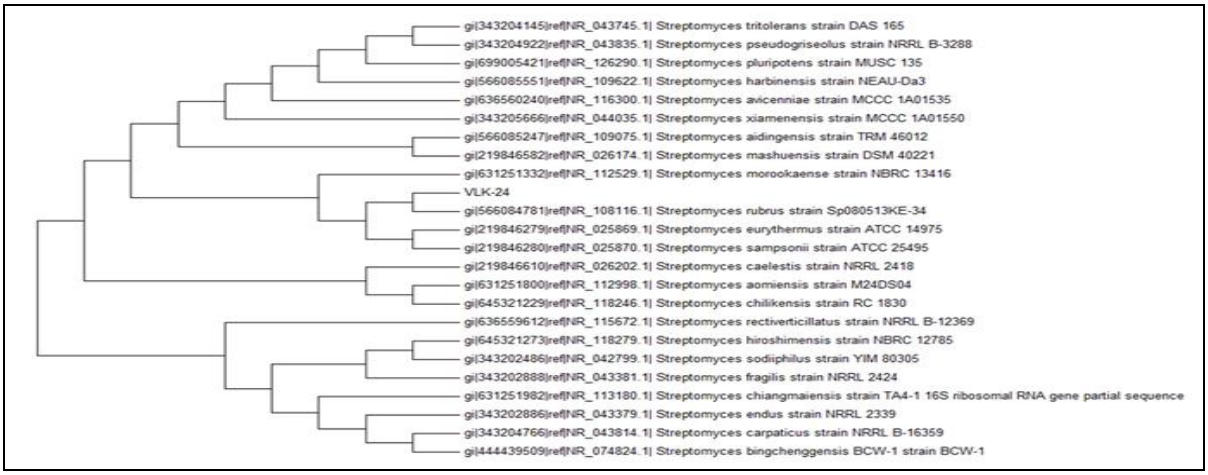

FIG. 5: MAXIMUM PARSIMONY TREE BASED ON PARTIAL 16S rRNA GENE SEQUENCE SHOWING RELATIONSHIP BETWEEN THE ISOLATE VLK-24 AND RELATED MEMBERS OF THE GENUS STREPTOMYCES

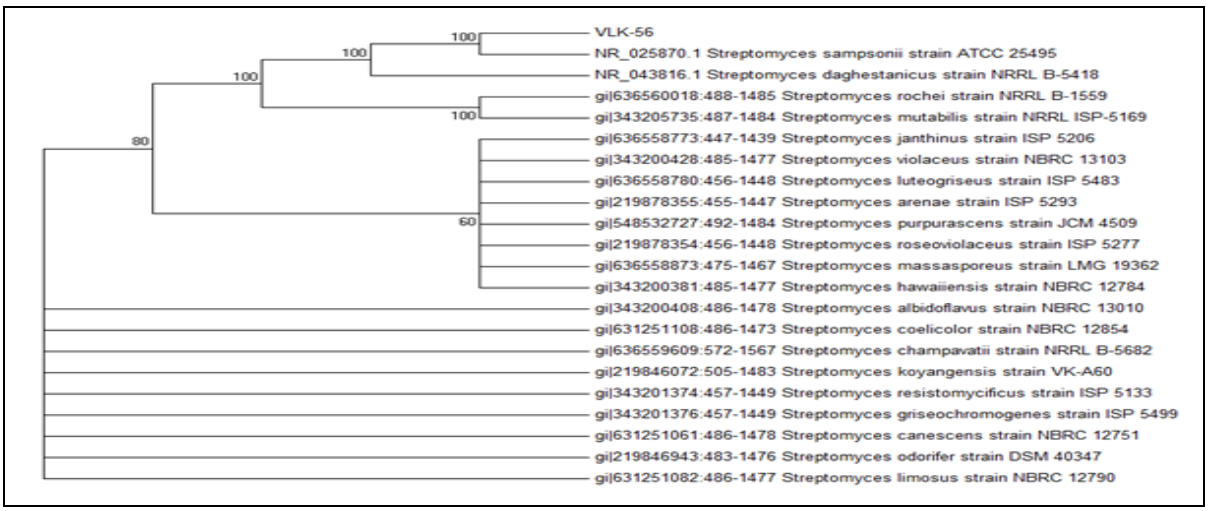

FIG. 6: MAXIMUM PARSIMONY TREE BASED ON PARTIAL 16S rRNA GENE SEQUENCE SHOWING RELATIONSHIP BETWEEN THE STRAIN VLK-56 AND RELATED MEMBERS OF THE GENUS STREPTOMYCES

Based on 16S rRNA gene sequences of the strains showed a close relation with *Streptomyces* sp. **Fig. 4-6**. The 16S rRNA sequences are deposited in the GenBank database of NCBI with the accession numbers MG309759 (*Streptomyces olivoverticillatus* VLK-15), MG309760 (*Streptomyces rubrus* VLK-24), and MF952650 (*Streptomyces sampsonii*-VLK-56).

The antimicrobial activity of the strains was evaluated by using ethyl acetate extract. The crude ethyl acetate extracts were highly effective against *Candida albicans*, *Staphylococcus aureus*, *Bacillus megaterium* followed by *B. subtilis*, *Escherichia coli*, and *Klebsiella pneumonia* **Fig. 7**.

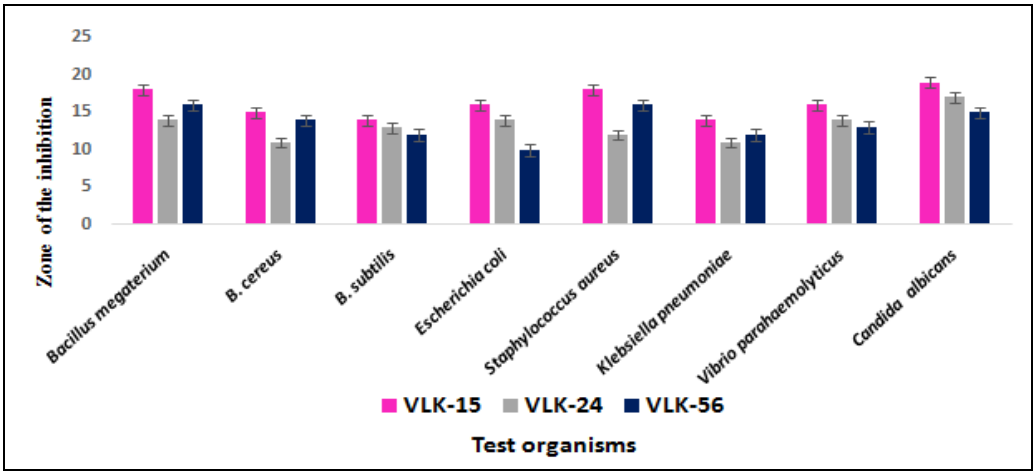

FIG. 7: ANTIMICROBIAL ACTIVITY OF ETHYL ACETATE EXTRACTS OF STRAINS AGAINST THE TEST ORGANISMS

CONCLUSION: The present findings revealed that the bioactive compounds produced by *Streptomyces* spp. possessed high antimicrobial activity. The potent actinobacteria *Streptomyces olivoverticillatus* VLK-15, *Streptomyces rubrus* VLK-24 and *Streptomyces sampsonii*-VLK-56 are reported from the mangrove ecosystem of the coastal region, Krishna district of Andhra Pradesh. It is evident from the study that mangrove habitats serve as a good source of potent actinobacteria with broad-spectrum antimicrobial activity.

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CONFLICTS OF INTEREST: The authors declare that there is no conflict of interest regarding this paper's publication.

REFERENCES:

1. Yang N and Song F: Bioprospecting of novel and bioactive compounds from marine actinomycetes isolated from south China sea sediments. Curr Microbiol 2018; 75: 142-49.
2. Gupte S, Kaur M and Kaur M: Novel approaches to developing new antibiotics. J BacteriolMycol 2017; 4: 00089.
3. Westh H, Zinn CS and Rosdahl VT: Sarisa Study Group: An international multicenter study of antimicrobial consumption and resistance in *Staphylococcus aureus* isolates from 15 hospitals in 14 countries. Microbial Drug Resistance 2004; 10: 169-76.
4. Xu DB, Ye WW, Han Y, Deng ZX and Hong K: Natural Products from Mangrove Actinomycetes. Mar Drugs 2014: 12: 2590-2613.
5. Hong K, Gao AH, Xie QY, Gao H, Zhuang L, Lin HP, Yu HP, Li J, Yao XS, Goodfellow M and Ruan JS: Actinomycetes for marine drug discovery isolated from mangrove soils and plants in China. Marine Drugs 2009; 7: 24-44.
6. Undabarrena A, Beltrametti F, Claverías FP, González M, Moore ER and Seeger M: Exploring the diversity and antimicrobial potential of marine actinobacteria from the comau fjord in Northern Patagonia, Chile. Front Microbiol 2016; 7: 1135.
7. Mangamuri U, Muvva V, Poda S, Naragani K, Munaganti RK, Chitturi B and Yenamandra V: Bioactive metabolites produced by *Streptomyces cheonanensis* VUK-A from Coringa Mangrove sediments: Isolation, structure elucidation and bioactivity.3 Biotech 2016; 6: 63.
8. Jose PA and Jebakumar SRD: Unexplored hyper saline habitats are sources of novel actinomycetes. Front Microbiol 2014; 5: 1-3.
9. Kiranmayi MU, Sudhakar P, Krishna N, Yellamanda B and Vijayalakshmi M: Taxonomic characterization of potential bioactive metabolite producing Actinomycetes from mangrove sediments of Coringa. J Phar Res 2011; 4: 4650-53.
10. Munaganti RK, Muvva V, Konda S, Naragani K, Mangamuri UK, Reddy DK and Akkewar DM: Antimicrobial profile of *Arthrobacter kerguelensis* VL-RK_09 isolated from Mango Orchards. Brazilian Journal of Microbiology 2016; 47: 1030-38.
11. Kumar S, Stecher G, Li M, Knyaz C and Tamura K: MEGA X: Molecular evolutionary genetics analysis across computing platforms. Molecular Biology and Evolution 2018; 35: 1547-49.
12. Bindu BSSNH, Muvva V, Munaganti RK, Naragani K, Konda S and Reddy DK: A study on production of antimicrobial metabolites by *Streptomyces lavendulocolor* VHB-9 isolated from Granite Mines. Brazilian Archives of Biology and Technology 2017; 60: 1-13.
13. Indupalli M, Muvva V, Mangamuri U, Kumar MR and Krishna N: Bioactive compounds from mangrove derived rare *Actinobacterium saccharomonosoraoceani* VJDS-3. 3 Biotech 2017; 8: 103.
14. Naragani K and Muvva V: Actinobacteria from Mangrove Habitats: Diversity and Antimicrobial Activity. European Journal of Biomedical and Pharmaceutical Sciences 2016; 3: 460-67.
15. Hensy WR: Bergey's manual of systematic bacteriology 9th edition. John G. Holt and Stanley T. Williams (EDS).Williams and Wilkins, Baltimore, Philadelphia, Hong Kong, London Munich, Sydney, Tokyo 1994.
16. Pridham TG and Gottlieb D: The utilization of carbon compounds by some Actinomycetales as an aid for species determination. J Bacteriol 1948; 56: 107-14.
17. Naragani K, Mangamuri U, Muvva V, Poda S and Munagant RK: Antimicrobial potential of Streptomyces cheonanensis VUK-A from Mangrove origin. International Journal of Pharmacy and Pharmaceutical Sciences 2016; 8: 53-57.

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HEALTH INSURANCE IN ANDHRA PRADESH: Dr. NTR VAIDYA SEVA HEALTH INSURANCE SCHEME

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Abstract*: Recently health care and health insurance and health care finances gained much attention in India. Various states in India and central government has been taken attention and putting grater efforts to introduce health care insurance for the poor and below poverty families for their quality medical care and providing financial protection against very high medical expenditures. The successfully running Dr.NTR Vaidya Seva Health Insurance scheme in Andhra Pradesh among the different schemes implementing in the various states and government of Andhra Pradesh earlier similar scheme. The Insurance scheme covered around 18 lakh families are benefited during 2014 to 2018 in 13 districts of the state. The scheme started with 938 procedures covered earlier and has been gradually extended to 1044 procedures. The majority of beneficiaries utilizing the scheme are illiterate and have a rural address. Since inception of the scheme (02.06.2014) to December 2018 – various Medical camps were held by the network hospitals in rural areas and total Surgeries/Therapies done by under this scheme is 1764855worth of more than 4,632 crore in which government is 437691 and corporate hospitals 1327164.*

Key words: NTR Vaidya Seva, *Health Insurance, Community Health insurance, Healthcare, Healthcare Expenditure.*

I. Introduction

Health is an important for all it is not personnel and also it has impact on society. Health is real wealth for everybody it is not only increase the quality of human life but also decreases both public and private expenditure on ill health or sickness and diseases. Health is a very significant constituent of Human resource development, and it also declared as a fundamental human right. Health care services can provide various factors to reduce infant mortality rate, death rate, raise life expectancy with keeping diseases under control. Economic growth of any country depends upon for ways by World development report 1993 they are: Reduction of production caused by illness of workers, it permits the natural recourses of that has been totally or practically increasable because of their diseases, it also effect the enrolment of children in schools, makes them better able to learn. It is frees for another use of resources that would otherwise has to be spent on treating illness. The economic gains are relatively better for poor people, who are in general most handicapped by ill health. Who stand to gain the most from the improvement of utilization of their personal resources.

I.

The out-of pocket expenditure/payments (OOPs) is defined as the direct expenditure which is paid by individuals for their health care when they are in services. The families spend less than 10% or more than 10% for their total family income which is measure the indication of catastrophic health expenditure [1]. The increase in catastrophic health expenditure it causes those families fall in poverty [2]. The total health expenditure for India was figured as Rs.4,83,259 crores for the years 2014-15 by the report of National Health account (NHA).

Total Health Expenditure (THE) includes of current and capital expenditures incurred by Government and private sources with External/Donor funds. Around 62.6% households are affected by OOPE was Rs. 3,02,425 cores during the year of 2014-15 In India [2]. Catastrophic health expenditure is more in rural India (25.3%) when compared with urban (17.5%) [3]. Out of pocket expenditure was 87% of the rural poverty occurs in poor states and this was 67% of richest states in India [4]. The poverty percentage is very high 3.5% in rural areas when compared to urban areas 2.5% [5,6]. A large proportion of the population sells their assets for their inpatient care [7,9]. Catastrophic out-of-pocket expenditure was reduced by the individual households with the help of their health insurance provide financial protection in the event of their ill health. Low income and developing countries health care finance is still mainly based on out-of pocket (OOP) payments, and the lack of prepayment mechanisms like insurance. In the absence of insurance, an illness not only reduces welfare directly and it also increases risk of economical failure due to their high treatment expenditures [9,10]. The historical timelines of health Insurance in India were given Table 1. It is evident that from 1907 to 2018 different schemes were implemented for better health for the people of India.

Table 1. Historical timeline of Health Insurance in India

Year	Name of the health insurance scheme
1907	First general insurance company
1952	Employees State Insurance Scheme Implemented (ESI Act 1948)
1954	Central Government Health Scheme
1973	General Insurance Corporation: 4 public insurers – National, New India, Oriental and United India
1999	Establishment of Insurance Regulatory Development Authority 100% Foreign Direct Investment In Health Insurance
2003	Yeshasvini Health Insurance, Karnataka
2007	De-tariffication of insurance
2007	Rajiv Arogyasri Scheme (RAS), Andhra Pradesh
2008	GOI 's Rashtriya Swasthya Bima Yojana (RSB Y)
2009	Kalaignar, Tamil Nadu
2010	RSB Y Plus, Himachal Pradesh and Vajpayee Arogyasri Scheme (VAS), Karnataka
2014	Arogyasri Scheme, Telangana
2014	Dr.NTR Vaidya Seva
2016	Dr.NTR Arogya Rasha
2018	Ayushman Bharat - India's National Health Protection Mission

Objectives of the study

1. To explore the current status of major health insurance schemes in India
2. To study the role of A.P state health insurance scheme (NTR Vaidya Seva) in Andhra Pradesh.

II. Methodology

The data collected from various sources such as scheme website, available assessment reports, and the data provided by the Dr. NTR Vaidya Seva Trust and research reports.

III Background

Current status of health financing including health insurance in India

Public health expenditure in India (total of central and state governments) remained constant at approximately 1.3% of gross domestic product (GDP) between 2008 and 2015 and increased marginally to 1.4% in 2016-17. Total public health expenditure in India (both central and state governments) is approximately 1.3% of GDP (gross domestic product) between2008-2015 and it's increased slightly 1.4% in

2016-2017. For the years 2018-19 it is estimated at 3.9% in which both public and private sectors. The total health expenditure allocation is estimated by Ministry of family welfare about 54.600 crore. **Table 2** shows the various states health related programmes and their coverage amount.

Table 2. Publicly financed health insurance schemes in India

State	Programme name	Primary care	Secondary care	Tertiary care	Maximum benefit (Rs.)
Maharashtra	Mahatma Jyotiba Phule Jan Arogya Yojana (MJPJAY)	No	No	Yes	1,50,000/
Gujarat	Mukhyamantri Amrutam Yojana (MAJ)	No	Partly	Yes	2,00,000/
Chhattisgarh	Sanjeevani Kosh	No	No	Yes	3,00,000/
Chhattisgarh	Chief Minister Child Heart Protection Scheme	No	No	Yes	1,80,000
Kerala	RSBY-CHIS (Comprehensive Health Insurance Scheme)	No	Yes	Yes	30,000/- (RSBY)+ 70,000/-
Karnataka	Yeshasvini Co-operative Farmers Healthcare Scheme	No	Yes	Yes	2,00,000/-
Andhra Pradesh	RAS (Rajiv Arogya Sri) Community Health Insurance Scheme (CHIS) upto 2014	No	Yes	Yes	2,00,000/
	Dr.NTR Vaidya Seva From 2014 onwards	No	Yes	Yes	2,50,000/
Telangana	Arogya Sri Community Health Insurance Scheme (CHIS)	No	Yes	Yes	2,00,000/-
Tamil Nadu	CM Health Insurance Scheme	No	Yes	Yes	1,00,000/-
Himachal Pradesh	RSBY plus	No	No	Yes	1,75,000
Meghalaya	Megha Health Insurance Scheme (MHIS)	Partial	Yes	Yes	1,60,000/-
Assam	Atal Amrit Abhiyan	No	Yes	Yes	2,00,000/-

The National Health Mission received the highest allocation at Rs.30,130 crore and constituted 55% of the total allocation. According to the National Family Health Survey 4 (2015- 16) (Ministry of Health and Family Welfare 2017), only 29% of households in India have one member covered under any health insurance scheme, be it public or private (20% women and 23% men). The top five states according to coverage are Andhra Pradesh (75%), Chhattisgarh (69%), Telangana (66%), Tamil Nadu (64%) and Tripura (58%). In-patient hospitalization expenditure in India has increased nearly 300% during the last ten years (National Sample Survey Office 2015). Household health expenditures include out-of-pocket expenditures (OOPE) (95%) and insurance (5%). According to the National Health Accounts (2014-15), total OOPE is 3.02 lakh crore. The highest OOPE is made towards purchasing medicines— 1.30 lakh crores. (43%), followed by private hospitals—86,189 crores (28%).

Government Sponsored Health Insurance Schemes and their Key Features

Revenue collection, risk pooling and purchasing of insurance scheme are three key functions for scheme performance and its functioning. The source of funds and collection mechanisms are important to pool up the revenue collection. While pooling funds refer to build up and running of funds, to ensure that financial risk to pay health care is borne by all and not by individuals who fall ill. Third function is purchasing care which refers to paying for health care. The insurer and their health insurance or the organizer of the scheme purchases services on behalf of a population. All these schemes broadly involve contracting with providers of designing an appropriate benefit package and making the choices around paying for them [11].

Dr. Nandamuri Taraka Rama Rao Vaidya Seva Health Insurance Scheme (Dr NTR Vaidya Seva Health Insurance Scheme) of Andhra Pradesh

According to Dr.Nandamuri Taraka Rama Rao Vaidya Seva Health Insurance Scheme is introduced Government of Andhra Pradesh to attainment of universal health coverage for BPL (Below Poverty Line) families. This scheme is helps to achieve equity, providing accountable and evidence-based good-quality health-care services in the state to assist poor families from catastrophic health expenditure. The scheme is a unique PPP model in the field of health insurance; tailor made to the health needs of poor patients and provides end-to-end cashless services for identified diseases under secondary and tertiary care through a network of service providers from Government and private sector.

The scheme is designed in such a way that the benefit in the primary care is addressed through free screening and outpatient consultation both in the health camps and in the network hospitals as part of scheme implementation. The IEC activity during the health camps, screening, counseling and treatment of common ailments in the health camps and out-patient services in network hospitals are supplementing the government health care system in preventive and primary care. The scheme is designed to get benefit for the people to identify for their primary care addressed through the free screening and outpatient consultation both in the health camps and network hospitals as part of scheme implementation. The main activity during the health camps, screening, counseling and treatment for common ailments are conducted by both government and network hospitals for preventive and primary care. The organization of the scheme and effective implementation, the state government established Dr.NTR vaidya seva trust. This trust works under the chairmanship of the Honorable Chief Minister and administrative chief executive officer who is an IAS or IPS officer. The other members are specialists in health care management's personals. The organization chart is given in Figure 1.

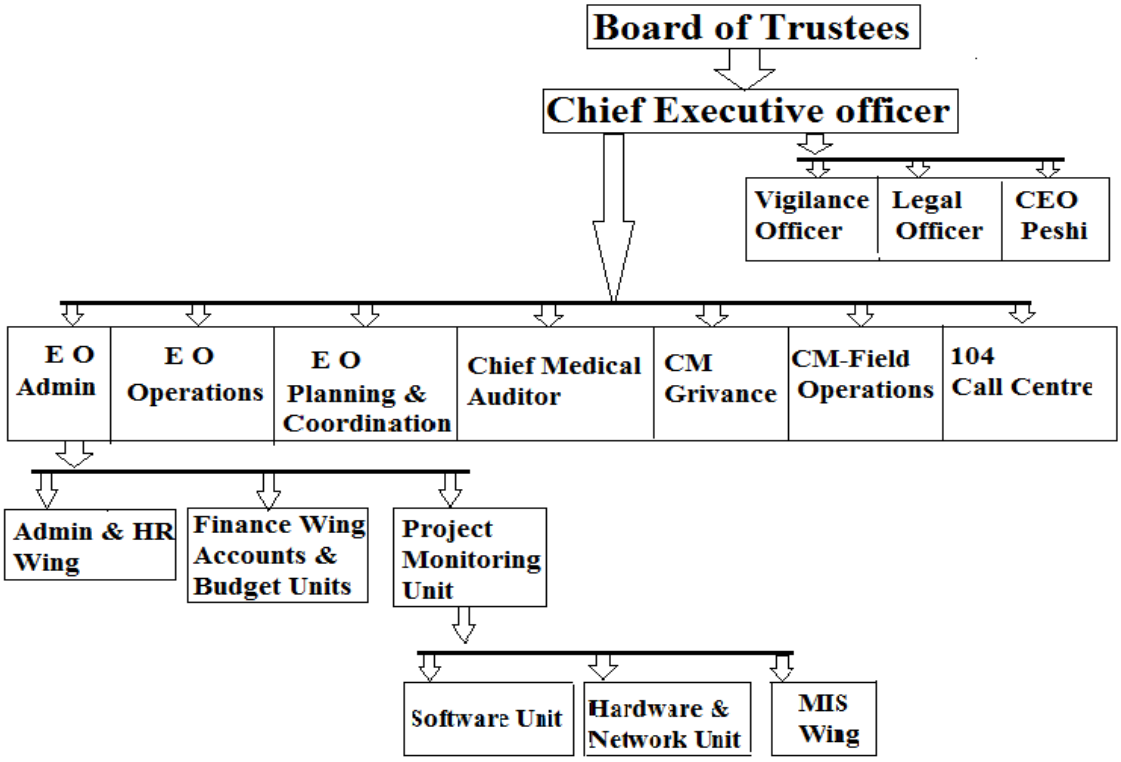

Figure 1.The organization chart of NTR Vaidya Seva

The main objectives are (i) To provide free quality hospital care for families who effected, by purchase of quality medical services from identified network of health care providers through a self-funded

reimbursement mechanism (serviced by Trust). (ii) To provide financial safety against the catastrophic health expenditures (iii) To strengthen the government hospitals through require areas of financing. (iv) To provide universal coverage of health for both urban and rural poor in the State of Andhra Pradesh.

Geometry of health insurance coverage

- Population coverage (Breadth of Universal Health coverage): The beneficiaries of this scheme are the members of Below Poverty Line (BPL) families. They are enumerated and photographed in White Ration Card linked with Adhar card and available in Civil Supplies Department database.
- Financial coverage (Height of Universal Health coverage): The scheme shall provide coverage for the services to the beneficiaries up to Rs.2.50 lakh per family per annum on floater basis. There shall be no co-payment under this scheme.
- Benefit Coverage (Depth of Universal Health coverage).

Out-Patient: The scheme is designed to get benefit in the primary care is addressed through free screening and outpatient consultation both in the health camps and network hospitals as part of scheme implementation.
In-patient: The scheme shall provide coverage for the 1044 "Listed Therapies" for identified diseases in the 29 categories – which are listed in www.ntrvaidyaseva.ap.gov.in

Package includes the following services:

- End-to-end cashless service offered through a network hospitals from the time of reporting of a patient till ten days post discharge medication, including complications if any up to thirty (30) days post-discharge, for those patients who undergo a "listed therapy(ies).
- Free outpatient evaluation of patients for listed therapies who may not undergo treatment for "listed therapies.
- All the pre-existing cases under listed therapies are covered under the scheme.
- Providing Food and Transportation for them.

Follow-up Services

Follow-up services are provided for a period of one year through fixed packages to all the patients whoever required for long term follow-up therapy. In order to get optimum benefit for the patients by the follow up service the procedure and avoid complications. Follow-up package for consultation, investigations, drugs etc., for one year for listed therapies are formulated by Technical committee of the Trust in consultation with specialists. More details we can find in www.ntrvaidyaseva.ap.gov.in.

Performance of the Scheme:

The impact of Dr.NTR vaidya seva scheme was evaluated by a rapid assessment, commissioned by the government of Andhra Pradesh. The aim of the present assessment was to explore the contribution of the scheme to the reduction of catastrophic health expenditure among the poor and to suggest ways by which relief of the scheme could be enhanced. This novel scheme was beginning to reach the BPL households in 13 districts of the Andhra Pradesh providing access to free secondary and tertiary healthcare to seriously ill poor people. An integrated model encompassing primary, secondary and tertiary care would be of larger number of families benefited below the poverty line and more cost-effective for the government. There is considerable potential for the government to build on this successful start and to strengthen equity of access and the quality of care provided by the scheme.

From Table 3 it is evident from the period 2nd June 2014 to March 2018, the total number of therapies preauthorized under NTR Vaidya Seva scheme is 17,64,855 and the expenditure was paid for 405.66, 814.49, 994.85, 1326.42 and 1089.79 cores for 2014 to 2018 respectively. Total amount paid for preauthorized therapies for all the years under this scheme is Rs.4631.21 Cores and average amount per

therapy is Rs.26,241. The preauthorize number is increasing every year from 2014 to 2017. There is a slight decrease in 2018 year due to another scheme government introduced Arogrya Rasha for the people. Total preauthorized therapies were done in Government and corporate hospitals are given in table 3 it is evident from the data both government and corporate hospitals preauthorized therapies are increased from 2014 to 2018 it is shown in Figure 2. The average expenditure was shared nearly 25 % government and 75 % corporate hospitals.

Table 3 Year wise preauthorized therapies and expenditure sharing

Year	Total number of Preauthorized	Preauthorized Amount paid from NTR Vaidya Seva Trust. Rs. Cores	Average amount Rs.	Govt	Corp	Sharing	
						Govt. %	Corp. %
2014	1,60,948	405.66	25205	43176	117772	26.83	73.17
2015	3,21,671	814.49	25320	83907	237764	26.08	73.92
2016	3,63,204	994.85	27391	82503	280701	22.72	77.28
2017	4,79,688	1326.42	27652	110342	369346	23.00	77.00
2018	4,39,344	1089.79	24805	117763	321581	26.80	73.20
Total	17,64,855	4631.21	26241	437691	1327164	24.80	75.20

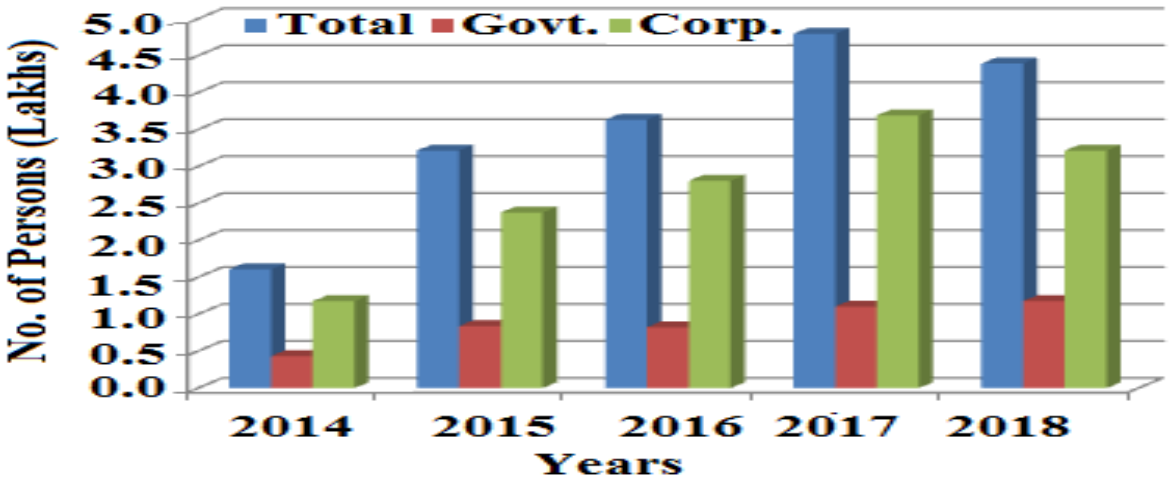

Figure 2 year wise performance of government and corporate hospitals

Table 4 District – wise total number of therapies preauthorized under NTR Vaidya Seva.

DISTRICT	2014	2015	2016	2017	2018	Total	Average per year	%
Anatapur	8522	18732	21309	28854	28997	106414	21282.8	6.03
Kurnool	10819	21196	24771	32013	33765	122564	24512.8	6.94
YSR Kadapa	9407	18765	20829	29244	29539	107784	21556.8	6.11
Chittoor	12238	23218	25478	36790	35503	133227	26645.4	7.55
Nellore	13404	26827	27034	39836	33233	140334	28066.8	7.95
Prakasam	11304	22844	26312	34819	30363	125642	25128.4	7.12
Guntur	17674	33265	38769	50416	46589	186713	37342.6	10.58
Krishna	16451	29857	32450	41964	35394	156116	31223.2	8.85
West Godavari	14450	27909	31898	40995	34022	149274	29854.8	8.46
East Godavari	18207	37094	45992	55398	52585	209276	41855.2	11.86
Vishakhapatnam	11842	25093	27561	36481	34528	135505	27101	7.68

Srikakulam	8587	18750	21018	27079	22326	97760	19552	5.54
Vizianagaram	8043	18121	19783	25799	22500	94246	18849.2	5.34
Total	160948	321671	363204	479688	439344	1764855	352971	

Table 4 shows 13 Districts and each district wise total number of preauthorized under NTR vaidya seva scheme from 2014 to 2018. It has been found that the total number of preauthorized that has been taken under this scheme is 17,64,855. The average of the each year about 3,52,971 therapies for all the districts are carried out. Srikakulam district average is low, where as highest average is East Godavari district 41855.2. Highest number of therapies preauthorized in East Godavari district 11.86 percent (209276), Guntur district 10.58 percent (186713) and Krishna district 8.85 percent (156116). Lowest number of therapies preauthorized in Vizinagaram district 5.34 percent (94246) and Srikakulam district 5.54 percent (97760).

In the Andhra Pradesh total 13 districts the gender and male, female, male child and female child are gradually increased from 2014 to 2018. Figure 3 shows the total therapies are highest for 2017 for male (260727) and female (178968). The gender therapies were given in Table 5.

Table 6 explains the category of the people for their therapies and average performance of the NTR Vaidya seva scheme. Maximum numbers of beneficiaries are BC category next followed by OC and SC category, very less in number by other category.

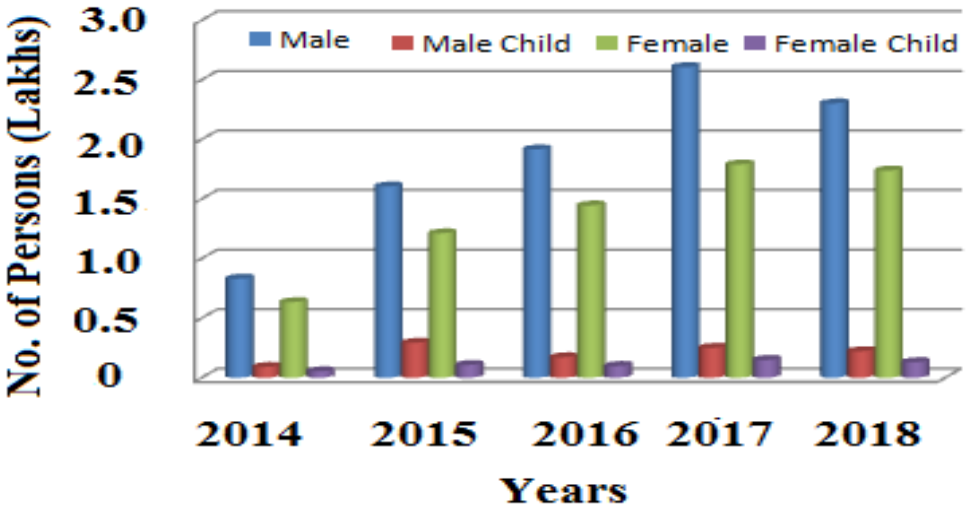

Figure 3 Gender therapies for the years 2014 to 2018.

Table 5 .The total gender therapies of the NTR Vaidya seva scheme.

Year	Male	Male Child	Female	Female Child
2014	83151	9027	63428	5342
2015	160504	29243	121215	10709
2016	191613	17162	144562	9867
2017	260727	25068	178968	14925
2018	230247	22202	173853	13042
Total	926242	102702	682026	53885

Table 6 Various Categories of NTR Vaidya Seva for the years 2014 to 2018.

Year	OC	BC	SC	ST	Minorities	Others
2014	40089	26981	79270	4348	10132	128
2015	79358	159733	52747	9265	20302	266

2016	88701	183604	58869	10112	21572	346
2017	114123	246164	76742	13138	29150	371
2018	99008	229994	70921	12592	26580	249
Total	**421279**	**846476**	**338549**	**49455**	**107736**	**1360**
Percentage	**23.87**	**47.96**	**19.18**	**2.80**	**6.10**	**0.08**

Conclusion

NTR Vaidya Seva Scheme is extremely popular in Andhra Pradesh State for the poor and BPL families. More than 17.64 lakh families are benefited during 2014 to 2018. The corporate hospitals are highest share when compare to government hospitals. The follow up service is also quite good in rating. But there is very less provision for outpatient treatment of everyday illnesses like BP, Blood sugar. Which affect the working expenditure for the patient and effect the families increases out of pocket expenses still continue. So, there is a coming up need to enlarge the scope of the scheme and better health care by plugging loopholes, in the interest of society and social well being of people, especially the poor and downtrodden. Without proper health care, the economic and social development of any country becomes impossible. In view of above key comments here and Government of Andhra Pradesh is planning to distribute free medicines for the poor people for everyday illnesses through health workers. However NTR Vaidya seva scheme is undoubtedly regarded as a boon to poor and downtrodden families in entire 13 districts. Who are otherwise vulnerable to a variety of diseases and efficient treatment due to heavy health expenditure.

References

[1] WHO-Health financing for universal coverage (2010) Out-of-pocket payments, user fees and catastrophic expenditure.

[2] Kumar D (2018) Does Household of India Face Out of Pocket Expenditure?. Health Econ Outcome Res 4: 153.

[3] van Doorslaer E (2006) Effect of payments for health care on poverty estimates in 11 countries in Asia: an analysis of household survey data. Lancet, summary.

[4] WHO (2011) World Health Survey, Website, Geneva: WHO.

[5] George Gotsadze AZ (2009) Household catastrophic health expenditure: Evidence from Georgia and its policy implications. BMC Health Services Res 9: 69.

[6] Garg CKA (2005) Reducing out-of-pocket expenditures to reduce income poverty - evidence from India. World Health Organization, Geneva 2-8.

[7] Peters, D., Yazbeck, A.S., Sharma R., Ramana, G.N.V., Pritchett, L, Wagstaff, A. (2002): *Better Health Systems for India's Poor. Findings, Analysis and Options,* Washington DC:World Bank.

[8] Dilip, T.R., and Duggal, R., (2002): *Incidence of non-fatal Health Outcomes and Debt in Urban India*, Draft paper presented for urban research symposium, 9-11 December 2002, at World Bank, Washington D.C.

[9] Wagstaff, A and E.van Doorslaer (2003): Catastrophe and Impoverishment in Paying for Health Care: With Applications to Vietnam 1993-98, *Health Economics*, 12: 921-934.

[10] Yellaiah, J. (2013): Health Insurance in India: Rajiv Aarogyasri Health Insurance Scheme in Andhra Pradesh, IOSR Journal of Humanities And Social Science, 8(1) 1 07-14.

[11] McIntyre, D. (2007), "Learning from Experience: Health care financing in low-and middle income countries", Global Forum for Health Research, Geneva, 2 940286-53-1.

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SONGS AND MUSIC IN SHAKESPEARE'S PLAYS

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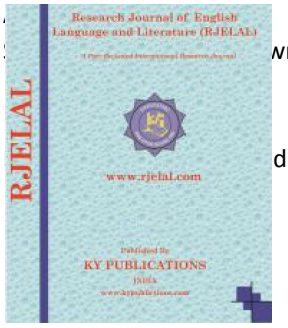

wn as a dramatist than a poet. His poetic gift is displayed not merely in his poems and sonnets but also in songs and the verses scattered in all his plays. His greatness lies in his command of rhythm and imagery.

drama may not be directly relevant to the content but wants them as part of entertainment. In his dramas The Tempest, As you like it, The Hamlet, etc. the songs are so deeply embedded in the text as dialogues that it is Unnecessary to stop the action to permit them to be performed. Shakespeare's use of instrumental music serves the demand of certain dramatic situations like rejoicing, mourning or supernatural interventions.

The soliloquies of his tragic heroes are best examples of greatest poetry. They are the spontaneous overflow of depressed souls. These poetic expressions win more than the desired effect. There was general enthusiasm for music in Elizabethan England. Thus Shakespeare's great combination of these skills gives his plays extraordinary power and richness.

Keywords: Shakespeare sonnets – songs – instrumental – music – stage music – magic music – called for songs – rejoicing – super natural inventions – love of music – casual musicians – musically train boys – songster.

The common reader of Shakespeare knows him as a dramatist and not as a poet because he is not aware of his sonnets and well-known poems, "Venus and Adonis" and "The Rape of Lucrece". Shakespeare's poetic gift is displayed not merely in his poems and sonnets but also in songs and the verses scattered in all his play. Some of the blank verses he uses in his tragedies can be compared with the blank verses of any of the greatest poet in the world. Shakespeare's greatness as a poet lies in his command of rhythm and imagery, use of the art of rhetoric, magic of words and above all his brilliant imagination.

Sometimes Shakespeare is highly metaphorical and that is why he is not always intelligible to the common reader, but in most of his plays, he uses a language which goes straight into the heart of the reader. His poetry is a superb means of dramatic expression. In his poem, "single nature's double name", the dramatist is like the phoenix and turtle, when he was writing plays he found instances for composing poems.

Music and songs in a drama may not be directly relevant to the realistic element or content of presentation, but the average play –goer wants them as part of entertainment. The dramatic conventions of the Elizabethan stage permitted the introduction of songs and instrumental music. Though the primary object was to please the public, Shakespeare's plays were so conceived that the songs and music could well save as essential elements in the structure of the play and convey

ideas which cannot be effectively done by speeches.,

> "The oddest thing about Shakespeare is that people sometimes forget that he wrote plays. There is so much poetry, philosophy and psychology in his writings that it is easy to forget that he wrote for the stage."

Helen Moeris observes-

In Richard two we will be fascinated by the psychology of Shakespeare's king and enchanted by the poetry of his speeches.

David Daiches observes -

"We all know that he was a great poet as well as a great dramatist and it is his combination of these two skills that gives his plays their extraordinary power and richness".

Though the primary object is to please the public, the song and music well serve as essential elements in the structure of the play. W.H Auden lists out some of those purposes that his poetry serves in his plays viz

(a) Dramatic relevance

(b) Tone of the drama

(c) Insight into the character

(d) Instruction

(e) State of mind of a character

As for dramatic relevance, some scenes of action are to be presented with a musical background. A banquet or a call to battle demands what is known as "stage music"

E.g.:- In the Tempest, Ariel sings to himself about his joy after he gets freedom....''

When the bee sucks, there suck I
In a cow slip's bill I lie;
Merrily, Merrily shall I live now''.

Other songs of Ariel are also dramatically significant. They set the course of events in the play.
These set of songs are so deeply embedded in the text as dialogues that it is unnecessary to stop the action to permit them to be performed ,for they are essentially a part of The Tempest , when Ariel sings,

"Come unto these yellow sands And then take hands ------

Ferdinand feels that the song is intended for him only and follows it.

Again Ariel sings.
"Full fathom five they father lies
of his bones are coral made"

The song entices Ferdinand to Prospero's cave.

Such "magic music "is employed to make someone fall asleep or fall in love or culminates in the marriage of lovers etc. It is brought about by a trap door device blow or behind the stage .In the Tempest the musical intervention is supernatural.

The tone of drama is impressively suggested by music. In films instrumental music sets off the mood and atmosphere of the scenes. It shows the heroine descent from the pedestal [as Hermione in the hunter's tale] the wedding masque in The Tempest besides serving a pure musical entertainment, prefigures a very happy return of voyage for the royal families.

"calm seas, auspicious gales,
And sail so expeditious that shall catch,
Your royal fleet far off"
Juno[Queen of Heaven] sings -------
"Hourly joys be still upon you !
Juno sings her blessings on you.''

Thus in The Tempest half dozen songs that are present serve important functions and a necessary dramatic device.

Musically trained boys and professional musicians done the roles especially in the last romances, thus enriching the musical content of the plays.

The songs rendered by Amiens, feste and Balthazar can be called "called-foe" songs, for they are sung at the request of another who wishes to hear some music. So, the characters become performers and so the emphasis is on the quality

and content of the musical numbers. Balthazar is asked by Duke pedro to entertain the group with a song, He sings-

"O good my Lord, tax not so bad a voice!!

To slander music any more than one more than once "

Likewise Amiens in "As you like it "sings at the request of Jaques

"under the greenwood tree

Who loves to lie with me"

Celebrates the simple, free life in the forest of Arden. The song brings us a picture of pastoral beauty exalted by the spirit of freedom that every creative enjoys.

The song -

"Blow Blow, thou winter wind
Thou art hot so unkind
As man's ingratitude
Thy tooth is not so keen
Because thou art not seen"

Tells us of man's cruelty to man and his selfishness which is more harmful than writer's harshness . Nature [the forest] beyond man's domain is indeed a blissful heaven for these who shun ambition and power over man.

A forester also sings when the countries hunt a deer -

"The horn ,the horn, the lusty horn,

Is not a thing to laugh to scorn." The horns of the deer with their familiar association with unfaithful wives provide a subject for some fun. The last song of hymn is a conventional marriage hymn.

"Wedding is great Juno's crown

O blessed bond of board and bed''!

Hyman invokes the presiding goddess of marriage to solemnize the wedding of the lovers.

Shakespeare's use of instrumental music serves the demand of certain dramatic situations like social rejoicing or mourning or supernatural interventions ,while casting magic spell etc as you ,see Cohen played to wake up Alone and Gonzalo [The tempest] from magic spell, when a message is conveyed to Jupiter [in Cymbeline] . In Antony and Cleopatra the doom of Antony is foretold by playing melancholy note.

Songs are also used for character – revelation. The Melancholy of Mariana [Measure for Measure], who bids a boy-singer to "please her woe" with Music, Mariana'fellings of sadness and frustration due to rejected love are reflected in the song –

"Take, O Take those lips away
Those so sweetly were forsworn
And those eyes, the break of day,
Lights that do mislead the morn'' etc.

This song to which she is listening is a poignant expression of her own battalion disappointment in love.

For a touchingly striking study of character, the song of Ophelia offers a fine example .In her madness wrought by grief, she sings songs before the king, queen and Horatio,

She sings,

''He is dead gone lady'
He is dead and gone,
At his head a grass green turf
At his heels a stone "

In the words of the king Claudius, Ophelia is struck by a battalion of sorrows, she foresees her death, she sings –

''He is gone, He is gone

And we cast away moon

Her profound grief over her father's death, and the absence of a decent burial for him, and her love for

hamlet and his rejection all went an expression in these songs.

The lines

''To the grave did not go '' and

''Tomorrow is saint –valentine's day '' reveal her chaste passion.

In Othello, Desdemona's willow song provides a picture of an innocent soul a waiting her doom with sublime helplessness. The sad songs of Ophelia strike a pathetic note indicating premonition of tragedy.

In hennery 4 part 1 the boy who played Glendower's daughter kate sings a song in welsh. In as you like it two boys sing besides Amiens.

These songs are not ''called for'' songs, but a spontaneous overflow of a depressed soul. These songs reveal the true mind and the sub-conscious feelings beneath the layers of the minds of Mariana, Ophelia and Desdemona.

Songs are hardly seen in the historical plays (exception Antony and Cleopatra – a Roman play). In tragedies the effect is profound .The songs in tragedies help the audience to identify themselves with characters. For example for the boy Lucius, Brutus sings in the tent wins our sympathy for him.

'' This is a sleepy tune: o murderous slumber

Lays't thou thy leaden mace upon my boy

That plays thee music?.....

A Shakespearean tragedy in never depressing '' says A.C Bradley. Along with other excellences his poetry is very enjoyable.

The soliloquies of his tragic heroes are best examples of greatest poetry. These poetic expressions emphatically convey the melancholy, and win more than the desired effect.

For example:-The agony of Othello after killing his wife

'My wife! My wife! What wife? I have no wife

O, insupportable, !O heavy now!

Methinks it should be now a huge eclipse".

'Othello has just done the deed and is undecided as to what he should do next .On mentioning the word wife'' he breaks down in a paroxysm of grief over the fact that now he has o wife.

And again Hamlet when he comes to know that his love Ophelia is no more and when he sees Laertes making an extravagant display of grief he bust out-------

''I loved Ophelia; forty thousand brothers
Could not, will all their quantity of love
Make up my sum''.

And his other famous soliloquies such as---

''To be or not to be ; that is the question ''

''O, that this too solid flesh would smell'--- a beautiful poetic utterances.

Elizabethan audience regarded plays as instructive .The acting of rebellion or abdication put dangerous ideas into the heads of the people. Censor was quick to crack down as such scenes. A play of Thomas More bore the instructions of censor on the script of play . Shakespeare knew the pulse of his audience The day before the ''Earl of Essex'' rebelled against queen Elizabeth , his friends bribed the players to act Richard 2 ,so that Londoners might actually see that it was possible to replace a weak and vacillating ruler by a strong and capable noble man . Shakespeare uses songs in his plays to instruct and instill great nationalism and patriotic feelings. Here is one song when England faced open war with spam in 1580.

''This England never did , not never shall , lie at the proud foot of a conqueror .....''

Shakespeare's plays are so familiar that they had an entry in to the Elizabethan song books. They are so written as to give plenty of freedom to the composer and at the same time to be relevant to their contexts and to contribute to the dramatic

effect. These songs are more thoughtful poems which can also stand alone.

For example : It was a lover and his lass (As you like it) was later set to music by Mocely .

There was general enthusiasm for music in Elizabethan England. The love of music was certainly not confined to the educated classes, casual musicians , wandering about the streets waiting for casual engagements to play at weeding or to entertain customers in taverns and in the houses of noble , were plenty . They played the flute , feste in The Twelfth night plays the pipe and taboo. Ariel in The Tempest is directed to play a tune on the taboo and pipe . Ophelia (in the first quarto) enters playing on a lute And in Julius Caesar , the instrument over which Lucius playing a 'sleepy tune '' and falls asleep is a lute.

Brother takes it from his lap, saying

''If thou dost nod , thou break'st thy instrument' .

A dreamer would be needed for the marches of soldiers .

For example : - As in Hamlet , a Danish March. In the Theales the normal places for music was a curtained box. Shakespeare himself had a very considerable knowledge of music. His poems and plays are full of allusions to the art. Instrumental music greatly contributed to the atmosphere the playwright wished to evoke.

The music employed in Shakespeare plays may be classed in three groups. 1) Fanfares 2)dancers and 3)songs. Fanfares include all the trumpet calls, alarms, retreats, tuckets as well as marches for drums. Music of this type is played in connection with battles, princes and kings. When Claudius drinks in Hamlet drums were played to indicate the sounds.

The songs

"It was a lover and his lass (As you like it) set by Morely

"6 mistress mine" (Twelfth Night) set by Morcely

"lawn as white as driven snow"(The winter's Tale)

Set by Dr.John Wilson,

''Full fathom five "set by Robert Johnson are famous.

It has been suggested that many of Shakespeare's songs were written to tunes already in existence, although those tunes may not be known to us now.

Musically train boys (choristers of royal chapel and st.paul's cathedral) and professional musicians donned these roles , thus enriching the musical context of the plays .

Shakespeare is not only a great poet, but also a great dramatist and a musician. He is one of the greatest songsters and it is his combination of all these skills that gives his plays their extraordinary power and richness.

Bibliography

1. William Shakespeare, The Tempest https ://books.google.co.in>books
2. The Hamlet, play by Andre Tchaikovsky and William Shakespeare
3. As you like it, William Shakespeare published in the first folio 1623.
4. Othello, book by William Shakespeare published in 1565.
5. Music in Shakespeare Tragedy by F.W.Stern fold.
6. Song in Shakespeare's plays – Shakespeare Oxford fellowship.
7. Shakespeare's Drinking songs 29 Mar 2013.
8. David Lindey Shakespeare and Music (2014).
9. Shakespeare's Song Book – by Ross W .Duffin.
10. Songs from Shakespeare's lays and popular songs of Shakespeare's time – Compiled and Edited by Tom Kines.

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THEMATIC ANALYSIS OF SATISH CHENDER'S TELUGU STORY "*IT IS WRONG.*" IN ENGLISH

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Abstract:

The short story "*Thappu*" in Telugu language by Satish Chender is translated as "IT IS WRONG." It talks about a thought provoking contemporary social issue. The issue of an inter caste marriage between a high caste disabled woman and a perfectly able man of low caste (outcaste), which is eventually supported by her mother but not by her community people in the village. It is a theme of true love and sacrifice being portrayed sensibly by the writer. The story answers to the questions such as who did wrong. Why did s/he do it? What is important for the people of her community? Is it life or pride? What did Veeralaksmi do for her daughter Malliswari who cannot walk on her own without any aid? Though we live in the 21th century where most of the people are able to transact globally without inhibitions some Indians are not able to understand the very purpose of life in the name of caste, religion, class and region. In this paper cause and effect of caste consciousness is argued.

Keywords: Love, discrimination, community counsel, sacrifice.

In this story, Satish Chender focuses on discrimination, alienation, communal pride and unity to break the relationship between Malliswari, a Kshathriya woman and Solomonraju an untouchable man(outcaste) lives in Peta (locality where untouchables live) in the name of caste. Malliswari is a differently abled woman, whereas Solomanraju a perfectly abled man.

Malliswari's father abandons Malliswari and her mother Veeralakshmi to keep up his Kshathriya pride. Veeralakshmi a single parent faces lot of troubles in bringing up of the disabled Malliswari. Malliswari and Solomonraju are schoolmates. As Malliswari could not en move for her personal needs without any aid, she takes help from her mother at home and a female friend who can carry her to the washroom and bring back to the class or the place wherever she is, but as a matter of fact, nobody is ready to help her except Solomonraju who is just three or four years old to her at the school.

This routine has gone until Malliswari becomes is years. After that she used to manage taking help from her female classmates who could help her involuntarily. But the

students, teachers and people of her community hate the relationship between Malliswari and Solomonraju because Solomonraju is an outcaste and who lives in "Peta" (a place where the out castes live). They murmur among themselves, "How dare he is to touch a Kshathriya woman, being a person from Peta?"(Satish Chender 844) Malliswari loves only two people in her life, they are her mother Veeralakshmi and Solomonraju, both serve her without any expectations. The relationship between Malliswari and Solomonraju is also approved by Veeralakshmi as she knows the difficulty and patience that her daughter requires to attend to her daily needs.

Malliswari, Solomonraju though become young, they have not lost their love and affection upon them. This bond of souls is misunderstood by Malliswari's father and the people of her community. Her father hated Malleswari and her mother because are on intimate terms with a low caste boy, as if it is an insult and damage to the pride of their clan "the Kshathriyas". Therefore, he deserts his wife and daughter and has gone off.

Malleswari one morning observing that her mother was reading the Bible, says, "Mother! Shall I tell you a secret? I've met Jesus in my dream." (843) Veeralakshmi curiously asked, "Jesus! Have you spoken to Him properly?" (843) In the course of their conversation about Jesus, Malleswari told her mother that she had taken for granted Jesus, and asked Him why didn't He give her the feet whereas, He has given eyes to the eyeless. She told her mother that Jesus replied, "You silly woman, I have given you the lap of a mother permanently. The mother's of others' carry them only for two or three years, whereas in your case you will ever be carried by your mother" (844). The words of Malleswari at that moment made Veeralakshmi understand the pain of disability being experienced by her daughter. Veeralakshmi thinks about the life of Malleswari after her death.

The very thought made her to take a fatal decision which will open the doors for her daughter's harmonious life with Solomonraju because the people of her clan will never allow her to marry Solomonraju. It shows Veeralakshmi's belief that Malliswari will be accepted by all the people in Peta unconditionally.

The people of her caste called on a community counsel to address the issue and to separate them under the headship of a gentleman by name Venkatapathiraju. In the community counselling, she confronts many people there and she even dares to ask her community people that she will marry anyone, if any one comes forward to take her as a maid for the son of any man. One person with great feeling of caste by name Kodipandhelaraju, who often spends most of his time in gambling and cock-fights insults her mother, "Veeralakshmi, why don't you also select a Johnraju for you like your daughter because your husband has already abandoned you both? Hasn't he?" (842)

This insult affected both Malliswari and Veeralakshmi to a great extent consequently made them to rethink whether to continue the relationship with Solomonraju or not. Malliswari thinks,"Of course, it's wrong! Wrong to ask a thing that cannot be asked! It is wrong to ask, taking for granted that it is my mother, whom I have asked! Asking mother to take me to my lover wherever, he asked me to come is wrong" (841). She then tries to discontinue her relationship with Solomanraju, who is the only source of relief whenever she gets upset in her life other than her mother at home. She thinks it is a wrong act which made her mother to face such a disgusting remark by the nincompoop of the community. Malliswari's mother Veeralakshmi on the other hand thinks for her daughter's new life after her death. The narrator here says, "Veeralakshmi felt that the words like a knife was directly pierced her heart like that of a cock that hits the opposite cock with its sharp knife tied to its leg in the fight." (842) Veeralakshmi having got all these ideas as possible outcomes of her daughter's life, ends her life leaving a letter for her daughter in the Bible, which is kept under the pillow in which she writes her that Solomonraju can only be the person who can carry her for the rest of her life.

Venkatapathiraju is a prominent person, a gentleman among their community; no one dares to question him or contradict him in his presence. Venkatapathiraju promises Veeralakshmi that his two acres out of four acres of fertile land shall also be given for her, if Malliswari keeps their pride in being away from Solomonraju and marry a man from their own community. A brief silence prevails there, and then Malleswari replies to Venkatapathiraju,"Uncle! I will marry a person from our own community", (843) not only that but also she says, "I will marry in this community only, and also in this village itself, as these people of our community have gathered to address the issue. I will marry the son of any of these persons, if comes forward. However, the person may be, may be a drunkard, an illiterate, a philanderer, anyone! Even if the groom comes not for me but for the land you have promised to gift us! I am ready to marry the person". (843)

Listening to her words, all the Kshatrya persons over there have left the place silently one after the other. She even dares to ask that she is ready to marry Vikramavarma the son of Venkatapathiraju. Venkatapathiraju abashed at her remark. Veeralakshmi immediately slaps Malleswari and says,"How dare you to expect his life, don't you have control on your expectations! Uncle Venkatapathi is like God for us!"(843) the narrator remarks soon after the words spoken by Veeralakshmi against her daughter's choice of marrying the son of Venkatapathiraju as "It is nothing but sheer violence, the violence that hurts even the people of great violence like the violence that takes place at giving birth to a child by a woman". (843)

The Paster opens the Bible kept under the pillow, finds to his surprise a letter addressed to Malleswari. In the letter Veeralakshmi writes, "You are gold. You will be carried for the rest of your life by Solomonraju only. I will not send you with Solomonraju to

his home as long as I am alive. I am therefore, going to live with Jesus, with lots of kisses....your mother." (846) With that letter mother goes to cemetery and daughter goes to Peta.

Satish chender concludes the story with the remark, talks about the need for unconditional love, affection and compassion towards fellow beings, as they are the real sources' of life not meanness and hatred toward people without reason but just for pride. All these qualities of life have been successfully addressed by him through the characters: Solomonraju and Veeralakshmi who show unconditional love and affection towards Malliswari. Venkatapathiraju though shows concern and humanity conducts the communal counsel for the issue. Kodipandhalaraju's disgusting remarks of Veeralakshmi shows how mean he is and also has full of community pride. Satish Chender advocates the contemporary social issue invoking the social responsibility of communal harmony and integrity, ended the story saying "the wrong of ethics has become the right of eternity" (846), reminding the axiom justice delayed but not denied.

References:

- Satish Chander. "*Thappu*" (2011), a short story translated as "It Is Wrong." by KoteswaraRao, K. V. 2019.
- Naveen Vasireddy, and Sivasankar Papineni, editors. *Paathikella Katha*-1990. Bangalore: MaNaSu Foundation, Vol.II, 2016 pp.841-846.
- Hornby, A.S. Hey, Leonie and Holloway, Suzanne, editors. Oxford Advanced Learner's Dictionary of Current English. Oxford, England: Oxford University Press, Edition 9, 2015.
- Gwynn, J. P. L. Assisted by Venkateswara Sastry, J. Oxford Telugu-English Dictionary.New Delhi: Oxford University Press, 2015.

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RESEARCH ARTICLE

Application of Krasnoselskii Theorem for Multivalued Operators

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ABSTRACT:
The purpose of this paper is to present some existence results of Krasnoselskii type in generalized Banach spaces for multivalued operators. We provide also an application to a Fredholm-Volterra type integral inclusions system and gives an example in support of our main result. 47H10, 54H25, 34B15 fixed point, generalized contraction multivalued operator, integral inclusion, Krasnoselskii's theorem.

KEY WORDS: Krasnoselskii Theorem**,** Fredholm-Volterra.

1. INTRODUCTION:

Let X be a nonempty set. A mapping $d: X \times X \to \mathbb{R}^m$ is called a vector-valued metric on X if the following properties are satisfied:
(a) $d(x,y) \geq 0$ for all $x, y \in X$; if $d(x,y) = 0$, then $x = y$;
(b) $d(x,y) = d(y,x)$ for all $x, y \in X$;
(c) $d(x,y) \leq d(x,z) + d(z,y)$ for all $x, y \in X$.
A set endowed with a vector-valued metric d is called generalized metric space. The notions of convergent sequence, Cauchy sequence, completeness, open subset and closed subset are similar to those for usual metric spaces.
We denote by $M_{mm}(\mathbb{R}_+)$ the set of all $m \times m$ matrices with positive elements and by I the identity $m \times m$ matrix. If $x, y \in \mathbb{R}^m$, $x = (x_1, \ldots, x_m)$ and $y = (y_1, \ldots, y_m)$, then, by definition:

$$x \leq y \text{ if and only if } x_i \leq y_i \text{ for } i \in \{1,2,\ldots,m\}.$$

Notice that we will make an identification between row and column vectors in $\mathbb{R}^m$.
Let (X,d) be a generalized metric space in Perov's sense. For $r := (r_1, \ldots, r_m) \in \mathbb{R}^m$ with $r_i > 0$ for each $i \in \{1,2,\ldots,m\}$, we denote by

$$B(x_0, r) := \{x \in X : d(x_0, x) < r\}$$

the open ball centered in x_0 with radius r and by

$$\tilde{B}(x_0, r) := \{x \in X : d(x_0, x) \leq r\}$$

the closed ball centered in x_0 with radius r.
For the proof of the main results we need the following theorems.

Theorem 1 *Let $A \in M_{mm}(\mathbb{R}_+)$. The following assertions are equivalents:*

(i) A *is convergent towards zero*;
(ii) $A^n \to 0$ *as* $n \to \infty$;
(iii) *The eigenvalues of* A *are in the open unit disc, i.e* $|\lambda| < 1$, *for every* $\lambda \in \mathbb{C}$ *with* $\det(A - \lambda I) = 0$;
(iv) *The matrix* $(I - A)$ *is nonsingular and*

$$(I - A)^{-1} = I + A + \ldots + A^n + \ldots; \tag{1.1}$$

(v) *The matrix* $(I - A)$ *is nonsingular and* $(I - A)^{-1}$ *has nonnegative elements*;
(vi) $A^n q \to 0$ *and* $qA^n \to 0$ *as* $n \to \infty$, *for each* $q \in \mathbb{R}^m$.

Definition 1 *([29]) Let* (X, d) *be a generalized metric space. A subset* C *of* X *is called compact if every open cover of* C *has a finite subcover. A set* C *of a topological space is said to be relatively compact if its closure is compact.*

Definition 2 *([23]) Let* X, Y *be two normed generalized spaces,* $K \subset X$ *and* $f: K \to Y$ *an operator. Then* f *is called:*
i) compact, if for any bounded subset $A \subset K$ we have that $f(A)$ is relatively compact (or equivalently $\overline{f(A)}$ is compact);
ii) complete continuous, if f is continuous and compact;
iii) with relatively compact range, if f is continuous and $f(K)$ is relatively compact.

For the case of multivalued operators in a generalized metric space (X, d) we recall some notions.
If $F: X \to P(X)$ is a multivalued operator then

$$Fix(F) := \{x \in X : x \in F(x)\}$$

is the fixed point set of the operator F.
Let us consider now the following sets of subsets of a metric space (X, d):

$$P(X) := \{Y \subseteq X : Y \text{isnonempty}\};$$
$$P_b(X) := \{Y \in P(X) : Y \text{isbounded}\};$$
$$P_{op}(X) := \{Y \in P(X) : Y \text{isopen}\};$$
$$P_{cl}(X) := \{Y \in P(X) : Y \text{isclosed}\};$$
$$P_{b,cl}(X) := \{Y \in P(X) : Y \text{isboundedandclosed}\};$$
$$P_{cp}(X) := \{Y \in P(X) : Y \text{iscompact}\}.$$

If X is a normed space, then we denote:

$$P_{cv}(X) := \{Y \in P(X) : Y \text{isconvex}\};$$
$$P_{cp,cv}(X) := P_{cp}(X) \cap P_{cv}(X).$$

Let (X, d) be a metric space and we define now the following generalized functionals:
(1) $D: P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$D(A, B) = \begin{cases} \inf\{d(a, b) | a \in A, b \in B\}, \text{if} A \neq \emptyset \neq B \\ 0, \text{ if} A = \emptyset = B \\ +\infty, \text{if} A = \emptyset \neq B \text{or} A \neq \emptyset = B, \end{cases}$$

where D is called the gap functional between A and B.
In particular, $D(x_0, B) = D(\{x_0\}, B)$ (where $x_0 \in X$) is called the distance from the point x_0 to the set B.
(2) $\delta: P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$\delta(A, B) = \begin{cases} \sup\{d(a, b) | a \in A, b \in B\}, \text{if} A \neq \emptyset \neq B \\ 0, \text{otherwise} \end{cases}$$

In particular $\delta(A) := \delta(A, A)$ is the diameter of set A.
(3) $\rho: P(X) \times P(X) \to \mathbb{R}_+ \cup \{+\infty\}$,

$$\rho(A,B)=\begin{cases}\sup\{D(a,B)|a\in A\},\text{if}A\neq\emptyset\neq B\\0,\text{if}A=\emptyset\\+\infty,\text{if}B=\emptyset\neq A,\end{cases}$$

where ρ is called the excess functional of A over B.
(4) $H:P(X)\times P(X)\to\mathbb{R}_+\cup\{+\infty\}$,

$$H(A,B)=\begin{cases}\max\{\rho(A,B),\rho(B,A)\},\text{if}A\neq\emptyset\neq B\\0,\text{if}A=\emptyset=B\\+\infty,\text{if}A=\emptyset\neq B\text{or}A\neq\emptyset=B,\end{cases}$$

where H is called the generalized Pompeiu-Hausdorff functional of A and B.
If (X,d) is a generalized metric space with $d:X\times X\to\mathbb{R}_+^m$, defined as

$$d(x,y):=\begin{pmatrix}d_1(x,y)\\\dots\\d_m(x,y)\end{pmatrix},$$

then we denote by $D(A,B):=\begin{pmatrix}D_{d_1}(A,B)\\\dots\\D_{d_m}(A,B)\end{pmatrix}$ the generalized gap functional on $P(X)$.
We will denote by H_{d_i} the Pompeiu-Hausdorff functional, generated by d_i, for $i\in\{1,\dots,m\}$.
We define $H:P(X)\times P(X)\to\mathbb{R}_+^m$ as

$$H(A,B):=\begin{pmatrix}H_{d_1}(A,B)\\\dots\\H_{d_m}(A,B)\end{pmatrix}.$$

The following lemma is well-known, see for example S. B. Nadler [10].

Lemma 1 *Let (X,d) be a metric space, $A,B\in P(X)$ and $q\in\mathbb{R}$, $q>1$. Then for every $a\in A$, there exists $b\in B$ such that $d(a,b)\leq qH_d(A,B)$.*

We have the following extension.

Lemma 2 *Let (X,d) be a generalized metric space, $A,B\in P(X)$ and $q\in\mathbb{R}$, $q>1$. Then for each $a\in A$, there exists $b\in B$ such that*

$$d(a,b)\leq qH(A,B)$$

Lemma 3 *Let $A,B\in P(X)$. We suppose that there exists $\eta\in(\mathbb{R}_+^m)^*$ such that*
(i) for each $a\in A$ there is $b\in B$ such that $d(a,b)\leq\eta$;
(ii) for each $b\in B$ there is $a\in A$ such that $d(a,b)\leq\eta$.
Then $H(A,B)\leq\eta$.

If (X,d) is a generalized metric space, with $d:X\times X\to\mathbb{R}_+^m$, then for $Y\in P(X)$, we will denote by $\overline{Y}$ the closure of Y with respect to d.
In this framework, a set $Y\in P(X)$ is said to be closed if and only if $Y=\overline{Y}$. Thus, a set Y is closed if and only if for any convergent (with respect to d) sequence $(x_n)_{n\in\mathbb{N}}\subset Y$ its limit belongs to Y.
We will denote by $P_{cl}^d(X)$ the set of all nonempty and closed (with respect to d) subsets of X.

Lemma 4 *Let $A\in P_{cl}^d(X)$ and $x\in X$. Then $D(x,A)=0$ iff $x\in A$.*

Definition 3 *([4]) Let (X,d) be a generalized metric space, $Y\subset X$ and $F:Y\to P_{b,cl}(X)$ be a multivalued operator. Then, F is called a multivalued $A-$contraction in the sense of Nadler if and only if $A\in M_{mm}(\mathbb{R}_+)$ is a matrix convergent to zero and*

$$H(F(x),F(y))\leq Ad(x,y),\text{forany}x,y\in Y.$$

If X, Y are two generalized metric spaces, we recall that a multivalued operator $F: X \to P(Y)$ is said to be:
lower semicontinuos (briefly l.s.c.) in x_0 if and only if for any open $U \subset X$ such that $F(x_0) \cap U \neq \emptyset$, there exists a neighborhood V for x_0 such that for any $x \in V$, we have that $F(x) \cap U \neq \emptyset$

Theorem 2 *([28]) Let $(X, \|\cdot\|)$ be a generalized Banach space, D a nonempty closed bounded convex subset of X and $T: D \to X$ such that:*
(i) $T = A + B$ with $A: D \to X$ completely continuous and $B: D \to X$
a generalized contraction, i.e. there exists a matrix $M \in M_{mm}(\mathbb{R}_+)$
convergent to zero, such that

$$\|B(x) - B(y)\| \leq M\|x - y\|$$

for all $x, y \in D$;
(ii) $A(x) + B(y) \in D$ for all $x, y \in D$.
Then T has at least one fixed point in D.

Theorem 3 *([16]) Let $(X, \|\cdot\|)$ be a generalized Banach space and $Y \in P_{cp,cv}(X)$. Assume that the operators $F: Y \to P_{b,cl,cv}(X)$, $G: Y \to P_{cp,cv}(X)$ satisfy the properties:*
(i) $F(y_1) + G(y_2) \subset Y$, for each $y_1, y_2 \in Y$;
(ii) F is a multivalued $A-$contraction mapping in Nadler's sense
(iii) G is l.s.c. and $G(Y)$ is relatively compact.
Then $F + G$ has a fixed point in Y.

2 Existence results for a system of operatorial inclusions

In this section we will prove Krasnoselskii's type fixed point theorems in generalized Banach spaces for multivalued operators.
Let's consider the following system of operatorial inclusions:

$$\begin{cases} x \in G_1(x,y,z) + G_2(x,y,z) + G_3(x,y,z) \\ y \in G_4(x,y,z) + G_5(x,y,z) + G_6(x,y,z) \\ z \in G_7(x,y,z) + G_8(x,y,z) + G_9(x,y,z) \end{cases} \tag{2.1}$$

where $G_1: Y \times W \times M \to P_{cp,cv}(Y)$, $G_2, G_3: Y \times W \times M \to P_{b,cl,cv}(Y)$, $G_4: Y \times W \times M \to P_{cp,cv}(W)$, $G_5, G_6: Y \times W \times M \to P_{b,cl,cv}(W)$, $G_7: Y \times W \times M \to P_{cp,cv}(M)$, $G_8, G_9: Y \times W \times M \to P_{b,cl,cv}(M)$, $Y, W, M \in P_{cp,cv}(X)$ such that for each $(x, y) \in Y \times W$

$$G_1(x,y,z) + G_2(x,y,z) + G_3(x,y,z) \subset Y$$
$$G_4(x,y,z) + G_5(x,y,z) + G_6(x,y,z) \subset W$$
$$G_7(x,y,z) + G_8(x,y,z) + G_9(x,y,z) \subset M.$$

We define

$$G(x,y,z) := \begin{pmatrix} G_1(x,y,z) \\ G_4(x,y,z) \\ G_7(x,y,z) \end{pmatrix}$$

$$F(x,y,z) := \begin{pmatrix} G_2(x,y,z) \\ G_5(x,y,z) \\ G_8(x,y,z \end{pmatrix}$$

and

$$J(x,y,z):=\begin{pmatrix} G_3(x,y,z) \\ G_6(x,y,z) \\ G_9(x,y,z \end{pmatrix}$$

where $G: Y \times W \times M \to P_{cp,cv}(Y \times W \times M)$ and $F, J: Y \times W \times M \to P_{b,cl,cv}(Y \times W \times M)$.
The system (2.1) is equivalent to

$$r \in F(r) + G(r) + J(r),$$

where $G: Z \to P_{cp,cv}(Z)$, $F: Z \to P_{b,cl,cv}(Z)$ and $Z := Y \times W \times M$.

Theorem 4 *Let* $(X, \|\cdot\|)$ *be a generalized Banach space,* $Y, W, M \in P_{cp,cv}(X)$ *and* $G_1: Y \times W \times M \to P_{cp,cv}(Y)$, $G_2, G_3: Y \times W \times M \to P_{b,cl,cv}(Y)$,
$G_4: Y \times W \times M \to P_{cp,cv}(W)$, $G_5, G_6: Y \times W \times M \to P_{b,cl,cv}(W)$, $G_7: Y \times W \times M \to P_{cp,cv}(M)$, $G_8, G_9: Y \times W \times M \to P_{b,cl,cv}(M)$, $Y, W, M \in P_{cp,cv}(X)$ satisfy the following conditions:
(i) $G_3(r_1) + G_2(r_2) + G_1(r_3) \subset Y$, $G_6(r_1) + G_5(r_2) + G_4(r_3) \subset W$, $G_9(r_1) + G_8(r_2) + G_7(r_3) \subset M$ for each $r_1, r_2, r_3 \in Y \times W \times M$;
(ii) G_1, G_3, G_5 are l.s.c and $G_1(r), G_3(r), G_5(r)$, for all $r \in Z$ are relatively compact;
(iii)

$$H(G_3(r_1), G_3(r_2)) \le b_{11}\|x_1 - x_2\| + b_{12}\|y_1 - y_2\| + b_{13}\|z_1 - z_2\|$$
$$H(G_6(r_1), G_6(r_2)) \le b_{21}\|x_1 - x_2\| + b_{22}\|y_1 - y_2\| + b_{23}\|z_1 - z_2\|$$
$$H(G_9(r_1), G_9(r_2)) \le b_{31}\|x_1 - x_2\| + b_{32}\|y_1 - y_2\| + b_{33}\|z_1 - z_2\|$$

where $S = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ is convergent to zero.
Then there exists at least one solution for the system (2.1).

Proof. We apply the fixed point theorem of Krasnoselskii for multivalued operators to the space X, endowed with the vector valued norm $\|\cdot\|_V$ defined by

$$\|(x,y,z)\|_V = \begin{pmatrix} \|x\| \\ \|y\| \\ \| z \| \end{pmatrix}$$

The operator G is l.s.c. This follows from the fact that G_1 ,G_4 and G_7 are l.s.c. From condition (ii) we obtain that $G(r)$ is relatively compact, where $r = (x, y, z)$.
We have to show that the operator F and J are S- contraction (multivalued S- contraction in Nadler's sense). Using the assumption (iii) we get that

$$\begin{pmatrix} H(G_2(x_1,y_1,z_1), G_2(x_2,y_2,z_1)) \\ H(G_5(x_1,y_1,z_1), G_5(x_2,y_2,z_1)) \\ H(G_7(x_1,y_1,z_1), G_7(x_2,y_2,z_1)) \end{pmatrix} \le \begin{pmatrix} b_{11}\|x_1 - x_2\| + b_{12}\|y_1 - y_2\| + b_{13}\|z_1 - z_2\| \\ b_{21}\|x_1 - x_2\| + b_{22}\|y_1 - y_2\| + b_{23}\|z_1 - z_2\| \\ b_{31}\|x_1 - x_2\| + b_{32}\|y_1 - y_2\| + b_{33}\|z_1 - z_2\| \end{pmatrix}$$
$$= \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \|x_1 - x_2\| \\ \|y_1 - y_2\| \\ \|z_1 - z_2\| \end{pmatrix}$$
$$= S \begin{pmatrix} \|x_1 - x_2\| \\ \|y_1 - y_2\| \\ \|z_1 - z_2\| \end{pmatrix}$$

and

$$\begin{pmatrix} H(G_3(x_1,y_1,z_1),G_3(x_2,y_2,z_1)) \\ H(G_6(x_1,y_1,z_1),G_6(x_2,y_2,z_1)) \\ H(G_9(x_1,y_1,z_1),G_9(x_2,y_2,z_1)) \end{pmatrix} \leq \begin{pmatrix} b_{11}\|x_1-x_2\|+b_{12}\|y_1-y_2\|+b_{13}\|z_1-z_2\| \\ b_{21}\|x_1-x_2\|+b_{22}\|y_1-y_2\|+b_{23}\|z_1-z_2\| \\ b_{31}\|x_1-x_2\|+b_{32}\|y_1-y_2\|+b_{33}\|z_1-z_2\| \end{pmatrix}$$
$$= \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \|x_1-x_2\| \\ \|y_1-y_2\| \\ \|z_1-z_2\| \end{pmatrix}$$
$$= S \begin{pmatrix} \|x_1-x_2\| \\ \|y_1-y_2\| \\ \|z_1-z_2\| \end{pmatrix}$$

or equivalently,

$$H_V(F(r_1),F(r_2)) \leq S\|r_1-r_2\|_V$$

$$H_V(J(r_1),J(r_2)) \leq S\|r_1-r_2\|_V$$

for $r_1=(x_1,y_1,z_1), r_2=(x_2,y_2,z_2)$.

Thus F is a multivalued S- contraction in Nadler's sense.

The invariance condition is satisfied since

$$G_1: Y\times W\times M \to P_{cp,cv}(Y)$$
$$G_4: Y\times W\times M \to P_{cp,cv}(W)$$
$$G_7: Y\times W\times M \to P_{cp,cv}(M)$$
$$G_2: Y\times W\times M \to P_{b,cl,cv}(Y)$$
$$G_5: Y\times W\times M \to P_{b,cl,cv}(W)$$
$$G_8: Y\times W\times M \to P_{b,cl,cv}(M)$$

and

$$G_3: Y\times W\times M \to P_{b,cl,cv}(Y)$$
$$G_6: Y\times W\times M \to P_{b,cl,cv}(W)$$
$$G_9: Y\times W\times M \to P_{b,cl,cv}(M)$$

which guarantee that

$$G: Y\times W\times M \to P_{cp,cv}(Y\times W\times M)$$
$$F: Y\times W\times M \to P_{b,cl,cv}(Y\times W\times M)$$
$$J: Y\times W\times M \to P_{b,cl,cv}(Y\times W\times M)$$

Thus we have that

$$G(r_3)=G(x_3,y_3,z_3)\in P_{cp,cv}(Z)$$
$$F(r_2)=F(x_2,y_2,z_2)\in P_{b,cl,cv}(Z)$$
$$J(r_1)=J(x_1,y_1,z_1)\in P_{b,cl,cv}(Z)$$

and

$$G(r_3)+F(z_2)+J(z_1)\in P_{cp,cv}(Z)$$

or equivalentely

$$\begin{pmatrix} G_1(x_1,y_1,z_1) \\ G_4(x_1,y_1,z_1) \\ G_7(x_1,y_1,z_1) \end{pmatrix} + \begin{pmatrix} G_2(x_2,y_2,z_1) \\ G_5(x_2,y_2,z_1) \\ G_8(x_2,y_2,z_1) \end{pmatrix} + \begin{pmatrix} G_3(x_2,y_2,z_1) \\ G_6(x_2,y_2,z_1) \\ G_9(x_2,y_2,z_1) \end{pmatrix} \in P_{cp,cv}(Z).$$

Thus Theorem 3 applies and guarantees the existence in $Y\times W\times M$ of at least one fixed point for $G+F+J$.

3 Applications:

The aim of this section is to present an application of Krasnoselskii's type fixed point theorem in generalized Banach spaces for a system of integral inclusions.

Theorem 5 *Let $I = [0, a]$ (with $a > 0$) be an interval of the real axis and consider the following system of integral equations:*

$$\begin{cases} x_1(t) & \in \lambda_{11}\int_0^t K_1(t,s,x_1(s),x_2(s),x_3(s))ds \\ & +\lambda_{12}\int_0^a L_1((t,s,x_1(s),x_2(s),x_3(s))ds + \lambda_{13}\int_0^a N_1((t,s,x_1(s),x_2(s),x_3(s))ds \\ x_2(t) & \in \lambda_{21}\int_0^t K_2(t,s,x_1(s),x_2(s),x_3(s))ds \\ & +\lambda_{22}\int_0^a L_2((t,s,x_1(s),x_2(s),x_3(s))ds + \lambda_{23}\int_0^a N_2((t,s,x_1(s),x_2(s),x_3(s))ds \\ x_3(t) & \in \lambda_{31}\int_0^t K_3(t,s,x_1(s),x_2(s),x_3(s))ds \\ & +\lambda_{32}\int_0^a L_3((t,s,x_1(s),x_2(s),x_3(s))ds + \lambda_{33}\int_0^a N_3((t,s,x_1(s),x_2(s),x_3(s))ds \end{cases} \quad (3.1)$$

for $t \in I$, where $\lambda_{ij} \in \mathbb{R}$, for $i, j \in \{1,2,3\}$.
We assume that:
(i) $L_1: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{cp,cv}(\mathbb{R}^n)$, $L_2: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{cp,cv}(\mathbb{R}^p)$, $L_3: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{cp,cv}(\mathbb{R}^q)$, $N_1: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{cp,cv}(\mathbb{R}^n)$, $N_2: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{cp,cv}(\mathbb{R}^p)$ and $N_3: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{cp,cv}(\mathbb{R}^q)$ are l.s.c. and integrably bounded.
(ii) $K_1: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{b,cl,cv}(\mathbb{R}^n)$, $K_2: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{b,cl,cv}(\mathbb{R}^p)$ and $K_3: I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q \to P_{b,cl,cv}(\mathbb{R}^q)$ are measurable and integrably bounded.
(iii) there exists a matrix $S = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \in M_3(\mathbb{R}_+)$

$$H_{\|\cdot\|_n}(K_1(t,s,u_1,u_2,u_3), K_1(t,s,v_1,v_2,v_3)) \le a_{11}\|u_1 - v_1\|_n + a_{12}\|u_2 - v_2\|_p + a_{13}\|u_3 - v_3\|_q$$
$$H_{\|\cdot\|_p}(K_2(t,s,u_1,u_2,u_3), K_2(t,s,v_1,v_2,v_3)) \le a_{21}\|u_1 - v_1\|_n + a_{22}\|u_2 - v_2\|_p + a_{23}\|u_3 - v_3\|_q$$
$$H_{\|\cdot\|_q}(K_3(t,s,u_1,u_2,u_3), K_2(t,s,v_1,v_2,v_3)) \le a_{31}\|u_1 - v_1\|_n + a_{32}\|u_2 - v_2\|_p + a_{33}\|u_3 - v_3\|_q$$

for each $(t,s,u_1,u_2,u_3), (t,s,v_1,v_2,v_3) \in I^2 \times \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^q$.
(iv)

$$|\lambda_{11}| \le \frac{R_1}{3(a(a_{11}\|\alpha - x_0^1\| + a_{12}\|\beta - x_2^0\| + a_{13}\|\gamma - x_3^0\|) + \delta(\int_0^t K_1(t,s,x_0^1(s),x_0^2(s),x_0^3(s)))ds}$$
$$|\lambda_{21}| \le \frac{R_2}{3(a(a_{11}\|\alpha - x_0^1\| + a_{12}\|\beta - x_2^0\| + a_{13}\|\gamma - x_3^0\|) + \delta(\int_0^t K_2(t,s,x_0^1(s),x_0^2(s),x_0^3(s)))ds}$$
$$|\lambda_{31}| \le \frac{R_3}{3(a(a_{11}\|\alpha - x_0^1\| + a_{12}\|\beta - x_2^0\| + a_{13}\|\gamma - x_3^0\|) + \delta(\int_0^t K_3(t,s,x_0^1(s),x_0^2(s),x_0^3(s)))ds}$$

(v)

$$|\lambda_{12}| \le \frac{R_1}{3M_{L_1}}, \text{where} M_{L_1} = \max_{t\in I}\int_0^a L_1(t,s,x_1(s),x_2(s),x_3(s))ds$$
$$|\lambda_{22}| \le \frac{R_1}{3M_{L_2}}, \text{where} M_{L_2} = \max_{t\in I}\int_0^a L_2(t,s,x_1(s),x_2(s),x_3(s))ds$$
$$|\lambda_{32}| \le \frac{R_1}{3M_{L_3}}, \text{where} M_{L_3} = \max_{t\in I}\int_0^a L_3(t,s,x_1(s),x_2(s),x_3(s))ds$$

$$|\lambda_{13}| \le \frac{R_1}{3M_{N_1}}, \text{where} M_{N_1} = \max_{t\in I}\int_0^a N_1(t,s,x_1(s),x_2(s),x_3(s))ds$$
$$|\lambda_{23}| \le \frac{R_1}{3M_{N_2}}, \text{where} M_{N_2} = \max_{t\in I}\int_0^a N_2(t,s,x_1(s),x_2(s),x_3(s))ds$$
$$|\lambda_{33}| \le \frac{R_1}{3M_{N_3}}, \text{where} M_{N_3} = \max_{t\in I}\int_0^a N_3(t,s,x_1(s),x_2(s),x_3(s))ds$$

with $R_1, R_2, R_3 > 0$.
Then, there exists $(x_1^0, x_2^0, x_3^0) \in C(I,\mathbb{R}^n) \times C(I,\mathbb{R}^p) \times C(I,\mathbb{R}^q)$ such that the system (3.1) has at least one solution $x^*: =$

$(x_1^*, x_2^*, x_3^*) \in \tilde{B}(x_1^0, R_1) \times \tilde{B}(x_2^0, R_2) \times \tilde{B}(x_3^0, R_3) \subset C(I, \mathbb{R}^n) \times C(I, \mathbb{R}^p) \times C(I, \mathbb{R}^q)$.

Proof. Let us denote $X_1 := \mathbb{R}^n$, $X_2 := \mathbb{R}^p$, $X_3 := \mathbb{R}^q$ and $x := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in C(I, X_1) \times C(I, X_2) \times C(I, X_3)$ we define the multivalued operators given by

$$G_1 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{cp,cv}(C(I, X_1))(x_1, x_2, x_3) \to G_1(x_1, x_2, x_3),$$

where $G_1(x_1, x_2, x_3)(t) := \lambda_{11} \int_0^a L_1((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_2 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{b,cl,cv}(C(I, X_1))(x_1, x_2, x_3) \to G_1(x_1, x_2, x_3),$$

where $G_2(x_1, x_2, x_3)(t) := \lambda_{12} \int_0^a N_1((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_3 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{b,cl,cv}(C(I, X_1))(x_1, x_2, x_3) \to G_1(x_1, x_2, x_3),$$

where $G_3(x_1, x_2, x_3)(t) := \lambda_{13} \int_0^a K_1((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_4 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{cp,cv}(C(I, X_2))(x_1, x_2, x_3) \to G_4(x_1, x_2, x_3),$$

where $G_4(x_1, x_2, x_3)(t) := \lambda_{21} \int_0^a L_2((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_5 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{b,cl,cv}(C(I, X_2))(x_1, x_2, x_3) \to G_5(x_1, x_2, x_3),$$

where $G_5(x_1, x_2, x_3)(t) := \lambda_{22} \int_0^a N_2((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_6 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{b,cl,cv}(C(I, X_2))(x_1, x_2, x_3) \to G_6(x_1, x_2, x_3),$$

where $G_6(x_1, x_2, x_3)(t) := \lambda_{23} \int_0^a K_2((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_7 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{cp,cv}(C(I, X_3))(x_1, x_2, x_3) \to G_7(x_1, x_2, x_3),$$

where $G_7(x_1, x_2, x_3)(t) := \lambda_{31} \int_0^a L_3((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_8 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{b,cl,cv}(C(I, X_3))(x_1, x_2, x_3) \to G_8(x_1, x_2, x_3),$$

where $G_8(x_1, x_2, x_3)(t) := \lambda_{32} \int_0^a N_3((t, s, x_1(s), x_2(s), x_3(s))ds$,

$$G_9 : C(I, X_1) \times C(I, X_2) \times C(I, X_3) \to P_{b,cl,cv}(C(I, X_3))(x_1, x_2, x_3) \to G_9(x_1, x_2, x_3),$$

where $G_9(x_1, x_2, x_3)(t) := \lambda_{33} \int_0^a K_3((t, s, x_1(s), x_2(s), x_3(s))ds$, The system (3.1) can be rewritten as a fixed point equation of the following form

$$x \in (J + F + G)(x)$$

where

$$J(x, y, z) = \begin{pmatrix} G_3(x, y, z) \\ G_6(x, y, z) \\ G_9(x, y, z) \end{pmatrix},$$

$$F(x, y, z) = \begin{pmatrix} G_2(x, y, z) \\ G_5(x, y, z) \\ G_8(x, y, z) \end{pmatrix}$$

and

$$G(x,y,z)=\begin{pmatrix}G_1(x,y,z)\\G_4(x,y,z)\\G_7(x,y,z)\end{pmatrix}$$

where

$$F\colon C(I,X_1)\times C(I,X_2)\times C(I,X_3)\to P_{b,cl,cv}(C(I,X_1)\times C(I,X_2)\times C(I,X_3))$$

$$J\colon C(I,X_1)\times C(I,X_2)\times C(I,X_3)\to P_{b,cl,cv}(C(I,X_1)\times C(I,X_2)\times C(I,X_3))$$

$$G\colon C(I,X_1)\times C(I,X_2)\times C(I,X_3)\to P_{cp,cv}(C(I,X_1)\times C(I,X_2)\times C(I,X_3)).$$

Obviously $x^*:=\begin{pmatrix}x_1^*\\x_2^*\\x_3^*\end{pmatrix}$ is a solution for our integral inclusions system if and only if is a fixed point for the operator $J+F+G$.

We need to show that the multivalued operators $G_1,G_2,G_3,G_4,G_5,G_6,G_7,G_8,G_9$ satisfy the assumptions of Theorem 4. We shall prove that $F=\begin{pmatrix}G_2\\G_5\\G_8\end{pmatrix}$ and $J=\begin{pmatrix}G_3\\G_6\\G_9\end{pmatrix}$ are multivalued S- contractions in Nadler's sense.

Let's denote $y:=(y_1,y_2,y_3)\in C(I,X_1)\times C(I,X_2)\times C(I,X_3)$, $r:=(r_1,r_2,r_3)\in C(I,X_1)\times C(I,X_2)\times C(I,X_3)$, $v_1\in G_3(x_1,x_2,x_3)$, $v_2\in G_6(x_1,x_2,x_3)$ and $v_3\in G_9(x_1,x_2,x_3)$.

Then $v_1,v_2,v_3\in C(I,X_1)\times C(I,X_2)\times C(I,X_3)$ and

$$v_1(t)\in\lambda_{11}\int_0^t K_1(t,s,x_1(s),x_2(s),x_3(s))ds$$
$$v_2(t)\in\lambda_{21}\int_0^t K_2(t,s,x_1(s),x_2(s),x_3(s))ds$$
$$v_3(t)\in\lambda_{31}\int_0^t K_3(t,s,x_1(s),x_2(s),x_3(s))ds$$

It follows that there are the operators f_3,f_6,f_9 the integrable selections for K_1, K_2 and K_3 such that

$$v_1(t)=\lambda_{11}\int_0^t f_3(s)ds$$
$$l_2(t)=\lambda_{21}\int_0^t f_6(s)ds$$
$$t_3(t)=\lambda_{31}\int_0^t f_9(s)ds$$

Since

$$H_{\|\cdot\|_n}(K_1(t,s,y_1,y_2,y_3),K_1(t,s,z_1,z_2,z_3))\le a_{11}\|y_1-r_1\|+a_{12}\|y_2-r_2\|+a_{13}\|y_3-r_3\|$$

$$H_{\|\cdot\|_p}(K_2(t,s,y_1,y_2,y_3),K_2(t,s,z_1,z_2,z_3))\le a_{21}\|y_1-r_1\|+a_{22}\|y_2-r_2\|+a_{23}\|y_3-r_3\|$$

and

$$H_{\|\cdot\|_q}(K_3(t,s,y_1,y_2,y_3),K_3(t,s,z_1,z_2,z_3))\le a_{31}\|y_1-r_1\|+a_{32}\|y_2-r_2\|+a_{33}\|y_3-r_3\|$$

we obtain that there exists

$$w_3\in K_1(t,s,r_1,r_2,r_3)$$
$$w_6\in K_2(t,s,r_1,r_2,r_3)$$
$$w_9\in K_3(t,s,r_1,r_2,r_3)$$

such that

$$\|f_3(t)-w_3\|\le a_{11}\|y_1-r_1\|+a_{12}\|y_2-r_2\|+a_{13}\|y_3-r_3\|$$
$$\|f_6(t)-w_6\|\le a_{21}\|y_1-r_1\|+a_{22}\|y_2-r_2\|+a_{23}\|y_3-r_3\|$$
$$\|f_9(t)-w_9\|\le a_{31}\|y_1-r_1\|+a_{32}\|y_2-r_2\|+a_{33}\|y_3-r_3\|.$$

Then the multivalued operators J_3,J_6,J_9 defined by

$$J_3(t)=K_1(t)\cap W_3(t)$$
$$J_6(t)=K_2(t)\cap W_6(t)$$
$$J_9(t)=K_3(t)\cap W_9(t),$$

where $K_1(t)=K_1(t,s,r_1,r_2,r_3)$ and

$$W_3(t)=\{w_3\,|\,\|f_3(t)-w_3\|\le a_{11}\|y_1-r_1\|+a_{12}\|y_2-r_2\|+a_{13}\|y_3-r_3\|\},$$

$K_2(t) = K_2(t, s, r_1, r_2, r_3)$ and

$$W_6(t) = \{w_6 | \|f_6(t) - w_6\| \leq a_{21}\|y_1 - r_1\| + a_{22}\|y_2 - r_2\| + a_{23}\|y_3 - r_3\|\},$$

$K_3(t) = K_3(t, s, r_1, r_2, r_3)$ and

$$W_9(t) = \{w_9 | \|f_9(t) - w_9\| \leq a_{31}\|y_1 - r_1\| + a_{32}\|y_2 - r_2\| + a_{33}\|y_3 - r_3\|\},$$

have nonempty values and are measurable.

Let f_{10}, f_{11}, f_{12} be three integrable selections for J_3,J_6 and J_9. Then

$$f_{10}(t) \in K_1(t, s, r_1, r_2, r_3)$$
$$f_{11}(t) \in K_2(t, s, r_1, r_2, r_3)$$
$$f_{12}(t) \in K_3(t, s, r_1, r_2, r_3)$$

and

$$\|f_3(t) - f_{10}(t)\| \leq a_{11}\|y_1 - r_1\| + a_{12}\|y_2 - r_2\| + a_{13}\|y_3 - r_3\|$$
$$\|f_6(t) - f_{11}(t)\| \leq a_{21}\|y_1 - r_1\| + a_{22}\|y_2 - r_2\| + a_{23}\|y_3 - r_3\|$$
$$\|f_9(t) - f_{12}(t)\| \leq a_{31}\|y_1 - r_1\| + a_{32}\|y_2 - r_2\| + a_{33}\|y_3 - r_3\|.$$

Let's define $v_2(t) = \lambda_{11}\int_0^t f_{10}(s)ds$, $l_2(t) = \lambda_{21}\int_0^t f_{11}(s)ds$ and $t_2(t) = \lambda_{31}\int_0^t f_{12}(s)ds$.

It follows that $v_2 \in G_3(r_1, r_2, r_3)$, $l_2 \in G_6(r_1, r_2, r_3)$, $t_2 \in G_9(r_1, r_2, r_3)$ and

$$\|v_1(t) - v_2(t)\| \leq |\lambda_{11}| \int_0^t \|f_3(s) - f_{10}(s)\|ds$$
$$\leq |\lambda_{11}| \int_0^t (a_{11}\|y_1 - r_1\| + a_{12}\|y_2 - r_2\| + a_{13}\|y_3 - r_3\|)ds$$
$$= |\lambda_{11}| \Big(\int_0^t a_{11}\|y_1(s) - r_1(s)\|e^{-\tau s} \cdot e^{\tau s}ds$$
$$+ \int_0^t a_{12}\|y_2(s) - r_2(s)\|e^{-\tau s} \cdot e^{\tau s}ds + \int_0^t a_{13}\|y_3(s) - r_3(s)\|e^{-\tau s} \cdot e^{\tau s}ds\Big)$$
$$\leq |\lambda_{11}| \Big(a_{11}\|y_1 - r_1\|_B \int_0^t e^{\tau s}ds + a_{12}\|y_2 - r_2\|_B \int_0^t e^{\tau s}ds$$
$$+a_{13}\|y_3 - r_3\|_B \int_0^t e^{\tau s}ds\Big)$$
$$= |\lambda_{11}| \Big(a_{11}\|y_1 - r_1\|_B \Big(\frac{1}{\tau}e^{\tau t} - \frac{1}{\tau}\Big) + a_{12}\|y_2 - r_2\|_B \Big(\frac{1}{\tau}e^{\tau t} - \frac{1}{\tau}\Big)$$
$$+a_{13}\|y_3 - r_3\|_B \Big(\frac{1}{\tau}e^{\tau t} - \frac{1}{\tau}\Big)\Big)$$
$$\leq |\lambda_{11}| \frac{e^{\tau t}}{\tau}(a_{11}\|y_1 - r_1\|_B + a_{12}\|y_2 - r_2\|_B + a_{13}\|y_3 - r_3\|_B)$$

$\|\cdot\|_B$ denotes the Bielecki-type norm on the generalized Banach space $C(I, X_1) \times C(I, X_2) \times C(I, X_3)$.

We obtain that

$$\|v_1 - v_2\|_B \leq \frac{|\lambda_{11}|}{\tau}(a_{11}\|y_1 - r_1\|_B + a_{12}\|y_2 - r_2\|_B + a_{13}\|y_3 - r_3\|_B).$$

In a similar way we get that

$$\|l_1 - l_2\|_B \leq \frac{|\lambda_{21}|}{\tau}(a_{21}\|y_1 - r_1\|_B + a_{22}\|y_2 - r_2\|_B + a_{23}\|y_3 - r_3\|_B).$$

and

$$\|t_1 - t_2\|_B \leq \frac{|\lambda_{31}|}{\tau}(a_{31}\|y_1 - r_1\|_B + a_{32}\|y_2 - r_2\|_B + a_{33}\|y_3 - r_3\|_B).$$

These inequalities can be rewritten in a vectorial form

$$\begin{pmatrix} \|v_1 - v_2\|_B \\ \|l_1 - l_2\|_B \\ \|t_1 - t_2\|_B \end{pmatrix} \leq \begin{pmatrix} \frac{|\lambda_{11}|}{\tau}(a_{11}\|y_1 - r_1\|_B + a_{12}\|y_2 - r_2\|_B + a_{13}\|y_3 - r_3\|_B) \\ \frac{|\lambda_{21}|}{\tau}(a_{21}\|y_1 - r_1\|_B + a_{22}\|y_2 - r_2\|_B + a_{23}\|y_3 - r_3\|_B) \\ \frac{|\lambda_{31}|}{\tau}(a_{31}\|y_1 - r_1\|_B + a_{32}\|y_2 - r_2\|_B + a_{33}\|y_3 - r_3\|_B) \end{pmatrix}$$
$$= \begin{pmatrix} \frac{|\lambda_{11}|}{\tau}a_{11} & \frac{|\lambda_{11}|}{\tau}a_{12} & \frac{|\lambda_{11}|}{\tau}a_{13} \\ \frac{|\lambda_{21}|}{\tau}a_{21} & \frac{|\lambda_{21}|}{\tau}a_{22} & \frac{|\lambda_{21}|}{\tau}a_{23} \\ \frac{|\lambda_{31}|}{\tau}a_{31} & \frac{|\lambda_{31}|}{\tau}a_{32} & \frac{|\lambda_{31}|}{\tau}a_{33} \end{pmatrix} \begin{pmatrix} \|y_1 - r_1\|_B \\ \|y_2 - r_2\|_B \\ \|y_3 - r_3\|_B \end{pmatrix}.$$

We get that

$$\begin{pmatrix} H(G_3(x_1,y_1,y_1,z_1),G_3(x_2,y_2,z_2)) \\ H(G_6(x_1,y_1,y_1,z_1),G_6(x_2,y_2,z_2)) \\ H(G_9(x_1,y_1,y_1,z_1),G_9(x_2,y_2,z_2)) \end{pmatrix} \leq [M] \begin{pmatrix} \|y_1-r_1\|_B \\ \|y_2-r_2\|_B \\ \|y_3-r_3\|_B \end{pmatrix}$$

where

$$[M]:= \begin{pmatrix} \frac{|\lambda_{11}|}{\tau}a_{11} & \frac{|\lambda_{11}|}{\tau}a_{12} & \frac{|\lambda_{11}|}{\tau}a_{13} \\ \frac{|\lambda_{21}|}{\tau}a_{21} & \frac{|\lambda_{21}|}{\tau}a_{22} & \frac{|\lambda_{21}|}{\tau}a_{23} \\ \frac{|\lambda_{31}|}{\tau}a_{31} & \frac{|\lambda_{31}|}{\tau}a_{32} & \frac{|\lambda_{31}|}{\tau}a_{33} \end{pmatrix}$$

Taking τ large enough it follows that the matrix $[M]$ is convergent to zero, and thus G_2 and G_4 are multivalued contractions in Nadler's sense. By Covitz- Nadler fixed point theorem we get that $x^0=(x_1^0,x_2^0,x_3^0)\in C(I,X_1)\times C(I,X_2)\times C(I,X_3)$ is a fixed point for $J(x,y,z)=\begin{pmatrix} G_3(x,y,z) \\ G_6(x,y,z) \\ G_9(x,y,z) \end{pmatrix}$.

Let $Y\times W\times M:=\tilde{B}(x_0^1,R_1)\times\tilde{B}(x_0^2,R_2)\times\tilde{B}(x_0^3,R_3)\subset C(I,X_1)\times C(I,X_2)\times C(I,X_3)$ with $R_1,R_2,R_3>0$.
The operators G_1 and G_3 are l.s.c. and $G_1(x,y,z)$, $G_4(x,y,z)$ and $G_7(x,y,z)$ are relatively compact.

We show that we can choose $R=\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}$, such that

$$J(\tilde{B}(x_0^1,R_1)\times\tilde{B}(x_0^2,R_2)\times\tilde{B}(x_0^3,R_3))\subseteq\tilde{B}\left(x_0^1,\tfrac{R_1}{3}\right)\times\tilde{B}\left(x_0^2,\tfrac{R_2}{3}\right)\times\tilde{B}\left(x_0^3,\tfrac{R_3}{3}\right).$$

We have to prove that

$$G_3(Y\times W\times M)\subseteq\tilde{B}\left(x_0^1,\tfrac{R_1}{3}\right)$$
$$G_6(Y\times W\times M)\subseteq\tilde{B}\left(x_0^2,\tfrac{R_2}{3}\right)\text{ and}$$
$$G_9(Y\times W\times M)\subseteq\tilde{B}\left(x_0^3,\tfrac{R_3}{3}\right).$$

We have to show that for each $u\in G_3(Y\times W\times M)$ implies that $u\in\tilde{B}\left(x_0^1,\frac{R_1}{3}\right)$ and for each $v\in G_6(Y\times W\times M)$ and $w\in G_9(Y\times W\times M)$ implies that $v\in\tilde{B}\left(x_0^2,\frac{R_2}{3}\right)$ and $v\in\tilde{B}\left(x_0^3,\frac{R_3}{3}\right)$.

Let $u\in G_3(Y\times W\times M)$, which is equivalent to the fact that there exist $\alpha\in\tilde{B}(x_0^1,R_1)$, $\beta\in\tilde{B}(x_0^2,R_2)$ and $\gamma\in\tilde{B}(x_0^3,R_3)$ such that $u\in G_3(\alpha,\beta,\gamma)$.

We will denote by $\|\cdot\|_{C_i}$ the supremum norm in $C(I,X_i)$, where $i\in\{1,2,3\}$.
It follows that

$$\|\alpha-x_0^1\|_{C_1}\leq R_1,$$
$$\|\beta-x_0^2\|_{C_2}\leq R_2$$
$$\|\gamma-x_0^3\|_{C_3}\leq R_3$$

and

$$u(t)\in\lambda_{11}\int_0^t K_1(t,s,\alpha(s),\beta(s),\gamma(s))ds, forall t\in[0,a].$$

Let $k_{\alpha,\beta,\gamma}\in L^1(I)$ and $k_{\alpha,\beta,\gamma}\in K_1(t,s,\alpha(s),\beta(s),\gamma(s))$ such that $u(t)=\lambda_{11}\int_0^t k_{\alpha,\beta,\gamma}(s)ds$.
We have to show that $\|u-x_0^1\|_{C_1}\leq\frac{R_1}{3}$.

Let $x_0^1\in G_3(x_0^1,x_0^2,x_0^3)$ which is equivalent to $x_0^1\in\lambda_{11}\int_0^t K_1(t,s,x_0^1(s),x_0^2(s),x_0^3(s))ds$. It follows that there is a mapping $g_{x_0^1}\in S_{K_1}$ $(i.e.\ g_{x_0^1}\in L^1$ and $g_{x_0^1}(s)\in K_1(t,s,x_0^1(s),x_0^2(s),x_0^3(s))$ such that $x_0^1(t)=\lambda_{11}\int_0^t g_{x_0^1}(s)ds$.

Let $v\in G_6(Y\times W\times M)$, $w\in G_9(Y\times W\times M)$ which is equivalent to the fact that there exist $\omega\in\tilde{B}(x_0^1,R_1)$, $\theta\in\tilde{B}(x_0^2,R_2)$ and $\mu\in\tilde{B}(x_0^3,R_3)$ such that $u\in G_6(\omega,\theta,\mu)$.
It follows that

$$\|\omega-x_0^1\|_{C_1}\leq R_1,$$

$$\|\theta - x_0^2\|_{C_2} \leq R_2$$
$$\|\mu - x_0^3\|_{C_3} \leq R_3$$

$$v(t) \in \lambda_{21} \int_0^t K_2(t,s,\psi(s),\phi(s),\varphi(s))ds, for\ \ each\ \ t \in [0,a].$$

Let $k_{\psi,\phi,\varphi} \in L^1$ and $k_{\psi,\phi,\varphi} \in K_2(t,s,\psi(s),\phi(s),\varphi(s))$ such that $v(t) = \lambda_{21} \int_0^t k_{\psi,\phi,\varphi}(s)ds$.

We have to show that $\|v - x_0^2\|_{C_2} \leq \frac{R_2}{3}$.

Let $x_0^2 \in G_6(x_0^1, x_0^2, x_0^3)$ which is equivalent to $x_0^2 \in \lambda_{21} \int_0^t K_2(t,s,x_0^1(s),x_0^2(s),x_0^3(s))ds$. It follows that there is a mapping $g_{x_0^2} \in S_{K_2}(i.e.\ g_{x_0^2} \in L^1$ and $g_{x_0^2}(s) \in K_2(t,s,x_0^1(s),x_0^2(s),x_0^3(s))$ such that $x_0^2(t) = \lambda_{21} \int_0^t g_{x_0^2}(s)ds$.
and

$$w(t) \in \lambda_{31} \int_0^t K_3(t,s,\rho(s),\xi(s),\kappa(s))ds, for\ \ each\ \ t \in [0,a].$$

Let $k_{\rho,\xi,\kappa} \in L^1$ and $k_{\rho,\xi,\kappa} \in K_2(t,s,\rho(s),\xi(s),\kappa(s))$ such that $w(t) = \lambda_{31} \int_0^t k_{\rho,\xi,\kappa}(s)ds$.

We have to show that $\|w - x_0^3\|_{C_3} \leq \frac{R_3}{3}$.

Let $x_0^3 \in G_9(x_0^1, x_0^2, x_0^3)$ which is equivalent to $x_0^3 \in \lambda_{31} \int_0^t K_3(t,s,x_0^1(s),x_0^2(s),x_0^3(s))ds$. It follows that there is a mapping $g_{x_0^3} \in S_{K_3}(i.e.\ g_{x_0^3} \in L^1$ and $g_{x_0^3}(s) \in K_3(t,s,x_0^1(s),x_0^2(s),x_0^3(s))$ such that $x_0^3(t) = \lambda_{31} \int_0^t g_{x_0^3}(s)ds$.

Since we know that

$$H_{\|\cdot\|_n}(K_1(t,s,\alpha,\beta,\gamma),K_1(t,s,x_0^1,x_0^2,x_0^3)) \leq a_{11}\|\alpha - x_0^1\| + a_{12}\|\beta - x_0^2\| + a_{13}\|\gamma - x_0^3\|$$
$$H_{\|\cdot\|_p}(K_2(t,s,\alpha,\beta,\gamma),K_2(t,s,x_0^1,x_0^2,x_0^3)) \leq a_{21}\|\alpha - x_0^1\| + a_{22}\|\beta - x_0^2\| + a_{23}\|\gamma - x_0^3\|$$
$$H_{\|\cdot\|_q}(K_3(t,s,\alpha,\beta,\gamma),K_3(t,s,x_0^1,x_0^2,x_0^3)) \leq a_{31}\|\alpha - x_0^1\| + a_{32}\|\beta - x_0^2\| + a_{33}\|\gamma - x_0^3\|$$

we get that there exists

$$\Delta_{\alpha,\beta,\gamma} \in K_1(t,s,x_0^1,x_0^2,x_0^3)$$
$$\Delta_{\psi,\phi,\varphi} \in K_2(t,s,x_0^1,x_0^2,x_0^3)$$
$$\Delta_{\rho,\xi,\kappa} \in K_3(t,s,x_0^1,x_0^2,x_0^3),,$$

such that

$$\left\|k_{\alpha,\beta,\gamma} - \Delta_{\alpha,\beta,\gamma}\right\| \leq a_{11}\|\alpha - x_0^1\|_{C_1} + a_{12}\|\beta - x_0^2\|_{C_2} + a_{13}\|\gamma - x_0^3\|_{C_3}$$
$$\left\|k_{\psi,\phi,\varphi} - \Delta_{\alpha,\beta,\gamma}\right\| \leq a_{21}\|\alpha - x_0^1\|_{C_1} + a_{22}\|\beta - x_0^2\|_{C_2} + a_{23}\|\gamma - x_0^3\|_{C_3}$$
$$\left\|k_{\rho,\xi,\kappa} - \Delta_{\alpha,\beta,\gamma}\right\| \leq a_{31}\|\alpha - x_0^1\|_{C_1} + a_{32}\|\beta - x_0^2\|_{C_2} + a_{33}\|\gamma - x_0^3\|_{C_3}.$$

Then the multivalued operators

$$J_{\alpha,\beta,\gamma}(t) = K_1(t) \cap \Lambda_{\alpha,\beta,\gamma}(t)$$
$$J_{\psi,\phi,\varphi}(t) = K_2(t) \cap \Lambda_{\psi,\phi,\varphi}(t)$$
$$J_{\rho,\xi,\kappa}(t) = K_3(t) \cap \Lambda_{\rho,\xi,\kappa}(t)$$

where $K_1(t) = K_1(t,s,x_0^1,x_0^2,x_0^3)$ and

$$\Lambda_{\alpha,\beta,\gamma}(t) = \left\{\Delta_{\alpha,\beta,\gamma} \middle| \left\|k_{\alpha,\beta,\gamma} - \Delta_{\alpha,\beta,\gamma}\right\| \leq a_{11}\|\alpha - x_0^1\|_{C_1} + a_{12}\|\beta - x_0^2\|_{C_2} + a_{13}\|\gamma - x_0^3\|_{C_3}\right\},$$

$K_2(t) = K_2(t,s,x_0^1,x_0^2,x_0^3)$ and

$$\Lambda_{\psi,\phi,\varphi}(t) = \left\{\Delta_{\psi,\phi,\varphi} \middle| \left\|k_{\psi,\phi,\varphi} - \Delta_{\psi,\phi,\varphi}\right\| \leq a_{21}\|\alpha - x_0^1\|_{C_1} + a_{22}\|\beta - x_0^2\|_{C_2} + a_{23}\|\gamma - x_0^3\|_{C_3}\right\}$$

$K_3(t) = K_2(t,s,x_0^1,x_0^2,x_0^3)$ and

$$\Lambda_{\rho,\xi,\kappa}(t) = \left\{\Delta_{\rho,\xi,\kappa} \middle| \left\|k_{\rho,\xi,\kappa} - \Delta_{\rho,\xi,\kappa}\right\| \leq a_{31}\|\alpha - x_0^1\|_{C_1} + a_{32}\|\beta - x_0^2\|_{C_2} + a_{33}\|\gamma - x_0^3\|_{C_3}\right\}$$

have nonempty values and are measurable.

By Kuratowski and Ryll Nardzewski'selection theorem there exist $k_{x_0^1} \in J_{\alpha,\beta,\gamma}(t)$, $k_{x_0^2} \in J_{\psi,\phi,\varphi}(t)$ $k_{x_0^3} \in J_{\rho,\xi,\kappa}(t)$ and

$$\left\|k_{\alpha,\beta,\gamma} - k_{x_0^1}\right\| \leq a_{11}\|\alpha - x_0^1\|_{C_1} + a_{12}\|\beta - x_0^2\|_{C_2} + a_{13}\|\gamma - x_0^3\|_{C_3}$$
$$\left\|k_{\psi,\phi,\varphi} - k_{x_0^2}\right\| \leq a_{21}\|\alpha - x_0^1\|_{C_1} + a_{22}\|\beta - x_0^2\|_{C_2} + a_{23}\|\gamma - x_0^3\|_{C_3}$$

$$\left\| k_{\rho,\xi,\kappa} - k_{x_0^3} \right\| \leq a_{31}\|\alpha - x_0^1\|_{C_1} + a_{32}\|\beta - x_0^2\|_{C_2} + a_{33}\|\gamma - x_0^3\|_{C_3}.$$

We make the following estimation

$$\begin{aligned}
&|u(t) - x_0^1(t)| \\
&= \left|\lambda_{11}\int_0^t k_{\alpha,\beta,\gamma}(s)ds - \lambda_{11}\int_0^t g_{x_0^1}(s)ds\right| \\
&\leq |\lambda_{11}|\int_0^t \left|k_{\alpha,\beta,\gamma}(s) - k_{x_0^1}(s)\right| ds + |\lambda_{11}|\int_0^t \left|k_{x_0^1}(s) - g_{x_0^1}(s)\right| ds \\
&\leq |\lambda_{11}|\int_0^t (a_{11}|\alpha(s) - x_0^1(s)| + a_{12}|\beta(s) - x_0^2(s)| + a_{13}|\gamma(s) - x_0^3(s)|)ds + \\
&|\lambda_{11}|\delta(G_3(x_0^1, x_0^2, x_0^3)) \\
&\leq |\lambda_{11}|a\left[a_{11}\|\alpha - x_0^1\|_{C_1} + a_{12}\|\beta - x_0^2\|_{C_2} + a_{13}\|\gamma - x_0^3\|_{C_3}\right] + |\lambda_{11}|\delta(G_3(x_0^1, x_0^2, x_0^3)).
\end{aligned}$$

Taking the $\sup_{t\in I}$ we get that

$$\|u - x_0^1\|_{C_1} \leq |\lambda_{11}|a(a_{11}\|\alpha - x_0^1\|_{C_1} + a_{12}\|\beta - x_0^2\|_{C_2} + a_{13}\|\gamma - x_0^3\|_{C_3}) + |\lambda_{11}|\delta(G_3(x_0^1, x_0^2, x_0^3)) \leq \frac{R_1}{3}.$$

In a similar way we get that

$$\|v - x_0^2\|_{C_2} \leq |\lambda_{21}|a(a_{21}\|\alpha - x_0^1\|_{C_1} + a_{22}\|\beta - x_0^2\|_{C_2} + a_{23}\|\gamma - x_0^3\|_{C_3}) + |\lambda_{21}|\delta(G_6(x_0^1, x_0^2, x_0^3)) \leq \frac{R_2}{3}.$$

and

$$\|w - x_0^3\|_{C_2} \leq |\lambda_{31}|a(a_{31}\|\alpha - x_0^1\|_{C_1} + a_{32}\|\beta - x_0^2\|_{C_2} + a_{33}\|\gamma - x_0^3\|_{C_3}) + |\lambda_{31}|\delta(G_9(x_0^1, x_0^2, x_0^3)) \leq \frac{R_3}{3}.$$

We show now that

$$G\left(\tilde{B}(x_0^1, R_1) \times \tilde{B}(x_0^2, R_2) \times \tilde{B}(x_0^3, R_3)\right) \subseteq \tilde{B}\left(0, \frac{R_1}{3}\right) \times \tilde{B}\left(0, \frac{R_2}{3}\right) \times \tilde{B}\left(0, \frac{R_3}{3}\right),$$

i.e.

$$G_1(Y \times W \times M) \subseteq \tilde{B}\left(0, \frac{R_1}{3}\right),$$

which is equivalent to, for each $u \in G_1(Y \times W \times M)$ there is an element $u \in \tilde{B}\left(0, \frac{R_1}{2}\right)$ and

$$G_4(Y \times W \times M) \subseteq \tilde{B}\left(0, \frac{R_2}{3}\right),$$

and

$$G_7(Y \times W \times M) \subseteq \tilde{B}\left(0, \frac{R_3}{3}\right),$$

which is equivalent to, for each $p \in G_4(Y \times W \times M)$ and $q \in G_7(Y \times W \times M)$ there is an element $p \in \tilde{B}\left(0, \frac{R_2}{3}\right)$ and $q \in \tilde{B}\left(0, \frac{R_3}{3}\right)$

Let $u \in G_1(Y \times W \times M)$, which is equivalent to the fact that there exist $\alpha \in \tilde{B}(x_0^1, R_1)$, $\beta \in \tilde{B}(x_0^2, R_2)$ and $\gamma \in \tilde{B}(x_0^3, R_3)$ such that $u \in G_1(\alpha, \beta, \gamma)$.

It follows that

$$u(t) \in \lambda_{13}\int_0^a L_1(t, s, \alpha(s), \beta(s), \gamma(s))ds.$$

Let $f^{\alpha,\beta,\gamma} \in S_{L_1}$ such that $u(t) = \lambda_{13}\int_0^a f^{\alpha,\beta,\gamma}(t, s)ds$. We have that

$$\|u(t)\| \leq |\lambda_{13}|\int_0^a \left|f^{\alpha,\beta,\gamma}(t, s)\right|_{\mathbb{R}^n} ds \leq |\lambda_{13}|M_{L_1}a \leq \frac{R_1}{3}.$$

Let $p \in G_4(Y \times W \times M)$, which is equivalent to the fact that there exist $\psi \in \tilde{B}(x_0^1, R_1)$, $\phi \in \tilde{B}(x_0^2, R_2)$ and $\varphi \in \tilde{B}(x_0^3, R_3)$ such that $p \in G_4(\psi, \phi, \varphi)$.

It follows that

$$p(t) \in \lambda_{23}\int_0^a L_2(t, s, \psi(s), \phi(s), \varphi(s))ds.$$

Let $f^{\psi,\phi,\varphi} \in S_{L_2}$ such that $p(t) = \lambda_{23}\int_0^a f^{\psi,\phi,\varphi}(t, s)ds$. We have that

$$\|p(t)\| \leq |\lambda_{23}|\int_0^a \left|f^{\psi,\phi,\varphi}(t, s)\right|_{\mathbb{R}^n} ds \leq |\lambda_{23}|M_{L_2}a \leq \frac{R_2}{3}.$$

Let $q \in G_7(Y \times W \times M)$, which is equivalent to the fact that there exist $\kappa \in \tilde{B}(x_0^1, R_1)$, $\xi \in \tilde{B}(x_0^2, R_2)$ and $\varphi \in \tilde{B}(x_0^3, R_3)$ such that $q \in G_3(\rho, \xi, \kappa)$.

It follows that

$$q(t) \in \lambda_{33} \int_0^a L_3(t,s,\rho(s),\xi(s),\kappa(s))ds.$$

Let $f^{\rho,\xi,\kappa} \in S_{L_3}$ such that $p(t) = \lambda_{33} \int_0^a f^{\rho,\xi,\kappa}(t,s)ds$. We have that

$$\|q(t)\| \leq |\lambda_{33}| \int_0^a \left|f^{\rho,\xi,\kappa}(t,s)\right|_{\mathbb{R}^n} ds \leq |\lambda_{33}| M_{L_3} a \leq \frac{R_3}{3}.$$

Thus, all the assumptions of Theorem 4 are satisfied. The conclusion follows by Theorem 4.
Next we give an example in support of our Theorem 4.

Example 1 *Let* $([0,1], \|\cdot\|)$ *be a generalized Banach space and*

$$G_3(t,s,x,y,z) = \frac{1}{3}\frac{xs|\sin y||\cos z|}{e^t(1+x^2)(1+sin^2y)(1+\cos^2 z)}$$
$$G_6(t,s,x,y,z) = \frac{1}{5}\frac{e^s(1+y^2)(1+\sin^2 x)(1+\cos^2 z)}{e^{t^2}(1+y^2)(1+\sin^2 x)(1+\cos^2 z)}$$
$$G_9(t,s,x,y,z) = \frac{1}{7}\frac{s^2|\cos z|+\sqrt{e^s(1+z^2)(1+\sin^2 y)(1+\cos^2 x)}}{e^t(1+z^2)(1+\sin^2 y)(1+\cos^2 x)}$$

then

$$\| G_3(t,s,x,y,z) - G_3(t,s,u,v,w) \| \leq \frac{1}{3}\frac{s}{t^2}$$
$$\| G_6(t,s,x,y,z) - G_6(t,s,u,v,w) \| \leq \frac{2}{5}\frac{e^s}{e^{t^2}}$$
$$\| G_9(t,s,x,y,z) - G_9(t,s,u,v,w) \| \leq \frac{1}{7}\frac{s^2}{e^t}$$

gives

$$S = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{5} & 0 \\ 0 & 0 & \frac{1}{7} \end{pmatrix} \to 0.$$

Also all the conditions of Theorem 4 satisfy and system 2.1 has unique solution (0,0,0).

Example 2
Let $([0,1], \|\cdot\|)$ be a generalized Banach space. Consider the system of integral equation 3.1. We take $n = p = q$ and

$$K_1(t,s,x,y,z) = \frac{x(s^2)s|\sin y(s^2)||\cos z(s^2)|}{e^t(1+x^2(s^2))(1+\sin^2 y(s^2))(1+\cos^2 z(s^2))}$$
$$K_2(t,s,x,y,z) = \frac{e^s y^2(s)(1+\cos^2 x(s))(1+\sin^2 z(s))}{e^{t^2}(1+y^2(s))(1+\sin^2 x(s))(1+\cos^2 z(s))}$$
$$K_3(t,s,x,y,z) = \frac{s^2|\cos z(s)|+\sqrt{e^s(1+z^2(s))(1+\sin^2 y(s))(1+\cos^2 x(s))}}{e^t(1+z^2(s))(1+\sin^2 y(s))(1+\cos^2 x(s))}$$

$$L_1(t,s,x,y,z) = \frac{x+y+z}{3t^2+3} + s$$
$$L_2(t,s,x,y,z) = \frac{t^2(x+y+z)}{3t^4+3} + s$$
$$L_3(t,s,x,y,z) = \frac{t^3(x+y+z)}{3t^5+3} + s$$

$$N_1(t,s,x,y,z) = \frac{t^2}{2(1+t^4)} + \frac{s^3\cos(sx)+e^s(2+\sin(x^4+y^4+z^4))}{e^{t^2}(2+\sin(x^4+y^4+z^4))}$$
$$N_2(t,s,x,y,z) = \frac{t^2}{2(1+t^4)} + \frac{s^3\cos(sy)+e^s(2+\sin(x^4+y^4+z^4))}{e^{t^2}(2+\sin(x^4+y^4+z^4))}$$
$$N_3(t,s,x,y,z) = \frac{t^2}{2(1+t^4)} + \frac{s^3\cos(sz)+e^s(2+\sin(x^4+y^4+z^4))}{e^{t^2}(2+\sin(x^4+y^4+z^4))}$$

then

$$\| K_1(t,s,x,y,z) - K_1(t,s,u,v,w) \| \leq \frac{1}{3}\frac{s}{t^2}$$
$$\| K_2(t,s,x,y,z) - K_2(t,s,u,v,w) \| \leq \frac{2}{5}\frac{e^s}{e^{t^2}}$$
$$\| K_3(t,s,x,y,z) - K_3(t,s,u,v,w) \| \leq \frac{1}{7}\frac{s^2}{e^t}$$

gives

$$S = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{5} & 0 \\ 0 & 0 & \frac{1}{7} \end{pmatrix} \to 0$$

Also all the conditions of Theorem 4 satisfied and provided a unique solution $(0,0,0)$ of system 3.1 .

REFERENCES:

1. R.P. Agarwal, *Contraction and approximate contraction with an application to multi-point boundary value problems*, J. Comput. Applied Math., 9 (1983), 315-325.
2. R. P. Agarwal, D. O'Regan and M. A. Taoudi, *Browder- Krasnoselskii type fixed point theorem in Banach space*, Fixed Point Theory Appl. 2010 (2010), Article ID 243716, doi:10.1155/2010/243716.
3. G. Allaire and S. M. Kaber, *Numerical Linear Algebra*, Texts in Applied Mathematics, vol. 55, Springer, New York, 2008.
4. A. Bucur, L. Guran, A. Petruşel, *Fixed points for multivalued operators on a set endowed with vector-valued metrics and applications*, Fixed Point Theory, 10 (2009), no.1, 19-34.
5. T. A. Burton, *A fixed-point theorem of Krasnoselskii*, Appl. Math. Lett. 11 (1998) 85–88.
6. T. A. Burton and I. K. Purnaras, *A Unification theory of Krasnoselskii for differential equations*, Nonlinear Anal. 89 (2013), 121-133.
7. A.D. Filip, A. Petruşel, *Fixed point theorems on spaces endowed with vector-valued metrics*, Fixed Point Theory and Applications, vol. 2010, Article ID 281381, 15 pages, 2009.
8. M. A. Krasnoselskii, *Two remarks on the method of successive approximations*, Uspehi Math. Nauk 10 (1955), 123- 127.
9. M. A. Krasnoselskii, Topological Methods in the Theory of Nonlinear Integral Equations, Pergamon Press, New York, 1964.
10. S.B. Nadler, *Multivalued contraction mappings*, Pacific J. Math., 30 (1969), 475-488.
11. D. O'Regan, *Fixed point theory for the sum of two operators*, Appl. Math. Lett. 9 (1996), 1-8.
12. D. O'Regan, N. Shahzad, R.P. Agarwal, *Fixed point theory for generalized contractive maps on spaces with vector-valued metrics*, Fixed Point Theory and Applications, Vol. 6, Nova Sci. Publ., New York, 2007, 143-149.
13. S. Park, *Generalizations of the Krasnoselskii's fixed point theorem*, Nonlinear Anal. 67 (2007), 3401-3410.
14. A. I. Perov, *On the Cauchy problem for a system of ordinary differential equations*, Pviblizhen. Met. Reshen. Differ. Uravn., 2 (1964), 115-134.
15. A. I. Perov, A.V. Kibenko, *On a certain general method for investigation of boundary value problems*, Izv. Akad. Nauk SSSR Ser. Mat., 30 (1966), 249-264 (in Russian).
16. I. R. Petre, A. Petruşel, *Krasnoselskii's theorem in generalized Banach spaces and applications*, Electronic J. Qualitative Theory Diff. Equ., 85 (2012), 1-20.
17. A. Petruşel, *Multivalued weakly Picard operators and applications*, Sci. Math. Japon., 59 (2004), 169-202.
18. R. Precup, *The role of matrices that are convergent to zero in the study of semilinear operator systems*, Math. Comput. Model., 49 (2009), 703-708.
19. R. Precup, A. Viorel, *Existence results for systems of nonlinear evolution equations*, Intern. J. Pure Appl. Math., 47 (2008), no.2, 199-206.
20. R. Precup, A. Viorel, *Existence results for systems of nonlinear evolution inclusions*, Fixed Point Theory, 11 (2010), no.2, 337-346.
21. Sh. Rezapour, P. Amiri, *Some fixed point results for multivalued operators in generalized metric spaces*, Computers and Math. with Appl., 61 (2011), 2661-2666.
22. I. A. Rus, *Principles and Applications of the Fixed Point Theory (in Romanian)*, Dacia, Cluj-Napoca, 1979.
23. I. A. Rus, A. Petrusel, G. Petrusel, *Fixed Point Theory*, Cluj University Press, Cluj-Napoca, 2008.
24. H. Schaeffer, *Uber die Methode der a priori- Schranken, Math. Ann.*, 129 (1955), pp. 415-416.
25. S. P. Singh, *Fixed point theorems for a sum of nonlinear operators*, Rend. Accad. Naz. Lincei 54 (1973), 1-4.
26. D. R. Smart, *Fixed Point Theorems*, Cambridge University Press, Cambridge, 1980.
27. R.S. Varga, *Matrix Iterative Analysis*, Springer Series in Computational Mathematics, Vol. 27, Springer, Berlin, 2000.
28. A. Viorel, *Contributions to the study of nonlinear evolution equations*, Ph.D. Thesis, Babes-Bolyai University, Cluj-Napoca, 2011.
29. A. C. Zaanen, *Riesz Spaces*, North-Holland Publishing Company, Amsterdam, Vol. 2, 1983.
30. E. Zeidler, *Nonlinear functional analysis and its applications*, I. Fixed- point theorems, Springer-Verlag, Berlin, 1993.

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APPLICATION OF FUZZY QUASI METRIC SPACES IN THE DOMAIN OF WORDS

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ABSTRACT
In this paper we prove Banach fixed point theorem in intuitionistic fuzzy quasi-metric space. The existence of a solution for a recurrence equation which appears in the average case analysis of Quicksort algorithms is obtained as an application. We generalize the results of Romaguera, Sapena and Tirado and also generalize several known results.

Mathematics subject classification :47H10, 54A40, 54H25, 68Q25, 68Q55

Keywords : fuzzy quasi-metric, , contraction mapping, fixed point, recursive equation.

1.1 Introduction

Fuzzy set theory was first introduced by Zadeh in 1965 to describe the situations where data are uncertain. Thenafter the concept of fuzzy sets was generalized as intuitionistic fuzzy set by Atanassov [1,2] in 1984, has a wide range of applications in various fields.
With the help of continuous t-norm the concept of fuzzy metric space was modified by Kramosil and Michalek [3] and George and Veeramani [4]. The concept of fuzzy quasi-metric space was introduced by Gregori and Romaguera [5] by generalizing the concept of fuzzy metric space given by Kramosil and Michalek [6].
The concept of intuitionistic fuzzy quasi-metric space was introduced by Tirado [7] by generalizing the notion of intuitionistic fuzzy metric space given by Alaca, Turkoglu and Yildiz [8] to the quasi-metric setting and gave intuitionistic fuzzy quasi-metric version of the Banach contraction principle. Our basic references are [9], [10], [11], [12], [13], [7].
In [11], Grabiec proved fuzzy versions of celebrated Banach fixed point theorem and Edelstein fixed point theorem. Romaguera, Sapena and Tirado [16] proved the Banach fixed point theorem in fuzzy quasi-metric spaces and applied the result to the domain of words.

1.2 Preliminaries

Definition 1.2.1. A binary operation $\star : [0,1] \times [0,1] \to [0,1]$ is a continuous $t-norm$ if it satisfies the following conditions:

(1) $\star$ is commutative and associative.
(2) $\star$ is continuous.
(3) $a \star 1 = a$ for all $a \in [0,1]$:
(4)$a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d$ with $a,b,c,d \in [0,1]$

Definition 1.2.2. A binary operation $\nabla : [0,1] \times [0,1] \to [0,1]$ is a continuous $t-conorm$ if it satisfies the following conditions:

(1) ∇ is commutative and associative.
(2) ∇ is continuous.
(3) $a \nabla 0 = a$ for all $a \in [0,1]$:
(4) $a \nabla b \leq c \nabla d$ whenever $a \leq c$ and $b \leq d$ with $a,b,c,d \in [0,1]$

Remark 1.2.1. The concept of triangular norms $(t-norm)$ and triangular conforms $(t-conorm)$ are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions

respectively. These concepts were originally introduced by Menger [17] in his study of statistical metric spaces. Several examples for these concepts were purposed by many authors.

Definition 1.2.3. A 5-tuple $(X, M, N, \star, \nabla)$ is said to be an intuitionistic fuzzy quasi-metric space if X is an arbitrary set, $\star$ is a continuous t-norm, ∇ is a continuous t-conorm and M and N are fuzzy sets on $X^2 \times [0,1)$ satisfying the following conditions:

(1) $M(x,y,t) + N(x,y,t) \leq 1,\ \forall\, x,y \in X$ and $t > 0$:
(2) $M(x,y,0) = 0\ \forall\ x,y \in X$:
(3) $M(x,y,t) = M(y,x,t) = 1,\ iff\ x = y\ \forall\, t > 0$:
(4) $M(x,y,t) \star M(y,z,s) \leq M(x,z,t+s)\ \forall\, x,y,z \in X$ and $s,t > 0$:
(5) for all $x,y \in X,\ M(x,y,.): [0,1) \to [0,1]$ is left continuous.
(6) $N(x,y,0) = 1\ \forall\, x,y \in X$:
(7) $N(x,y,t) = N(y,x,t) = 0\ iff\ x = y\ \forall\, t > 0$:
(8) $N(x,y,t)\ \nabla\ N(y,z,s) \geq N(x,z,t+s)\ \forall\, x,y,z \in X$ and $s,t > 0$:
(9) for all $x,y \in X, N(x,y,.): [0,1) \to [0,1]$ is right continuous.

In this case, we say that $(M, N, \star, \nabla)$ is an intuitionistic fuzzy quasi-metric (an ifqm) on X. If in addition M and N satisfy $M(x,y,t) = M(y,x,t)$ and $N(x,y,t) = N(y,x,t)$ for all $x,y \in X$ and $t > 0$ then $(M, N, \star, \nabla)$ is called intuitionistic fuzzy metric on X and $(X, M, N, \star, \nabla)$ is called an intuitionistic fuzzy metric space.

Example 1.2.1. Let (X,d) be a quasi-metric space. Define $t-norm\ \ a \star b = min\{a,b\}$ and $t-conorm\ \ a\ \nabla\ b = max\{a,b\}$ and for all $x,y \in X$ and $t > 0$, $M_d(x,y,t) = \frac{t}{t+d(x,y)}$ and $N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$.

Then $(X, M, N, \star, \nabla)$ is an intuitionistic fuzzy quasi-metric space. We call this intuitionistic fuzzy quasi-metric (M, N) induced by the metric d the standard intuitionistic fuzzy-quasi metric. Furthermore it is easy to check that $(M_d)^{-1} = M_{d^{-1}}$, $(M_d)^i = M_{d^i}$, $(N_d)^{-1} = N_{d^{-1}}$, $(N_d)^i = N_{d^i}$. The topology τ generated by d coincides with the topology τ_{MN_d} generated by the induced intuitionistic fuzzy quasi-metric $(M, N, \star, \nabla)$.

Remark 1.2.2. It is clear that if $(X, M, N, \star, \nabla)$ is an ifqm-space then $(X, M, \star)$ is a fuzzy quasi-metric space. Conversely if $(X, M, \star)$ is a fuzzy quasi-metric space on X, then $(X, M, 1-M, \star, \nabla)$ is an ifqm-space where $a\ \nabla\ b = 1 - [(1-a) \star (1-b)]$ for all $a,b \in [0,1]$. If $(M, N, \star, \nabla)$ is an ifqm on X, then $(M^{-1}, N^{-1}, \star, \nabla)$ is also an ifqm on X where M^{-1} and N^{-1} are the fuzzy sets in $X \times X \times (0,1)$ defined by

$$M^{-1}(x,y,t) = M(y,x,t) \text{ and } N^{-1}(x,y,t) = N(y,x,t).$$

Moreover if we denote M^i and N^s, the fuzzy sets on $X^2 \times [0,1)$ given by

$$M^i(x,y,t) = min\{M(x,y,t), M^{-1}(x,y,t)\}$$

and

$$N^s(x,y,t) = max\{N(x,y,t), N^{-1}(x,y,t)\}.$$

Then $(M^i, N^s, \star, \nabla)$ is an intuitionistic fuzzy metric on X. In order to construct a suitable topology on an ifqm-space $(X, M, N, \star, \nabla)$ it seems natural to consider balls $B(x,r,t)$ defined similarly to Park [16] and Alaca, Turkoglu and Yildiz [1] by

$B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r. N(x,y,t) < r\ \forall\ x \in X\}\ r \in (0,1)$ and $t > 0$:

Then one can prove as in Park [16] that the family of sets of the form

$\{B(x,r,t): x \in X, 0 < r < 1, t > 0\}$ is a base for the topology $\tau_{M,N}$ on X.

Definition 1.2.4. Let $(X, M, N, \star, \nabla)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}_n\ in\ X$ is called a Cauchy if for each $\epsilon \in (0,1)$ and each $t >$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$, $N(x_n, x_m, t) < \epsilon$

whenever $n, m \geq n_0$. We say that $(X, M, N, \star, \nabla)$ is complete if every Cauchy sequence is convergent.

Definition 1.2.5. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy metric space $(X, M, N, \star, \nabla)$ is said to be converges to a point $x \in X$ if and only if $\lim_{n\to\infty} M(x, x_n, t) = 1$ and $\lim_{n\to\infty} N(x, x_n, t) = 0$ for all $t > 0$.

Definition 1.2.6. Let $(X, M, N, \star, \nabla)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}_n$ in X is called G −Cauchy if for each $p \in N$ and each $t > 0$,
$\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_{n+p}, t) = 0$.
We say that $(X, M, N, \star, \nabla)$ is G −complete if every G −Cauchy sequence is convergent.

1.3. The Banach Fixed Point Theorem In Intuitionistic Fuzzy Quasi-Metric Space

Definition 1.3.1. A B −contraction on an intuitionistic fuzzy metric space $(X, M, N, \star, \nabla)$ is a self mapping f on X such that there is a constant $k \in (0, 1)$ satisfying

$$M(f(x), f(y), kt) \geq M(x, y, t)$$

and $N(f(x), f(y), kt) \leq N(x, y, t)$ for all $x, y \in X,\ t > 0$.

Theorem A. Let $(X, M, N, \star, \nabla)$ be a G −complete intuitionistic fuzzy metric space and $f : X \to X$ be a self-map such that

$$M(f(x), f(y), kt) \geq M(x, y, t)$$

and $N(f(x), f(y), kt) \leq N(x, y, t)$,
for all $x, y \in X,\ t > 0$ with $k \in (0, 1)$. Then f has a unique fixed point.
Generalizing in a natural way the notations of G-completeness and B-contraction to intuitionistic fuzzy quasi-metric spaces we introduce the following concepts.

Definition 1.3.2. A sequence $\{x_n\}_n$ in an intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is said to be G −Cauchy if it is a G −Cauchy sequence in the intuitionistic fuzzy metric space $(X, M^i, N^s, \star, \nabla)$.

Definition 1.3.3. An intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is called G −bicomplete if the intuitionistic fuzzy metric space $(X, M^i, N^s, \star, \nabla)$ is G-complete. In this case we say that $(M, N, \star, \nabla)$ is a fuzzy quasi-metric on X.

Definition 1.3.4. A B −contraction on an intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is a self mapping f on X such that there is a constant $k \in (0, 1)$ satisfying $M(f(x), f(y), kt) \geq M(x, y, t)$**.** Let $(X, M, N, \star, \nabla)$ be a G −bicomplete intuitionistic fuzzy quasi-metric space such that $\lim_{n\to\infty} M(x, y, t) = 1$ and $\lim_{n\to\infty} N(x, y, t) = 0$
for all $x, y \in X$. Then every B −contraction on X has a unique fixed point and $N(f(x), f(y), kt) \leq N(x, y, t)$, for all $x, y \in X, t > 0$.
The number k is called a contraction constant of f.

Theorem 1.3.1

Proof. Let $f : X \to X$ be a B −contraction on X with contraction constant $k \in (0, 1)$. Then

$$M(f(x), f(y), kt) \geq M(x, y, t)$$

and

$$N(f(x), f(y), kt) \leq N(x, y, t),$$

for all $x, y \in X, t > 0$.

It immediately follows that

$$M^i(f(x), f(y), kt) \geq M^i(x, y, t) \text{ and}$$
$$N^s(f(x), f(y), kt) \leq N^s(x, y, t),$$

for all $x, y \in X, t > 0$: Hence f is a B −contraction on the G −complete fuzzy metric space $(X, M^i, N^s, \star, \nabla)$ and by $Theorem\ A$, f has a unique fixed point.

Definition 1.3.4. A generalized B −contraction on an intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is a self mapping f on X such that there is a constant $k \in (0,1)$ satisfying

$$M(f(x), f(y), kt) \geq min\{M(x,y,t), M(x,f(x),t), M(y,f(y),t)\}$$
$$and \quad N(f(x), f(y), kt) \leq max\{N(x,y,t), N(x,f(x),t), N(y,f(y),t)\}$$

for all $x, y \in X,\ t > 0$.

The number k is called a contraction constant of f .

Proof. Let $f : X \to X$ be a weak B −contraction on X with contraction constant $k \in (0,1)$. Then

$$M(f(x), f(y), kt) \geq min\{M(x,y,t), M(x,f(x),t), M(y,f(y),t)\}$$
$$and \quad N(f(x), f(y), kt) \leq max\{N(x,y,t), N(x,f(x),t), N(y,f(y),t)\}$$

for all $x, y \in X, t > 0$.

It immediately follows that

$$M^i(f(x), f(y), kt) \geq min\{M^i(x,y,t), M^i(x,f(x),t), M^i(y,f(y),t)\}$$
$$and$$
$$N^s(f(x), f(y), kt) \leq max\{N^s(x,y,t), N^s(x,f(x),t), N^s(y,f(y),t)\}$$

for all $x, y \in X, t > 0$: Hence f is a weak B −contraction on the G −complete fuzzy metric space $(X, M^i, N^s, \star, \nabla)$ and by $Theorem\ A$, f has a unique fixed point.

1.4. G-Bicompleteness In Non-Archimedean Intuitionistic Fuzzy Quasi-Metric Space

Definition 1.4. 1. An intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$ is called a non-Archimedean intuitionistic fuzzy quasi-metric space if $(M, N, \star, \nabla)$ is a non-Archimedean intuitionistic fuzzy quasi-metric on X, that is, $M(x,y,t) \geq min\{M(x,z,t), M(z,y,t)\}$

$$and\ \ N(x,y,t) \leq max\{M(x,z,t), M(z,y,t)\},$$

for all $x, y, z \in X$ and $t > 0$.

Lemma 1.4.1. Each G −Cauchy sequence in a non-Archimedean intuitionistic fuzzy quasi-metric space is a Cauchy sequence.

Proof. Let$\{x_n\}$ be a G −Cauchy sequence in the non-Archimedean intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$, then for each $t > 0$, we have $\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_{n+1}, t) = 0$, which implies that, for each $\epsilon \in (0,1)$, there is $n_0 \in N$ such that

$M^i(x_n, x_{n+1}, t) > 1 - \epsilon$ and $N^s(x_n, x_{n+1}, t) < \epsilon$, for each $n \geq n_0$,

Now let $m > n \geq n_0$. Then $m = n + j$, for some $j \in N$. So

$$M^i(x_n, x_m, t) \geq min\begin{Bmatrix} M^i(x_n, x_{n+1}, t), M^i(x_{n+1}, x_{n+2}, t), \\ \dots, M^i(x_{n+j-1}, x_{n+j}, t) \end{Bmatrix}$$
$$> 1 - \epsilon$$

and

$$N^s(x_n, x_m, t) < max\begin{Bmatrix} N^s(x_n, x_{n+1}, t), N^s(x_{n+1}, x_{n+2}, t), \\ \dots, N^s(x_{n+j-1}, x_{n+j}, t) \end{Bmatrix}$$
$$< \epsilon.$$

We conclude that $\{x_n\}$ is a Cauchy sequence in $(X, M, N, \star, \nabla)$.

Theorem 1.4.1. Each bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space is G −bicomplete.

Proof. Let $\{x_n\}$ be a G −Cauchy sequence in the bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space $(X, M, N, \star, \nabla)$: By $Lemma$ 4.1, $\{x_n\}$ is a Cauchy sequence in $(X, M, N, \star, \nabla)$. Hence there is $x \in X$ such that

$\lim_{n\to\infty} M(x, x_n, t) = 1$ and $\lim_{n\to\infty} N(x, x_n, t) = 0$, for all $t > 0$.

We conclude that

$(X, M^i, N^s, \star, \nabla)$ is G −complete, that is, $(X, M, N, \star, \nabla)$ is G −bicomplete.

Corollary 1.4.2. Each complete non-Archimedean intuitionistic fuzzy metric space is G-complete.

1.5. Application To The Domain Of Words

Let Σ be a non-empty alphabet. Let Σ^∞ be the set of all finite and infinite sequences ("words") over Σ, where we adopt the convention that the empty sequence ϕ is an element of Σ^∞. The symbol $\sqsubseteq$ denote the prefix order on Σ^∞ , that is, $x \sqsubseteq y \Leftrightarrow x$ is a prefix of y. Now, for each $x \in \Sigma^\infty$ denote by $l(x)$ the length of x. Then $l(x) \in [1, \infty)$ whenever $x \neq \phi$ and $l(\phi) = 0$. For each $x, y \in \Sigma^\infty$ let $x \sqcap y$ be the common prefix of x and y. Thus the function $d_\sqsubseteq$ defined on $\Sigma^\infty \times \Sigma^\infty$ by

$$d_\sqsubseteq(x,y) = \begin{cases} 0 & if\ x \sqsubseteq y \\ 2^{-l(x \sqcap y)} & otherwise \end{cases}$$

is a quasi-metric on Σ^∞ (We adopt the convention that $2^\infty = 0$). Actually,
$d_\sqsubseteq$ is a non-Archimedean quasi-metric on Σ^∞ and the non-Archimedean quasi metric $(\Sigma^\infty)^s$ is the Baire metric on Σ^∞, that is, $(\Sigma^\infty)^s(x,x) = 0$ and $(\Sigma^\infty)^s(x,y) = 2^{-l(x \sqcap y)}$ for all $x, y \in \Sigma^\infty$ such that $x \neq y$. It is well known that $(\Sigma^\infty)^s$ is complete. From this fact it is clear that Σ^∞ is bicomplete. The quasi-metric Σ^∞ , which was introduced by Smyth [20], will be called the Baire quasi-metric. Observe that condition $d_\sqsubseteq(x,y) = 0$ can be used to distinguish between the case that x is a prefix of y and the remaining cases.

Example 1.5.1. Let $d_\sqsubseteq$ be a (non-Archimedean) quasi-metric on a set X and let $M_{d_\sqsubseteq}$ and $N_{d_\sqsubseteq}$ are fuzzy sets in $X \times X \times [0,1)$ given by

$$M_{d_\sqsubseteq}(x,y,t) = \frac{t}{t + d_\sqsubseteq(x,y)} \quad \text{and}$$

$$N_{d_\sqsubseteq}(x,y,t) = \frac{d_\sqsubseteq(x,y)}{t + d_\sqsubseteq(x,y)}$$

for all $x, y \in X$ and $t > 0$. Then $(M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ is a (non-Archimedean) intuitionistic fuzzy quasi-metric on X, where $\wedge$ denotes the continuous

t −norm and $\vee$ denotes the continuous t −conorm given by $a \wedge b = min\{a,b\}$ and $a \vee b = max\{a,b\}$.
It is clear that $\tau_{MN_{d_\sqsubseteq}} = \tau_{d_\sqsubseteq}$ and that $(M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ is bicomplete if and only if $(X, d_\sqsubseteq)$ is bicomplete.

Proposition 1.5.1. $(\Sigma^\infty, M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ is a G −bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space.

Proposition 1.5.2. $(\Sigma^\infty, M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ is a G −bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space.

The intuitionistic fuzzy non-Archimedean quasi-metric $(M_{d_\sqsubseteq}, N_{d_\sqsubseteq}, \wedge, \vee)$ is
given by $M_{d_{\sqsubseteq 1}}(x,y,0) = 0$ and $N_{d_{\sqsubseteq 0}}(x,y,0) = 1$ for all $x, y \in \Sigma^\infty$.
$M_{d_{\sqsubseteq 1}}(x,y,t) = 1$ and $N_{d_{\sqsubseteq 0}}(x,y,t) = 0$ if x is a prefix of y and $t > 0$, $M_{d_{\sqsubseteq 1}}(x,y,t) = 1 - 2^{-l(x \sqcap y)}$ and $N_{d_{\sqsubseteq 0}}(x,y,t) = 2^{-l(x \sqcap y)}$
if x is not a prefix of y and $t \in (0,1)$,
$M_{d_{\sqsubseteq 1}}(x,y,t) = 1$ and $N_{d_{\sqsubseteq 0}}(x,y,t) = 0$ if x is not a prefix of y and $t > 1$.
$Proposition$ 4.5.2 allows us to apply any of the $Proposition$ 4.5.1 and $Theorem$ 4.3.1 to the complexity analysis of quicksort algorithm, to show, in direct way, the existence and uniqueness of solution for the following recurrence equation:

$$T(1) = 0 \text{ and } T(n) = \frac{2(n-1)}{n} + \frac{n+1}{n} T(n-1),\ n \geq 2$$

The average case analysis of Quicksort is discussed in [14] (see also [6]), where the above recurrence equation is obtained
Consider as an alphabet Σ the set of non-negative real numbers, that is,
$\Sigma = [0,1)$. We associate to T the functional $\Phi : \Sigma^\infty \to \Sigma^\infty$ given by $(\Phi(x))_1 = T(1)$ and $(\Phi(x))_n = \frac{2(n-1)}{n} + \frac{n+1}{n} x_{(n-1)},\ n \geq 2$

If $x \in \Sigma^{\infty}$ has length $n < 1$, we write $x = x_1x_2x_3 \ldots x_n$, and if x is an infinite word we write $x = x_1x_2x_3 \ldots x_n \ldots$ Next we show that Φ is a B −contraction on the G −bicomplete non-Archimedean intuitionistic fuzzy quasi-metric space $(\Sigma^{\infty}, M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ with contraction constant $\frac{1}{2}$.

To this end, we first note that, by construction,

we have $l(\Phi(x)) = l(x) + 1$

for all $x \in \Sigma^{\infty}$ (in particular $l(\Phi(x)) = 1$ whenever $l(x) = 1$).

Furthermore, it is clear that $x \sqsubseteq y \Leftrightarrow \Phi(x) \sqsubseteq \Phi(y)$,

and consequently $\Phi(x \sqcap y) \sqsubseteq \Phi(x) \sqcap \Phi(y)$, for all $x, y \in \Sigma^{\infty}$.

Hence $l(\Phi(x \sqcap y)) \sqsubseteq l(\Phi(x) \sqcap \Phi(y))$, for all $x, y \in \Sigma^{\infty}$.

From the preceding observations we deduce that for all $x, y \in X$, if x is a prefix of y, then $M_{d_{\sqsubseteq}}\left(\Phi(x), \Phi(y), \frac{t}{2}\right) = M_{d_{\sqsubseteq}}(x, y, t) = 1$

and $N_{d_{\sqsubseteq}}\left(\Phi(x), \Phi(y), \frac{t}{2}\right) = N_{d_{\sqsubseteq}}(x, y, t) = 0.$

and if x is not a prefix of y, then for all $t > 0$,

$$M_{d_{\sqsubseteq}}\left(\Phi(x), \Phi(y), \frac{t}{2}\right) = \frac{\frac{t}{2}}{\left(\frac{t}{2}\right)+2^{-l(\Phi(x) \sqcap \Phi(y))}}$$

$$\geq \frac{\frac{t}{2}}{\left(\frac{t}{2}\right)+2^{-l(\Phi(x \sqcap y))}}$$

$$\geq \frac{\frac{t}{2}}{\left(\frac{t}{2}\right)+2^{-(l(x \sqcap y)+1)}}$$

$$\geq \frac{t}{t+2^{-(l(x \sqcap y))}}$$

$$\geq M_{d_{\sqsubseteq}}(x, y, t)$$

and

$$N_{d_{\sqsubseteq}}\left(\Phi(x), \Phi(y), \frac{t}{2}\right) = \frac{2^{-l(\Phi(x) \sqcap \Phi(y))}}{\left(\frac{t}{2}\right)+2^{-l(\Phi(x) \sqcap \Phi(y))}}$$

$$\leq \frac{2^{-l(\Phi(x \sqcap y))}}{\left(\frac{t}{2}\right)+2^{-l(\Phi(x \sqcap y))}}$$

$$\leq \frac{2^{-(l(x \sqcap y)+1)}}{\left(\frac{t}{2}\right)+2^{-(l(x \sqcap y)+1)}}$$

$$\leq \frac{2^{-(l(x \sqcap y))}}{t+2^{-(l(x \sqcap y))}}$$

$$\leq N_{d_{\sqsubseteq}}(x, y, t)$$

Therefore, Φ is a B −contraction on $(\Sigma^{\infty}, M_{d_{\sqsubseteq}}, N_{d_{\sqsubseteq}}, \wedge, \vee)$ with contraction constant $\frac{1}{2}$. So, by $Theorem$ 1.3.1, Φ has a unique fixed point $z = z_1z_2z_3 \ldots$, which is obviously the unique solution to the recurrence equation T, that is, $z_1 = 0$ and

$$z_n = \frac{2(n-1)}{n} + \frac{n+1}{n} z_{(n-1)}, \; n \geq 2 .$$

REFERENCES

1. Atanassov, K.T.: New operations defined over the intuitionistic fuzzy set, Fuzzy sets and systems, 61 (1994), 137-142.
2. Atanassov, K., Intuitionistic fuzzy sets, Fuzzy sets and systems, 20 (1986),87-96.
3. Kramosil, I., Michalek, J., Fuzzy metric and statistical metric spaces, Kybernetika, 11 (1975), 336-344.
4. George, A., Veeramani, P., On some results in fuzzy metric spaces, Fuzzy Sets and Systems 64 (1994), 395-399.
5. Gregori, V., Romaguera, S., Fuzzy quasi-metric spaces, Appl. Gen. Topology, 5 (2004), 129-136.
6. Kramosil, I., Michalek, J., Fuzzy metric and statistical metric spaces, Kybernetika, 11 (1975), 336-344.

7. Tirado, P., Contractive maps and complexity analysis in fuzzy quasi metric spaces, Universidad Politecnica De Valencia Ph.D. thesis, (2008).
8. Alaca C., Turkoglu D. and Yildiz C., Fixed point in intuitionistic fuzzy metric spaces, Chaos Solitons and Fractals 2006-09, 1073-78.
9. Dubosis, D., Prade, H., Fuzzy sets: Theory and application to policy analysis and information systems. Plenum Press, New York, 1980.
10. George, A., Veeramani, P., On some results in fuzzy metric spaces, Fuzzy Sets and Systems 64 (1994), 395-399.
11. Grabiec, M., Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988), 385-389.
12. Grabiec, M., Cho, Y.J., Radu, V., On non symmetrical topological and probabilistic structure, Nova Science Publishers, Inc., New York, 2006.
13. Gregori, V., Romaguera, S., Fuzzy quasi-metric spaces, Appl. Gen. Topology, 5 (2004), 129-136.
14. Ali,J.,Imdad,M. and Bahuguna,D.: Common fixed point theorems in Menger spaces with common property (E.A), Comput. Math. Appl., 60(12) (2010), 3152–3159.
15. Aliouche,A.: Common fixed point theorems of Gregus type for weakly compatible mappings satisfying generalized contractive conditions, J. Math. Anal. Appl., 341 (2008), 707-719.
16. Romaguera, S, Sapena, A, Tirado, P., The Banach fixed point theorem in fuzzy quasi-metric spaces with application to the domain of words. Topol. Appl.. 154, 2196–2203 (2007).
17. EL. Naschie, MS., The two slit experiment as the foundation of E-infinity of high energy physics., Chaos, Solitions and Fractals, 25(2005), 509–514.
18. Fang, J. X., On fixed point theorems in fuzzy metric spaces, Fuzzy sets and Systems, 46 (1992), 107-113.
19. Grabiec, M., Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988), 385-389.
20. Hadzic, O, Pap, E., Fixed Point Theory in Probabilistic Metric Spaces, Kluwer Academic, Dordrecht (2001)
21. Ha K.S., Cho Y.J., and White A., Strictly convex and strictly 2-convex 2-normed spaces, Mathematica Japonica, 33 (3)(1988) 375-384..
22. Jachymski, J., On probabilistic ψ −contractions on Menger spaces. Nonlinear Anal. TMA. 73, 2199–2203 (2010).
23. Jungck, G., Compatible mappings and common fixed points, Int. J. Math. Sci., 9(1986), 771–779.
24. Jungck, G. and Rhoades, B.E., Some fixed point theorems for compatible maps, Int. J.Math. Math. Sci., 16 (1993), 417-428.
25. Karapinar, E, Romaguera, S, Tas, K., Fixed points for cyclic orbital generalized contractions on complete metric spaces. Cent. Eur. J. Math.. 11, 552–560 (2013).
26. Klement, E.P., Operations in fuzzy sets on approximate approach, Inform. Sci., 27 (1984), 221-232.
27. Klement, E.P., Mesiar, R., Pap, E., Triangular Norma tends in logic, vol. 8, Kluwer Academic Publisher, Dordrecht, 2000.
28. Kramosil, I., Michalek, J., Fuzzy metric and statistical metric spaces, Kybernetika, 11 (1975), 336-344.

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Research Article

THE SUM OF ith SMALLEST PARTS OF $r-partitions$ of n

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ABSTRACT

George E Andrews [1] derived generating function for the number of smallest parts of *partitions* of positive integer *n*. Hanuma Reddy [2] defined i^{th} smallest part and derived a relation between the ith smallest and i^{th} greatest parts of *partitions* of *n* in general form. Here we derive generating function for the sum of the ith smallest parts of $r-partitions$ of *n.*

Key Words:

partitions, *r-partitions*, smallest parts of *partition* and i^{th} smallest parts of $partition$ of positive integer *n.*

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INTRODUCTION

A $partition$ of a positive integer n is a finite non increasing sequence of positive integers $\lambda_1,\lambda_2,...,\lambda_r$ such that $\sum_{i=1}^{r}\lambda_i = n$ and is denoted by $n=(\lambda_1,\lambda_2,...,\lambda_r)$, $n=\lambda_1+\lambda_2+\lambda_3+...\lambda_r$ or $\lambda=\left(\lambda_1^{f_1},\lambda_2^{f_2},\lambda_3^{f_3},...\right)$ when λ_1 repeats f_1 times, λ_2 repeats f_2 times and so on. The λ_i are called the parts of the *partition*. In what follows λ stands for a *partition* of n, $\lambda=(\lambda_1,\lambda_2,...,\lambda_r),\lambda_1\geq\lambda_2\geq...\geq\lambda_r$. The set of all *partitions* of n is represented by $\xi(n)$ by and its cardinality $p(n)$.

If $1\leq r\leq n$ then $\xi_r(n)$ is the set of *partition*s of *n* with *r* parts and its cardinality is denoted by $p_r(n)$. A *partition* of *n* with exactly *r* parts is called $r-partition$ of *n*. We define

$$p_r(n)=\begin{cases}0 & \text{if } r=0 \text{ or } r>n\\ \text{number of } r-partitions \text{ of } n & \text{if } 0<r\leq n\end{cases}$$

$spt(n)$ denotes the number of smallest parts including repetitions in all *partitions* of n. $spt_i(n)$ denotes the number of ith smallest parts including repetitions in all *partitions* of n. $r-spt_i(n)$ denotes the number of ith smallest parts in all $r-partitions$ of *n.* The number of $partitions$ of n with least part greater than or equal to k is represented by $p(k,n)$.

Existing generating functions are given below.

Function	Generating function
$p_r(n)$	$\frac{q^r}{(q)_r}$
$p_r(n-k)$	$\frac{q^{r+k}}{(q)_r}$
number of divisors	$\sum_{n=1}^{\infty}\frac{q^n}{(1-q^n)}$
sum of divisors	

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$$\sum_{n=1}^{\infty}\frac{n.q^n}{\left(1-q^n\right)} \qquad (1.1.1)$$

where $(q)_k=\prod_{n=1}^{k}\left(1-q^n\right)$ for $k>0$, $(q)_k=1$ for $k=0$ and $(q)_k=0$ for $k<0$.

and $(a)_n=(a;q)_n=(1-a)(1-aq)\left(1-aq^2\right)...\left(1-aq^{n-1}\right)$

(1.2) Theorem: The generating function for the sum of smallest parts of the $partitions$ of n is

$$\sum_{n=1}^{\infty}sum\ spt(n)q^n=\frac{1}{(q)_\infty}\sum_{n=1}^{\infty}\frac{nq^n(q)_{n-1}}{\left(1-q^n\right)}$$

Proof : From [4] we have the sum of smallest parts $sum\ spt(n)$ of the $partitions$ of a positive integer n is

$$sum\ spt(n)=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}k\ \ p(k,n-tk)+\sigma(n)$$

where $\sigma(n)$ is sum of positive divisors of n.

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}k\ \ p_r(k,n-tk)$$

First replace $k+1$ by k, then replace n by $n-tk$ in [3]

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}k\,p_r\left(n-tk-r(k-1)\right)+\sigma(n)$$

Hence the generating function for the sum of smallest parts of the $partitions$ of a positive integer n is

$$\sum_{n=1}^{\infty}sum\ spt(n)q^n=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{kq^{r+tk+r(k-1)}}{(q)_r}+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

from (1.1.1)

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}\sum_{r=1}^{\infty}\frac{kq^{tk+rk}}{(q)_r}+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}kq^{tk}\left[\sum_{r=1}^{\infty}\frac{\left(q^k\right)^r}{(q)_r}\right]+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

$$=\sum_{k=1}^{\infty}\sum_{t=1}^{\infty}kq^{tk}\left[1+\sum_{r=1}^{\infty}\frac{\left(q^k\right)^r}{(q)_r}-1\right]+\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}$$

$$=\sum_{k=1}^{\infty}\frac{kq^k}{\left(1-q^k\right)}\prod_{r=0}^{\infty}\left(\frac{1}{1-q^rq^k}\right)$$

$$=\sum_{k=1}^{\infty}\frac{kq^k}{\left(1-q^k\right)}\prod_{r=0}^{\infty}\left(\frac{1}{1-q^{r+k}}\right)$$

$$=\frac{1}{(q)_\infty}\sum_{k=1}^{\infty}\frac{kq^k(q)_{k-1}}{\left(1-q^k\right)}$$

$$=\frac{1}{(q)_\infty}\sum_{n=1}^{\infty}\frac{nq^n(q)_{n-1}}{\left(1-q^n\right)}$$

$$\sum_{n=1}^{\infty}sum\ spt(n)q^n=\frac{1}{(q)_\infty}\sum_{n=1}^{\infty}\frac{nq^n(q)_{n-1}}{\left(1-q^n\right)}\ \blacksquare$$

(1.3) Theorem: The sum $r-spt(n)$ of ith smallest parts of $r-partitions$ of n such that i^{th} smallest part is first part $\left(i.e\ \lambda_1\ \text{as}\ i^{th}\text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty}sum\left(r-spt_i(n)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}$$

$$\left.+\ldots+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\right\}\ \text{for } i=r \qquad (1.3.1)$$

Proof: Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu^\alpha\right)$ be any $r-partition$ of n with all parts equal.

We know that the sum of the smallest parts of $r-partitions$ of n such that the smallest part is the first part $\left(i.e\ \lambda_1\ \text{as smallest part}\right)$ and having k as smallest part is $k\beta$

$$\text{where}\ \beta=\begin{cases}1 & \text{if}\ \ \frac{n}{r}=k\\ 0 & \text{otherwise}\end{cases} \qquad (1.3.2)$$

The generating function for the sum of smallest parts of $r-partitions$ of n such that smallest part as the first part $\left(i.e\ \lambda_1\ \text{as smallest part}\right)$

$$\sum_{n=1}^{\infty}\left(r-spt_1(n)\right)q^n=\sum_{k=1}^{\infty}k.q^{kr}==\frac{q^r}{\left(1-q^r\right)^2}\quad \text{for } r=1$$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2}\right)$ be any $r-partition$ of n with two distinct parts.

Subtracting μ_2 from each λ_i for $i=1$ to r, we get $n_1=\left(\mu_1^{(1)}\right)^{\alpha_1}$ where $n_1=n-r\mu_2$, $r_1=r-\alpha_2$ and $\mu_1^{(1)}=\mu_1-\mu_2$

The sum of the smallest parts of $r_1-partitions$ of n_1 such that smallest part is the first part and having *k* as the smallest part is $k\beta_1$.

$$\text{where } \beta_1=\begin{cases}1 & \text{if } r_1 \mid n_1\\ 0 & \text{otherwise}\end{cases} \text{ and } \qquad n=n_1+r\mu_2,$$

$\mu_1=k-\mu_2.$

Now the sum of second smallest parts of $r-partitions$ of n such that the second smallest part is the first part and having *k* as the smallest part is $k\beta_1$.

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n-r\mu_2}{r_1}=k-\mu_2\\ 0 & \text{otherwise}\end{cases}$$

Therefore the generating function for the sum of the second smallest parts of $r-partitions$ of n such that the second smallest part as the first part $\left(i.e\ \lambda_1 \text{ as second smallest part}\right)$ is

$$\sum_{n=1}^{\infty} sum\left(r-spt_2\left(n\right)\right)q^n=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k-\mu_2=1}^{\infty} kq^{\mu_2 r+(k-\mu_2)r_1}$$

$$=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_1=1}^{\infty}\left(\mu_1+\mu_2\right)q^{\mu_1 r_1+\mu_2 r}$$

$$=\left[\sum_{\mu_2=1}^{\infty}\mu_2 q^{\mu_2 r}\sum_{\mu_1=1}^{\infty}\sum_{r_1=1}^{r-1}q^{\mu_1 r_1}+\sum_{\mu_2=1}^{\infty}q^{\mu_2 r}\sum_{\mu_1=1}^{\infty}\sum_{r_1=1}^{r-1}\mu_1 q^{\mu_1 r_1}\right]$$

$$=\left[\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\right]$$

for $r=2$ (1.3.4)

Continuing this process, we get the generating function for the sum of ith smallest parts of $r-partitions$ of n such that i^{th} smallest part as first part $\left(i.e\ \lambda_1 \text{ as } i^{th}\text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty} sum\left(r-spt_i\left(n\right)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}$$

$$\left.+.....+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\right\} \text{ for } i=r \quad \blacksquare$$

(1.4) Theorem: The generating function for the sum of ith smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} sum\left(r-spt_i\left(n\right)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{\left(q\right)_{r_i}}+1\right)\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{\left(q\right)_{r_i}}+1\right)$$

$$\left.+...+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{\left(q\right)_{r_i}}+1\right)\right\}$$

Proof: The sum of smallest parts of $r-partitions$ of n having k as a smallest part is

$$k\sum_{i=0}^{\infty}p_{r-1-i}\left[n-\left(k-1\right)r-1-i\right]+k\beta$$

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$$

The generating function for the sum of the smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} sum\left(r-spt\left(n\right)\right)q^n=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{kq^{r-1-i+(k-1)r+1+i}}{\left(q\right)_{r-1-i}}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{kq^{kr}}{\left(q\right)_{r-1-i}}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\sum_{i=0}^{\infty}\frac{\left(q^r+2q^{2r}+3q^{3r}+...\right)}{\left(q\right)_{r-1-i}}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\sum_{i=1}^{r-1}\frac{1}{\left(q\right)_i}+\frac{q^r}{\left(1-q^r\right)^2}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\left(\sum_{i=1}^{r-1}\frac{1}{\left(q\right)_i}+1\right)$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\left(\sum_{r_1=1}^{r-1}\frac{1}{\left(q\right)_{r_1}}+1\right)$$

From theorems (3.3.1) and (3.3.4) in [5], the sum of second smallest parts (without λ_1) of $r-partitions$ of n with least part k is

$$\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\sum_{k=1}^{\infty} k.p_{r_1-1-i}\left(n-\mu_l r-\left(k-\mu_l-1\right)r_1-1-i\right)$$

The generating function for the sum of second smallest parts (without λ_1) of $r-partitions$ of n is

$$\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\frac{kq^{r_1-1-i+\mu_l r+\left(k-\mu_l-1\right)r_1+1+i}}{\left(q\right)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\frac{kq^{\mu_l r+kr_1-\mu_l r_1}}{\left(q\right)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=\mu_l+1}^{\infty}\frac{kq^{\mu_l\left(r-r_1\right)+kr_1}}{\left(q\right)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\left(\mu_l+\mu_{l-1}\right)q^{\mu_l\left(r-r_1\right)+\left(\mu_l+\mu_{l-1}\right)r_1}}{\left(q\right)_{r_1-1-i}}$$

where $k=\mu_l+\mu_{l-1}$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\left(\mu_l+\mu_{l-1}\right)q^{\mu_l r+\mu_{l-1}r_1}}{\left(q\right)_{r_1-1-i}}$$

$$=\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\mu_l q^{\mu_l r}q^{\mu_{l-1}r_1}}{\left(q\right)_{r_1-1-i}}+\sum_{\mu_l=1}^{\infty}\sum_{i=0}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_{l-1}=1}^{\infty}\frac{\mu_{l-1}q^{\mu_l r}q^{\mu_{l-1}r_1}}{\left(q\right)_{r_1-1-i}}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{1}{\left(q\right)_{r_2}}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{1}{\left(q\right)_{r_2}}$$

$$=\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{1}{\left(q\right)_{r_2}}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{1}{\left(q\right)_{r_2}}$$

The generating function for the sum of second smallest parts of $r-partitions$ of n are equal to

$$\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\quad \text{for } r=2$$

Therefore the generating function for the sum of second smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty}sum\left(r-spt_2\left(n\right)\right)q^n=$$

$$\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{\left(q\right)_{r_2}}+1\right)+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{\left(q\right)_{r_2}}+1\right)$$

Similarly the generating function for the sum of third smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty}sum\left(r-spt_3\left(n\right)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\left(\sum_{r_3=1}^{r_2-1}\frac{1}{\left(q\right)_{r_3}}+1\right)\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\left(\sum_{r_3=1}^{r_2-1}\frac{1}{\left(q\right)_{r_3}}+1\right)$$

$$\left.+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)^2}\left(\sum_{r_3=1}^{r_2-1}\frac{1}{\left(q\right)_{r_3}}+1\right)\right\}$$

By induction, we get the generating function for the sum of the ith smallest parts of $r-partitions$ of n which is given by

$$\sum_{n=1}^{\infty}sum\left(r-spt_i\left(n\right)\right)q^n$$

$$=\left\{\frac{q^r}{\left(1-q^r\right)^2}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{\left(q\right)_{r_i}}+1\right)\right.$$

$$+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)^2}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{\left(q\right)_{r_i}}+1\right)$$

$$\left.+\ldots+\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)^2}\left(\sum_{r_i=1}^{r_{i-1}}\frac{1}{\left(q\right)_{r_i}}+1\right)\right\}$$

References

1. Andrews G. E. (1998), The Theory of *Partitions*, *Cambridge University Press*, Cambridge. MR **99c:** 11126.
2. Hanuma Reddy K. (2010), A Study of *r – partitions,* submitted to Acharya Nagarjuna University, awarded of Ph.D. in Mathematics.
3. Hanuma Reddy K, Manjusree A (2015), The number of smallest parts in the *partitions* of *n*, International Journal in IT and Engineering, vol.03, Issue-03, ISSN:2321-1776.
4. Manjusree A. (2017), A Study of *r –jthover partitions,* submitted to Acharya Nagarjuna University, awarded of Ph.D in Mathematics.

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The number of ith smallest parts of $r-partitions$ of n

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Abstract: George E Andrews [1] derived generating function for the number of smallest parts of *partitions* of positive integer *n*. Hanuma Reddy [2] defined i^{th} smallest part and derived a relation between the ith smallest and i^{th} greatest parts of *partitions* of *n* in general form. Here we derive generating function for the number of the ith smallest parts of $r-partitions$ of *n.*

Keywords: *partitions*, *r-partitions*, smallest parts of *partition* and i^{th} smallest parts of *partition* of positive integer *n.*

Subject classification: 11P81 Elementary theory of *partitions*.

Introduction:

A *partition* of a positive integer n is a finite non increasing sequence of positive integers $\lambda_1,\lambda_2,...,\lambda_r$ such that $\sum_{i=1}^{r}\lambda_i = n$ and is denoted by $n=(\lambda_1,\lambda_2,...,\lambda_r), n=\lambda_1+\lambda_2+\lambda_3+...\lambda_r$ or $\lambda=(\lambda_1^{f_1},\lambda_2^{f_2},\lambda_3^{f_3},...)$ when λ_1 repeats f_1 times, λ_2 repeats f_2 times and so on. The λ_i are called the parts of the *partition*. In what follows λ stands for a *partition*of n, $\lambda=(\lambda_1,\lambda_2,...,\lambda_r),\lambda_1\geq\lambda_2\geq...\geq\lambda_r$. The set of all *partitions* of n is represented by $\xi(n)$ by and its cardinolity $p(n)$.

If $1\leq r\leq n$ then $\xi_r(n)$ is the set of *partition*s of *n* with *r* parts and its cardinality is denoted by $p_r(n)$. A *partition* of *n* with exactly *r* parts is called $r-partition$ of *n*. We define

$$p_r(n)=\begin{cases}0 & \text{if } r=0 \text{ or } r>n\\ \text{number of } r-partitions \text{ of } n & \text{if } 0<r\leq n\end{cases}$$

$spt(n)$ denotes the number of smallest parts including repetitions in all *partitions* of n. $spt_i(n)$ denotes the number of ith smallest parts including repetitions in all *partitions* of n. $r-spt_i(n)$ denotes the number of ith smallest parts in all $r-partitions$ of *n.*The number of *partitions* of n with least part greater than or equal to k is represented by $p(k,n)$.

1.1 Existing generating functions are given below.

Function	Generating function
$p_r(n)$	$\frac{q^r}{(q)_r}$
$p_r(n-k)$	$\frac{q^{r+k}}{(q)_r}$
number of divisors	$\sum_{n=1}^{\infty}\frac{q^n}{(1-q^n)}$
sum of divisors	$\sum_{n=1}^{\infty}\frac{n.q^n}{(1-q^n)}$ (1.1.1)

where $(q)_k=\prod_{n=1}^{k}(1-q^n)$ for $k>0$, $(q)_k=1$ for $k=0$ and $(q)_k=0$ for $k<0$.

and $(a)_n=(a;q)_n=(1-a)(1-aq)(1-aq^2)...(1-aq^{n-1})$

1.2Theorem: If $k\in N$ and $1\le k\le\left[\frac{n}{r}\right]$, then the number $f_r^{\,i}(k,n)$ of $r-$*partitions* of n with k as i^{th} smallest part is

i) $f_r(k,n)=p_{r-1}\left[n-(k-1)r-1\right]+\beta$ for $i=1$

where $\beta=\begin{cases}1 & \text{if } \frac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$

ii) If $i>1$

$$f_r^{\,i}(k,n)=\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}...\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}p_{r_{i-1}-1}\left[\begin{matrix}(n-r\mu_l-r_1\mu_{l-1}-...-r_{i-2}\mu_{l-i+2})\\ -(k-1)(r-\alpha_l...-\alpha_{l-i+2})-1\end{matrix}\right]$$

$$+\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}...\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}\beta_{i-1}\qquad\text{for } i>1$$

where $\beta_{i-1}=\begin{cases}1 & \text{if } \frac{n-r\mu_l-r_1\mu_{l-1}-...-r_{i-2}\mu_{l-i+2}}{r_{i-1}}=k-\mu_l...-\mu_{l-i+2}\\ 0 & \text{otherwise}\end{cases}$

Proof :

(i) For $i=1$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2},,\ldots,\mu_{l-1}^{\alpha_{l-1}},k^{\alpha_l}\right)$ be any $r-$*partition* of n with *l* distinct parts.

Put $t=1$in theorem 1.2 in [3], we get the number of $r-$*partitions* of *n* with *k* as smallest part is

$$f_r(k,n)=p_{r-1}(k,n-k)+\beta$$

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$$

First replace $k+1$ by k,r by $r-1,$ then replace n by $n-k$ in theorem 1.3 in [3], we get

$$=p_{r-1}\left[n-(k-1)r-1\right]+\beta \quad (1.2.1)$$

(ii) For $i>1$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)$

$$=\left(\mu_1^{\alpha_1},\ldots,\mu_{l-i}^{\alpha_{l-i}},\mu_{l-i+1}^{\alpha_{l-i+1}},\mu_{l-i+2}^{\alpha_{l-i+2}},\ldots,\mu_{l-1}^{\alpha_{l-1}},\mu_l^{\alpha_l}\right) \quad (1.2.2)$$

be any $r-$*partition* of n with *l* distinct parts. Subtracting μ_l from λ_i for $i=1$ to r, we get

$$n_1=(\lambda_1^{(1)},\lambda_2^{(1)},\ldots,\lambda_{r_1}^{(1)})=\left(\left(\mu_1^{(1)}\right)^{\alpha_1},\ldots,\left(\mu_{l-i}^{(1)}\right)^{\alpha_{l-i}},\left(\mu_{l-i+1}^{(1)}\right)^{\alpha_{l-i+1}},\left(\mu_{l-i+2}^{(1)}\right)^{\alpha_{l-i+2}},\ldots,\left(\mu_{l-1}^{(1)}\right)^{\alpha_{l-1}}\right)$$

$$\text{where } n_1=n-r\mu_l, r_1=r-\alpha_l \text{ and } \mu_\varphi^{(1)}=\mu_\varphi-\mu_l\,\forall\,\varphi \qquad (1.2.3)$$

From (1.2.1) we have the number of r_1-*partitions* of n_1 having smallest element *k* is

$$p_{r_1-1}\left[n_1-(k-1)r_1-1\right]+\beta_1$$

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n_1}{r_1}=k\\ 0 & \text{otherwise}\end{cases}$$

$$=p_{r_1-1}\left[(n-r\mu_l)-(k-1)r_1-1\right]+\beta_1 \quad (1.2.4)$$

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n-r\mu_l}{r_1}=k-\mu_l\\ 0 & \text{otherwise}\end{cases}$$

In (1.2.2), the part μ_l may vary from 1 to $\mu_{l-1}-1$ and r_1 may vary from 1 to $r-1$(if $\mu_l=\mu_{l-1}$ or $r_1=r,$ the *partition*(1.2.2) does not have *l* distinct parts.

It contradicts our assumption for $\mu_l>\mu_{l-1}$.)

Therefore the number of $r-$*partitions* of *n* with second smallest part *k* is $f_r^{\,2}(k,n)$

$$f_r^{\,2}(k,n)=\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}p_{r_1-1}\left[(n-r\mu_l)-(k-1)r_1-1\right]+\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\beta_1 \qquad (1.2.5)$$

Continuing this process in(1.2.3), we get

$$n_h=(\lambda_1^{(h)},\lambda_2^{(h)},\ldots,\lambda_{r_h}^{(h)})=\left(\left(\mu_1^{(h)}\right)^{\alpha_1},\left(\mu_2^{(h)}\right)^{\alpha_2},\ldots,\left(\mu_{l-h-1}^{(h)}\right)^{\alpha_{l-h-1}},\left(\mu_{l-h}^{(h)}\right)^{\alpha_{l-h}}\right)$$

where $n_0=n, n_h=n_{h-1}-r_{h-1}\mu_{l-h+1}, r_0=r,\ r_h=r_{h-1}-\alpha_{l-h+1}$ and $\mu_\varphi^{(h)}=\mu_\varphi^{(h-1)}-\mu_{l-h+1}\forall\varphi$

From (1.2.1), we have the number of r_h-*partition* of n_h having smallest part k is

$$p_{r_h-1}\left[n_h-(k-1)r_h-1\right]+\beta_h$$

$$\text{where } \beta_h=\begin{cases}1 & \text{if } r_h\mid n_h\\ 0 & \text{otherwise}\end{cases}$$

Hence the number $f_r^{\,i}(k,n)$ of $r_{i-1}-$*partitions* of n_{i-1} with i^{th} smallest part k as

$$f_r^{\,i}(k,n)=p_{r_{i-1}-1}\left[n_{i-1}-(k-1)r_{i-1}-1\right]+\beta_{i-1}$$

$$\text{where } \beta_{i-1}=\begin{cases}1 & \text{if } r_{i-1}\mid n_{i-1}\\ 0 & \text{otherwise}\end{cases}$$

$$=\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\cdots\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}p_{r_{i-1}-1}\left[\begin{matrix}(n-r\mu_l-r_1\mu_{l-1}-\ldots-r_{i-2}\mu_{l-i+2})\\ -(k-1)(r-\alpha_l\ldots-\alpha_{l-i+2})-1\end{matrix}\right]$$

$$+\sum_{r_1=1}^{r-1}\sum_{\mu_l=1}^{\infty}\cdots\sum_{r_{i-2}=1}^{r_{i-3}-1}\sum_{\mu_{l-i+2}=1}^{\infty}\beta_{i-1}$$

$$\text{where } \beta_{i-1}=\begin{cases}1 & \text{if } \dfrac{n-r\mu_l-r_1\mu_{l-1}-\ldots-r_{i-2}\mu_{l-i+2}}{r_{i-1}}=k-\mu_l\ldots-\mu_{l-i+2}\\ 0 & \text{otherwise}\end{cases}$$

This completes the proof ■

1.3Theorem: The generating function for the number of ith smallest parts of $r-$*partitions* of n such that i^{th} smallest part as first part $(i.e\ \lambda_1$ as i^{th} smallest part$)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_i(n)\right)q^n=\frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{(1-q^{r_{i-1}})}\quad \text{for } i=r\quad (1.3.1)$$

Proof: Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=(\mu^r)$ be any $r-$*partition* of n with all equal parts.

We know that β is the number of smallest parts of $r-$*partitions* of n such that smallest part is the first part which is k $(i.e\ \lambda_1$ as smallest part$)$.

$$\text{where } \beta=\begin{cases}1 & \text{if } \dfrac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}\quad (1.3.2)$$

The generating function for the number of smallest parts of $r-partitions$ of n such that smallest part is the first part $\left(i.e\ \lambda_1 \text{ as smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_1(n)\right)q^n=\sum_{k=1}^{\infty}q^{kr}=\frac{q^r}{\left(1-q^r\right)} \text{ for } r=1 \quad (1.3.3)$$

Let $n=(\lambda_1,\lambda_2,\ldots,\lambda_r)=\left(\mu_1^{\alpha_1},\mu_2^{\alpha_2}\right)$ be any $r-partition$ of n with two distinct parts.

Subtracting μ_2 from each λ_i for $i=1$ to r, we get

$$n_1=\left(\mu_1^{(1)}\right)^{\alpha_1} \text{ where } n_1=n-r\mu_2,\quad r_1=r-\alpha_2 \text{ and } \mu_1^{(1)}=\mu_1-\mu_2$$

The number of smallest parts of $r_1-partitions$ of n_1 such that the smallest part is the first part and having k as a smallest part is β_1

$$\text{where } \beta_1=\begin{cases}1 & \text{if } r_1 \mid n_1\\ 0 & \text{otherwise}\end{cases}$$

Since $n=n_1+r\mu_2$ and $\mu_1=k-\mu_2$, the number of second smallest parts of $r-partitions$ of n such that second smallest part is the first part and having k as a smallest part is β_1

$$\text{where } \beta_1=\begin{cases}1 & \text{if } \dfrac{n-r\mu_2}{r_1}=k-\mu_2\\ 0 & \text{otherwise}\end{cases}$$

Therefore the generating function for the number of second smallest parts of $r-partitions$ of n such that second smallest part is the first part $\left(i.e\ \lambda_1 \text{ as second smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_2(n)\right)q^n=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k-\mu_2=1}^{\infty}q^{\mu_2 r+(k-\mu_2)r_1}$$

$$=\sum_{\mu_2=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{\mu_1=1}^{\infty}q^{\mu_1 r_1+\mu_2 r}$$

$$=\sum_{\mu_2=1}^{\infty}q^{\mu_2 r}\sum_{\mu_1=1}^{\infty}\sum_{r_1=1}^{r-1}q^{\mu_1 r_1}$$

$$=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)} \text{ for } r=2 \quad (1.3.4)$$

Continuing this process, we getthe generating function for the number of ith smallest parts of $r-partitions$ of n such that i^{th} smallest part as first part $\left(i.e\ \lambda_1 \text{ as } i^{th}\text{smallest part}\right)$ is

$$\sum_{n=1}^{\infty}\left(r-spt_i(n)\right)q^n=\frac{q^r}{\left(1-q^r\right)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{\left(1-q^{r_1}\right)}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{\left(1-q^{r_2}\right)}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{\left(1-q^{r_{i-1}}\right)} \text{ for } i=r \ \blacksquare$$

1.4Theorem: The number of smallest parts of $r-partitions$ of n having k as a smallest part is

$$\sum_{i=0}^{\infty} p_{r-1-i}\left[n-(k-1)r-1-i\right]+\beta$$

where $\beta=\begin{cases}1 & \text{if } r\mid n\\ 0 & \text{otherwise}\end{cases}$

Proof: From (1.2.10), the number of $r-partitions$ of n with the smallest part k is

$$f_r(k,n)=p_{r-1}\left[n-(k-1)r-1\right]+\beta$$

Fix $k\in\{1,2,...,n\}$.For $1\le i\le r$ the number of $r-partitions$ of n with the $(r-i)$ smallest parts each being k is the number of $r-partitions$ of $n-(r-i)k$. Summing over $i=1$ to r we get the total number of $r-partitions$ of n with$k$ as the smallest parts .

This number $\sum_{i=0}^{\infty} p_{r-1-i}\left[n-(k-1)r-1-i\right]+\beta$ where $\beta=\begin{cases}1 & \text{if } r\mid n\\ 0 & \text{otherwise}\end{cases}$ ■

1.5Theorem: The generating function for the number of ith smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} r-spt_i(n)q^n=\frac{q^n}{(1-q^n)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{(1-q^{r_{i-1}})}\left(\sum_{r_i=1}^{r_{i-1}-1}\frac{1}{(q)_{r_i}}+1\right)$$

Proof: From theorem1.4, we have the number of smallest parts of $r-partitions$ of n having k as a smallest part is

$$\sum_{i=0}^{\infty} p_{r-1-i}\left[n-(k-1)r-1-i\right]+\beta \text{ where } \beta=\begin{cases}1 & \text{if } \frac{n}{r}=k\\ 0 & \text{otherwise}\end{cases}$$

The generating function for the number of smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} r-spt(n)q^n=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{q^{r-1-i+(k-1)r+1+i}}{(q)_{r-1-i}}+\frac{q^r}{(1-q^r)}$$

$$=\sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\frac{q^{kr}}{(q)_{r-1-i}}+\frac{q^r}{(1-q^r)}$$

$$=\sum_{i=0}^{\infty}\frac{(q^r+q^{2r}+q^{3r}+...)}{(q)_{r-1-i}}+\frac{q^r}{(1-q^r)}$$

$$=\frac{q^r}{(1-q^r)}\sum_{i=1}^{r-1}\frac{1}{(q)_i}+\frac{q^r}{(1-q^r)}$$

$$= \frac{q^r}{(1-q^r)}\left(\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right)$$

$$= \frac{q^r}{(1-q^r)}\left(\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right)$$

$$= \frac{q^r}{(1-q^r)}\left(\sum_{r_1=1}^{r-1}\frac{1}{(q)_{r_1}}+1\right)$$

From theorem 1.4 and theorem1.6, we get the number of second smallest parts of $r-partitions$ of n with second least part $k \neq \lambda_1$ is

$$f_r{}^2(k,n) = \sum_{r=1}^{\infty}\sum_{\mu_0=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{i=1}^{\infty} p_{r_1-1-i}\left[(n-r\mu_0)-(k-1)r_1-1-i\right]$$

The generating function for the number of second smallest parts $\neq \lambda_1$ of $r-partitions$ of n

$$\sum_{r=1}^{\infty}\sum_{\mu_0=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\frac{q^{r_1-1-i+r\mu_0+(k-1)r_1+1+i}}{(q)_{r_1-1-i}}$$

$$= \sum_{r=1}^{\infty}\sum_{\mu_0=1}^{\infty}\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\frac{q^{r\mu_0+kr_1}}{(q)_{r_1-1-i}}$$

$$= \sum_{\mu_0=1}^{\infty}\sum_{r=1}^{\infty} q^{r\mu_0}\left[\sum_{r_1=1}^{r-1}\sum_{k=1}^{\infty}\sum_{i=1}^{\infty}\frac{q^{kr_1}}{(q)_{r_1-1-i}}\right]$$

$$= \frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}\right) \qquad (1.5.1)$$

From(1.3.4) the generating function for the number of second smallest parts of $r-partitions$ of n with second smallest part equal to λ_1 is

$$\frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})} \quad \text{for } r=2 \qquad (1.5.2)$$

From (1.5.1) and (1.5.2) we get the generating function for the number of second smallest parts of $r-partitions$ of n which is given by

$$\sum_{n=1}^{\infty} r-spt_2(n)q^n = \frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}\right)+\frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}$$

$$= \frac{q^r}{(1-q^r)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\left(\sum_{r_2=1}^{r_1-1}\frac{1}{(q)_{r_2}}+1\right) \qquad (1.5.3)$$

By induction, the generating function for the number of ith smallest parts of $r-partitions$ of n is

$$\sum_{n=1}^{\infty} r-spt_i(n)q^n = \frac{q^n}{(1-q^n)}\sum_{r_1=1}^{r-1}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_{i-1}=1}^{r_{i-2}-1}\frac{q^{r_{i-1}}}{(1-q^{r_{i-1}})}\left(\sum_{r_i=1}^{r_{i-1}-1}\frac{1}{(q)_{r_i}}+1\right)\blacksquare$$

1.7 Corollary:

The generating function for the number of $k's$ as smallest part of

$r-partitions$ of n is

$$q^{kr}\left[\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right]$$

1.8 Corollary: The generating function for the number of $k's$ as smallest parts of *partitions*of *n* is

$$\sum_{r=1}^{\infty} q^{kr}\left[\sum_{i=1}^{r-1}\frac{1}{(q)_i}+1\right]$$

1.9 Corollary: The generating function for the number of $r-partitions$ of n having i distinct integers is

$$\sum_{r_1=1}^{\infty}\frac{q^{r_1}}{(1-q^{r_1})}\sum_{r_2=1}^{r_1-1}\frac{q^{r_2}}{(1-q^{r_2})}\cdots\sum_{r_i=1}^{r_{i-1}-1}\frac{q^{r_i}}{(1-q^{r_i})}$$

1.10 Corollary: The generating function for the number of smallest parts of $r-partitions$ of n which are multiples of c is

$$\sum_{n=1}^{\infty} c\left(r-spt_i(n)\right)q^n = \frac{q^{cr}}{(1-q^{cr})}\left[\sum_{r_1=1}^{r-1}\frac{1}{(q)_{r_1}}+1\right]$$

References

1. Andrews G. E. (1998), The Theory of *Partitions*, *Cambridge University Press*, Cambridge. MR **99c:** 11126.
2. B.Hanuma Reddy K. (2010), A Study of $r-partitions$, submitted to AcharyaNagarjuna University, awarded of Ph.D. in Mathematics.
3. Hanuma Reddy K, Manjusree A (2015), The number of smallest parts in the *partitions* of *n*, International Journal in IT and Engineering, vol.03, Issue-03, ISSN:2321-1776.

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On Semicircular Extreme-value distribution

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Abstract

This paper is aimed at construction of Semicircular Extreme-value distribution for modeling semicircular data by applying simple projection method on Extreme-value distribution. It is extended it to the l-axial Extreme-value distribution by a simple projection for modeling any arc of arbitrary length.

Keywords: characteristic function, semi circular models, l-axial data, projection, trigonometric moments.

AMS Subject Classification: 60E05, 62H11

1. INTRODUCTION

In some of the cases the directional / angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004), Jones (1968) and Byoung et al(2008). For example, when a sea turtle emerges from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data is available in Ugai et al (1977).

Guardiola (2004) obtained the semicircular normal distribution by using a simple projection and Byoung et at (2008) developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Phani et al (2013) constructed some semicircular distributions by applying inverse stereographic projection. In this paper the **Semicircular Extreme-value distribution** (**SCEVD**) is developed by projecting Extreme-value distribution over a semicircular segment. The graphs of the density and distribution functions for various values of parameters are plotted. The characteristic function is also derived. This model is extended to l-axial model.

2. SEMICIRCULAR EXTREME-VALUE DISTRIBUTION

A random variable X on the real line is said to have Extreme-value Distribution with location parameter γ and scale parameter $\lambda>0$, if the probability density function and cumulative distribution function of X for $x,\gamma\in\mathbb{R}$ and $\lambda>0$ are given respectively by

$$f(x)=\frac{1}{\lambda}\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\exp\left(-\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right),$$

where $\lambda>0$, and $\gamma,\ x\in\mathbb{R}$ (2.1)

$$F(x)=\exp\left(-\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right)\ . \qquad (2.2)$$

Simple projection is defined by

mapping $x=v\tan(\theta)$ or $\theta=\tan^{-1}\left(\frac{x}{v}\right),\ v>0\in\mathbb{R}$.

Application of this simple projection on Extreme – value distribution results to a Semicircular Extreme-value distribution.

Definition:

A random variable X_{SC} on the Semicircle is said to have the Semicircular Extreme-value distribution with location parameter μ scale parameter $\sigma>0$ denoted by **SCEVD** (σ,μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta)=\frac{1}{\sigma}\sec^2(\theta)\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\},$$

where, $\mu=\frac{\gamma}{v}$, $\sigma=\frac{\lambda}{v}>0$ (2.3)

$$G(\theta)=\exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\},\ \text{where } \sigma>0,\ -\frac{\pi}{2}<\theta<\frac{\pi}{2} \qquad (2.4)$$

Hence the proposed new model **SCED**(σ,μ) is a Semicircular model.

Graphs of the probability density and cumulative distribution functions of the Semicircular Extreme-value distribution for various values of σ and μ

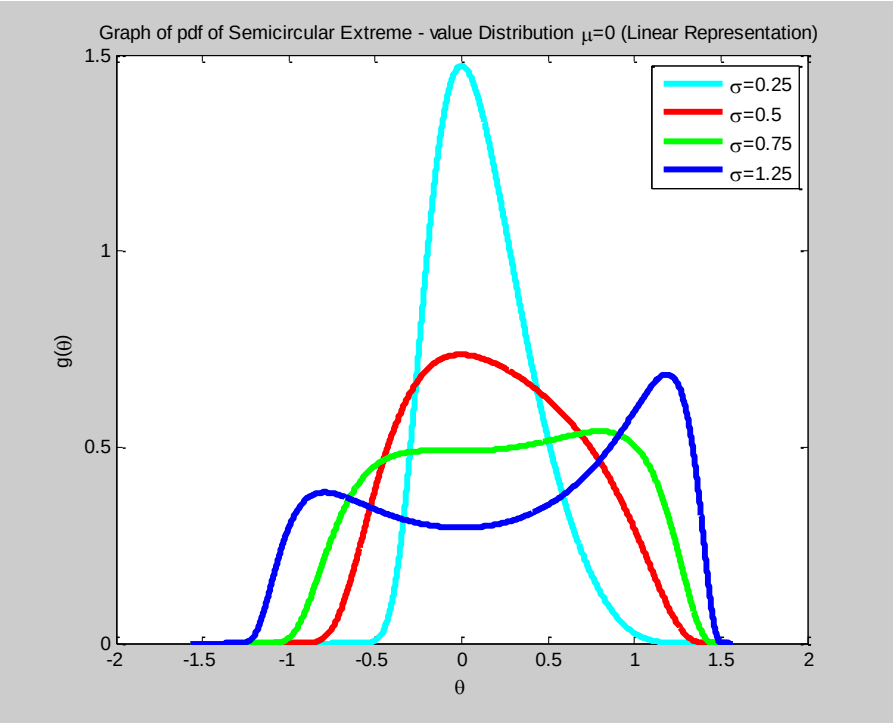

Fig1. Graph of pdf of Semicircular Extreme – value Distribution (Linear Representation)

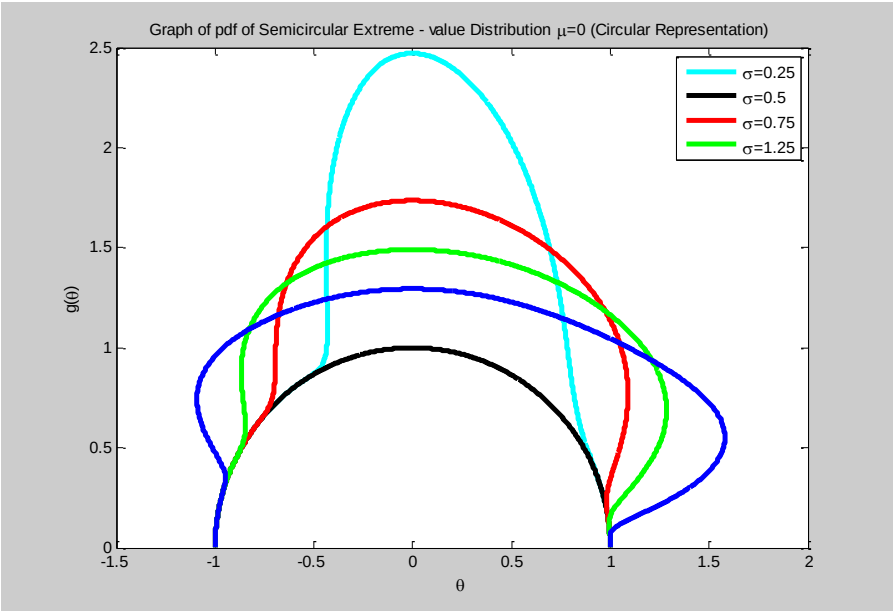

Fig2. Graph of pdf of Semicircular Extreme – value Distribution (Circular Representation)

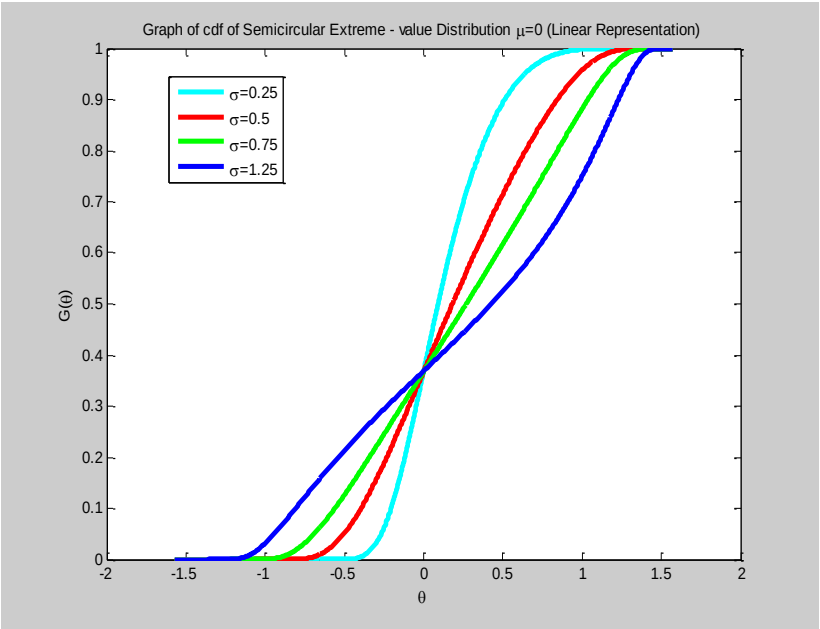

Fig3. Graph of cdf of Semicircular Extreme – value Distribution

3. The Characteristic function of Semicircular Extreme-value distribution

The characteristic function of a semicircular model with probability density function $g(\theta)$ is defined as

$$\Phi_{X_{SC}}(p)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta} g(\theta)\,d\theta \ , p\in\mathbb{Z}.$$

$$\Phi_{X_{SC}}(p)=\frac{1}{\sigma}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta}\sec^2(\theta)\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\}d\theta \ , p\in\mathbb{Z} \qquad (3.1)$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function. Population characteristics are also studied using first two trigonometric moments.

Graphs of the characteristic function of Semicircular Extreme -value distribution for various values of parameter are plotted.

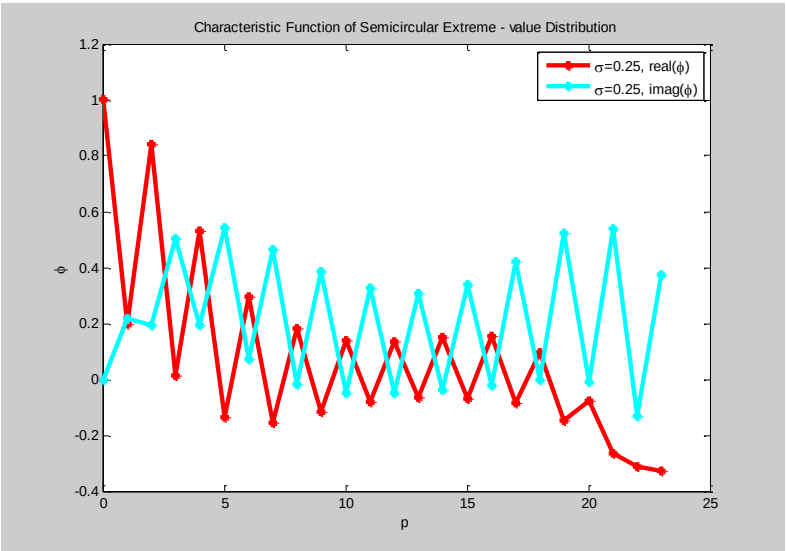

Fig4. Graph of Characteristic function of Semicircular Extreme – value Distribution

The expressions for mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions are available in Mardia and Jupp (2000). These characteristics for the Semicircular Extreme - value distribution are also based on their respective trigonometric moments and can be expressed in terms of trigonometric moments α_p and β_p which are presented here.

From the population characteristics, it can be observed that with increasing value of scale parameter σ , the circular variance gradually decrease, the distribution is negatively skewed and remained platykurtic

Table 3.1 Population Characteristics of Semicircular Extreme – value model

	$\sigma = 0.25$	$\sigma = 0.5$	$\sigma = 0.75$	$\sigma = 1.25$
Trigonometric Moments				
α_1	0.1995	0.1074	-0.0183	-0.1525
α_2	0.8426	0.6299	0.4812	0.1586
β_1	0.2176	0.3461	0.3727	0.2671
β_2	0.1926	0.2203	0.2488	0.3660
Resultant Length				
ρ_1	0.2952	0.3624	0.3732	0.3076
ρ_2	0.8643	0.6673	0.5417	0.3989
Mean Direction μ_0	0.8288	1.2700	-1.5218	-1.0518
Circular Variance v_0	0.7048	0.6376	0.6268	0.6124
Circular Standard Deviation				
σ_0	1.5622	1.4249	1.4041	1.5356
	0.5400	0.8994	1.1074	1.3558
Central Trigonometric Moments				
α_1^*	0.2952	0.3624	0.3732	0.3076
α_2^*	0.1188	-0.3947	-0.5032	-0.3958
β_1^*	0	0	0	0
β_2^*	-0.8561	-0.5381	-0.2005	-0.0493
Skewness γ_1^0	-1.4467	-1.0568	-0.4040	-0.0856
Kurtosis γ_2^0	0.2239	-1.0131	-1.3299	-0.8442

4 Extension to l -axial distribution

The newly proposed model is extended to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l = 1, 2, ...,$. So it is desirable to extend the Semicircular Extreme-value distribution to its *l* – axial model. To construct the l -axial Extreme-value distribution, the density function of Semicircular Extreme-value distribution is considered and the transformation $\phi = 2\theta/l$, $l = 1, 2, ...,$ is used. The probability density function of ϕ is given by

$$g(\phi) = \frac{l}{2\sigma}\sec^2\left(\frac{l\phi}{2}\right) e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}} e^{-e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}}$$

$$\text{where} \quad -\frac{\pi}{l} < \phi < \frac{\pi}{l} \qquad (4.1)$$

Case (1) When $l=2$, the probability density function (4.1) is the same as the probability density function of Semicircular **Extreme-value distribution .**

Case (2) When $l=1$, the probability density function (4.1) is the same as that of **Stereographic Extreme-value Distribution** [Phani et al., 2012] which is circular distribution.

If X has Extreme-value with scale parameter λ and location parameter $\gamma=0$, then $Y=e^X$ has a **log-Extreme-value distribution** with the same parameters. So, using $y=v\tan(\theta)$, a probability density function is

$$g(\theta)=\frac{2}{\lambda\sin(2\theta)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan(\theta)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan(\theta)\right)\right)}} \qquad 0<\theta<\frac{\pi}{2}, \qquad (4.2)$$

Suppose $\phi=\frac{4\theta}{l}$, $l=1,2,...,$ then ϕ has a probability density function ,

$$g(\phi)=\frac{1}{2\lambda\sin\left(\frac{l\phi}{2}\right)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{l\phi}{4}\right)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{l\phi}{4}\right)\right)\right)}},\ 0<\theta<\frac{2\pi}{l} \qquad (4.3)$$

It is said that ϕ follows the l **- axial log-Extreme-value distribution**.

Case (1): When l =1,

$$g(\phi)=\frac{1}{2\lambda\sin\left(\frac{\phi}{2}\right)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{4}\right)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{4}\right)\right)\right)}},\ 0<\theta<2\pi \qquad (4.4)$$

It is called as a **Circular log-Extreme-value distribution**.

Case (2): When l = 2 ,

$$g(\phi)=\frac{1}{2\lambda\sin(\phi)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{2}\right)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{2}\right)\right)\right)}},\ 0<\theta<\pi \qquad (4.5)$$

It is called as a **Semicircular log-Extreme-value distribution**.

Case (3): When l = 4 ,

$$g(\phi)=\frac{1}{2\lambda\sin(2\phi)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan(\theta)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan(\theta)\right)\right)}},\ 0<\theta<\frac{\pi}{2} \qquad (4.6)$$

REFERENCES

[1] Abramowitz, M. and Stegun, I.A. (1965). Handbook of Mathematical Functions, *Dover, New York*.

[2] Byoung, J.A and Hyoung M.K.(2008). A New Family of Semicircular Models: The Semicircular Laplace Distributions, Communications of the Korean Statistical Society Vol.15, pp. 775-781.

[3] Gradshteyn and Ryzhik (2007). Table of Integrals, series and products, 7th edition, Academic Press.

[4] Guardiola, J.H. (2004). The Semicircular Normal Distribution, Ph.D Dissertation, Baylor University, Institute of Statistics.

[5] Jammalamadaka S. Rao and Sen Gupta, A. (2001). Topics in Circular Statistics, *World Scientific Press, Singapore.*

[6] Jones, T.A. (1968). Statistical Analysis of orientation data, Journal of Sedimentary Petrology, 38, 61-67.

[7] Kim,H.M.(2008). New family of the t distributions for modelling semicircular data, Communications of the Korean Statistical Society.

[8] Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, *John Wiley, Chichester.*

[9] Ugai.S.,K., Nishijima, M. and Kan,T.(1977), Characteristics of raindrop size and raindrop shape, *Open symposium* URSI *Commission F*, 225-230.

[10] Phani Y, Girija S.V.S., and Dattatreya Rao A.V. (2013). On Construction of Stereographic Semicircular Models, Journal of Applied Probability and Statistics. Vol 8, No. 1, pp. 75-90.

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On Discrete Wrapped Cauchy Model

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Abstract
Modeling angular data throws many challenges in practical situations. Good number of circular / semicircular models are developed for modeling continuous circular / angular data. Scant attention was paid in analysis of discrete angular data, in particular, construction of discrete angular models for fitting angular data is not touched so far. Hence an attempt is made to develop method for constructing Discrete *l* - axial models and study their population characteristics.

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1. Introduction

In some of the cases the directional / angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004), Jones (1968) and Byoung et al (2008). For example, in search of a nesting site on dry land sea turtles come out from the ocean, a random variable with points on a semicircle is well adequate for modeling such data. Also, if an aircraft is lost and its departure and its initial headings are identified, a semicircular random variable is adequate for modeling such axial / angular data. And some more examples of semicircular data are presented in Ugai et al (1977). Phani et al (2013) derived certain semicircular distributions by employing inverse stereographic projection.

Good number of circular / semicircular models are developed for modeling continuous circular / angular data. Dattatreya rao et al (2007) constructed wrapped versions of Lognormal, Logistic, Weibull and Extreme - value distributions. Scant attention was paid in analysis of discrete angular data, in particular, construction of discrete angular models for fitting angular data is not touched so far. Hence an attempt is made to develop method for constructing Discrete *l* - axial models and study their population characteristics.

For statistical analysis of discrete directional data existing linear distributions will not yield suitable results. Srihari et al (2018a; 2018b; 2018c) derived the Discrete Wrapped Exponential Distribution, the Wrapped Negative Binomial model and the Wrapped Logarithmic Distribution respectively. Here an attempt is made to construct a new discrete circular model by applying the method of discretization for existing continuous circular model. By considering the Wrapped Cauchy Distribution, discretization is employed to transform it as Discrete Wrapped Cauchy distribution which is a circular model. The probability mass function, distribution function and characteristic function of the new discrete circular model are obtained and population characteristics are studied . Discrete Wrapped *l* - axial Cauchy distribution by extending the Discrete Wrapped Cauchy distribution is derived

which is suitable for modeling discrete axial data.

2. Circular Distributions

A circular distribution is a probability distribution whose total probability is defined on the unit circle. Since each point on the unit circle represents a direction, it is a way of assigning probabilities to different directions. The range of a circular random variable θ measured in radians, can be taken as $[0, 2\pi)$ or $[-\pi, \pi)$.

Circular distributions are of two types, they may be discrete, assigning probabilities to a finite number of directions or continuous assigning probabilities to a infinite number of directions. One of the construction methods of circular models is wrapping a linear distribution around a unit circle. Here Wrapping methodology for the construction of Discrete Circular model is discussed.

2.1. Discrete Wrapped Circular Random Variables

If *X* is a discrete random variable on the set of integers, then reduction modulo $2\pi m\left(m \in \mathbf{Z}^{+}\right)$ wraps the integers on to the group of mth roots of unity which is a sub group of unit circle.

$$\theta = 2\pi x \left(\mathrm{mod}\, 2\pi m\right)$$

Since θ contains a finite number of elements they are denoted by $\theta = \left\{ \frac{2\pi r}{m} \Big/ r = 0,1,2,\ldots m-1 \right\}$ which is lattice on the unit circle.

2.2. Probability Mass Function

Suppose if θ is a wrapped discrete circular random variable then probability mass function of θ is denoted by $Pr\left(\theta = \frac{2\pi r}{m}\right)$ which satisfies the following properties

1. $Pr\left(\theta = \frac{2\pi r}{m}\right) \geq 0$

2. $\sum_{r=0}^{m-1} Pr\left(\theta = \frac{2\pi r}{m}\right) = 1$

3. $Pr(\theta) = Pr(\theta + 2\pi l)$ For any integer l (i.e Pr is periodic)

2.3. Distribution Function

Suppose if θ is a wrapped discrete circular random variable then distribution function of θ is denoted by $F_w(\theta)$ and it is defined as

$$F_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right) \quad \text{where } k = 0,1,...m-1$$

In mathematics, discretization is the process of transforming continuous functions, models, variables, and equations into its discrete counterparts. This process is usually carried out to model a given discrete phenomenon depending on the practical situation.

3. Discretization of a Continuous Distribution

In this paper, we adopt discretization from Min – Zhen Wang and Kunio Shimizu (2014) to construct discrete circular models by transforming existing continuous circular models. The process is as follows

Consider a continuous circular distribution with probability density function (pdf) $f(\theta)$ on the circle to construct a probability mass function $Pr\left(\theta = \frac{2\pi r}{m}\right)$ on a set of the equally spaced points for $r = 0,1,2...m-1$ with a fixed integer m equal or greater than 2.

$Pr\left(\theta = \frac{2\pi r}{m}\right) = f\left(\frac{2\pi r}{m}\right) \Big/ \sum_{r=0}^{m-1} f\left(\frac{2\pi r}{m}\right)$ is probability mass function(pmf) of respective discrete circular model. $m = 1$ is excluded as it is degenerate.

4. Discrete Wrapped Cauchy Distribution

The probability density function of wrapped Cauchy distribution is defined as

$$f(\theta) = \frac{1}{2\pi}\frac{1-\rho^2}{1+\rho^2-2\rho\cos(\theta-\mu)} \quad \text{where } \mu, \sigma \text{ are parameters}$$
$$\text{and } \rho = e^{-\sigma}(\sigma > 0) \text{ and } 0 \le \theta, \mu < 2\pi \qquad (4.1)$$

Now the probability mass function of the Discrete Wrapped Cauchy Distribution is defined as

$$g_W(\theta) = Pr\left(\theta = \frac{2\pi r}{m}\right) = \frac{\frac{1}{2\pi}\frac{1-\rho^2}{1+\rho^2-2\rho\cos(\theta-\mu)}}{\sum_{r=0}^{m-1}\frac{1}{2\pi}\frac{1-\rho^2}{1+\rho^2-2\rho\cos(\theta-\mu)}}$$

$$= \frac{\frac{1}{1+\rho^2-2\rho\cos(\theta-\mu)}}{\sum_{r=0}^{m-1}\frac{1}{1+\rho^2-2\rho\cos(\theta-\mu)}}$$

(4.2)

$$\text{Consider } \sum_{r=0}^{m-1}\frac{1}{1+\rho^2-2\rho\cos(\theta-\mu)} = \frac{1}{1+\rho^2-2\rho\cos\mu} + \frac{1}{1+\rho^2-2\rho\cos\left(\frac{2\pi}{m}-\mu\right)} + \ldots$$

$$+ \frac{1}{1+\rho^2-2\rho\cos\left(\frac{2\pi(m-1)}{m}-\mu\right)}$$

$$= \frac{m\left(1-\rho^{2m}\right)}{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m\cos m\mu\right)}$$

$$\text{Hence } g_W(\theta) = Pr\left(\theta = \frac{2\pi r}{m}\right) = \frac{1}{m}\,\frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m\cos m\mu\right)}{\left(1-\rho^{2m}\right)\left(1+\rho^2-2\rho\cos(\theta-\mu)\right)}$$

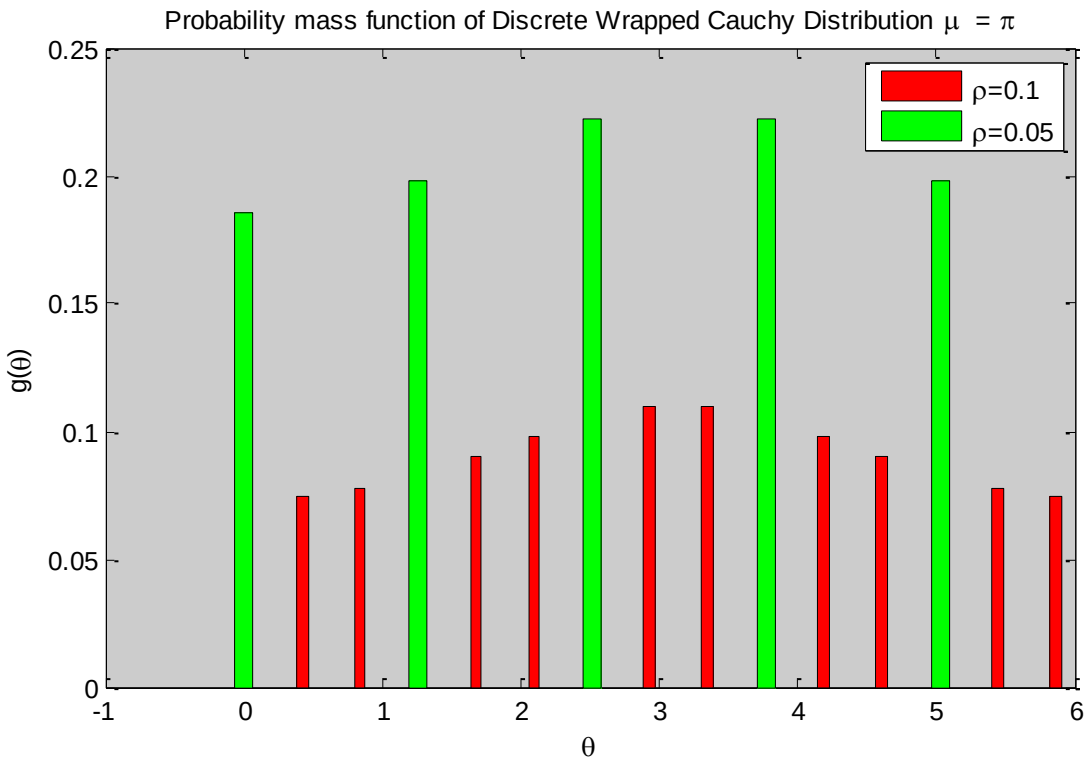

Fig 1 Graph for Probability mass function of Discrete Wrapped Cauchy Distribution

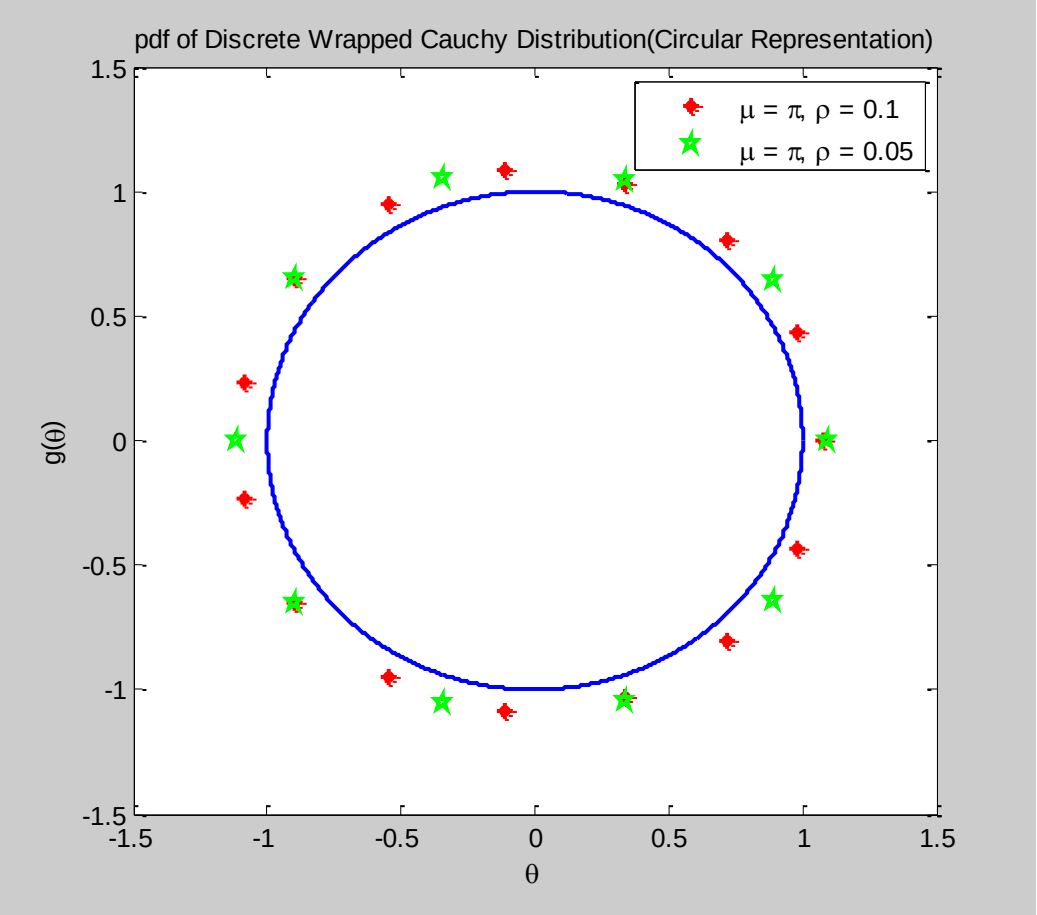

Fig 2 Graph for Probability mass function of Discrete Wrapped Cauchy Distribution (Circular representation)

5. Distribution Function of Discrete Wrapped Cauchy Distribution

The Distribution function of Discrete Wrapped Cauchy Distribution is defined as

$$G_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right) = \sum_{r=0}^{k} \left(\frac{\dfrac{1}{1+\rho^2 - 2\rho\cos\left(\dfrac{2\pi r}{m} - \mu\right)}}{\displaystyle\sum_{r=0}^{m-1} \frac{1}{1+\rho^2 - 2\rho\cos(\theta - \mu)}} \right)$$

$$\text{where } 0 \le k \le m-1$$

(5.1)

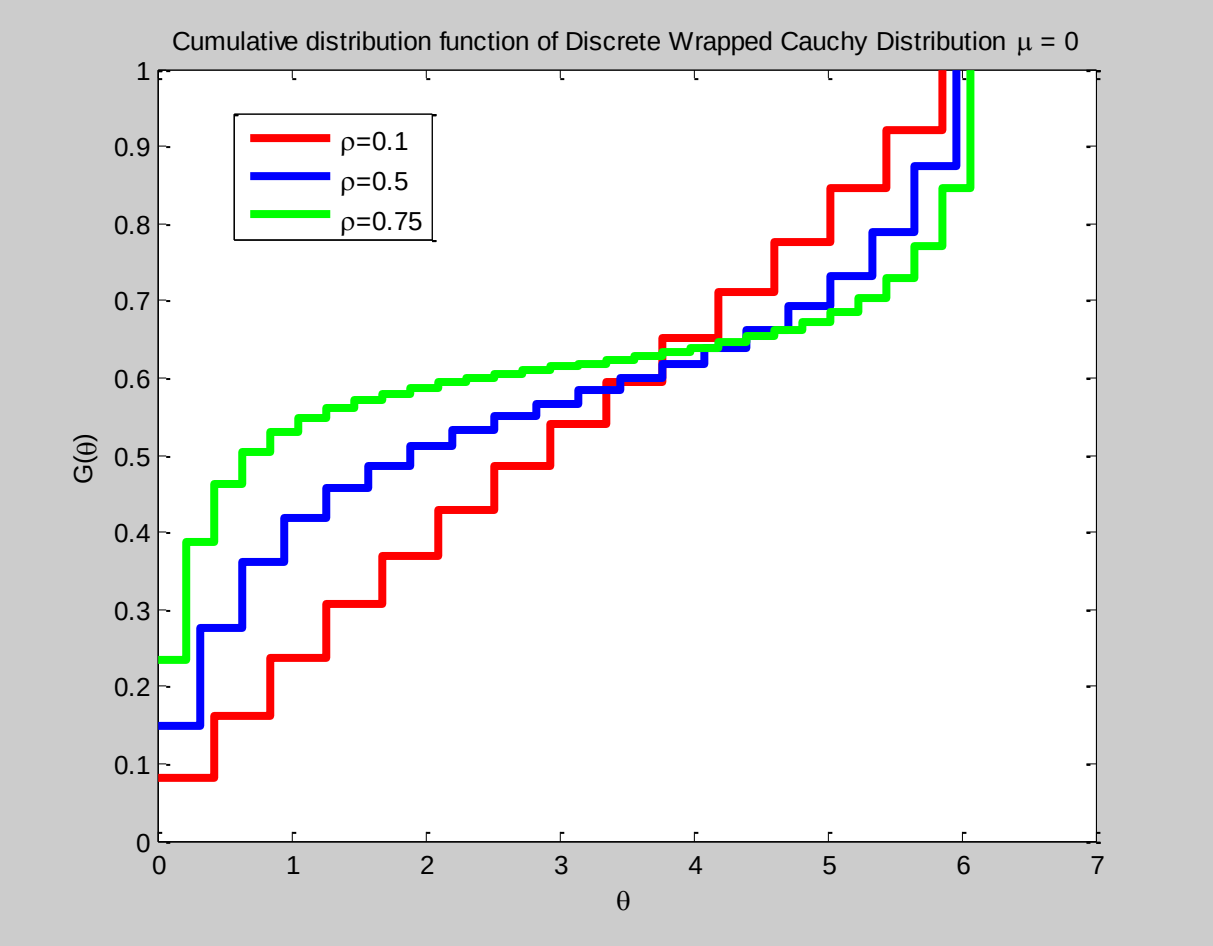

Fig 3 Graph for Cumulative distribution function of Discrete Wrapped Cauchy Distribution

6. Characteristic function of the Discrete Wrapped Cauchy Distribution

The characteristic function of the Discrete Wrapped Cauchy Distribution is defined as $\varphi_\theta(p) = E(e^{ip\theta})$ where $p \in ¢$

$$= \sum_{r=0}^{m-1} e^{ip\theta} Pr\left(\theta = \frac{2\pi r}{m}\right)$$

$$= \frac{1}{m} \frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m \cos m\mu\right)}{\left(1-\rho^{2m}\right)} \sum_{r=0}^{m-1} \frac{e^{ip\theta}}{1+\rho^2-2\rho\cos(\theta-\mu)}, \quad m \in ¥ \tag{6.1}$$

The real and imaginary parts α_p and β_p respectively of the characteristic function are known as p^{th} trigonometric moments and population characteristics are evaluated using first two trigonometric moments.

$$\alpha_p = \frac{1}{m} \frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m \cos m\mu\right)}{\left(1-\rho^{2m}\right)} \sum_{r=0}^{m-1} \frac{\cos p\theta}{1+\rho^2-2\rho\cos(\theta-\mu)}, \quad m \in ¥ \tag{6.2}$$

$$\beta_p = \frac{1}{m}\frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m\cos m\mu\right)}{\left(1-\rho^{2m}\right)}\sum_{r=0}^{m-1}\frac{\sin p\theta}{1+\rho^2-2\rho\cos(\theta-\mu)},\ m\in ¥ \tag{6.3}$$

Clearly $\rho_p = \sqrt{\alpha_p^2+\beta_p^2}$ and $\mu_p = \tan^{-1}\left[\frac{\beta_p}{\alpha_p}\right]$

Now μ_1 represents mean direction and $\rho_1(0\le\rho_1\le 1)$ represents concentration towards the mean direction

In general $\mu_1=\mu$ and $\rho_1=\rho$

With first two trigonometric moments population characteristics are evaluated using MATLAB.

7. Discrete Wrapped *l* - axial Cauchy Distribution

We extend the above **Discrete Wrapped Cauchy Distribution which is a circular model** to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$. So it is desirable to extend the **Discrete Wrapped Cauchy Distribution** to construct the **Discrete Wrapped *l* - axial Cauchy Distribution**, we consider the density function of **Discrete Wrapped Cauchy Distribution** and use the transformation $\phi=\frac{\theta}{l}$, $l=1,2,...,$. The probability mass function of ϕ is given by

$$g_W(\phi) = Pr\left(\phi=\frac{2\pi r}{lm}\right)=\frac{l}{m}\frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m\cos ml\mu\right)}{\left(1-\rho^{2m}\right)\left(1+\rho^2-2\rho\cos\left(l\left(\phi-\mu\right)\right)\right)}\ ,\ m\in ¥\ ,$$

$$0\le\phi,\mu<\frac{2\pi}{l},\ \rho=e^{-\sigma}(\sigma>0)\text{ and }l=1,2,.. \tag{7.1}$$

It is coined as **Discrete Wrapped *l* - axial Cauchy Distribution**

Case (1) When $l=1$, the probability density function is

$$g_W(\phi) = Pr\left(\phi=\frac{2\pi r}{m}\right)=\frac{1}{m}\frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m\cos m\mu\right)}{\left(1-\rho^{2m}\right)\left(1+\rho^2-2\rho\cos\left(\phi-\mu\right)\right)}$$

$$m\in ¥,\ 0\le\phi<2\pi,\ \rho=e^{-\sigma}(\sigma>0) \tag{7.2}$$

It is called the **Discrete Wrapped Circular Cauchy Distribution**.

Case (2) When $l=2$, the probability density function is the **Discrete Wrapped Semicircular Cauchy Distribution**

$$g_W(\phi)=Pr\left(\phi=\frac{\pi r}{m}\right)=\frac{2}{m}\frac{\left(1-\rho^2\right)\left(1+\rho^{2m}-2\rho^m\cos 2m\mu\right)}{\left(1-\rho^{2m}\right)\left(1+\rho^2-2\rho\cos\left(2\left(\phi-\mu\right)\right)\right)}\ ,\ m\in ¥\ ,$$

$$0\le\phi,\mu<\pi,\ \rho=e^{-\sigma}(\sigma>0) \qquad (7.3)$$

Conclusion: A new discrete circular model is constructed which was not figured out so far. Its pdf , cdf and characteristic function are derived and population characteristics are studied. This discrete circular model is extended to its l – axial model.

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References

1. Byoung, J. A., and Hyoung-Moon K. (2008). A New Family of Semicircular Models: The Semicircular Laplace Distributions, Communications of the Korean Statistical Society. Vol.15 (5), 775-781.

2. DattatreyaRao, A.V., RamabhadraSarma, I. and Girija, S.V.S. (2007). On Wrapped Version of Some Life Testing Models, Comm. Statist., - Theor. Meth. 36, 11, pp.2027-2035.

3. Gaurdiola, J.H. (2004). The Semicircular Normal Distribution, Ph.D Dissertation, Baylor University, Institute of Statistics.

4. Girija, S.V.S "Construction of New Circular Models",VDM VERLAG, Germany. ISBN 978-3-639-27939-9, 2010.

5. Girija, S.V.S , A.V.DattatreyaRao and G.V.L.N.Srihari ,"On Wrapped Binomial Model Characteristics", Horizon Research PublicationsVolume2(7),pp.231-234,2014(a).

6. Girija, S.V.S, A.V.DattatreyaRao and G.V.L.N.Srihari, "On Characteristic Function of Wrapped Poisson Distribution", *International Journal of Mathamatical archive* Volume5(5),pp.168-173,2014(b).

7. Girija, S.V.S, A.V.DattatreyaRao and Y.Phani, "*On Stereographic Lognormal Distribution",* International Journal of Advances in Applied Sciences (IJAAS),Vol. 2 No.3, pp.125-132,2013(a).

8. Mardia, K.V. and P.E.Jupp, “Directional Statistics” 2nd Edition , *Wiley, New York 2001.*

9. Min – Zhen Wang and Kunio Shimizu, “Discrete Cardioid Distribution”, presented in Conference on Advances and Applications in Distribution Theory, The Institute of Statistical Mathematics, Tokyo, Japan,2014.

10. Phani, Y., Girija, S.V.S. and Dattatreya Rao, A.V. (2013). On Construction of Stereographic Semicircular Models, Journal of Applied Probability and Statistics, Vol. 8, no. 1, May 2013, pp. 75-90.

11. Rao S. Jammalamadaka and A.Sen Gupta, “Topics in Circular Statistics, *World Scientific Press”, Singapore,2014.*

12. Srihari G.V.L.N ,Girija S.V.S, and Dattatreya Rao A.V., *"On Discrete Wrapped Exponential distribution-Characteristics"*, International Journal of Scientific Research In Mathematical and Statistical Sciences, volume 5(2),pp.57-64(2018a).

13. Srihari G.V.L.N ,Girija S.V.S, and Dattatreya Rao A.V., *"On Wrapped Negative Binomial Model"*, International Journal of Mathematical archive, Volume 9(5),pp.1-6(2018b).

14. Srihari G.V.L.N ,Girija S.V.S, and Dattatreya Rao A.V. *"On Characteristics of Wrapped Logarithmic Model"*, International Journal of Current Trends In Science and Technology, volume 8(4), pp.20413-20419, (2018c).

15. Ugai, S., Kato, K., Nishijima, M. and Kan, T.(1977). Characteristics of raindrop size and raindrop shape, Open Symposium URSI Commission F,225-230.

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On Stereographic Semicircular Quasi Lindley Distribution

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ABSTRACT: In this paper, we made an attempt to construct a new Semicircular model, we call this as "**StereographicSemicircular Quasi Lindleydistribution**," by applying modified inverse Stereographic projection on Quasi Lindley distribution (Rama and Mishra (2013)) for modeling semicircular data. Probability density and cumulative distribution functions of said model are derived and their graphs are plotted for various values of parameters. The first two trigonometric moments are derived and the proposed model is extended for *l*-axial data also.
Keywords: Semicircular models, inverse stereographic projection, trigonometric moments.
AMS Subject Classification: 60E05, 62H11.

1. Introduction

Angular data/ circular data are very common in the areas of biology, geology, meteorology, earth science, political science, Economics, computer science, etc. Full circular models are prevalent at most of the text books (Fisher, 1993; Jammalamadaka and Sen Gupta, 2001; Mardia and Jupp, 2000). In some of the cases the angular data does not require full circular models for modeling; this fact is noted in Guardiola (2004), Jones (1968), Byoung et al (2008) and Phani et al (2013). For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data are available in Ugai et al (1977).

Guardiola (2004) obtained the semicircular normal distribution by using a simple projection and Byoung et al (2008) developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Phani et al (2013) constructed some semicircular distributions by applying modified inverse Stereographic projection, Girija et al (2014) developed a family of semicircular Logistic distributions by applying simple projection. In this paper we develop new Semicircular model coined as Stereographic Semicircular Quasi Lindley distribution by applying modified inverse stereographic projection on Quasi Lindley distribution. The graphs of the density function and distribution function for various values of parameters are plotted. We derive the first two trigonometric moments of the proposed model to evaluate population characteristics.

2. Methodology of modified Inverse Stereographic Projection

Modified Inverse Stereographic Projection is defined by a one to one mapping given by $T(\theta) = x = v\tan\left(\frac{\theta}{2}\right)$, where $x \in (-\infty, \infty)$, $\theta \in (-\pi, \pi)$, $v \in ¡^{+}$. Suppose x is randomly chosen on the interval $(-\infty, \infty)$. Let $F(x)$ and $f(x)$ denote the Cumulative distribution and probability density functions of the linear random variable X respectively. Then $T^{-1}(x) = \theta = 2\tan^{-1}\left\{\frac{x}{v}\right\}$ by Minh and Farnum (2003) is a random point on the unit circle. Let $G(\theta)$ and $g(\theta)$ denote the Cumulative distribution and probability density functions of this random point θ respectively. Then $G(\theta)$ and $g(\theta)$ can be written in terms of $F(x)$ and $f(x)$ using the following Theorem.

Theorem 2.1: For $v > 0$,

i) $G(\theta) = F\left(v\tan\left(\frac{\theta}{2}\right)\right)$

ii) $$g(\theta) = v\left(\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{2}\right) f\left(v\tan\left(\frac{\theta}{2}\right)\right)$$

If a linear random variable X has a support on $¡$, then θ has a support on $(-\pi, \pi)$ and if X has a support on $¡^{+}$, then θ has a support on $(0, \pi)$. These means that, after the Inverse Stereographic Projection is applied, we can deal circular data if the support of X is on $¡$ and we can handle semicircular data if the support of X is on $¡^{+}$.

3. Stereographic Semicircular Quasi Lindley Distribution

Here a linear model, Quasi Lindley distribution is considered and by inducing modified inverse stereographic projection, a stereographic semicircular model is defined.

Definition: A random variable X on the real line is said to have Quasi Lindley Distribution with scale parameter $\alpha > 0$, shape parameter $\sigma > 0$ and location parameter α if the probability density and cumulative distribution functions of X are respectively given by

$$f(x;\alpha,\sigma) = \frac{\alpha}{(1+\sigma)}(\sigma + \alpha x)\exp(-\alpha x),\ \sigma > -1,\ 0 < \alpha \text{ and } 0 < x < \infty \tag{3.1}$$

and

$$F(x;\alpha,\sigma)=1-\exp(-\alpha x)\left[1+\frac{\alpha x}{(\sigma+1)}\right],\ \sigma>-1,\ 0<\alpha \text{ and } 0<x<\infty \tag{3.2}$$

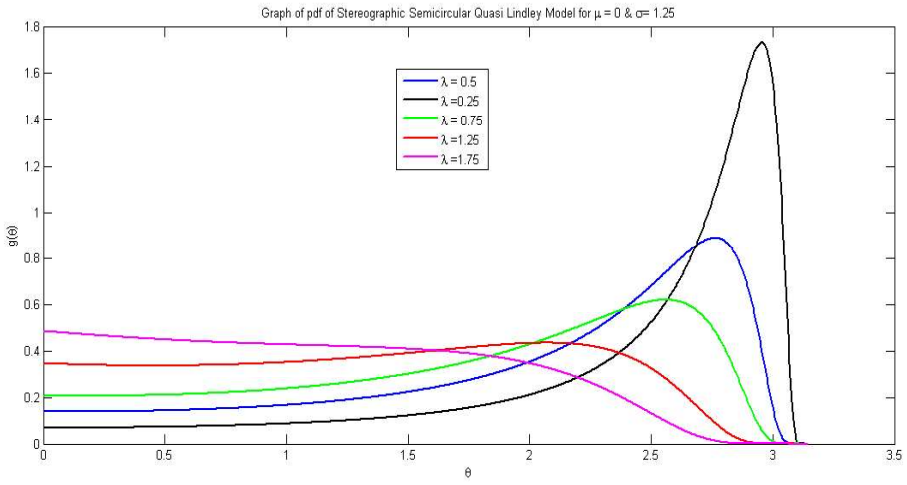
Then by applying modified inverse Stereographic projection defined by a one to one mapping $x=v\tan\left(\frac{\theta}{2}\right), v\in\mathbf{R}^{+}$, which leads to a Semicircular Model on unit semicircle. We call this model as Stereographic Semicircular Quasi Lindley Distribution.

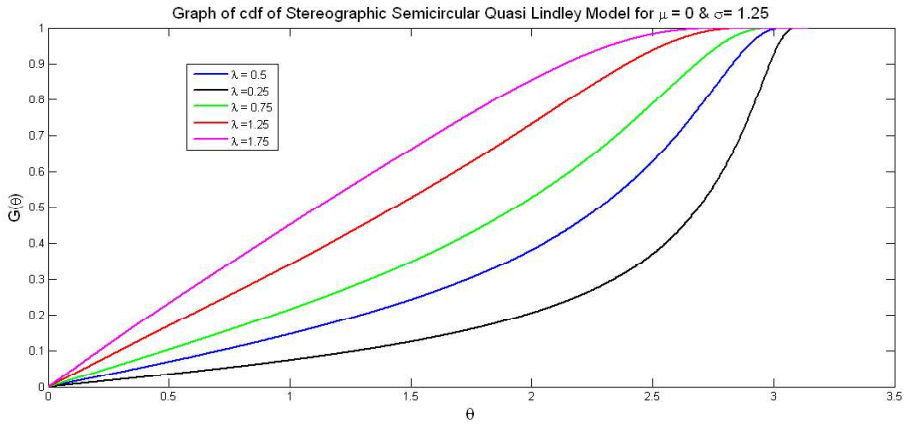
Definition: A random variable X_{SC} on the Semicircle is said to have the Stereographic Semicircular Quasi Lindley distribution with shape parameter $\sigma>-1$, location parameter μ and scale parameter $\sigma>0$ denoted by **SSCQLD**(σ,λ,μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta)=\frac{\lambda\sec^2\left(\frac{\theta-\mu}{2}\right)}{2(\sigma+1)}\left(\sigma+\lambda\tan\left(\frac{\theta-\mu}{2}\right)\right)\exp\left(-\lambda\tan\left(\frac{\theta-\mu}{2}\right)\right),$$
$$\text{where } 0\le\mu,\theta<\pi,\ \lambda=v\alpha>0 \text{ and } \sigma>-1 \tag{3.3}$$

$$G(\theta)=1-\exp\left(-\lambda\tan\left(\frac{\theta-\mu}{2}\right)\right)\left[1+\frac{\lambda\tan\left(\frac{\theta-\mu}{2}\right)}{(\sigma+1)}\right]$$
$$\text{where } 0\le\mu,\theta<\pi,\ \lambda=v\alpha>0 \text{ and } \sigma>-1 \tag{3.4}$$

Graphs of pdf and cdf of Stereographic Semicircular Quasi Lindley Distribution for various values of parameters





3.Trigonometric moments of Stereographic Semicircular Quasi Lindley Distribution

It is customary to derive the trigonometric moments when a new distribution is proposed. Without loss of generality here we assume that $\mu = 0$, in (3.3) .The trigonometric moments of the distribution are given by $\{\varphi_p : p = 0, \pm 1, \pm 2, \pm 3, \ldots\}$, where $\varphi_p = \alpha_p + i\beta_p$, with $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$ being the pth order cosine and sine moments of the random angle θ, respectively.

Theorem 4.1 Under the pdf of Stereographic Semicircular Quasi Lindley Distribution with $\mu = 0$, the first four $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$, $p = 1, 2, 3, 4$, are given as follows:

$$\alpha_1 = 1 - \frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right) - \frac{\lambda^2}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\ -1,0,\frac{1}{2}\end{matrix}\right),$$

$$\beta_1 = \frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}0\\ 0,0,\frac{1}{2}\end{matrix}\right) + \frac{\lambda^2}{\sqrt{\pi}(\sigma+1)} G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right),$$

$$\alpha_2 = 1 + \frac{4\lambda\sigma}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{3}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right) + G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]$$

$$-\frac{4\lambda^2}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\ -\frac{1}{2},0,\frac{1}{2}\end{matrix}\right) + G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\ -1,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$\beta_2 = \frac{2\lambda\sigma}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle| \begin{matrix} 0 \\ 0,0,\frac{1}{2} \end{matrix} \right) - 2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle| \begin{matrix} -1 \\ 0,0,\frac{1}{2} \end{matrix} \right) \right]$$

$$+ \frac{2\lambda^2}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle| \begin{matrix} -\frac{1}{2} \\ -\frac{1}{2},0,\frac{1}{2} \end{matrix} \right) - 2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle| \begin{matrix} -\frac{3}{2} \\ -\frac{1}{2},0,\frac{1}{2} \end{matrix} \right) \right]$$

where $\int_0^\infty x^{2\nu-1}\left(u^2+x^2\right)^{Q-1} e^{-\mu x}dx = \frac{u^{2\nu+2Q-2}}{2\sqrt{\pi}\Gamma(1-Q)} G_{13}^{31}\left(\frac{\mu^2u^2}{4}\middle| \begin{matrix} 1-\nu \\ 1-Q-\nu,0,\frac{1}{2} \end{matrix} \right)$ (4.1)

for $|\arg u\pi| < \frac{\pi}{2}, \operatorname{Re}\mu > 0$ and $\operatorname{Re}\nu > 0$ and $G_{13}^{31}\left(\frac{\mu^2u^2}{4}\middle| \begin{matrix} 1-\nu \\ 1-Q-\nu,0,\frac{1}{2} \end{matrix} \right)$ is called as

Meijer's **G-**function (Gradstein and Ryzhik, 2007, formula no. 3.389.2).

Proof:

$$\varphi_{\mathrm{p}} = \int_0^\pi e^{ip\theta} g(\theta)\,d\theta = \int_0^\pi \cos(\mathrm{p}\theta)\, g(\theta)\,d\theta + i\int_0^\pi \sin(\mathrm{p}\theta)\, g(\theta)\,d\theta$$

$$= E\left(\cos(\mathrm{p}\theta)\right) + i\,E\left(\sin(\mathrm{p}\theta)\right) = \alpha_{\mathrm{p}} + i\,\beta_{\mathrm{p}} \quad \text{for } \mathrm{p} = 0, \pm1, \pm2, \pm3, \ldots,$$

For the first cosine and sine moments, use the transformations $x = \tan\left(\frac{\theta}{2}\right)$, $\cos\theta = 1 - \frac{2x^2}{1+x^2}$ and $\sin\theta = \frac{2x}{1+x^2}$,the results α_1 and β_1 follows by the integral formula **(4.1)**

Now $\mathrm{E}\left(\cos(\mathrm{p}\theta)\right) = \frac{\lambda}{2(\sigma+1)}\int_0^\pi \cos(\mathrm{p}\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$ and

$\mathrm{E}\left(\sin(\mathrm{p}\theta)\right) = \frac{\lambda}{2(\sigma+1)}\int_0^\pi \sin(\mathrm{p}\theta)\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$

$$\alpha_1 = \frac{\lambda}{2(\sigma+1)}\int_0^\pi \cos\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$= \frac{\lambda}{(\sigma+1)}\int_0^\infty \left[1 - \frac{2x^2}{1+x^2}\right](\sigma+\lambda x)e^{-\lambda x}dx$$

$$= 1 - \frac{2\lambda\sigma}{(\sigma+1)}\int_0^\infty \frac{x^2}{1+x^2}e^{-\lambda x}dx - \frac{2\lambda^2}{(\sigma+1)}\int_0^\infty \frac{x^3}{1+x^2}e^{-\lambda x}dx$$

$$= 1 - \frac{2\lambda\sigma}{(\sigma+1)}\int_0^\infty x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx - \frac{2\lambda^2}{(\sigma+1)}\int_0^\infty x^{2(2)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$=1-\frac{2\lambda\sigma}{(\sigma+1)}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]-\frac{2\lambda^2}{(\sigma+1)}\left[\frac{1}{2\sqrt{\pi}}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$\alpha_1=1-\frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)-\frac{\lambda^2}{\sqrt{\pi}(\sigma+1)}G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)$$

$$\beta_1=\frac{\lambda}{2(\sigma+1)}\int_0^{\pi}\sin\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{2\lambda}{(\sigma+1)}\int_0^{\infty}\left(\frac{x}{1+x^2}\right)(\sigma+\lambda x)e^{-\lambda x}dx$$

$$=\frac{2\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2(1)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx+\frac{2\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$\beta_1=\frac{\lambda\sigma}{\sqrt{\pi}(\sigma+1)}G\left(\frac{\lambda^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)+\frac{\lambda^2}{\sqrt{\pi}(\sigma+1)}G\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)$$

To obtain second cosine and sine moments α_2 and β_2, we use the transformations $x=\tan\left(\frac{\theta}{2}\right)$, $\cos 2\theta=1+\frac{8x^4}{\left(1+x^2\right)^2}-\frac{8x^2}{\left(1+x^2\right)}$ and $\sin 2\theta=\frac{4x}{\left(1+x^2\right)}-\frac{8x^3}{\left(1+x^2\right)^2}$, the results of α_2 and β_2 follows by the same integral formula of α_1.

$$\alpha_2=\frac{\lambda}{2(\sigma+1)}\int_0^{\pi}\cos 2\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\left[1+\frac{8x^4}{\left(1+x^2\right)^2}-\frac{8x^2}{\left(1+x^2\right)}\right](\sigma+\lambda x)e^{-\lambda x}dx$$

$$=1+\frac{8\lambda}{(\sigma+1)}\int_0^{\infty}\frac{x^4}{\left(1+x^2\right)^2}(\sigma+\lambda x)e^{-\lambda x}dx-\frac{8\lambda}{(\sigma+1)}\int_0^{\infty}\frac{x^2}{\left(1+x^2\right)}(\sigma+\lambda x)e^{-\lambda x}dx$$

$$=1+\frac{8\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{5}{2}\right)-1}\left(1+x^2\right)^{-1-1}e^{-\lambda x}dx+\frac{8\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2(3)-1}\left(1+x^2\right)^{-1-1}e^{-\lambda x}dx$$

$$-\frac{8\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx-\frac{8\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2(2)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$\alpha_2=1+\frac{4\lambda\sigma}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{3}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)+G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]$$

$$-\frac{4\lambda^2}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)+G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\-1,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$\beta_2=\frac{\lambda}{2(\sigma+1)}\int_0^{\pi}\sin 2\theta\sec^2\left(\frac{\theta}{2}\right)\left(\sigma+\lambda\tan\left(\frac{\theta}{2}\right)\right)e^{-\lambda\left(\tan\left(\frac{\theta}{2}\right)\right)}d\theta$$

$$=\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\left[\frac{4x}{\left(1+x^2\right)}-\frac{8x^3}{\left(1+x^2\right)^2}\right](\sigma+\lambda x)e^{-\lambda x}dx$$

3. $$=\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\frac{4x}{\left(1+x^2\right)}(\sigma+\lambda x)e^{-\lambda x}dx-\frac{\lambda}{(\sigma+1)}\int_0^{\infty}\frac{8x^3}{\left(1+x^2\right)^2}(\sigma+\lambda x)e^{-\lambda x}dx$$

4. $$\frac{4\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2(1)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx+\frac{4\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{3}{2}\right)-1}\left(1+x^2\right)^{0-1}e^{-\lambda x}dx$$

$$-\frac{8\lambda\sigma}{(\sigma+1)}\int_0^{\infty}x^{2(2)-1}\left(1+x^2\right)^{-1-1}e^{-\lambda x}dx-\frac{8\lambda^2}{(\sigma+1)}\int_0^{\infty}x^{2\left(\frac{5}{2}\right)-1}\left(1+x^2\right)^{-1-1}e^{-\lambda x}dx$$

$$\beta_2=\frac{2\lambda\sigma}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}0\\0,0,\frac{1}{2}\end{matrix}\right)-2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-1\\0,0,\frac{1}{2}\end{matrix}\right)\right]$$

$$+\frac{2\lambda^2}{\sqrt{\pi}(\sigma+1)}\left[G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{1}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)-2G_{13}^{31}\left(\frac{\lambda^2}{4}\middle|\begin{matrix}-\frac{3}{2}\\-\frac{1}{2},0,\frac{1}{2}\end{matrix}\right)\right]$$

Higher-order moments can be obtained similarly.

The first two trigonometric moments are sufficient for calculating population characteristics.

5. Stereographic -l-axial Quasi Lindley distribution

We extend the above Stereographic Semicircular model to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$. So it is desirable to extend the Stereographic Semicircular Quasi Lindley distribution to construct the Stereographic-l -axial Quasi Lindley distribution.We considers the density function of Stereographic Semicircular Quasi Lindley distribution and use the transformation $\theta=2\phi/l$, $l=1,2,...,$. The probability density function of Quasi Lindley distribution in ϕ is given by

$$g(\phi)=\frac{\lambda\sec^2\left(\frac{\phi}{2}\right)}{2(\sigma+1)}\left(\sigma+\lambda\tan\left(\frac{\phi}{2}\right)\right)\exp\left(-\lambda\tan\left(\frac{\phi}{2}\right)\right),$$

where $0 \le \mu, \phi < \pi,\ \lambda = v\alpha > 0$ and $\sigma > -1$

Then the corresponding Stereographic-l -axial Quasi Lindley distribution is

$$g(\theta) = \frac{\lambda l \sec^2\left(\frac{l\theta}{4}\right)}{4(\sigma+1)}\left(\sigma + \lambda \tan\left(\frac{l\theta}{4}\right)\right)\exp\left(-\lambda \tan\left(\frac{l\theta}{4}\right)\right),$$

$$0 < \theta < \frac{2\pi}{l},\ \sigma > -1, \lambda > 0 \text{ and } l = 1, 2, .. \qquad (5.1)$$

We call it as **Stereographic -l -axial Quasi Lindley distribution.**

Case (1) When $l = 1$, in the probability density function (4.1), we get the density function

$$g(\theta) = \frac{\lambda \sec^2\left(\frac{\theta}{4}\right)}{4(\sigma+1)}\left(\sigma + \lambda \tan\left(\frac{\theta}{4}\right)\right)\exp\left(-\lambda \tan\left(\frac{\theta}{4}\right)\right),$$

$$0 < \theta < 2\pi,\ \sigma > -1, \lambda > 0 \qquad (5.2)$$

We call it as **Stereographic CircularQuasi Lindley distribution**.

Case (2) When $l = 2$, the probability density function (5.1) is the same as that of **Stereographic Semicircular Quasi Lindley Distribution.**

6.Conclusion

In this paper, we discussed semicircular distribution induced by modified inverse stereographic projection on Quasi Lindley Distribution. The density and distribution function of Stereographic semicircular Quasi Lindley distribution admit explicit forms, as do trigonometric moments.As this distribution is asymmetric, suitable for modeling asymmetrical directional data.

7. References

Bakouch HS, Al-Zahrani BM, Al-Shomrani AA, Marchi VAA, Louzada F (2012). An extended Lindley distribution. J. Korean Stat. Soc. 41(1);75-85.

Byoung Jin Ahn and Hyoung – Moon Kim (2008), A New Family f Semicircular Models: The Semicircular Laplace Distributions, Communications of the Korean Statistical Society, 15 (5);775 – 781.

Gradshteyn and Ryzhik (2007). Table of Integrals, series and products, 7th edition, Academic Press.

Guardiola, J.H. (2004). The Semicircular Normal Distribution, Ph.D Dissertation, Baylor University, Institute of Statistics.

Jammalamadaka S. Rao and Sen Gupta, A. (2001), Topics in Circular Statistics, *World Scientific Press, Singapore.*

Minh, Do Le and Farnum, Nicholas R. (2003), Using Bilinear Transformations to Induce Probability Distributions, *Communication in Statistics – Theory and Methods,* 32(1); 1 – 9.

Mardia, K.V. (1972), Statistics of Directional Data, *Academic Press, New York.*

Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, *John Wiley, Chichester.*

Phani.Y, Girija S.V.S., and Dattatreya Rao A.V. (2013). On Construction of Stereographic Semicircular Models, Journal of Applied Probability and Statistics. 8(1); 75-90.

Rama S and Mishra A (2013). A quasi Lindley distribution. African Journal of Mathematics and Computer Science Research.6(4);64-71

Toshihiro Abe, Kunio Shimizu and Arthur Pewsey, (2010), Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection, *J. Japan Statist. Soc.,* 40 (1), 45-61.

Ugai. S.K., Nishijima, M. and Kan, T. (1977), Characteristics of raindrop size and raindrop shape, *Open symposium* URSI *Commission F,* 225-230.

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ON THE RISING SUN OFFSET PEARSON TYPE II MODEL

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ABSTRACT

The circular models based on the Rising Sun function are motivated by purely mathematical considerations as a smoothing function and possible application. This work takes a further step in this direction using several mathematical tools such as Real Analysis along with MATLAB and are applied to enlarge the horizon of Mathematical Statistics. Here an attempt is made to construct new circular model using the Rising Sun function on the Offset Pearson Type II model and both linear and circular representations of graphs of pdf are plotted using MATLAB.

Mathematics Subject Classification 2000*: 60E10.*

Keywords: *Circular model, trigonometric moments, Rising Sun function, characteristic function, Offset Pearson Type II model.*

1. INTRODUCTION

The available methods of generating circular models are wrapping a linear model, offsetting a bivariate linear model and applying stereographic projection on a linear model. The Rising Sun function [Van Rooij and Schikoff (1982), p.10] smoothens the existing curve and many bumps disappear. This may lead to the effect of increasing the smoothing the curve in density estimation. Girija (2010) proposed a new method of generating circular model by using the Rising Sun function (**RSF)** and Radhika.*et.al* (2013) derived circular models based on the Rising Sun function motivated by the mathematical significance of the Rising Sun function behind the construction of circular models, here an attempt is made to construct a new circular model named The Rising Sun Offset Pearson Type II Model and various properties are discussed.

2. CONSTRUCTION OF CIRCULAR MODELS USING THE RISING SUN FUNCTION

The Circular Distribution [Jammalamadaka and Sengupta (2001)] is defined as under

In the continuous case $g:[0,\ 2\pi)\to\mathbb{R}$ is the probability density function of a circular distribution if and only if g has the following basic properties

- $g(\theta)\geq 0,\quad \forall\theta$ (2.1)
- $\int_0^{2\pi} g(\theta)\,d\theta = 1$ (2.2)

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- $g(\theta) = g(\theta + 2k\pi)$ (2.3)

for any integer k (i.e., g is periodic) (Mardia,1972)

It may be noted that the circular distribution is a probability distribution whose total probability is concentrated on the unit circle $\{(\cos\theta, \sin\theta) / 0 \le \theta < 2\pi\}$ in the plane which satisfies the properties (2.1) through (2.3).

If $G(\theta)$ denotes the cdf of the r.v., the characteristic function of the circular model is given by

$$\varphi_\theta(t) = E\left(e^{it\theta}\right) = \int_0^{2\pi} e^{it\theta} dG(\theta) = \rho_t e^{i\mu_t} \quad t \in \mathbb{R} \tag{2.4}$$

It is known that whenever $\varphi(t) \neq 0$, $e^{2\pi it} = 1$ (Mardia 1972 p. 41). This suggests that the function $\varphi(t)$ should only be defined for integer values of *t*. accordingly the characteristic function $\varphi(p) = \varphi_p$ is defined by

$$\varphi_\theta(p) = E\left(e^{ip\theta}\right) = \int_0^{2\pi} e^{ip\theta} dF(\theta) = \rho_p e^{i\mu_p} \qquad p \in \mathbb{Z} \tag{2.5}$$

Clearly, $\varphi_0 = 1$, $\overline{\varphi}_p = \varphi_{-p}$.

Trigonometric moments [Jammalamadaka and Sengupta (2001)]

The value of the characteristic function φ_p at an integer *p* is called the pth trigonometric moment of θ. The real and the imaginary parts of φ_p are denoted by α_p and β_p respectively. We can also view these trigonometric moments in terms of

$$\alpha_p = E(\cos p\theta),\ \beta_p = E(\sin p\theta),\ p \in \mathbb{Z}$$

The Rising Sun function (RSF) of a bounded function $f : [a,b] \to \mathbb{R}$ is defined by

$$f_\Theta(x) = \sup\{f(t) : x \le t \le b\} \tag{2.6}$$

It is easy to show that

- when f is nonnegative then f_Θ is nonnegative
- when f is continuous then f_Θ is continuous
- f_Θ is monotonically decreasing , hence $f_\Theta = f$ when f is decreasing and f_Θ is the smallest monotonically decreasing function such that $f_\Theta = f$.

Imagine the Rising Sun on *x* – axis. Then $\{(x,y) \in \mathbb{R}^2 : y \ge f_\Theta(x)\}$ is illuminated by the sun whereas $\{(x,y) \in \mathbb{R}^2 : y < f_\Theta(x)\}$ is covered by darkness. The set $\{(x, f(x)) : f(x) = f_\Theta(x)\}$ is the collection of those points of the graph of f that receive light from the sun.

If *f* is continuous on [*a*,*b*] then for any *k* in the range of f_Θ, $S = \{x \in [a,b] : f_\Theta(x) = k\}$ is a closed and bounded interval **[**Van Rooij and Schikoff (1982)].

The Rising Sun Lemma 2.1: [Van Rooij and Schikoff (1982)]: Let $f : [a,b] \to \mathbb{R}$ be a continuous function. Then $E = \{x \in (a,b) : f_\Theta(x) > f(x)\}$ is open. If (α, β) is a component of *E* then
$f_\Theta(\beta) = f(\beta)$ and $f_\Theta(\alpha) = f(\alpha)$ for $\alpha \neq a$, and f_Θ is constant in that interval.
The well known Lebesgue theorem is also proved using the Rising Sun function.

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A new construction procedure of a class of Circular Models using RSF is obtained in the following theorem and an illustration is also included in this section. These distributions are named as 'Rising Sun Circular models'.

Theorem 2.2: [Girija (2010)]: If g is the pdf and *G* is the cdf of a random variable of a circular distribution then the Rising Sun function g_Θ , gives rise to the pdfg_c of a circular model. The distribution function of g_c is given by

$$G_c = \begin{cases} \frac{1}{\mathrm{K}}\left[\theta_1\ g(\theta_1) + G(\theta) - G(\theta_1)\right] & \text{for } \theta_1 < \theta \\ \frac{1}{\mathrm{K}}\left[\theta\ \mathrm{g}(\theta_1)\right] & \text{for } \theta_1 \geq \theta \end{cases} \tag{4.2.2}$$

Analogous to Theorem 2.1 the Rising Sun lemma for circular data is presented as follows

The Circular Rising Sun Lemma 2.3 [Radhika et al (2013)]: Let $g:[0,2\pi] \to \mathbb{R}$ be a continuous function. Then $E = \{\theta \in (0,2\pi) : g_\Theta(\theta) > g(\theta)\}$ is open. If (α, β) is a component of *E* then $g_\Theta(\beta) = g(\beta)$ and $g_\Theta(\alpha) = g(\alpha)$ for $\alpha \neq 0,$ and g_Θ is constant in that interval. g_C is the normalized function of g_Θ, hence β represents the mode of the circular model at which both g_Θ and g_C have maximum value. Hence component of g_C as well as g_Θ is (0, β) and g_C is the pdf of circular model known as Rising Sun circular model.

3. THE RISING SUN OFFSET PEARSON TYPE II MODEL

The Offset Pearson Type II (OP-II) distribution is derived from the Bivariate Pearson Type II distribution [Balakrishnan and Chin (2008), p. 371]. Radhika *et.al* (2013) derived the Offset Pearson Type II model.

The pdf $g(\theta)$ and cdf $G(\theta)$ of the Offset Pearson Type II model for the Bivariate Pearson Type II distribution with parameters *q*> 1 and ρ where $|\rho| < 1$ are respectively given by

$$g(\theta) = \frac{\sqrt{1-\rho^2}}{2\pi\,(1 - \rho \sin 2\theta)} \text{ where } \theta \in [0, 2\pi) \tag{3.1}$$

and

$$G(\theta) = \begin{cases} \frac{1}{2\pi}\left\{\tan^{-1}\left(\frac{\tan\theta - \rho}{\sqrt{1-\rho^2}}\right) - \tan^{-1}\left(\frac{-\rho}{\sqrt{1-\rho^2}}\right)\right\}, \theta \in \left[0, \frac{\pi}{2}\right) \\ \frac{1}{2\pi}\left\{\pi + \tan^{-1}\left(\frac{\tan\theta - \rho}{\sqrt{1-\rho^2}}\right) - \tan^{-1}\left(\frac{-\rho}{\sqrt{1-\rho^2}}\right)\right\}, \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \\ \frac{1}{2\pi}\left\{2\pi + \tan^{-1}\left(\frac{\tan\theta - \rho}{\sqrt{1-\rho^2}}\right) - \tan^{-1}\left(\frac{-\rho}{\sqrt{1-\rho^2}}\right)\right\}, \theta \in \left(\frac{3\pi}{2}, 2\pi\right) \end{cases} . \tag{3.2}$$

The Rising Sun function of the Offset Pearson Type II distribution is defined as

$$\begin{aligned} g_\Theta(\theta) &= \mathrm{Sup}\left(g(t) : \theta \leq t < 2\pi\right) \\ &= \mathrm{Sup}\left(\frac{1}{2\pi}\ \frac{\sqrt{1-\rho^2}}{(1 - \rho \sin 2t)} \text{ where } \theta \in [0, 2\pi)\ : \theta \leq t < 2\pi\right) \end{aligned} \tag{3.3}$$

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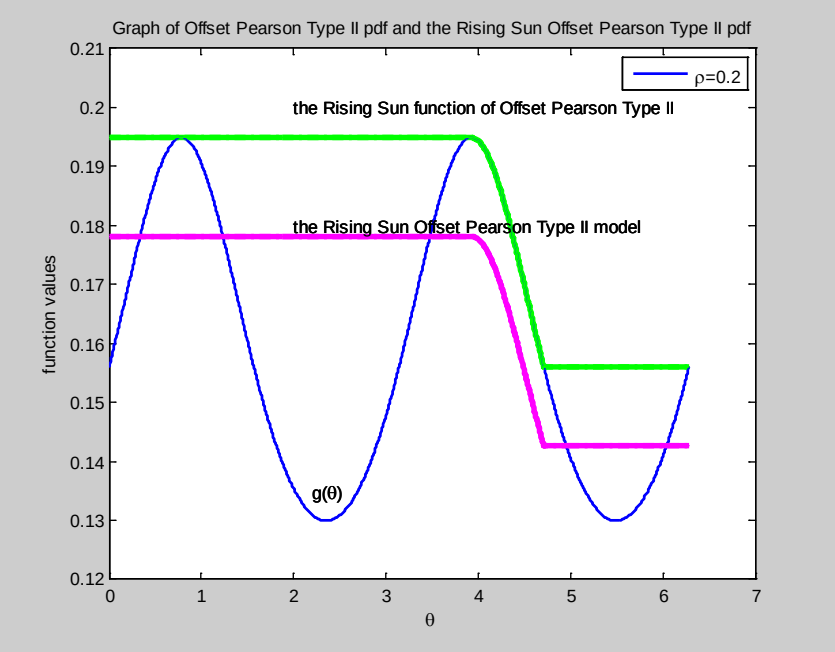
Normalizing this function with the constant $K_1 = \int_0^{2\pi} g_\Theta(\theta)\ d\theta$ the pdf of the Rising Sun Offset Pearson Type II distribution (RSOP-II) is obtained.

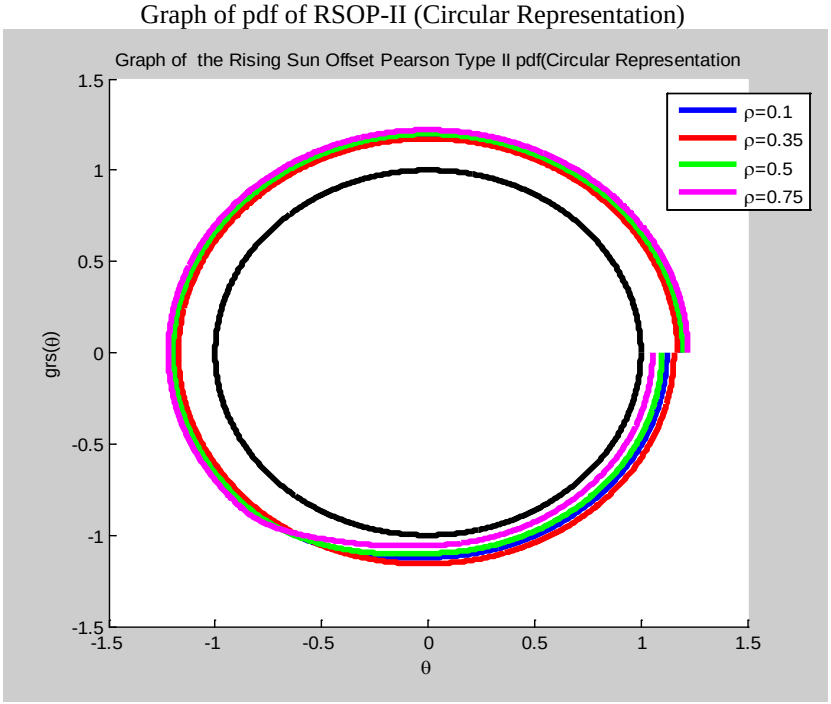
$$g_c(\theta) = \frac{\text{Sup}\left(\frac{1}{2\pi} \frac{\sqrt{1-\rho^2}}{(1-\rho \sin 2t)} \text{ where } \theta \in [0, 2\pi) : \theta \le t < 2\pi \right)}{\sqrt{\int_0^{2\pi} g_\Theta(\theta)\ d\theta}} \tag{3.4}$$

The graphs of pdfof the Rising Sun Offset Pearson Type II distribution (RSOP-II) are plotted and population characteristics are studied using MATLAB.

Graph of pdf of RSOP-II (Linear Representation)



Graph of pdf of RSOP-II (Circular Representation)



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REFERENCES

1. Balakrishnan, N. And Chin – Diew Lai (2008), Continuous Bivariate Distributions Springer.
2. DattatreyaRao, A.V., Ramabhadra Sarma, I. and Girija, S.V.S. (2007), On Wrapped Version of Some Life Testing Models, *Comm Statist,-Theor.Meth*., 36, issue # 11, pp.2027-2035.
3. Girija, S.V.S., (2010), Construction of New Circular Models, VDM – VERLAG, Germany.
4. Jammalamadaka S. Rao and Sen Gupta, A. (2001), Topics in Circular Statistics, *World Scientific Press, Singapore.*
5. Mardia. K. V. (1972a). *Statistics of Directional Data*. Academic Press, London.
6. Mardia, K.V. and Jupp, P.E. (2000), Directional Statistics, *John Wiley, Chichester.*
7. Radhika, A.J.V., Girija, S.V.S. & DattatreyaRao, A.V. (2013a).*On Univariate Offset Pearson Type II Model – Application To Live Data*,International Journal of Mathematics and Statistics Studies, Vol.1, No. 1, 2013, pp. 1-9.
8. Radhika, A.J.V., Girija, S.V.S. & DattatreyaRao, A.V. (2013b). *On Rising Sun von Mises and Rising Sun Wrapped Cauchy Circular Models*, Journal of the Applied Mathematics, Statistics and Informatics (JAMSI), Vol. 9, No. 2, pp. 61 – 67.
9. Van Rooij, A.C.M. and Schikof, W.H. (1982). A Second Course on Real Functions, *Cambridge University Press, Cambridge*

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THE DISCRETE RISING SUN WRAPPED CAUCHY MODEL

By

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ABSTRACT

The circular models based on the Rising Sun function are motivated by purely mathematical considerations as a smoothing function and possible application. This work takes a further step in this direction using several mathematical tools such as Real Analysis along with MATLAB and are applied to enlarge the horizon of Mathematical Statistics. All the available circular / angular models are continuous distributions. Scant attention was made on construction and applications of discrete circular / angular models. Circular model using the Rising Sun function on continuous Wrapped Cauchy distribution is available in literature. Here an attempt is made to construct new discrete circular model by applying the methodology of discretization on the Rising Sun Wrapped Cauchy distribution and the population characteristics are evaluated using MATLAB.

Keywords: Circular Model, Trigonometric Moments, Rising Sun Wrapped Cauchy Distribution, Discretization, Characteristic Function.

INTRODUCTION

The available methods of generating circular models are wrapping a linear model, offsetting a bivariate linear model, and applying stereographic projection on a linear model. Rao, Sarma, and Girija (2007) contributed wrapped version of some life testing models and these are continuous circular distributions. Sarma, Rao, and Girija (2009, 2011) worked on characteristic function of circular models.

The Rising Sun function (Van Rooij & Schikhof, 1982), smoothens the existing curve and many bumps disappear. This may lead to the effect of increasing the smoothing the curve in density estimation. Girija (2010) has proposed a new method of generating circular model by using the Rising Sun function (RSF).

The Wrapped Cauchy distribution was introduced by Levy (1939) and has been studied by Wintner (1947). McCullagh (1996) showed that Wrapped Cauchy distribution can be obtained by mapping Cauchy distribution onto the circle by the transformation. Radhika, Girija, and Rao (2013) derived the Rising Sun Wrapped Cauchy model motivated by the mathematical significance of the Rising Sun function behind the construction of circular models.

All the available circular/ angular models are continuous distributions. Scant attention was made on construction and applications of discrete circular/ angular models. Glancing the literature, it is observed that, Srihari, Girija, and Rao (2018) have derived Discrete Wrapped Exponential distribution and Srinivas, Srihari, and Girija (2018) have published a work on Discrete Wrapped Lindley Distribution. Girija, Srihari, and Srinivas (2019) have constructed a Discrete Wrapped Cauchy Model.

The circular models based on the Rising Sun function are motivated by purely mathematical considerations as a smoothing function and possible application. This work takes a further step in this direction using several mathematical tools such as

Real Analysis along with MATLAB and are applied to enlarge the horizon of Mathematical Statistics. Rising Sun function has high mathematical significance. Here an attempt is made to construct a new discrete circular model named the Discrete Rising Sun Wrapped Cauchy distribution and its population characteristics are studied using MATLAB.

1. Construction of Circular Models Using the Rising Sun Function

The Circular Distribution (Jammalamadaka & Sengupta, 2001) is defined as under.

In the continuous case g: [0, 2π) → R is the probability density function of a circular distribution if and only if g has the following basic properties.

$$g(\theta) \geq 0, \forall \theta \in [0, 2\pi) \quad (1)$$

$$\int_0^{2\pi} g(\theta)\, d\theta = 1 \quad (2)$$

$$g(\theta) = g(\theta + 2k\pi) \quad (3)$$

for any integer k (i.e., g is periodic) (Mardia, 1975).

It may be noted that the circular distribution is a probability distribution whose total probability is concentrated on the unit circle $\{(\cos\theta, \sin\theta) / 0 \leq \theta < 2\pi\}$ in the plane, which satisfies the properties (1) through (3).

If G(θ) denotes the cdf of the r.v., the characteristic function of the circular model is given by,

$$\varphi_\theta(t) = E\left(e^{it\theta}\right) = \int_0^{2\pi} e^{it\theta}\, dG(\theta) = \rho_t e^{i\mu_t} \quad t \in R \quad (4)$$

It is known that whenever $\varphi(t) \neq 0$, $e^{2\pi it} = 1$ (Mardia, 1975). This suggests that the function φ(t) should only be defined for integer values of t. Accordingly, the characteristic function $\varphi(p) = \varphi_p$ is defined by,

$$\varphi_\theta(p) = E\left(e^{ip\theta}\right) = \int_0^{2\pi} e^{ip\theta}\, dF(\theta) = \rho_p e^{i\mu_p} \quad p \in Z \quad (5)$$

Clearly, $\varphi_0 = 1, \quad \overline{\varphi}_p = \varphi_{-p}$

Trigonometric moments (Jammalamadaka & Sengupta, 2001)

The value of the characteristic function φ_p at an integer p is called the pth trigonometric moment of θ. The real and the imaginary parts of φ_p are denoted by α_p and β_p, respectively. These trigonometric moments can be viewed in terms of

$$\alpha_p = E(\cos p\theta), \beta_p = E(\sin p\theta), p \in Z$$

The Rising Sun function (RSF) of a bounded function f: [a, b] → R is defined by,

$$f_\Theta(x) = \sup\{f(t) : x \leq t \leq b\} \quad (6)$$

It is easy to show that

- when f is nonnegative then f_Θ is nonnegative
- when f is continuous then f_Θ is continuous
- f_Θ is monotonically decreasing, hence $f_\Theta = f$ when f is decreasing and f_Θ is the smallest monotonically decreasing function such that $f_\Theta = f$.

Imagine the Rising Sun on x – axis. Then $\{(x, y) \in R^2: y \geq f_\Theta(x)\}$ is illuminated by the sun whereas $\{(x, y) \in R^2: y < f_\Theta(x)\}$ is covered by darkness. The set $\{(x, f(x)): f(x) = f_\Theta(x)\}$ is the collection of those points of the graph of f that receive light from the sun.

If f is continuous on [a, b] then for any k in the range of f_Θ, S = {x ∈ [a, b]: $f_\Theta(x)=k$} is a closed and bounded interval (Van Rooij & Schikoff, 1982).

The Rising Sun Lemma 2.1: (Van Rooij & Schikoff, 1982): Let f: [a, b] → R be a continuous function. Then E = {x ∈ (a, b): $f_\Theta(x)>f(x)$} is open. If (α, β) is a component of E then $f_\Theta(\beta) = f(\beta)$ and $f_\Theta(\alpha) = f(\alpha)$ for $\alpha \neq a$, and f_Θ is constant in that interval.

The well known Lebesgue theorem is also proved using the Rising Sun function.

A new construction procedure of a class of Circular Models using RSF is obtained in the following theorem and an illustration is also included in this section. These distributions are named as 'Rising Sun Circular models'.

Theorem 2.2: (Girija, 2010): If g is the probability density function and G is the cumulative distribution function of a random variable of a circular distribution then the Rising Sun function g_Θ, gives rise to the Probability Density Function g_c of a circular model. The distribution function of g_c is given by,

$$G_c = \begin{cases} \frac{1}{K}\left[\theta_1\, g(\theta_1) + G(\theta) - G(\theta_1)\right] & \text{for } \theta_1 < \theta \\ \frac{1}{K}\left[\theta\ g(\theta_1)\right] & \text{for } \theta_1 \geq \theta \end{cases} \quad (7)$$

Analogous to Theorem 2.1, The Rising Sun lemma for circular data is presented as follows.

The Circular Rising Sun Lemma 2.3 (Radhika et al., 2013): Let g: [0, 2π) → R be a continuous function. Then E = {θ∈(0, 2π): $g_\Theta(\theta) > g(\theta)$} is open. If (α, β) is a component of E then $g_\Theta(\beta) = g(\beta)$ and $g_\Theta(\alpha) = g(\alpha)$ for $\alpha \neq 0$, and g_Θ is constant in that interval. g_c is the normalized function of g_Θ, hence β represents the mode of the circular model at which both g_Θ and g_c have maximum value. Hence component of g_c as well as g_Θ is (0, β) and g_c is the Probability Density Function of circular model known as Rising Sun circular model.

2. The Rising Sun Wrapped Cauchy Model

The Wrapped Cauchy distribution was introduced by Levy (1939) and has been studied by Wintner (1947). McCullagh (1996) showed that Wrapped Cauchy distribution can be obtained by mapping Cauchy distribution onto the circle by the transformation $x \leftrightarrow 2\tan^{-1} x$.

The Probability Density Function of the Wrapped Cauchy distribution (Mardia & Jupp, 2000) is

$$g(\theta) = \left(\frac{1}{2\pi}\left(1 + 2\sum_{p=1}^{\infty}\rho^p \cos p(\theta - \mu)\right)\right) \quad (8)$$

where $\rho = \pi e^{-a}, a > 0$ and $\theta \in [0, 2\pi)$

We shall denote this distribution by WC (μ, ρ). It follows from considering the real part of the geometric series $\sum_{p=1}^{\infty}\rho^p e^{-ip(\theta-\mu)}$ that reduces to

$$g(\theta) = \frac{(1-\rho^2)}{2\pi\left(1+\rho^2 - 2\rho\cos(\theta-\mu)\right)} \quad (9)$$

In particular, the mean direction is μ(mod 2π) and the mean resultant length is ρ and this distribution is unimodal and symmetric about μ.

The Rising Sun function of the Wrapped Cauchy distribution is derived by (Radhika et al., 2013) as

$$g_\Theta(\theta) = \sup\left(\frac{1}{2\pi}\left(\frac{(1-\rho^2)}{(1+\rho^2 - 2\rho\cos(t-\mu))}\right) : \theta \leq t < 2\pi\right) \quad (10)$$

Normalizing this function with the constant $K_2 = \int_0^{2\pi} g_\Theta(\theta)\ d\theta$, the pdf of the Rising Sun Wrapped Cauchy distribution (RSWC) is obtained.

$$g_c(\theta) = \frac{\sup\left(\frac{1}{2\pi}\left(\frac{(1-\rho^2)}{(1+\rho^2-2\rho\cos(t-\mu))}\right):\theta \le t < 2\pi\right)}{\int_0^{2\pi} g_\Theta(\theta)\ d\theta} \qquad (11)$$

The graphs of Probability Density Function of the Wrapped Cauchy, Rising Sun of Wrapped Cauchy, and Probability Density Function of Rising Sun Wrapped Cauchy models are plotted in Figure 1.

3. Discrete Circular Random Variables

If X is a discrete random variable on the set of integers, then reduction modulo 2π m($m \in Z^+$) wraps the integers on to the group of mth roots of unity which is a sub group of unit circle.

$\theta = 2\pi x (\text{mod } 2\pi m)$

Since θ contains a finite number of elements they are denoted by $\theta = \left\{\frac{2\pi r}{m} \Big/ r = 0,1,2,...m-1\right\}$ which is lattice on the unit circle.

3.1 Probability Mass Function

Suppose if θ is a wrapped discrete circular random variable then probability mass function of θ is denoted by $Pr\left(\theta = \frac{2\pi r}{m}\right)$,

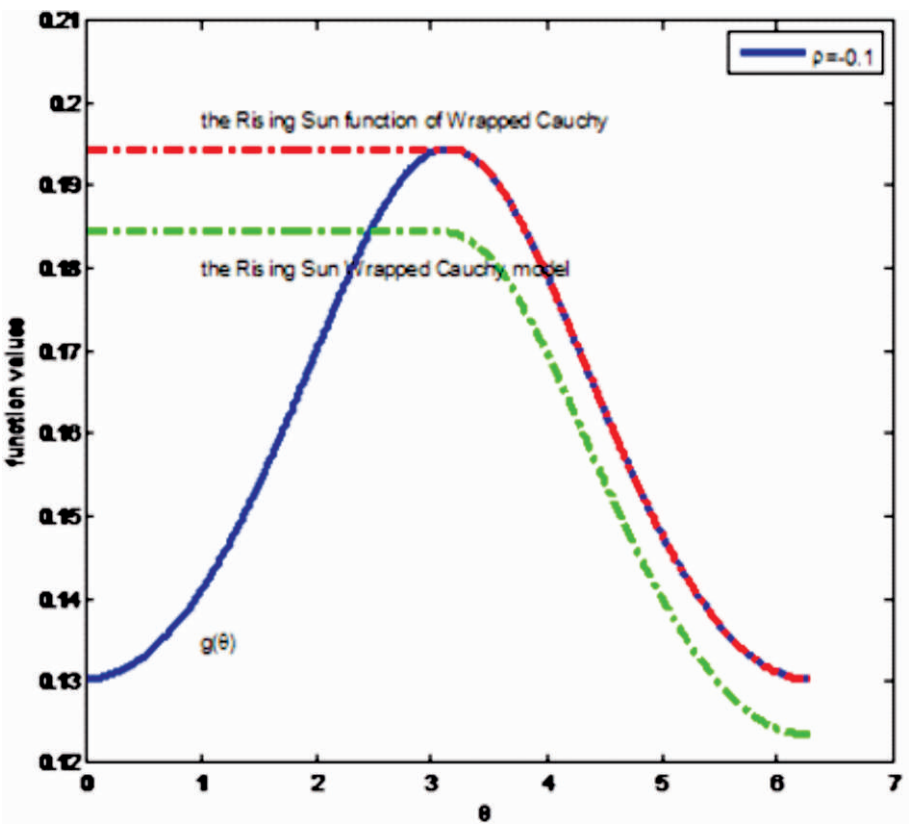

Figure 1. Graph of the Wrapped Cauchy Probability Density Function and the Rising Sun Wrapped Cauchy Probability Density Function

which satisfies the following properties.

$$Pr\left(\theta = \frac{2\pi r}{m}\right) \geq 0$$

$$\sum_{r=0}^{m-1} Pr\left(\theta = \frac{2\pi r}{m}\right) = 1$$

$$Pr\left(\theta = \frac{2\pi r}{m}\right) = Pr(\theta + 2\pi l) \text{ for any integer } l \text{ (ie Pr is periodic)}$$

3.2 Distribution Function

Suppose if θ is a wrapped discrete circular random variable then distribution function of θ is denoted by $G_w(\theta)$ and it is defined as,

$$G_w(\theta) = \sum_{r=0}^{k} Pr\left(\theta = \frac{2\pi r}{m}\right) \qquad \text{where } k = 0, 1, \ldots, m-1$$

4. Discrete Rising Sun Wrapped Cauchy Model

The process of Discretization is usually carried out to model a given discrete phenomenon depending on the practical situation.

Discretization (Wang & Shimizu, 2014) is revisited to construct discrete circular models by transforming existing continuous circular models. The process is as follows.

Consider a continuous circular distribution with probability density function (pdf) $g(\theta)$ on the circle to construct a probability mass function $Pr\left(\theta = \frac{2\pi r}{m}\right)$ on a set of the equally spaced points for r= 0, 1, . . . , m-1 with a fixed integer m equal or greater than 2.

$Pr\left(\theta = \frac{2\pi r}{m}\right) = g\left(\frac{2\pi r}{m}\right) \Big/ \sum_{r=0}^{m-1} g\left(\frac{2\pi r}{m}\right)$ is probability mass function (pmf) of respective discrete circular model.

m = 1 is excluded as it degenerates.

The characteristic function of the Discrete circular random variable is defined as,

$$\varphi_\theta(p) = E(e^{ip\theta}) \text{ where } p \in Z$$

$$= \sum_{r=0}^{m-1} e^{ip\theta} Pr\left(\theta = \frac{2\pi r}{m}\right)$$

The probability density function of the Rising Sun Wrapped Cauchy model equation (11) is discretized to construct the Discrete Rising Sun Wrapped Cauchy model (DRSWC) on unit circle. The probability mass function of the Discrete Rising Sun Wrapped Cauchy model (DRSWC) is,

$$g_{drs}(\theta) = Pr\left(\theta = \frac{2\pi r}{m}\right)$$

$$= \frac{\sup\left(\frac{1}{2\pi}\left(\frac{(1-\rho^2)}{(1+\rho^2-2\rho\cos(t-\mu))}\right) : \theta \leq t < 2\pi\right)}{\int_0^{2\pi} g_\Theta(\theta)\, d\theta}$$

$$= \frac{\sup\left(\frac{1}{2\pi}\left(\frac{(1-\rho^2)}{(1+\rho^2-2\rho\cos(t-\mu))}\right) : \theta \leq t < 2\pi\right)}{\sum_{r=0}^{m-1} \int_0^{2\pi} g_\Theta(\theta)\, d\theta}$$

$$= \frac{\sup\left(\frac{1}{2\pi}\left(\frac{(1-\rho^2)}{(1+\rho^2-2\rho\cos(t-\mu))}\right):\theta \le t < 2\pi\right)}{\sum_{r=0}^{m-1}\sup\left(\frac{1}{2\pi}\left(\frac{(1-\rho^2)}{(1+\rho^2-2\rho\cos(t-\mu))}\right):\theta \le t < 2\pi\right)},$$

$$\theta = \frac{2\pi r}{m},\ t = \frac{2\pi k}{m} < 2\pi,\ r \le k \in \mathrm{N}$$

The graphs of the Probability Mass Function of Discrete Rising Sun Wrapped Cauchy model both linear and circular representations are plotted here through MATLAB in Figures 2 and 3, respectively.

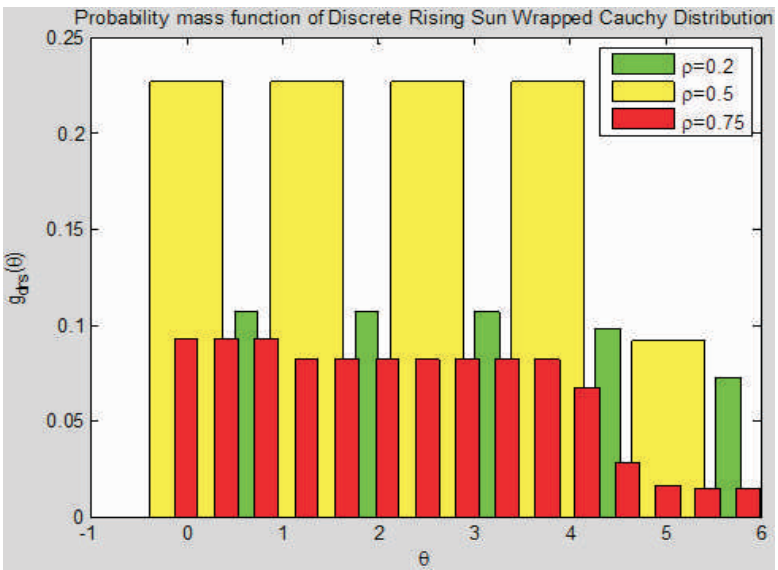

Figure 2. Graph of Probability Mass Function of Discrete Rising Sun Wrapped Cauchy Model (Linear Representation)

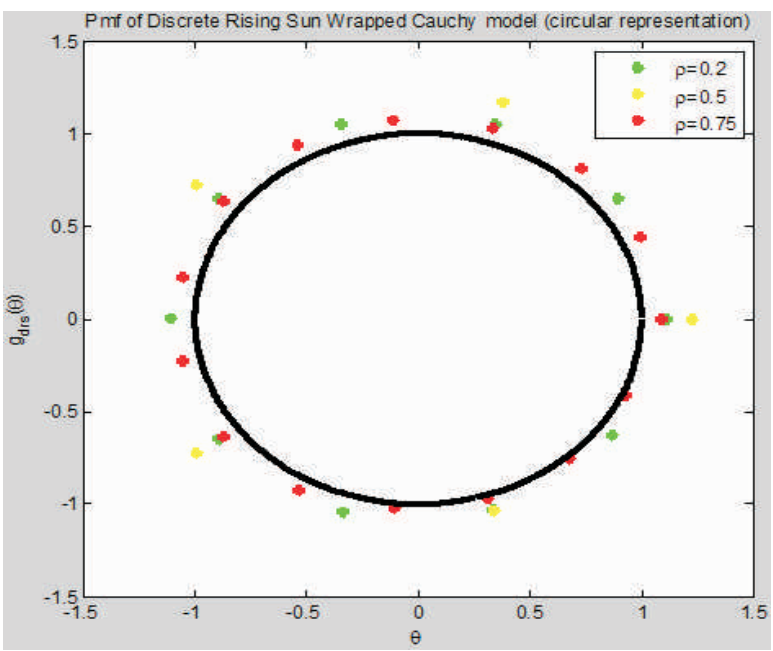

Figure 3. Graph of Probability Mass Function of Discrete Rising Sun Wrapped Cauchy Model (Circular Representation)

5. Characteristic Function

The Characteristic function of the Wrapped discrete circular random variable θ is defined as,

$$\varphi_\theta(p) = \sum_{r=0}^{m-1} e^{ip\theta} Pr(\theta = \frac{2\pi r}{m}) = \rho_p e^{i\mu_p} \qquad \text{where} \quad p \in \mathbf{Z}$$
$$= \alpha_p + i\beta_p$$

These trigonometric moments α_p, β_p in terms of $\alpha_p = E(\cos p\theta)$, $\beta_p = E(\sin p\theta)$ and these are related as,

$$\rho_p = \sqrt{\alpha_p^2 + \beta_p^2} \text{ and } \mu_p = \tan^{-1}\left(\frac{\beta_p}{\alpha_p}\right)$$

6. Central Trigonometric Moments

The pth Central trigonometric moment of θ is defined as, $\varphi_\theta^*(p) = E[e^{ip(\theta-\mu)}] = \alpha^*_p + i\beta^*_p$

Now the Skewness for the circular distribution is defined as $\gamma_1 = \frac{\beta_2^*}{(V_o)^{\frac{3}{2}}}$ and kurtosis for the Circular distribution is defined as, $\gamma_2 = \frac{\alpha_2^* - (1-V_o)^4}{(V_o)^2}$.

Conclusion

The work contemplated is a construction of new discrete circular model. Discrete Rising Sun Wrapped Cauchy model is derived which is useful for analysis of discrete angular or periodic data.

References

[1]. Girija, S. V. S. (2010). *Construction of New Circular Models*. VDM Verlag, Germany.

[2]. Girija, S. V. S., Srihari, G. V. L. N., & Srinivas, R. (2019). On Discrete Wrapped Cauchy Model. *IISTE-Journal of Mathematical Theory and Modeling*, 9(4), 15-23. DOI: 10.7176/MTM.

[3]. Jammalamadaka, S. R., & Sengupta, A. (2001). *Topics in Circular Statistics* (Vol. 5). World Scientific.

[4]. Levy, P. (1939). L'addition des variables aléatoires définies sur une circonférence. *Bulletin de la Société mathématique de France*, 67, 1-41.

[5]. Mardia, K. V. (1975). Statistics of directional data. *Journal of the Royal Statistical Society: Series B (Methodological)*, 37(3), 349-371. https://doi.org/10.1111/j.2517-6161.1975.tb01550.x

[6]. Mardia, K. V., & Jupp, P. E. (2000). *Directional Statistics*. John Wiley and sons.

[7]. McCullagh, P. (1996). Möbius transformation and Cauchy parameter estimation. *The Annals of Statistics*, 24(2), 787-808.

[8]. Radhika, A. J. V., Girija, S. V. S., & Rao, A. D. (2013). On rising Sun von mises and rising Sun wrapped Cauchy circular models. *Journal of Applied Mathematics, Statistics and Informatics*, 9(2), 41-55.

[9]. Rao, D. A. V., Sarma, R. I., & Girija, S. V. S. (2007). On wrapped version of some life testing models. *Communications in Statistics—Theory and Methods*, 36(11), 2027-2035. https://doi.org/10.1080/03610920601143832

[10]. Sarma, R. I., Rao, D. A. V., & Girija, S. V. S. (2009). On characteristic functions of wrapped half logistic and binormal distributions. *International Journal of Statistics and Systems*, 4(1), 33-45.

[11]. Sarma, R. I., Rao, D. A. V., & Girija, S. V. S. (2011). On characteristic functions of the wrapped lognormal and the wrapped Weibull distributions. *Journal of Statistical Computation and Simulation*, 81(5), 579-589. https://doi.org/10.1080/00949650903436547

[12]. Srihari, G. V. L. N., Girija, S. V. S., & Rao, D. A. V. (2018). On Discrete Wrapped Exponential distribution-Characteristics. *International Journal of Scientific Research In Mathematical and Statistical Sciences*, 5(2), 57-64.

[13]. Srinivas, R., Srihari, G. V. L. N., & Girija, S. V. S. (2018). On construction of Discrete Wrapped Lindley Distribution. *i-manager's Journal on Mathematics*, 7(4), 10. DOI:10.26634/jmat.7.4.15874

[14]. Van Rooij, V. A. C., & Schikhof, W. H. (1982). *A Second Course on Real Functions*. Cambridge University Press, Cambridge.

[15]. Wang, & Shimizu, K. (2014). Discrete Cardioid Distribution. *Conference on Advances and Applications in Distribution Theory*. The Institute of Statistical Mathematics, Tokyo, Japan.

[16]. Wintner, A. (1947). On the shape of the angular case of Cauchy's distribution curves. *The Annals of Mathematical Statistics*, 18(4), 589-593.

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ANALYTICAL STUDY OF SIO_2-WATER BASED NANO FLUID FLOW IN A ROTATING FRAME WITH THERMO PHORESIS

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ABSTRACT

The problem of unsteady MHD free convection flow of SIO_2- H_2O nano-fluid through a porous medium bounded by a vertical semi-infinite permeable flat plate with constant heat source and convective boundary condition in a rotating frame of reference is performed, theoretically. The plate is assumed to oscillate in time with steady frequency, so that the solutions of the boundary layer are the same oscillatory types. The entire system rotates about the axes normal to the plate. The dimensionless governing equations for this investigation are solved analytically using perturbation approximation. The effects of various parameters on the flow, heat and mass transfer characteristics are discussed through graphs.

KEYWORDS: Convective Flow, Porous, Rotating Frame, Nano Fluid & Mass Transfer

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Original Article

INTRODUCTION

The exponential growth of the study of nano fluids made the latest technology more convenient and user friendly. As the Nano fluids carries the metallic particle, the heat transfer and diffusion together is an interesting phenomenon with wide application in the field of biomedical, drug delivery etc. There are many applications of hydromagnetic flow of non-Newtonian fluids in a rotating body in metrology, geographic, turbomachinery, astrophysical and several other areas. The study of Nano fluid attracted many researches since last few decades ([12], [13], and [14]) because of its vital applications. In addition, it has many applications in biomedical such as blood flow in capillaries, dialysis of blood in artificial kidney and flow in blood oxygenation. Rotating flows of MHD fluids have many applications in meteorology, geophysics, turbo machinery and many other fields. Such flows in the presence of a magnetic field are significant because of their geophysical and astrophysical importance and invested by many researchers ([7], [8]). The transport phenomena in porous media have been of continuing interest for the past decades. This is due to its wide applications in solar receiver devices, building thermal insulation, heat exchangers, energy storage units, ceramic processing, and catalytic reactors to name a

few. Furthermore, nanofluids have been also used in porous media due to their superior thermal characteristics. Porous media increase the contact surface area between liquid and solid surface, and, on the other hand, nano particles dispersed in nanofluid enhance the effective thermal conductivity. On the other hand, utilizing porous media in heat exchangers is another technique to augment of thermal efficiency. Porous media by providing high surface area contact will ameliorate heat transfer rate in ducts. The natural convection of boundary layer of a porous medium Nano fluid problems were made by ([3], [9]). Over last decades, numerous works have reported ([1], [6], [11]) on the thermal conductivity of nano fluids than that of the conventional heat transfer fluids with the reason of random motion of nanoparticles. The heat source/sink effects in thermal convection are significant, where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Khan and Aziz [5] represented the boundary layer flow of a Nano fluid past, a vertical surface with a constant heat flux. Generally, it was known that heat and mass fluxes were created from temperature and concentration gradient, respectively. However, heat flux is actually can existed due to the concentration gradient which is known as Soret effect. Same goes to the mass flux, where the flux occurred by the temperature gradient and is called Dufour effect. Configuration involving the heat and mass transfer with Soret and Dufour effects is an important subject due to a wide range of applications such as the solidification of binary alloys, groundwater pollutant migration, chemical reactors, geosciences multi-component melts, oil-reservoirs, isotope separation, and in mixture between gases. Generally, the effects of diffusion of matter caused by temperature gradients (Soret effect) and diffusion of heat caused by concentration gradients (Dufour effect) can be become influential when the temperature and concentration gradients are very large and discussed by many researchers recently ([2], [4], [10]).

Present paper deals the study to analyze the development of unsteady, free convection flow of a Nano fluid, past a moving vertical permeable semi-infinite flat plate in a rotating frame of reference. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency, in the presence of a transverse magnetic field.

MATHEMATICAL MODEL AND SOLUTION OF THE PROBLEM

We consider an unsteady and three-dimensional flow of an electrically conducting incompressible Nano fluid past a semi-infinite vertical permeable plate. The flow is assumed to be in the x* – direction, which is taken along the plate in the upward direction and z* axis is normal to the plate.

- A uniform external magnetic field B_0 is taken to be acting along the z* axis.
- It is assumed that there is no applied voltage, which implies the absence of an electric field.
- It is assumed the whole system is rotated with a constant vector Ω about the z*-axis.
- The fluid is grey, absorbing emitting but not scattering medium.
- The radiation heat flux in x*-direction is considered negligible in comparison that in the z*- direction.
- Due to semi-infinite plate surface assumption, the flow variables are functions of z and time t only.
- Assumed that the regular fluid and the suspended nano particles are in thermal equilibrium, and no slip occurs between them.

Boundary layer approximations, the boundary layer equations governing the flow, temperature and concentration along with the Boussinesq are:

$$\frac{\partial w^*}{\partial z^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} - 2\Omega v^* = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u^*}{\partial z^{*2}} + \frac{[\rho\beta]_{nf}}{\rho_{nf}} g(T - T_\infty) - \frac{1}{\rho_{nf}}\sigma B_0^2 u^* - \frac{\mu_{nf}}{\rho_{nf}}\frac{u^*}{k} \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} + w^*\frac{\partial v^*}{\partial z^*} + 2\Omega u^* = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 v^*}{\partial z^{*2}} - \frac{1}{\rho_{nf}}\sigma B_0^2 v^* - \frac{\mu_{nf}}{\rho_{nf}}\frac{v^*}{k} \tag{3}$$

$$\frac{\partial T}{\partial t^*} + w^*\frac{\partial T}{\partial t^*} = \alpha_{nf}\frac{\partial^2 T}{\partial z^{*2}} - \frac{Q^*(T - T_\infty)}{[\rho Cp]_{nf}} \tag{4}$$

$$\frac{\partial C}{\partial t^*} + w^*\frac{\partial C}{\partial z^*} = D_B\frac{\partial^2 C}{\partial z^{*2}} - K_l(C - C_\infty) + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial z^{*2}} \tag{5}$$

The boundary conditions for the problem are given by

$$u^*(z^*, t^*) = 0,\ v^*(z^*, t^*) = 0,\ T = T_\infty,\ C = C_\infty \text{ for } t^* \le 0 \text{ and any } z^* \tag{6}$$

$$\left.\begin{array}{l} u^*(\infty, t^*) = U_0\left[1 + \frac{\varepsilon}{2}(e^{in^*t^*} + e^{-in^*t^*})\right],\ v^*(\infty, t^*) = 0, -k_{nf}\frac{\partial T}{\partial z^*} = h_f(T_w - T_\infty), C = C_w \\ \text{at } z^* = 0 \\ u^* \to 0, v^* \to 0, T \to T_\infty, C \to C_\infty \text{ as } z^* \to \infty \end{array}\right\} \text{for } t^* \ge 0 \tag{7}$$

Here, u*, v* and w* are the velocity components along the x*, y* and z* axis, respectively.

The properties of nanofluids which are given by (Oztop and Abu-Nada [15])

$$\begin{aligned} &\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, (\rho Cp)_{nf} = (1-\phi)(\rho Cp)_f + \phi(\rho Cp)_s, \\ &(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \mu_{nf} = \frac{\mu_f}{[1-\phi]^{2.5}}, \alpha_{nf} = \frac{K_{nf}}{(\rho Cp)_{nf}}, \\ &\frac{K_{nf}}{K_f} = \left[\frac{K_s + 2K_f - 2\phi(K_f - K_s)}{K_s + 2K_f + 2\phi(K_f - K_s)}\right] \end{aligned} \tag{8}$$

The thermo physical properties of the base fluid (water) and other nanofluids are shown in Table 1.

Table 1: Nano Fluids and their Thermo Physical Characteristics (Mohammad Mehdi Keshtekar Et Al. [16])

Nanofluid	Density(ρ)	Specific Heat(Cp)	Thermal Conductivity(k)	$\beta X10^{-5}$
SiO_2	3970	765	0.63	36.0
Pure water	997.1	4179	0.613	21

The solution of Equation (1) is considered as

$$w^* = -w_0 \tag{9}$$

Introducing dimensionless variables in the following manner:

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{U_0}, z = \frac{z^* U_0}{\nu_f}, t = \frac{t^* U_0^2}{\nu_f}, n = \frac{\nu_f n^*}{U_0^2}, S = \frac{w_0}{U_0}, R = \frac{2\Omega \nu_f}{U_0^2}, Q_H = \frac{Q^* \nu_f^2}{K_f U_0^2},$$
$$\Pr = \frac{\nu_f}{\alpha_f}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \psi = \frac{C - C_\infty}{C_w - C_\infty}, M = \frac{\sigma B_0^2 \nu_f}{\rho_f U_0^2}, K = \frac{k \rho_f U_0^2}{\nu_f^2}, Kr = \frac{K_l \nu_f}{U_0^2}, Sc = \frac{\nu_f}{D_B}, \tag{10}$$
$$Sr = \frac{D_m k_T (T_w - T_\infty)}{T_m \nu_f (C_w - C_\infty)}, U_0^3 = g \beta_f (T_w - T_\infty) \nu_f$$

After reducing dimensionless form and the solutions are given by

$$H(z,t) = (1 - B_2) e^{-d_7 z} + B_2 e^{-d_1 z} + \frac{\varepsilon}{2}\left\{e^{-d_8 z}\, e^{\text{int}} + e^{-d_9 z}\, e^{-\text{int}}\right\} \tag{11}$$

$$\theta(z,t) = C_1 e^{-d_1 z} \tag{12}$$

$$\psi(z,t) = (1 + B_1) e^{-d_4 z} - B_1 e^{-d_1 z} + \frac{\varepsilon}{2}\left\{d_8 e^{\text{int}} + d_9 e^{-\text{int}}\right\} \tag{13}$$

Where $H(z,t) = u(z,t) + iv(z,t)$

RESULT ANALYSIS

A theoretical study on the effect of nano particle SiO_2 on MHD free convection flow along a vertical semi-infinite flat plate with constant heat source, soret effect and chemical reaction when the plate oscillates in time t in the presence of a rotating frame of reference has been performed. The effects of nanoparticles on the velocity, the temperature and the concentration profiles as well as on the skin friction coefficient, the local Nusselt number and the local Sherwood number are discussed. Figure 1 (a-c) shows the effect of suction parameter S on the fluid velocity, temperature and concentration for SiO_2 nano particles with $\phi = 0.15$. As an output of figures, it is understandable that the velocity, temperature and concentration of the fluid across the momentum boundary layer decreases by increasing suction parameter *S* (>0). This indicates the usual fact that the suction stabilizes the boundary growth. These consequences are obviously supported from the physical point of view. Figure 2 (a-c) shows the nano fluid velocity, temperature and concentration profiles for different values of heat generation parameter Q_H with SiO_2 nanoparticles. From the graphs, it is clear that there is a decrease in the nanofluid velocity and

temperature profiles and reverse flow happened in case of concentration with an increasing in Q_H. These profiles satisfy the far field boundary conditions, asymptotically. This is due to the fact that when heat is absorbed, the buoyancy forces decrease which retard the flow rate and thereby give rise to a decrease in the velocity and temperature profiles. Figure 3 (a-c) illustrates the effect of the nanoparticle volume fraction ϕ on the velocity, temperature and concentration profiles for SiO_2 nanoparticles. It is clear that as the nanoparticle volume fraction increases, the nanofluid velocity and concentration decreases and the fluid temperature increases with an increasing value of volume fraction parameter ϕ. Hence, the thermal boundary layer thickness increases and tends asymptotically to zero as the distance increase from the thermal boundary layer. This agrees with the physical behavior that, when the volume fraction of SiO_2 increases, the thermal conductivity increases, and then the thermal boundary-layer thickness increase. Figure. 4 (a-c) demonstrates the effect of the convective parameter γ on the fluid velocity, temperature and concentration. As an output of figures, it is observed that the velocity is increasing, and in the case of the temperature and the concentration, it happened in reverse. From the temperature distributionwe have, the fluid within the thermal boundary layer increases with increasing values of γ. Figure 5 (a-c) shows the variation of Prandtl number of nanofluids on velocity, temperature and concentration with $\phi = 0.15$. The Prandtl number of nanofluids is found to decrease in velocity and temperature and increase in concentration with an increasing of Prandtl number. The influence of rotation R parameter on the nanofluid velocity profiles for various values of SiO_2 nanoparticles with $\phi = 0.15$(nanofluid) is shown in figure.6. From the figure, we see that an increasing in R leads to decreasing in the values of the velocity across the momentum boundary layer. Figure.7 demonstrates the effect of magnetic field parameter M on the fluid velocity SiO_2 nanoparticles. It is clear from figure that the velocity distribution across the momentum boundary layer reduces with an increase in the magnetic field parameter *M* and decreases asymptotically, to zero at the edge of the hydrodynamic boundary layer. Hence, the hydrodynamic boundary layer thickness decreases as the magnetic field parameter M increases, the local velocity also decreases. The variation in the concentration boundary layer of the flow field is shown in figure.8. Due to the change in the Soret number, Sr curves are with Sr = 1,2,3 and 4, respectively. Comparing the curves of the said figure, it is observed that a growing Soret number increases the concentration profiles as well as concentration boundary layer thickness.

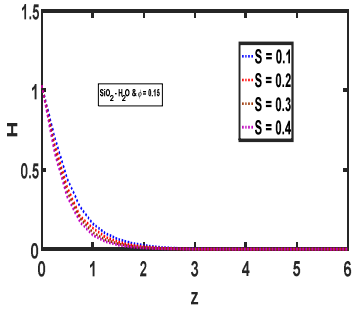

Figure 1(a): Suction Parameter S Verses H

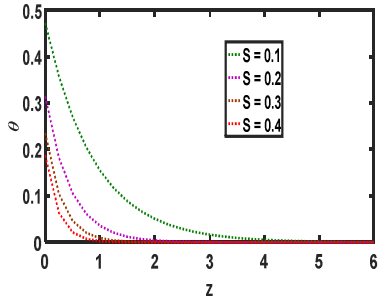

Figure 1(b): Suction Parameter S Verses Θ

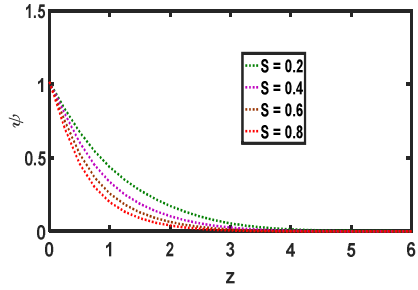

Figure 1(c): Suction Parameter S Verses Ψ

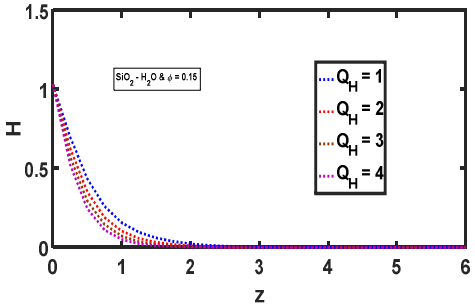

Figure 2(a): Heat Generation Parameter Q_H Verses H

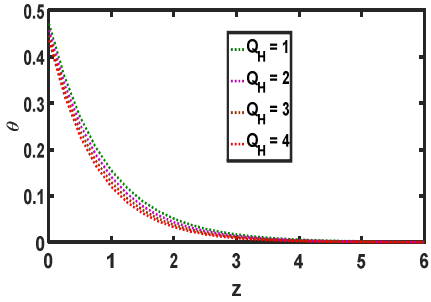

Figure 2(b): Heat Generation Parameter Q_H Verses Θ

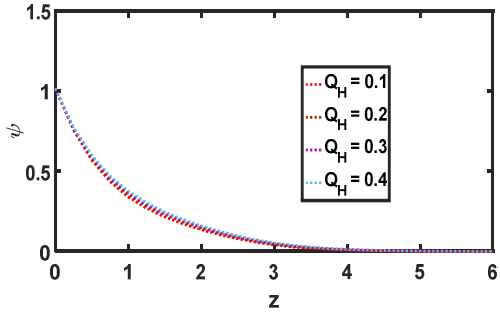

Figure 2(c): Heat Generation Parameter Q_H Verses Ψ

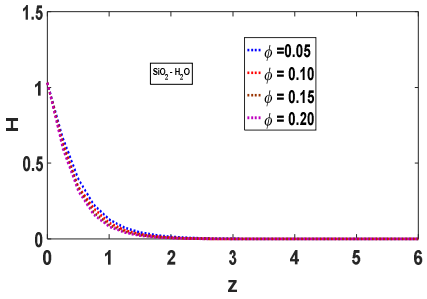

Figure 3(a): Volume Fraction Parameter Φ Verses H

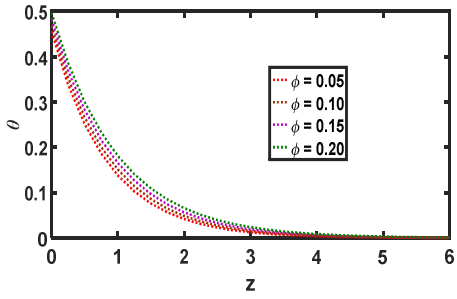

Figure 3(b): Volume Fraction Parameter Φ Verses Θ

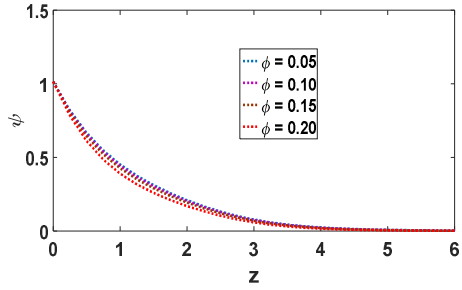

Figure 3(c): Volume Fraction Parameter Φ Verses Ψ

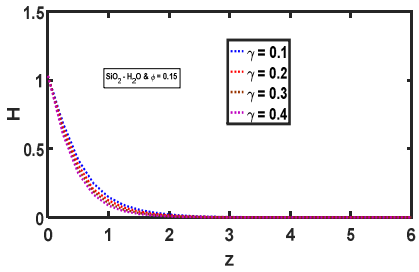

Figure 4(a): Convective Parameter Γ Verses H

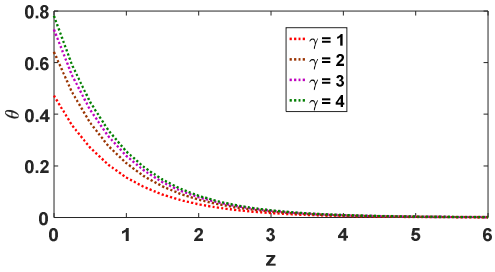

Figure 4(b): Convective Parameter Γ Verses Θ

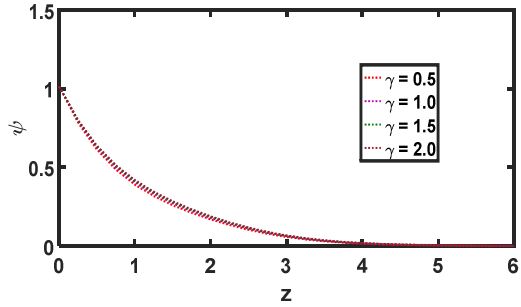

Figure 4(c): Convective Parameter Γ Verses Ψ

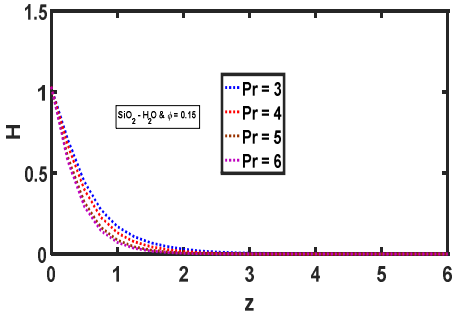

Figure 5(a): Prandtal Number Pr Verses H

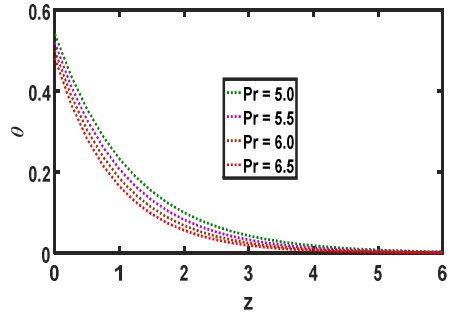

Figure 5(b): Prandtal Number Pr Verses Θ

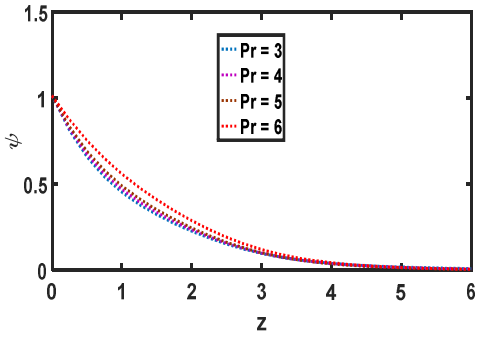

Figure 5(c): Prandtal Number Pr Verses Ψ

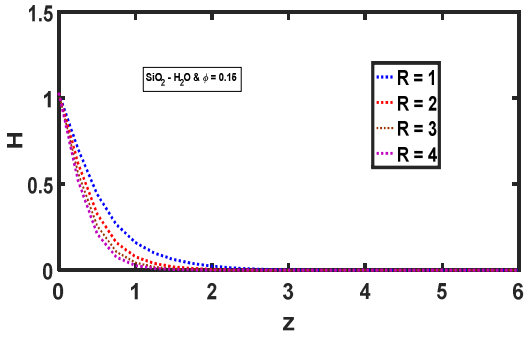

Figure 6: Rotation Parameter R Verses H

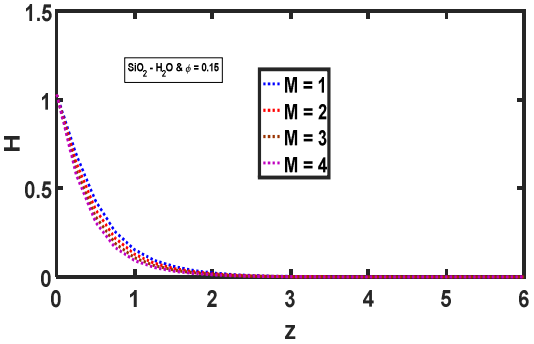

Figure 7: Magnetic Parameter M Verses H

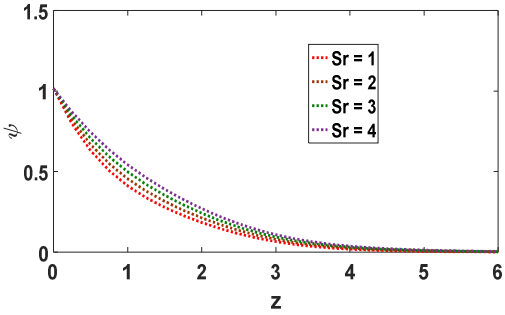

Figure 8: Soret Number Sr Verses

CONCLUSIONS

In this work, we have studied and analyzed the development of the unsteady free convection flow of a SiO_2-water nanofluid past a moving vertical permeable semi-infinite flat plate, in a rotating frame of reference. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency, in the presence of a transverse magnetic field. The effects of various parameters on velocity, temperature and concentration profiles are discussed through graphs. The following are the conclusions derived from the present study.

- As S increases, the SiO_2-water nanofluid velocity decreases.
- As the M increases, the SiO_2-water nanofluid velocity decreases.
- The nanofluid velocity and concentration decreases, and the fluid temperature increases with an increasing value of volume fraction parameter ϕ.
- There is a decrease in the SiO_2-water nanofluid velocity and temperature profiles, and reverse flow happened in case of concentration with an increasing in Q_H.

REFERENCES

1. *Boungiorno, J. (2009). A benchmark study of thermal conductivity of nanofluids. J. Appl. Phys., 106: paper 094312.*
2. *Buongiorno, J. (2006). Convective transport in nanofluids. J Heat Transfer, 128, 240–250.*
3. *Gorla, R.S.R., Chamkha, A.J.(2011). Natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with ananofluid. J. Modern Phys.,2, 62–71.*
4. *Krishna, V. M. (2016). Heat Transfer Enhancement by using ZnO Water Nanofluid in a Concentric Tube Heat Exchanger under Forced Convection Conditions. International Journal of Innovations in Engineering and Technology, 7(4), 177-184.*
5. *Khan, W.A., Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. Int. J. Heat Mass Transfer, 53, 2477–83.*
6. *Khan, W. A., Aziz, A.(2011). Natural convection flow of a nanofluid over a vertical plate with uniform surface heat flux. Int J ThermSci, 50, 1207–14.*
7. *Makinde, O.D., Mutuku, W.N. (2014). Hydromagnetic thermal boundary layer of nanofluids over a convectively heated flat plate with viscous dissipationand Ohmic heating. UPB Sci Bull Ser A, 76, 181–192.*
8. *QASHQAEI, A., & ASL, R. G. (2015). Numerical Modeling And Simulation Of Copper Oxide Nanofluids Used In Compact Heat Exchangers. International Journal of Mechanical Engineering, 4 (2), 1, 8.*
9. *Herdricha, G., Auweter-Kurtza, M., Fertiga, M., Nawaza, A., Petkowa. D. (2006). MHD Flow Control For Plasma Technology Applications, 80, 1167–1173.*
10. *Seddeek, M. A., Afify, A. A., Hanaya, M. A. (2009). Similarity Solutions For Steady MHD Falkner-Skan Flow and Heat Transfer Over a Wedge by Considering the Effect of Variable Viscosity and Thermal Conductivity.Applications and Applied Mathematics, 4, 301–313.*
11. *Abd El-Gaied, S. M., Hamad, M. A. A. (2013). MHD Forced Convection Laminar Boundary Layer Flow of Alumina-Water Nanofluid over a Moving Permeable Flat Plate with Convective Surface Boundary Condition. Hindawi Publishing Corporation, Journal of Applied Mathematics,2013, 1-8.*
12. *Md Abdullah Al Mahbub., Nasrin Jahan Nasu., Shomi Aktar., Zillur Rahman., (2013). Soret-Dufour Effects on the MHD Flow and Heat Transfer of Microrotation Fluid over a Nonlinear Stretching Plate in the Presence of Suction.Applied Mathematics, 4, 864-875.*
13. *Lee,S., Choi,S. U. S., Li.S., Eastman, J. A. (1999).Measuring thermal conductivity of fluids containing oxide Nano particles, ASME J. Heat Transfer, 121, 280–289.*
14. *Ellahi,R., Raza, V., Vafai, K., Weinberg,S. L. (2012). Series solutions of non-Newtonian nanofluids with Reynold's model and Vogel's model by means of the homotopy analysis method, Math. Comput. Model. 55, 1876–1891.*
15. *Fakour M, Rahbari A and Khodabandeh E 2018 Nanofluid thin film flow and heat transfer over an unsteady stretching elastic sheet by LMS J. Mech. Sci. Tech. 32 177–83*

16. *Abolbashari, M.H., Freidoonimehr, N., Rashidi, M.M. (2015). Analytical modeling of entropy generation for Casson nano-fluid flow induced by a stretching surface Adv. Powder Technol., 6, 542–552*

17. *Oztop, H.F., Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. Int J Heat Mass Transf, 29, 1326–1336.*

18. *Mohammad Mehdi Keshtekar, Ahmad Khalueir, Hadi Ameri Fard. (2014). Effects Of Thermal Radiation, Viscous Dissipation, Variable Magnetic Field And Suction On Mixed Convection Mhd Flow Of Nano Fluid Over A Non-Linear Stretching Sheet.IOSR Journal of Engineering, 4, 44-54.*

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(51) International classification :A23K 50/80
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(32) Priority Date :NA
(33) Name of priority country :NA
(86) International Application No :NA
Filing Date :NA
(87) International Publication No : NA
(61) Patent of Addition to Application Number :NA
Filing Date :NA
(62) Divisional to Application Number :NA
Filing Date :NA

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(57) Abstract :
In underwater communications systems, wireless sensor grids are the enabled technology, and they combine wireless sensor grids. Underwater communication mainly carried out by a series of nodes for transmitting data to the nearby base station for the monitoring and control station in the shore area between its nodes and sink node, the sensed data. The major problems in the oceanic area are under water pollution. Underwater contamination causes acidification, eutrophication, plastic waste, water noise and toxins. This pollution is currently identified by means of human monitoring. Automated and smart monitoring systems are necessary to detect the occurrence of this pollution. The proposed model describes the smart sensor-based monitoring system which identifies pollution in the groundwater and alerts it to occur. With the monitoring system you can detect contamination by automatically and intelligently using the temperature sensor, humidity sensor, pressure sensor and chemical sensor. The system efficiency is tested and the results show that it is better than the process of human monitoring

No. of Pages : 18 No. of Claims : 5

Summary of Invention

The crucial aim of the research is towards track the consistency of the water samples by developing smart water quality monitoring system capable of detecting four different physical parameters: pH, temperatures, humidity, water conductivity and evaluating the derived importance of the listed parameters using the correct machine learning method. With the assistance of Arduino dependent sensors, various water samples are checked, and their values are obtained from different measurements. A quick forest binary classifier provides more considerable efficiency testing to verify the precision and efficacy of the method when forecasting the water quality. A system device consumes demonstrated its value using delivering reliable results in detecting water quality dependent on physical parameters.3 This device could be applied for real-time water monitoring elucidation. This could detect the chemical water parameters.

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We Claim

What is claim is

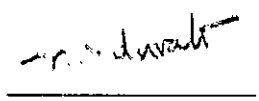
1) Proposed intelligent system consist a modules namely underground water quality monitoring. In water quality monitoring various sensors are involved to check the quality of water in terms of Hydrogen ions (pH), humidity, Temperature, and water conductivity. In quality water distribution management system it used to monitor

2) The water quality monitoring module as claimed in claim 1, receives all the real time information from various sensors to check the quality of water in terms of suitable to living begin or not. All the sensor information's are automatically processed for efficient observation, monitoring and controlling thought hardware platform.

3) The quality water distribution management system as claimed in claim 1, receives only the quality water from the claim 1 and distribute it to the consumer in proper way. Providing accurate water consumption levels, smart meter billing, reducing wastage of water in water-pipeline and finding the capacity of stored water level are monitored by hardware platform.

4) As claimed in claim 2 and claim 3, the observed real time sensor values are monitored and controlled by secure mobile application for the use of remote accessing.

5) As claimed in claim 2 and claim 3, the observed real time sensor values are also stored system for data storage and data processing for present and future need.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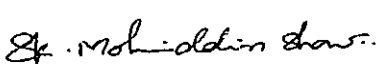
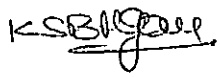

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(Dr. SHAIK MOHIDDIN SHAW) (Dr. K.S. BALAMURUGAN)

(Dr. J L RAMA PRASAD) (DR. G.DHARMAIAH) (SREELATHA SANKURU)

(NADIMPALLI UDAYA BHASKARA VARMA) (KRANTHI KUMAR KOTHA)

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FIELD OF INVENTION

This invention is a device for tracking the water contamination that is used to measure water constantly, based on the four physical parameters, such as temperature, Humidity, pH, and Pressure properties. The ARDUINO-UNO is mounted on the four sensors which detect water parameters. Extracted sensory data were passed to the WHO (World Health Organization) software package according to the normal values. The proposed method of machine innovation would expertly measure water limitations using the fast wood binary classification to decide whether or not the measured water sample will drink based on the determined results.

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BACKGROUND OF INVENTION

The impact of water is beyond any human thing's description. Water conservation is a major concern with the exponential growth of the worldwide population, especially in production, agriculture and other industries. Most people around the world struggled behind drinking water. Per year, many people die of all lethal water-borne diseases. A research found that nearly five million deaths are mainly caused by unregulated water use.

Analysis of the WHO shows that almost 1.4 million child deaths are stopped by providing them with drinking water. The key purpose of this project remains the introduction of an intelligent water quality management network to help us monitor many physical parameters of drinking water rather than manual processes. Many computational studies have been conducted in recent years to create smart systems for water parameter specification and monitoring.

It is suggested to track the quality and delivery of water in-road control network based on sensor nodes. Their conceptual design, optical sensors are based on the lightweight, low-cost, electro-chemical pipe frames and sensors for this design. This approach is suitable for broad scale water consumers, water generators and colonial water plan classifications. For water protection control of both publisher and customer, a broker less software framework is built. The pH, temperature and oxygen measured from water samples are analyzed, leading to a proportional reversal to the relationship between them.

There are four basic sensors, such as ph, temperature and humidity, based on an industrial water quality control system. The goal of the research remains the development by the wireless framework of an intelligent water quality monitoring system. Four physical constraints are determined using four independent Arduino Uno equipped sensors: temperature, humidity and *pH of various water samples.*

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Object of Invention

- Microcontroller (Arduino)
- Humidity Sensor
- Pressure sensor
- Chemical sensor
- Temperature sensor
- RF Transmitter

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Summary of Invention

The crucial aim of the research is towards track the consistency of the water samples by developing smart water quality monitoring system capable of detecting four different physical parameters: pH, temperatures, humidity, water conductivity and evaluating the derived importance of the listed parameters using the correct machine learning method. With the assistance of Arduino dependent sensors, various water samples are checked, and their values are obtained from different measurements. A quick forest binary classifier provides more considerable efficiency testing to verify the precision and efficacy of the method when forecasting the water quality. A system device consumes demonstrated its value using delivering reliable results in detecting water quality dependent on physical parameters.3 This device could be applied for real-time water monitoring elucidation. This could detect the chemical water parameters.

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Detailed Description of Invention

The detailed description of various exemplary embodiments of the disclosure is described herein with reference to the accompanying drawings. It should be noted that the embodiments are described herein in such details as to clearly communicate the disclosure. However, the amount of details provided herein is not intended to limit the anticipated variations 10 of embodiments; on the contrary, the intention is to cover all modifications, equivalents, and alternatives falling within the spirit and scope of the present disclosure as defined by the appended claims.

It is also to be understood that various arrangements may be devised that, although not explicitly described or shown herein, embody the principles of the present disclosure. Moreover, all statements herein reciting principles, aspects, and embodiments of the present disclosure, as well as specific examples, are intended to encompass equivalents thereof

A real-time remote water quality observing device is invented to monitor the quality of water of a static water reservoir or a water distribution system of an urban or sub-urban area. The water quality data can be saved locally in the device and also the water quality data can be visualizing locally by a display unit that is attached with the device. As well as the data can be visualizing from any location through central server using any remote monitoring device through remote login.

In one embodiment of the present invention, a water quality monitoring unit is illustrated in the figure 1. The real-time water quality monitoring unit comprises a sensing unit for measuring a physical condition parameter of water producing measured data of said monitoring physical parameters. The measuring components are the sensors which are used for sensing 30 water quality parameters (pH, pressure, temperature, humidity) of underground water. The sensing unit includes a pH sensor, a humidity sensor, a pressure sensor, a temperature sensor. The said sensors are fixed the controller. Said measured physical condition parameter of said water can be pH, humidity, pressure, temperature of underground water being used for producing said monitoring data accordingly. All the data coming from the sensors like Temperature, Pressure, pH and humidity data are sent to the main processor. After collecting the data by the

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Main Processor Unit, it saved the data locally in local storage unit in CSV format as a backup, displayed the measured value and data in attached Display Unit and send the data to a central server (201) via transmitting unit. Furthermore, the local monitoring unit comprises all the sensor data coming from different sensors, a main processor unit, display, local storage unit and a transmitting unit. The water quality data can be seen from any remote monitoring device through remote login. This key water quality misbrands are best in use for classifying/defining water quality as well as governing a distributed water supply network/pipeline network/ static reservoir, and could simply be measured by respective monitoring units. Moreover, said monitoring unit can be a sensor, especially a (online) membrane sensor, further more specifically monitoring real-time water quality of said water online

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Detailed Description of Drawings

(1) Figure (i) shows the Block Diagram

BATTERY/POWER SUPPLY:

A power supply unit (or PSU) converts mains AC to low-voltage regulated DC power for the internal components.

(2) Figure (ii) shows Micro Controller

A microcontroller is a compact integrated circuit designed to govern a specific operation in an embedded system. A typical microcontroller includes a processor, memory and input/output (I/O) peripherals on a single chip.

(3) Figure (iii) shows the Temperature Sensor

An electronic device for the temperature sensor measures the temperature of your environment and converts your input data into electronic data to record, track or signal temperature changes.

(4) Figure (iv) shows the Humidity Sensor

A sensor is a moisture device for water vapour detection and measurement. There was a mistake. These humidity sensors accurately measure the dew point and absolute humidity based on our robust, capacitive technology.

(5) Figure (v) shows the Pressure Sensor

By converting pressure to an analogue electrical signal, a pressure sensor works. During the steam age, demand for pressure measurement instruments increased. There was a misunderstanding. We now use pressure transducers and pressure switches to electronically measure pressure.

(6) Figure (vi) shows the Chemical Sensor

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Chemical sensors are measuring instruments which convert chemical or physical properties of a particular analyte into a measurable signal, the magnitude of which is usually proportional to the analyser 's concentration

(7) Figure (vii) shows the RF Transmitter

The transmitter is the generator of the radiofrequency current that is supplied to the transmitter spool. The transmitter is RF. This creates a signal that excites protons in the field of photography. There was a misunderstanding. This energy is transmitted in the form of a quick, intense RF explosion at the resonant frequency of hydrogen.

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Different Embodiment of Invention

Economical

This system is economical as compared to many of the existing manual solutions. It saves a lot of money and time of the government.

Real time monitoring

This system works in real time to monitor the quality of underground water. The system enables to a real time view of data using wireless data transmission and its store the monitor data's for future use.

Requires no human supervision: -

This system requires almost no human supervision. All of these sensors are continually monitor the underground water quality and send to the monitor data to server using RF transmitter.

Effective

Monitoring offers reliable data needed for sound water quality management decisions now and in the future. Water quality control is used to warn us about established, potential and developing problems; to determine whether or not drinking water requirements are met with and to safeguard other useful water uses. so this system is very important and effective.

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Applications of Invention

ADVANTAGES

1. Due to automation, the parameter management time is reduced.
2. For average citizens this is economically available.
3. Low maintenance.
4. Prevention of water diseases.

APPLICATIONS

1. This system is used in domestic and industrial applications.
2. Water supply agencies.
3. To determine the cause for water diseases in the health department.

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Drawings

BLOCK DIAGRAM

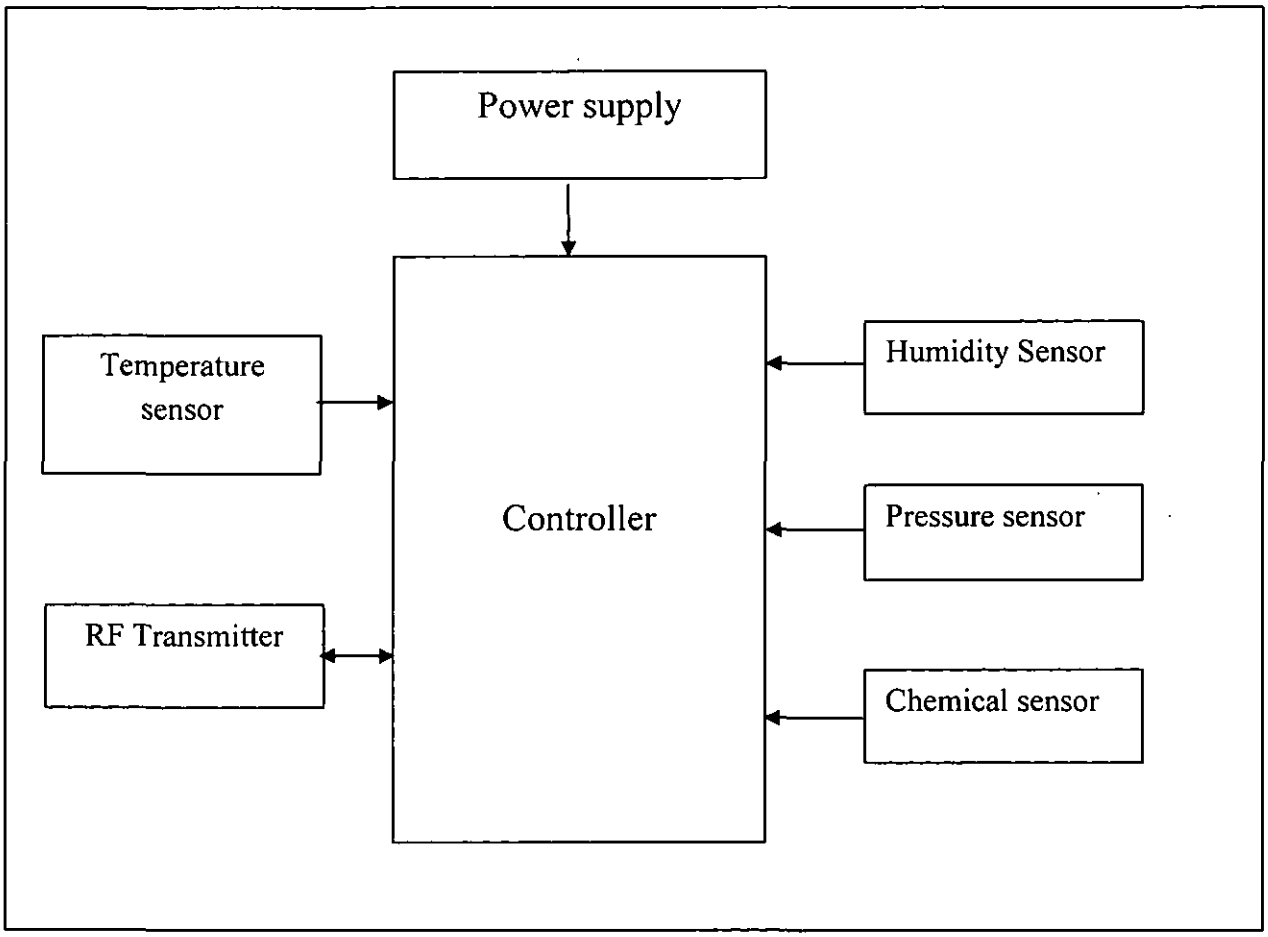

Figure (i) shows the Block Diagram

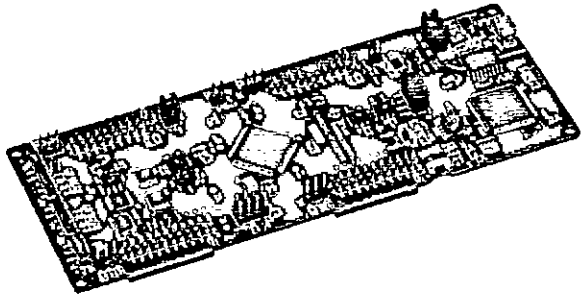

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Figure (ii) shows Micro Controller

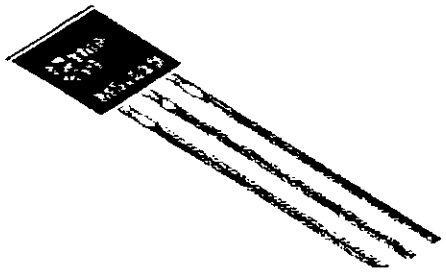

Figure (iii) shows the Temperature Sensor

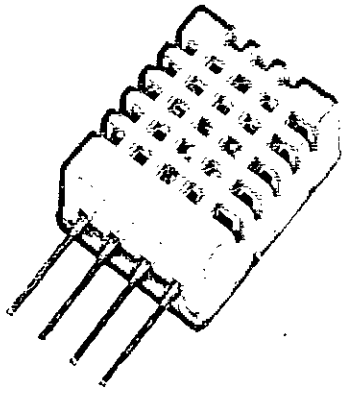

Figure (iv) shows the Humidity Sensor

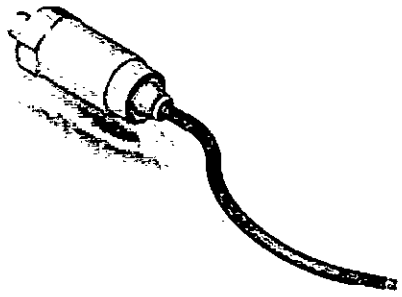

Figure (v) shows the Pressure Sensor

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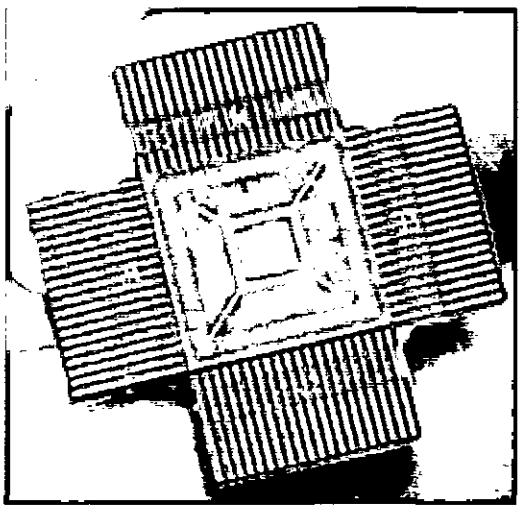

Figure (vi) shows the Chemical Sensor

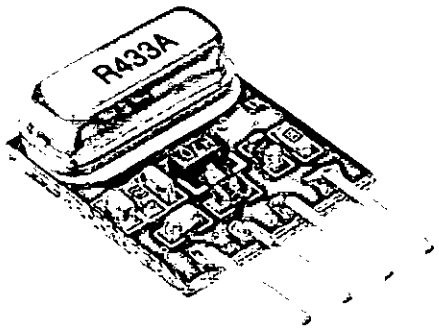

Figure (vii) shows the RF Transmitter

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NewAppl. (D) 89136-F-9
P.V. 12/10/20

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"FORM 1 THE PATENTS ACT 1970 (39 of 1970) and THE PATENTS RULES, 2003 **APPLICATION FOR GRANT OF PATENT** (See section 7, 54 and 135 and sub-rule (1) of rule 20)	(FOR OFFICE USE ONLY) 700301568

	Application No.	202041044264
	Filing date:	12/10/2020
	Amount of Fee paid:	1750/-
	CBR No:	34169
	Signature:	P.V. 12/10/20
1. APPLICANT'S REFERENCE / IDENTIFICATION NO. (AS ALLOTTED BY OFFICE)		

2. TYPE OF APPLICATION [Please tick (✓) at the appropriate category]

Ordinary (✓)		Convention ( )		PCT-NP ( )	
Divisional ( )	Patent of Addition (✓)	Divisional ( )	Patent of Addition ( )	Divisional ( )	Patent of Addition ( )

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3B. CATEGORY OF APPLICANT [Please tick (✓) at the appropriate category]

Natural Person ( )	Other than Natural Person		
	Small Entity ( )	Startup ()	Others ( )

4. INVENTOR(S) [Please tick (✓) at the appropriate category]

Are all the inventor(s) same as the applicant(s) named above?	Yes (✓)	No ( )

If "No", furnish the details of the inventor(s)

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5. TITLE OF THE INVENTION: UNDER WATER POLLUTION MONITORING SYSTEM TO PREVENT FATALITIES OF AQUA ECOSYSTEM.

~~6. AUTHORISED REGISTERED PATENT AGENT(S)~~	~~IN/PA-No.~~	
	~~Name~~	
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Name	SREELATHA SANKURU, LECTURER, MATHEMATICS,

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~~8. IN CASE OF APPLICATION CLAIMING PRIORITY OF APPLICATION FILED IN CONVENTION COUNTRY, PARTICULARS OF CONVENTION APPLICATION~~

~~Country~~	~~Application Number~~	~~Filing date~~	~~Name of the Applicant~~	~~Title of the invention~~	~~IPC (as classified in the convention country)~~

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~~9. IN CASE OF PCT NATIONAL PHASE APPLICATION, PARTICULARS OF INTERNATIONAL APPLICATION FILED UNDER PATENT CO-OPERATION TREATY (PCT)~~	
~~International application number~~	~~International filing date~~
~~10. IN CASE OF DIVISIONAL APPLICATION FILED UNDER SECTION 16, PARTICULARS OF ORIGINAL (FIRST) APPLICATION~~	
~~Original (first) application No.~~	~~Date of filing of original (first) application~~
~~11. IN CASE OF PATENT OF ADDITION FILED UNDER SECTION 54, PARTICULARS OFMAIN APPLICATION OR PATENT~~	
~~Main application/patent No.~~	~~Date of filing of main application~~

12. DECLARATIONS

(i) Declaration by the inventors

(In case the applicant is an assignee: the inventors may sign herein below or the applicant may upload the assignment or enclose the assignment with this application for patent or send the assignment by post/electronic transmission duly authenticated within the prescribed period).

We, the above-named inventors are the true & first inventors for this Invention and declare that the applicants herein are our assignee or legal representative.

(a) Date: 10.10.2020

(b) Signature: 

(c) Name: Dr. Ch. BABY RANI

(a) Date:10.10.2020

(b) Signature: _____

(c)Name: Dr. NALLAPATI VEDAVATHI

(a) Date: 10.10.2020

(b) Signature: 

(c) Name: Dr. Y. UDAYA KUMAR

(a) Date: 10.10.2020

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

(b) Signature:

(c) Name: Dr. SHAIK MOHIDDIN SHAW

(a) Date: 10.10.2020

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(c) Name: Dr. K.S. BALAMURUGAN

(a) Date: 10.10.2020

(b) Signature:

(c) Name: Dr. J L RAMA PRASAD

(a) Date: 10.10.2020

(b) Signature:

(c) Name: Dr. G. DHARMAIAH

(a) Date: 10.10.2020

(b) Signature:

(c) Name: SREELATHA SANKURU

(a) Date: 10.10.2020

(b) Signature:

(c) Name: NADIMPALLI UDAYA BHASKARA VARMA

(a) Date: 10.10.2020

(b) Signature:

(c) Name: KRANTHI KUMAR KOTHA

12-Oct-2020/89136/202041044264/Form 1

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(ii) Declaration by the applicant(s) in the convention country

(In case the applicant in India is different than the applicant in the convention country: the applicant in the convention country may sign herein below or applicant in India may upload the assignment from the applicant in the convention country or enclose the said assignment with this application for patent or send the assignment by post/electronic transmission duly authenticated within the prescribed period)

I/We, the applicant(s) in the convention country declare that the applicant(s) herein is/are my/our assignee or legal representative.

(a) Date

(b) Signature(s)

(c) Name(s) of the signatory

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(iii) Declaration by the applicants

We the applicants hereby declares that: -

We are in possession of the above-mentioned invention.

- ☐ The provisional/complete specification relating to the invention is filed with this application - NA
- ☐ The invention as disclosed in the specification uses the biological material from India and the necessary permission from the competent authority shall be submitted by us before the grant of patent to us.
- ☐ There is no lawful ground of objection(s) to the grant of the Patent to us - NA
- ☐ We are the true &first inventors.
- ☐ We are the assignee or legal representative of true & first inventors
- ☐ The application or each of the applications, particulars of which are given inParagraph-8, was the first application in convention country/countries in respect of my/our invention(s) - NA
- ☐ We claim the priority from the above-mentioned application(s) filed in convention country/countries and state that no application for protection in respect of the invention had been made in a convention country before that date by me/us or by any person from which We derive the title.
- ☐ Our application in India is based on international application under Patent Cooperation Treaty (PCT) as mentioned inParagraph-9 - NA
- ☐ The application is divided out of Our application particulars of which is given in Paragraph-10 and pray that this application may be treated as deemed to have been filed on DD/MM/YYYY under section 16 of the Act.
- ☐ The said invention is an improvement in or modification of the invention particulars of which are given inParagraph-11 – NA

13. FOLLOWING ARE THE ATTACHMENTS WITH THE APPLICATION

(a) Form 2

Item	Details	Fee	Remarks
Complete/ provisional specification) #	No. of pages One		
No. of Claim(s)	No. of claims Five and No. of pages - Two		
Abstract	No. of pages - One		
No. of Drawing(s)	No. of drawings and No. of pages – Seven and Five		

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In case of a complete specification, if the applicant desires to adopt the drawings filed with his provisionalspecificationasthedrawingsorpartofthedrawingsforthecompletespecification underrule13(4), the number of such pages filed with the provisional specification are required to be mentioned here.

(b) Complete specification (in conformation with the international application)/as amended before the International Preliminary Examination Authority (IPEA), as applicable(2copies).

(c) Sequence listing in electronic form

(d) Drawings (in conformation with the international application)/as amended before the International Preliminary Examination Authority (IPEA), as applicable (2copies).

(e) Priority document(s) or a request to retrieve the priority document(s) from DAS (Digital Access Service) if the applicant had already requested the office of first filing to make the priority document(s) available to DAS.

(f) Translation of priority document/Specification/International Search Report/International Preliminary Report on Patentability.

(g) Statement and Undertaking on Form3

(h) Declaration of Inventorship on Form5

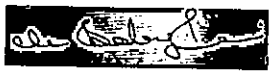
(i) Power of Authority

(j).....

Total fee ₹4,500.00 in Demand Draft No. 211703 Date 07.10.2020 Drawn on Axis Bank, Nanganallur Branch.

We hereby declare that to the best of our knowledge, information and belief the fact and matters slated herein are correct and We request that a patent may be granted to us for the said invention.

Dated this Tuesday of 10th October.2020

(a) Signature: 

(b) Name: Dr. Ch. BABY RANI

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12-Oct-2020/89136/202041044264/Form 1

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(b) Name: NADIMPALLI UDAYA BHASKARA VARMA

(a) Signature:

(b) Name: KRANTHI KUMAR KOTHA

To,

The Controller of Patents

The Patent Office, at Guindy, Chennai – 600 032.

Note: -

* Repeat boxes in case of more than one entry.
* To be signed by the applicant(s) or by authorized registered patent agent otherwise where mentioned.
* Tick (✓)/cross (x) whichever is applicable/not applicable in declaration inparagraph-12.
* Name of the inventor and applicant should be given in full, family name in the beginning.
* Strike out the portion which is/are not applicable.
* For fee: See First Schedule";

12-Oct-2020/89136/202041044264/Form 1

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700301569

FORM 2
THE PATENT ACT 1970
(39 of 1970)
&
The Patents Rules, 2003
PROVISIONAL/COMPLETE SPECIFICATION
(See Section 10 and Rule 13)

COMPLETE

UNDER WATER POLLUTION MONITORING SYSTEM TO PREVENT FATALITIES OF AQUA ECOSYSTEM

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12-Oct-2020/89136/202041044264/Form 2(Title Page)

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The following specification particularly describes the invention

and the manner in which it is to be performed

12-Oct-2020/89136/202041044264/Form 2(Title Page)

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FORM 9
THE PATENT ACT, 1970
(39 of 1970)
&
The Patents Rules, 2003
REQUEST FOR PUBLICATION
[See Section 11A[2]; rule 24A]

CBR : 34169
DATE : 12/10/2020
AMT : 2750/-
P.V
12/10/20

1. Name, Address and Nationality of the Applicants

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12-Oct-2020/89136/202041044264/Form 9

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2. To be Signed by the Applicant Or his authorized registered Patent Agent	hereby request for early publication of our application for Patent No.....dated ..... under section 11A(2) of the Act. Dated this .....day of .....20.

2. Name of the Natural Persons Has signed

(a) Signature: 

(b) Name: Dr. Ch. BABY RANI

(a) Signature: _____

(b) Name: Dr. NALLAPATI VEDAVATHI

(a) Signature: 

(b) Name: Dr. Y. UDAYA KUMAR

(a) Signature:

(b) Name: Dr. SHAIK MOHIDDIN SHAW

(a) Signature:

(b) Name: Dr. K.S. BALAMURUGAN

12-Oct-2020/89136/2020410442 64/Form 9

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(d) Name: Dr. J L RAMA PRASAD

(a) Signature:

(b) Name: Dr. G. DHARMAIAH

(a) Signature:

(b) Name: SREELATHA SANKURU

(a) Signature:

(b) Name: NADIMPALLI UDAYA BHASKARA VARMA

(a) Signature:

(b) Name: KRANTHI KUMAR KOTHA

To

The Controller of Patents,
The Patent Office,
At Guindy, Chennai 600 032.

Note:- For Fee: See First Schedule

12-Oct-2020/89136/202041044264/Form 9

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QOS Modelling for Software Service Improvement using Adaptive Learning

Chinnam Subbarao, I. Ramesh Babu,

ABSTRACT: ***In this paper, we discussed about quality of service (QoS) related to software implementation for outsourced applications. Quality of service is key challenge to produce distributed software services and mainly focused on relative service provide based on possessions usage of distributed environment. In this work, we presented effective QOS Machine Learning approach to calculate and predict quality of service with respect to outside environment conditions. We also presented depth analysis on relative and successive correlations with enhanced performance of resource classification. Improve and maintain quality of service for dynamic resource utilization and perform application level semantic data relation. The objective of research work is to provide a hybrid learner's resolution that increases the precision while keeping prototype complication passable. Our proposed experimental results give more accurate and efficiency with respect to traditional approaches.***

Keyword: Adaptive Learning, Quality of service (QoS), Control Primitive, Hybrid Multi-Leaners, Cloud-based Software, Linear Regression and Machine Learning.

I. INTRODUCTION

The flexibility of Cloud-Based Software is responsible for a model change in the way we handle and constantly develop cloud-based application solutions. However, it would be a challenge for application technicians and Cloud-Based Software technicians to estimate the extensive difference of behaviors that application facilities can understanding while operating on a distributed and on-request atmosphere like Cloud-Based Software. It is principally hard we may predict the highly effective variations in quantity of work and the run time requirements of cloud-based application solutions. Its reality indicates that it gets more complicated to ensure the Quality of Support (QoS) once technological innovation cloud- view point's resolutions. The strategy of off-line and guide organization techniques for QoS are simple complicated if possible exertion to accomplish. Through such viewpoint in attention, the important issue, that facility suppliers expression is in what way to handle run time QoS by automatic climbing to the finest collection of device principles on-the-fly. Identifying several essential difficulties of QoS acting in the Cloud-Based Software, which have not been or have only been partly regarded in past perform.

Some of the major traditional approaches for escalation of QoS are introduced and they are analytical or simulated oriented and semi trained or dynamic software services. Semi supervised methodologies focus on flexible and powerful applications for primitive relations in connection to quality facility, which implies the application changes based to the QoS variations.

Revised Manuscript Received on October 05, 2019.

Chinnam Subbarao, Research Scholar, CSE, Acharya Nagarjuna University, Guntur.

Dr. I. Ramesh Babu, Research Supervisor, CSE, Acharya Nagarjuna University, Guntur

However, existing collection and disconnected circumstances to increase eminence of facility that can be reliable with virtual semi dynamic operations. So in this paper, we extend software process improvement i.e. Enhanced and Dynamic QOS Machine Learning Modeling to support dynamic data service processing in distributed computing based server applications.

The Research article is structured as follow: Segment II deliver summary of related theory in Cloud based Software Service Implementation. Section III states the Implementation of QoS adaptive learning approach. In Segment IV, relates the demonstration of our projected method against various other schemes. Lastly, Segment V designates the conclusions of this paper.

II. RELATED THEORY

Frequently, administrations and multi-level utilizations can have numerous imitations for different resolutions in the cloud. For instance, benefit separation, stack regulating and so forth. Along these lines we accept that every level in a multi-levels application, comprising of solid administrations S_1, S_2, ... S_i, can have different imitations conveyed on various Virtual Machines even PMs. In this effort, we suggest to the imitations of durable organizations as direction occurrences: the kth profit circumstance of the lth durable organization is specified by S_{kl}. Numerous administration examples are conveyed on a cloud programming stack working on VM that can be format utilizing different device handles. These device handles can be whichever shared among the administration occasions running on a Virtual Machines or particular to one administration example (e.g., strings of an administration case).

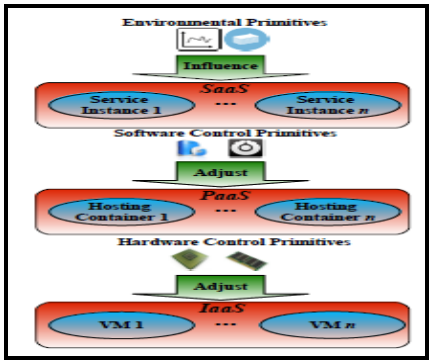

Figure 2: Cloud based software service primitives

The populaces in cloud plug in as the important influences of a QoS presentation. Deprived offail of all inclusive statement, we decay the idea of natives into 2 noteworthy areas: these are Environmental Primitive (EP) and Control Primitive (CP). Control Primitives are the inward device handles and can be any programming or equipment that can be overseen by the cloud suppliers to help QoS. In particular, programming device natives are programming strategies and

the significant designs in cloud, for example, the quantity of strings in string group of administration/application, the cradle dimension and capacity adjusting strategies and so on. Though, equipment control natives are computational assets, for example, memoryand CPU. As appeared in Figure 2, programming and equipment device natives depend on the IaaS and PaaS deposits individually. Specifically, it is non-unimportant to study programming device natives while displaying QoS in the cloud as they have been appeared to be very significant highlights for QoS. Then again, Environmental Primitives allude to the outer jolt that reason flow and vulnerabilities in the cloud; for cases, workload and unusual approaching information and so on. On the off chance that the cloud supplier can control the nearness of the boost, at that point these can be considered as control natives. To enhance precision and avoid clamors, choosing the correct natives as sources of info is basic for QoS demonstrating in the cloud. In any case, the trouble is that the crude sources of info, which are significant and valuable for displaying QoS, have a tendency to be energetic. In such setting, conceivable contributions of a QoS prototype can be the natives that have a tendency to specifically impact the QoS, it can likewise incorporate the natives that have a place with the co-found administration cases and the co-facilitated VMs; Explicitly, all conceivable natives assistances for demonstrating the QoS traits of an administration occasion shape a space, which is named conceivable significant group space.

III. QOS ADAPTIVE LEARNING APPROACH

Main progression of the projected approach is demonstrated in Fig.2, the administration cases run on different virtual services with primary machines are overseen by a devoted instance of middleware (MI), which is joined to the root area of PM. Every MI is individual-versatile as the criticism circle turns ceaselessly to retain methodologies are refreshed. Projected approach is intended for web service oriented situations; the main offline planning is to characterize the present administration occurrences.

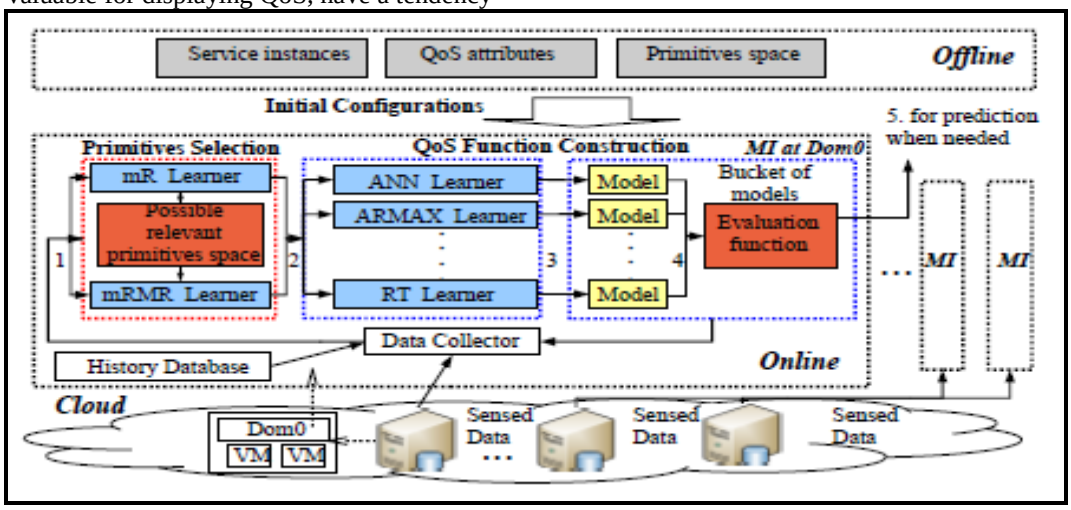

Figure 2: Overview of the proposed approach working with distributed computing based software services.

Inside the input circle, Data Collector constantly screens and supplies info tests of service quality and natives from the administration occurrences / random virtual machines run on primary machines, and those from alternate PMs within the sight of utilitarian reliance. It can be accomplished by getting to the distributed computing to pre-defined environments. It is important that the displaying interim can be processed with the inspecting interim. Upon each demonstrating interim, quality is an administration occurrence, every single chronicled datum is passed with native's choice stage for figuring out which and when natives correspond with service quality at organization running time. There are two main steps to reveal quality of service utilization in web services. They are Hybrid learner for software services, QOS function for adaptive multi-learner. We segment the natives that give distinctive parts of data on different service quality applications; it conveys about p + 1 allotments where p is equivalent to the quantity of natives in the immediate group space; in staying single parcel alludes to the circuitous crude space. Procedure for hybrid web services learned as follows.

Algorithm 1: Basic representative process communicate to hybrid data processing

Input: Given Vector B relates to QoS associated with direct primitive parameters.

Initialization: direct primitives with respect to associated attributes

Independent attributes representations for different attribute parameters.

the line records of the chosen relevant attributes in matrix $SP_k^{ij}(t)$

k (t)

1: begin primitives selection

2: Cdirect := ;, Cindirect := ;,

3: Cdirect := argmax $\Phi(G,Y)$ based on above equations

4: Cindirect: =argmax $\Phi(G,Y)$ connected with above equivalences.

5: end primitives selection

Particularly, main objective functions relate to our proposed application the subsequent deviations will seem centered on dissimilar purposes:

$$\frac{\sum_{N\in S}^{k} V(A,B)}{1+\sum_{N,N'\in S} V(A,A')}$$

Multi-predictive parameter sequences relate to provide web services.

$$\frac{\sum_{N\in S}^{k} V(A,B)\times(n-1)}{k^2-k+2\times\sum_{N,N'\in S} V(A,A')}$$

Average service utilization via software based distributed computing services

Where A0 is vector representation of different types of relative & primitive attributes, already discussed in above sections. It is defined to filter un-relevant and predefined attributes and process quickly. These parameters give maximal relevance for different attributes as follows:

$$\max \Phi(G,Y), \Phi = \sum_{N\in G}^{n} V(A,B), s,t.V(A,B) > 0$$

Where G signifies direct primary space relates and supports for other direct parameter sequences to support for both relevant and irrelevant primitives. Machine learning calculations contain effective nature in different levels of quality of service work capacity maintenance and evaluation of adaptive services to explore finest index data. So that, Introduce a versatile multi-apprentices method for refreshing QoS function.

This section describes performance evaluation; we experimentally define multi-learner data representation to access software services with respective learning model sequences. To increase accuracy in advance of service implementation based on intensive data against software primitives. We implement this application in standard computer configurations. To take out the impedance caused by displaying, we designated single CPU center and 1.2GB RAM to Dom0, that has a tendency to be adequate. To reenact QoS obstruction caused by the VMs while not debilitating assets, we run 3 co-facilitated VMs on every PM; the rest of the assets are equitably designated to the co-facilitated VMs. We discussed classification accuracy with respect to different resources like CPU, Memory, Mean, Prediction Accuracy and Time analysis of projected approach in terms of different services utilized by clients. As specified in above sections, we pre-process the data related distributed computing oriented resources to process dissimilar consistency client's data.

To reproduce a sensible workload inside the limit of our test bed, we change the quantity of customers relatively concurring to the FIFA98 capability that is packed in the mode that the change of a day in the pattern relates to 200 secs for our situation. This setup can create up to 400 parallel solicitations; we trust that such pressure is sensible and sufficiently substantial to reenact QoS impedance in an open cloud. The examining and demonstrating interims are both 119secsthrough the aggregate of 499 interims where the initial 150 interims utilize a static what's more, steady capability slant going for giving some basic information to the displaying; while the back 350 interims take after the FIFA98 drift. This arrangement can create one original example for every interim for refreshing the model.

IV. EXPERIMENTAL ANALYSIS

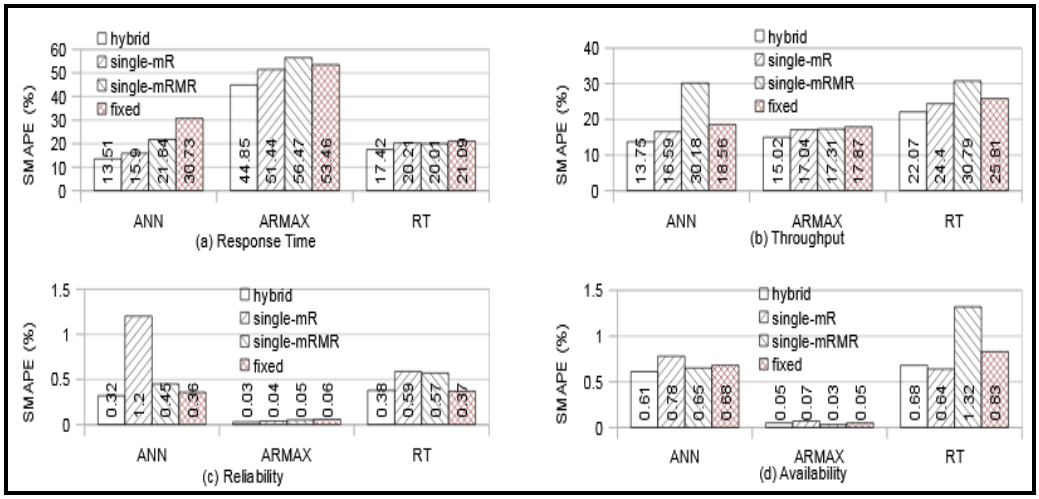

Figure 3:Precision of distinct QOS facilityconstraints with distinctqualities.

To survey our cross breed multi-students (indicated as HYBRID) for natives purpose, we contrast its impact on exactness and three other choice systems, these are:

solitary learner with mR (meant as SOLITARY - MR), solitary student with mRMR (signified as SOLITARY-M RMR) and the Settled strategy that statistical utilizes assured natives (memory and CPU for our situation) as information sources. For entire circumstances, we relate3 broadly utilized knowledge intentions (i.e., RT, ARMAX and ANN) for QoS work preparing under entire QoS properties. Fig. 3(a) demonstrates the outcome for reaction time. Specifically, the diminishment on expectation mistake varieties from 3.93% (12.61% to 14.8% in contradiction of SOLITARY-MR on ANN) to 16.33% (12.61% to 31.337% in contradiction of SECURE on ANN). Comparative outcome can be seen in Fig. 3(b) that demonstrates the precision for production. The HYBRID is better than the others for all calculations with change running from 1.99% (14.67% to 16.89% in contradiction of SOLITARY - MR on ARMAX) to 15.33% (14.45% to 29.81% in contradiction of SOLITARY - M RMR on ANN). The precision for unwavering quality and accessibility are outlined in Fig. 3(c) and 3(d). The HYBRID is over superior to the others on ARMAX and ANN for unwavering quality; it is additionally the finest solitary on ANN for accessibility.

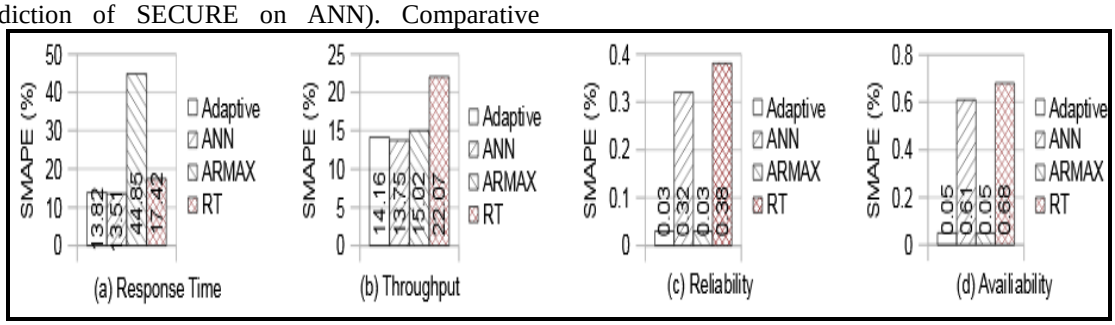

Figure 4: Accuracy of different learning based procedures with respect to QOS.

Fig. 4 delineates cases of the genuine and anticipated QoS regards for all the considered QoS qualities. Because of constrained space, we have utilized an occasion of the administration named Search Item by Category as the illustration.

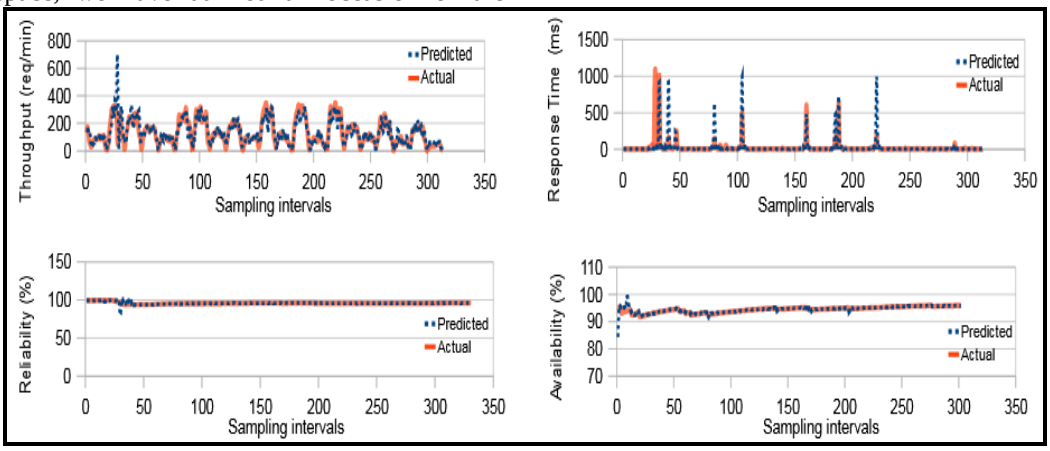

Figure 5: Predictive QOS standards for distributed software atmosphere.

Fig. 5-6 and table 1 show that the CPU utilization in resource provisioning in distributed computing with different services .

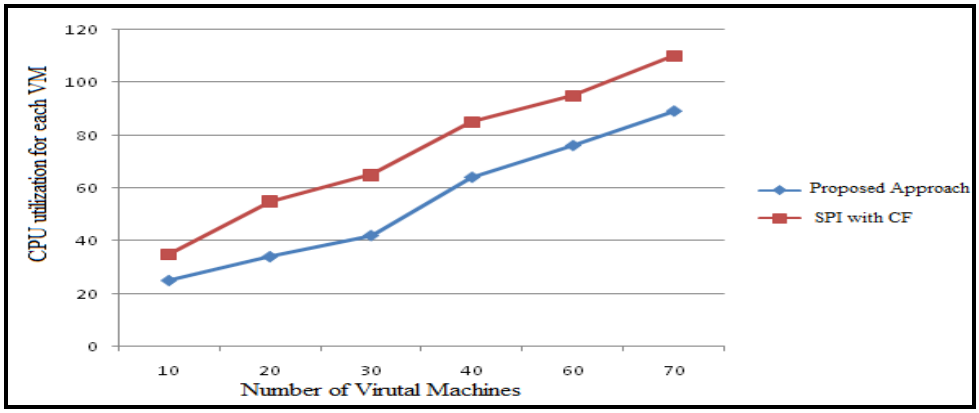

Figure 6 CPU Utilization based on different virtual services in distributed computing.

Table 1: CPU Utilization values for different VMs with respect to memory utilization and time.

No. of VMs	Proposed Approach	SPI with Memory scenarios	Memory (for different VMs)	Time Efficiency
10	25	35	3457	13
20	34	55	4587	22
30	42	65	7458	33
40	64	85	10524	42
60	76	95	22563	53
70	89	110	32145	68

Memory usage for different services specified in Fig. 7, it shows different memory storage with different services based on virtual machine readings.

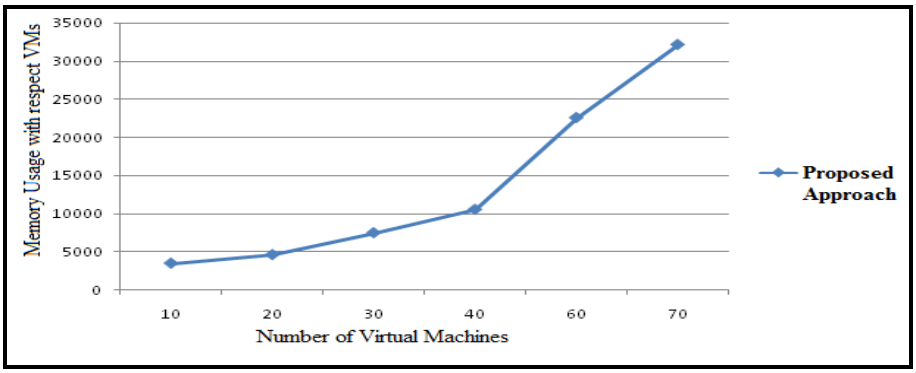

Figure 7: Memory interpretation for dissimilar virtual machine conditions.

Execution time for different services from different virtual machines loading and running different services for different users may appear in Fig. 8 with different resource operations.

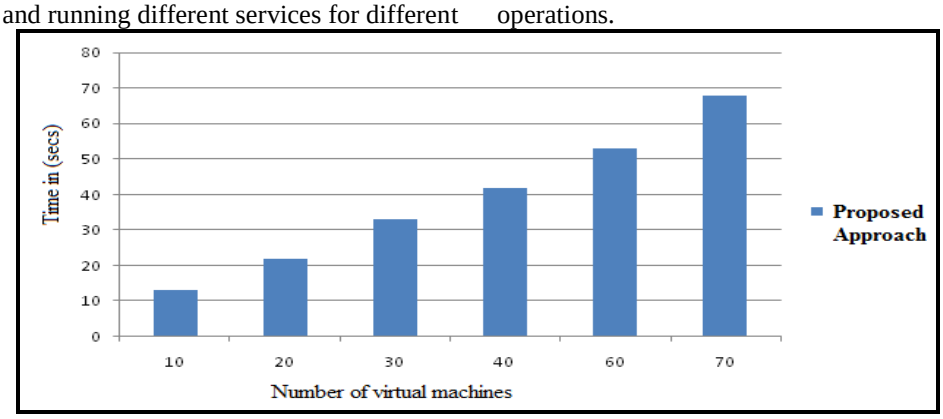

Figure 8: Time efficiency for virtual machine utilization based on different services.

Finally, methods related to neural networks with an ideal and basic measure yield predominant forecast precision than mathematical Linear Regression presentations. To accomplish more classification accuracy in projection, the plotted graph models to be prepared by the creation information in a customary interim; along these lines have recursive data in arbiter relations. Be that as it may, since the preparation of Neural network configuration methodologies take noteworthy time, the recurrence of the preparation ought to be resolved in light of the fundamental asset utilization conduct of the application in outsourced distributed computing based software services.

V. CONCLUSION

Quality of service is key challenge in cloud to produce distributed software services and mainly focused on relative service provide based on possessions usage of distributed environment. In this work, we projected and advanced enriched and self- adaptive framework to improve delivery quality in distributed computing based software application developments. To track dynamic updates from relates to define service quality and interference, for that we use both multi attribute primitive learners to train different services running on distributed manner to satisfy accuracy. We define different learning procedures significantly increase QoS with their attributes based on fluctuative parameters. The objective of research work is to provide a hybrid learner's resolution that increases precision while keeping prototype difficulty passable and dynamic service utilization of different

attributes with aggressive and technical data representations. Our proposed experimental results give more accurate and efficient results when compare to formal procedures with different parameters like CPU utilization, memory and time efficiency in different applications.

REFERENCES

1. R. Bahsoon and T. Chen, "Self-adaptive and sensitivity-aware QoS modeling for the cloud" In the 8th SEAMS, pp. 43 –52, May 2013.
2. S. Chase, S.B. Jeffrey and H.C. Lim, "Automated Control for Elastic Storage," In 10th Conference on Autonomic Computing, 2010.
3. C. J. Stone, R. A. Olshen, J. H. Friedman and L. Breiman, "Classification and Regression Trees", Wadsworth, Belmont, 1984.
4. H. Ghanbari et al, "Tracking Adaptive Performance Models Using Dynamic Clustering of User Classes", In 2nd conference on performance engineering, 2011.
5. T. JinandM. Arlitt, "A workload characterization study of the 1998 world cup web site," IEEE Network, 14(3), pp. 30 –37, May 2000.
6. Y. Jiang et al, "Self-adaptive Cloud Capacity Planning" In 9th IEEE International Conference on Services Computing, 2012.
7. G.C. Reinsel, G.M. Jenkins and G.Box, "Time series analysis: forecasting and control", third edition. Prentice-Hall, 1994.
8. E. Frank and I.H. Witten, "Data mining: practical machine work learning tools and techniques", Morgan Kaufmann: Los Altos, CA, 2005.
9. B.E. Flores, "A pragmatic view of accuracy measurement in forecasting," Omega (Oxford), vol. 14, no. 2, pp. 93-98, 1986.

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CHALLENGES OF STUDENT- CENTRED LEARNING – A NOTE

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Introduction

Education enables individuals and societies to become all round thinkers by providing knowledge, ability, skill and attitude, thereby it strengthen their problem solving ability. In order to achieve this, learners should be taught in a way that enables them to really engage themselves in the process. In teacher-centred method, student put all their focus on the teacher. In the teacher-centred method during activities in classroom, the only participants are teachers. In teacher-centred method, the primary sources of responsibility for the learning path in the hands of students. Learning is not a spectator sport. Students do not learn much just by sitting in class listening to teachers, memorizing pre-packaged assignments, and spitting out answers. They must talk about what they are learning, write about it, relate it to past experiences, and apply it to their daily lives. They must make what they learn part of themselves knowledge are teachers.

Student-centred method on the other hand, enables students to put all their focus on their knowledge. Student-centred method also known as learner-centred is a method of teaching in which the focus of instruction is shifted from the teacher to the students. In original usage, student-centred learning aims to develop learner autonomy and independence by putting

The concept of student–centred learning has been credited as early as 1905 to Hayward and in 1956 to Dewey's work 1. Carl Rogers, the father of client–centred counselling, is associated with expanding this approach into a general theory of education2. The term student–centred learning was also associated with the work of Piaget and with Malcolm Knowles3, He describes the shift in power from the expert teacher to the student learner, driven by a need for a change in the traditional environment where in this 'so-called educational atmosphere, students become passive, apathetic and bored'. In the School system, the concept of child–centred education has been derived, in particular, from the work of Froebel and the idea that the teacher should not 'interfere with this process of maturation, but act as a guide4.

Student–Centred Learning

Student-centred involves providing opportunities for students to meaningfully talk and listen, write, read, and reflect on the content, ideas, issues, and concerns of an academic subject. The student-centred learning approach is diametrically opposed, in its ethos, to the philosophy underlying the conventional method of learning. By its very nature, student-centred learning allows students to shape their own learning paths and places upon them the responsibility to actively participate in making their educational process a meaningful one. The student-centred learning experience is not a passive one, as it is based on the premise that "student passivity does not support or enhance..... Learning' and that it is precisely "active learning" which helps students to learn in dependently. Within student-centred learning, students are given options in shaping their courses and in choosing particular units within their study program. Some proponents of student-centred learning assert that rather than devoting so much effort to teaching students what to think, student-centred learning is based on the idea of teach them how to think'5. According to Weimer (2002) there are five practices that need to change to achieve learning teaching: (a) the functions of content, (b)the role of the teacher, (c)the responsibility for learning, (d) the processes and purposes of evaluation, and (e) the balance of power

Kember (1997) described two broad orientations in teaching: the teacher centred/content oriented conception and the student centred/learning oriented conceptions. In a very useful breakdown of

these orientations he supports many other authors views in relation to student–centred view including: that knowledge is constructed by students and that the lecturer is a facilitator of learning rather than a presenter of information. Choice in the area of the learning is emphasised by Burnard, as he interprets Rogers' ideas of student–centeredness as 'students might not only choose what to study, but how and why that topic might be an interesting one to study6. He also emphasises Rogers' belief that students' perceptions of the world were important, that they were relevant and appropriate. This definition therefore emphasises the concept off students having 'choice' in their learning.

Harden and Crosby (2000)7 describe teacher–centred learning strategies as the focus on the teacher transmitting knowledge, from the expert to the novice. In contrast, they describe student–centred learning as focusing on the students' learning and 'what students do to achieve this, rather than what the teacher does'. This definition emphasises the concept of the student 'doing'.

Other authors articulate broader, more comprehensive definitions. Lea et al. (2003)8 summarises some of the literature on student–centred learning to include the followings tenets:

1. 'the reliance on active rather than passive learning,
2. an emphasis on deep learning and understanding,
3. increased responsibility and accountability on the part of the student,
4. an increased sense of autonomy in the learner
5. an interdependence between teacher and learner,
6. mutual respect within the learner teacher relationship,
7. and a reflexive approach to the teaching and learning process on the part of both teacher and learner.'

Gibbs (1995)9 draws on similar concepts when he describes student–centred courses as those that emphasise: learner activity rather than passivity; students' experience on the course outside the institution and prior to the course; process and competence, rather than content; where the key decisions about learning are made by the student through negotiation with the teacher. According to Brandes and Ginnis (1986) 10 the main principles of student–centred learning are:

a) The learner has full responsibility for her/his learning
b) Involvement and participation are necessary for learning
c) The relationship between learners is more equal, promoting growth, development
d) The teacher becomes a facilitator and resource person
e) The learner experiences confluence in his education
f) The learner sees himself differently as a result of the learning experience.

Characteristics of Student-Centred Teaching

According to Weimer and Maryellen there are five characteristics of student-centred teaching.

1. student-centred teaching engages students in the hard, messy work of learning.

2. student-centred teaching includes explicit skill instruction.

3. student-centred teaching encourages students to reflect on what they are learning and how they are learning it.

4. student-centred teaching motivates students by giving them some control over learning processes.

5. student-centred teaching encourages collaboration

One come across many challenges in the implementation of student-centred in in relation to the required standard they are expected to achieve. Based on the analysis of students and teachers response to questionnaire it was found that they face different problems while implementing student-centred method. The challenges were lack of interest and confidence, lack of teachers and

students interest, class size, students feel discomfort when they work with others. In addition to this, from the analysis of teachers' interview, it was found that the students lack the skill of expressing their idea in English and students show a disciplinary problem when they are engaged in various techniques of student centred method. With the recent changes in educational laws and new classroom management opportunities, students are becoming more actively engaged in instruction. Student-centred classroom management is rare because it requires, "a willingness to give up some control and let students take some ownership, which requires guts as well as talent". Teachers have a tremendous impact on a child's life and their success starts in a student centred classroom.

School administrators need to support student-centred management and provide training and skills to their teachers because teachers and their students will have higher outcomes and success. Because classrooms are diverse and have students of all learning styles and learning needs, teachers need to implement individualized student-centred classroom management techniques to ensure all students will be successful. Teacher-centred classroom management, such as using classroom wide behavior management systems, or following scripted lessons are not effective ways to meet the needs of individual students.

Implications for curriculum design

In relation to curriculum design, student–centeredness includes the idea that students have choice in what to study, how to study. However, to what extent can this be carried out in the structures of today's Universities? Modularisation, which will be expected in all European undergraduate courses by 2006, provides a structure that allows students an element of choice in what modules they study. Donnelly and Fitzmaurice (2005) in their chapter in this collection on 'Designing Modules for Learning' highlight the importance of attempting to focus on the needs of the students at the early stage of curriculum design. Choice in the curriculum is not without its difficulties and Edwards argues about the dangers of individuality in the concept of the social learner and how this can in a seemingly contradictory way lead to disempowerment (2001).

One student–centred approach to curriculum design, Problem–Based Learning (PBL), allows for some choice within a programme of areas that students may study. It allows students to set some of their own learning objectives/outcomes, dependent on prior knowledge. ProblemBased Learning, through the use of problems/issues/triggers, encourages the students to develop their own learning goals, thereby filling in the gaps in their knowledge or understanding (Boud and Feletti 1997). This element of choice or control is referred to in many of the definitions of student–centred learning. This aspect of responsibility aligns with the Lea et al. (2003) view that student–centred learning involves 'increased responsibility and accountability on the part of the student'. These approaches are more controlled by the teacher in their presentation and outcome (Davis and Harden 1999). However, they are useful in addressing the active learning aspect of student–centred learning. Other approaches to curriculum design also support the idea of student choice and activity in learning, for example, the systems–based approach, resource–based learning, and experiential/ personal relevance approach (Toohey 2000).

A growing practice in course design internationally is the writing of learning outcomes/objectives focusing on what the student will be able to do, rather than on the content being covered by the teacher (UCD Centre for Teaching and Learning 2005). This practice is an example of the move towards student–centred learning in the curriculum and helps to shift the emphasis on the learner as opposed to a coverage model by the teacher. Donnelly and Fitzmaurice (2005) re-iterate the importance of this shift in emphasis. This is also reflected in Gibbs' (1995) definition, i.e. an emphasis on the process and competence, rather than content..

Implications for teaching/learning methods

The University of Glasgow (2004) identified four main strategies in a study on student–centred learning practices in their University. The first strategy was to make the student more active in

acquiring knowledge and skills and might include exercises in class, fieldwork, use of CAL (computer assisted learning) packages etc. The second strategy was to make the student more aware of what they are doing and why they are doing it. A third strategy is a focus on interaction, such as the use of tutorials and other discussion groups. The final strategy is the focus on transferable skills. This last strategy is not mentioned in other definitions of the student–centred learning but does look beyond the immediate course requirements to other benefits to the student in later employment.

The effectiveness and critiques of student–centred learning

The use of student–centred learning appears to be reflective of today's society where choice and democracy are important concepts. Lea et al. (2003) reviewed several studies on student–centred learning and found that overall it was an effective approach. A six-year study in Helsinki, which compared traditional and activating instruction, found that the activating group developed better study skills and understanding, but were slower in their study initially11. Equally, Hall and Saunders found that students had increased participation, motivation and grades in a first year information technology course (1997). In addition, 94% of the students would recommend it to others over the more conventional approach 12. Students in a UK University elaborated on the impact of student–centred learning on them, i.e. they felt there was more respect for the student in this approach, that it was more interesting, exciting, and it boosted their confidence13.

Student–centred learning, despite its popularity, is not without its critics. The main critique of student–centred learning is its focus on the individual learner. In addition, there are some difficulties in its implementation, i.e. the resources needed to implement it, the belief system of the students and staff, and students' lack of familiarity with the term.

Conclusion:

The changing demographics of the student population and the more consumer/client–centred culture in today's society have provided a climate where the use of student–centred learning is thriving. The interpretation of the term 'student–centred learning' appears to vary between authors as some equate it with 'active learning', while others take a more comprehensive definition including: active learning, choice in learning, and the shift of power in the teacher–student relationship. It is used very commonly in the literature and in University policy statements, but this has not necessarily transferred into practice.

Student–centred learning is not without some criticism but in general it has been seen to be a positive experience, for example, Edwards (2001) emphasises the value of student–centred learning: 'Placing learners at the heart of the learning process and meeting their needs, is taken to a progressive step in which learner–centred approaches mean that persons are able to learn what is relevant for them in ways that are appropriate. Waste in human and educational resources is reduced as it suggested learners no longer have to learn what they already know or can do, nor what they are uninterested14.

Today, information and communications technology (ICTs) is increasingly being employed in classrooms across the globe. Owing to this, educators face growing challenges as they teach a very "different" and "wireless" generation of students who use technology which is evolving every day. With over 50 percent of India below 25 years of age, the need for a skill set that adds to their digital profile is of utmost importance.

Consequently, the need for active learners who have acquired the skills of problem-solving, independent thinking and autonomous learning, as well as the ability to work cooperatively has increased dramatically. Therefore, teaching methods, pedagogies and educational goals must be directed at producing individuals who have faith in their abilities and who will work at developing their capabilities throughout their lives.

It is important to note that in the 21st century, learning is not "one size fits all" approach. This era has become more about developing skills than just

absorbing information. The most important skills is self-development, which includes knowing one's strengths and weaknesses, and finding ways to mitigate gaps in personal development. This can be achieved by implementing student-centred learning practices, thereby letting students take charge of their learning. This method prepares individuals to take responsibilities and makes them independent. It helps develop the characteristics of lifelong learning – motivation, self-evaluation, time management and skills to access information. The 21st Century skills comprise the 4cs namely, (a)critical thinking, (b)creativity; (c) communication and (d)collaboration.

Student-centred learning is an approach in which students influence the content, activities, materials and pace of learning. This learning model places the students at the centre of the learning process. The instructor provides students with opportunities to learn independently, and from one another, and facilitates the development of skills that they need to do so effectively.

References:

1. O'Sullivan, M. (2003). The reconceptualisation of learner-centred approaches: A Nambian case study, *International Journal of Educational Dedvelopment*.
2. Burnard, P. (1999). Carl Rogers and postmodernism: Challenged in nursing and health sciences. Nursing and Health Sciences 1, 241–247. Geraldine O'Neill and Tim McMahon 37.
3. Rogoff, B. (1999). Cognitive development through social interaction: Vgotsky and Piaget. In P. Murphy (Ed.), Learners, Learning and Assessment. London: Open University Press.
4. Rogers, C. R. (1983a). As a teacher, can I be myself? In Freedom to Learn for the 80's. Ohio: Charles E. Merrill Publishing Company.
5. Simon, B. (1999). Why no pedagogy in England? In J. Leach and B. Moon (Eds.), Learners and Pedagogy. London: Sage Publications.
6. Taylor, P. G. (2000). Changing Expectations: Preparing students for Flexible Learning. The International Journal of Academic Development 5(2), 107–115.
7. Rogers, C. R. (1983b). The politics of education. In Freedom to Learn for the 80's. Ohio: Charles E. Merrill Publishing Company
8. Harden, R. M. and J. Crosby (2000). AMEE Guide No 20: The good teacher is more than a lecturerthe twelve roles of the teacher. Medical Teacher 22(4), 334–347.
9. Lea, S. J., D. Stephenson, and J. Troy (2003). Higher Education Students' Attitudes to Student Centred Learning: Beyond 'educational bulimia'. Studies in Higher Education 28(3), p.322.
10. Gibbs, G. (1995). Assessing Student Centred Courses. Oxford: Oxford Centre for Staff Learning and Development.
11. Brandes, D. and P. Ginnis (1986). A Guide to Student Centred Learning. Oxford: Blackwell. Bredo, E. (1999). Reconstructing educational psychology. In P. Murphy (Ed.), Learners, Learning and Assessment. London: Open University Press.
12. Broadfoot, P. (1999). Assessment and the emergence of modern society. In B. Moon and P. Murphy (Eds.), Curriculum in Context. London: Sage Publications
13. Lonka, K. and K. Ahola (1995). Activating instruction: How to foster study and thinking skills in Higher Education. European Journal of Psychology of Education 10, 351–368.
14. Hall, J. and P. Saunders (1997). Adopting a student-centred approach to management of learning. In C. Bell, M. Bowden, and A. Trott (Eds.), Implementing Flexible Learning. London: Kogan Page
15. Lee et al, 2003, *Op.Cit.*
16. Edwards, R. (2001). Meeting individual learner needs: power, subject, subjection. In C. Paechter, M. Preedy, D. Scott, and J. Soler (Eds.), Knowledge, Power and Learning. London: SAGEp.37.

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अम्बटि. विद्यासागरः और डा. डि. यन्. दीक्षितः

प्रस्तावना

मन्दाकिनी विजयसारथेः कृतिषु प्रौढतरं काव्यम्। मन्दं अकति इति ञिच्छति इति। मन्दाकिनी शब्दस्य यौगिकाऽर्थे सत्यपि गङ्गायाः नामसु मन्दाकिनी शब्दः रूढः। गङ्गायाः उपनदीषु भागीरथी, मन्दाकिनी इत्यादि नामाभिः शाखा नदीनां प्रसिद्धिः तासां हिमालयेषु तत्र तत्र संगमः सङ्गमस्थलानां प्रयागशब्देन व्यवहारः अद्यापि प्रसिद्धः एव देवप्रयागः , रुद्रप्रयागः, कर्णप्रयागः इत्यत्र यथा -- यावत् अलहाबाद् नगरे यमुनायाः सङ्गमः तावत्पर्यन्तं बहवः प्रयागाः वर्तन्ते। बदरिका क्षेत्रे गङ्गायाः नाम मन्दाकिनी प्रसिद्धा। तथापि मन्दाकिनी शब्दः समग्रायाः गङ्गायाः पर्यायतया रूढः।

मन्दाकिनी शब्दोत्पत्तिः महौत्कृष्टत्वम् ----

संस्कृतवाङ्मये समग्र गङ्गापर्यायतया मन्दाकिनी शब्दः बहुषु काव्येषु प्रयुक्तः दृश्यते। यथा नैषधीयचरिते ---

अश्रान्त श्रुतिपाठपूतरसनाविर्भूतभूरिस्तवा
जिह्वाब्रह्ममुखोद्यविघ्नितनस्वर्ग क्रिया र्कलिना।
पूर्वं गाधिसुतेन सामिघटिता मुक्ता नु मन्दाकिनी
यत्प्रासाददुकूलवल्लिरनिलादोलैरखेलद्दिति।। [1]

रामायणेपि मन्दाकिनी शब्दः गङ्गा पर्यायतया प्रसिद्धः विशेषतया आकाशात् हरशिरसि अवतरन्ती गङ्गा मन्दाकिनी नाम्ना काव्येषु व्यवहृता। गङ्गायाः अवतरण कथा संस्कृतवाङ्मये प्रथमतया बालकाण्डे श्रीमद्रामायणे पञ्चसु सर्गेषु वर्णिता।
सूर्य वंशजः सगरः अश्वमेधेन इष्टवान् तस्य एकस्यां भार्यायां षष्टिसहस्र पुत्राः द्वितीयस्यां एकः पुत्रश्च बभूवुः।
तेषु द्वितीयस्याः पुत्रः नाम्ना कर्मणापि असमुञ्जसः पित्र सज्यात् बहिष्कृतः। तदानीं तस्य पत्नी अन्तर्वन्ती आसीत्। तस्याः अंशुमाक्षः नामकः। सगरस्य षष्टिः पुत्र सहस्राणि यागाश्वरक्षकाः यागाश्वमन्वयुः। इन्द्रोऽपि सगरस्य शततमं अश्वमेधं सान्तरायं कर्तुं यज्ञाश्वं जहार पाताले नारायणावतारस्य कपिलस्याश्रमे गोपायितवान् च। सगरकुमाराः अश्वं अदृष्टवा भूमिं खनित्वा पातालं ययुः कपिलाश्रमे अश्वं ददृष्टुः कपिलमेव अश्वस्य हर्तरं मत्वा तर्जयामास। ततः कुपितेन कपिलेन ते भस्मसात्कृतः।

अभ्यथावन्त संक्रुद्धास्तिष्ठ तिष्ठेति चाब्रुवन्।
अस्माकं त्वं हि तुरगं यज्ञियं हृतवानसि।। [1]
दुर्मेधस्त्वं हि संप्राप्तान्विद्धि नः सगरात्मजान्।

तेषां पारलौकिकाः क्रियाः कर्तुं कस्यापि अवकाशः नासीत् अनन्तरं सगरस्य पौत्रः अंशुमान् कपिलाश्रं गत्वा कपिलं प्रसाद्य अश्वमनीय यज्ञसमाप्तिं कृतवान्। तत्र सलिलस्य अलभ्यत्वात् सलिलक्रियाः कर्तुमपि अशुंमान् न पारितवान्। अशुंमान् दिलीपः भगीरथः इति क्रमेण सर्वे सगरवंशजाः गङ्गां पालमानेतुं तपस्यां चक्रुः। तेषु अंशुमतः पौत्रः भगीरथः एव सफलो बभूव। वतरिष्यमाणां गङ्गां धर्तुं शिव एव समर्थः इति ज्ञात्वा भगीरथः शिवं प्रसाद्य गङ्गां धारयितुं प्रार्थितवान्। प्रथमं शिवजटाजूटे अवतीर्णे गङ्गा शिवशिरसि गर्वेण विहरणं कृतवती शिवेन सा जटाजूटे बद्धा शिवोपि भगीरथ मनुसृत्य सप्तधा गङ्गां बिन्दुसरसि पातयामास। भागीरथ मनुसरन्ती गङ्गा क्रमेण पातालं प्रविश्य त्रिपथगा बभूव। इमां कथां आधारीकृत्य केवल गङ्गावतरण वर्णनात्मकं वर्णनमेव आदाय विजयसारथिः मन्दाकिनी काव्यं रचितवान्। कविना ईषत् परिवर्तिता श्रीमद्रामायण शता मूलकथा स्वकाव्ये अभिवर्णिता। मन्दाकिन्याः रचनोद्देशः कविना स्वयं एवमाविष्कृतः।

[1]अभिजानन्त्येव समेऽपि सुमनसो विद्वांसः शङ्करजगन्नाथनीलकण्ठ प्रमुखा महापुरुषाः भगवतीं भागीरथीं स्तुत्वा तस्या अनुग्रहविशेषेण परमांप्रतिष्ठाम् अवापुरिति। तेषाम् अनुकरणेन अनुसरणेन च वयं आत्मनः कृतार्थतां मन्यामहे। ऐहिकामुष्मिकफल- धात्रीति गङ्गेयं बहुपुराणेषु बहुधाऽजेगीयत। भगवत्या स्सुरसरितः पममावतरणकथा गीतेषु ग्रथिता मया। औचित्यदृशा किञ्चिदिव परिवर्तितापि कथा। काव्येऽस्मिन् विषयेयं एव सन्दब्धोस्ति प्रादान् मन्दाकिनी मात्मसुतां परिवहराजः सागराय। सङ्गस्तु मुत्कण्ठ तेसागरोऽपितया। कल्लोलैरभ्रङ्कषैः तां बलात् ग्रहीतुं काऽमोपि शासनात् भगवतो विभाय। वेलेयं न कदाऽपि भवताति- क्रम्यतामिति तं भगवानपि बलवद् आदिदेश। गङ्गाया बलवद- वतारणेन लोकस्यापकृतिर्भूयसीभवेदित्युभयधा सन्त्रस्त इव सागरः सौवादुद्यमात् व्यरंसीत्। विरहवेदनाम सहमानोऽपि सोऽयं किं कर्तव्यताविमूढेन मनसा क्षणमिवविचिन्त्य भगीरथं स्वपरमसखं राजप्रतिनधिम् आकार्य तं प्रार्थयामा सैवम् सखे। भवता कथमपि प्रेयसी मम सेयं सुरसरित् आनीयतामिति सोऽपि भगीरथः स्वमनोरथं सफलमेवमन्वानः कपिलेन शापाग्निना भस्मसात् कृतान् स्ववंशीयान् गङ्गाजलाप्लावनेन परलोकम् अवापयितुं सागराभीष्ट- ञ्चसफलं कर्तुं तामानेतुं समुद्यतः सागरानुमत्या स्वर्लोकमयासीत्। आतिर्थयविधिना सत्कृतेन भगीरथेन प्रार्थिता कथितमात्मप्रियस्य सागरस्य सन्देशमन्यमुदन्दञ्च समाकर्ण्य गाढं निःश्वस्य च तं प्रत्येयमवोचतं सा सुरनदी --- भ्रात अहमपि सागरं प्राणेश्वरमुपगन्तुमुत्कण्ठ्या समुदीक्षे --- प्रत्युतमत्प्रवाहसंपातं भूमाता न कदाऽपिसोढु शक्नुयात्। यदि नाम भगवान् गिरिशो मां वोढुं नियतमभ्युपागमिष्यत् तदा ममावतरणं निर्बाधम् अभिविष्यत् अतः कार्यमिदं साधयितुं तमेव शिवम् अभ्यर्थयस्व। तदा भगीरथेन तेन संप्रार्थितो भगवान् सदाशिवः सुपर्वापगां दरीधर्तुमुररीचकार। सैषा परिवहमण्डलादात्ममन्दिरादवततार शिवशिरसि सङ्गोचिर्तनैव स्त्रोतसा। फालानलेन विपानलेन चाशीविपानलेन पोप्लुष्यमाणवपुः सोपि सुरधुनी-शिशिरवारिस्पर्शमात्रेण अननुभूतपूर्वं सुखं किमप्यवाप्नुवालः तां सर्वदा निजकपर्द एव बन्दीकर्तुं यावदियेष तावदेवास्य भावमवगम्य वर्धित दुर्भरधाराबहुलेन स्त्रोतः स्वरूपेण विज्रंभमाणा तस्य जटाजूटमयीं कारां निर्भिद्य बहिर्निर्गमत्। अवतीर्य च भुविसप्तधाविभक्तैः स्त्रोत्रोभिः सकलमपि भूमीतलं पुनती भगीरथ पथमनुरुन्धाना तपोवनानि सादरमभिनन्दन्ती, वनानि सिञ्चन्ती, भूधरान् भिन्दन्ती, क्षेत्राणि तर्पयन्ती, ग्रामान् पुराणि पल्लीश्च रञ्जयन्ती प्रकृतिमखिलां पुलकयन्ती, सागरद्वितीभिरापगाभिर्मलन्ती पुण्यानिकिरन्ती दुरितान्य-मनुदन्ती ऊष्मणं चूषन्ती तीर्थानि सार्थयन्ती परासून् भगीरथवंशान् लेखलोकमवापयन्ती प्रजामखिलामाबालगोपालमामोदयन्तीचान्तेसागरेण प्रियेण मिलित्वा तदन्तरङ्गे निलीना भवति। इदमप्यपरमवधेयमस्ति सादरं सुमनसाम् यत् अधुना विविधा सुभाषासु गीतस्यैव संरंभो दृश्यते। प्रतिभाषं अपि नवीना साहिती प्रक्रिया नूत्नेन पथा नरीनृत्यते। सारस्वते सर्वतः कोऽपि सरसस्समुन्नमेषो देदिवीति। लोकगीतनाम्ना प्रचलितस्य गीतमयस्यैव साहितीविशेषस्य साम्राज्यं प्रचलति। संस्कृते वचन प्रक्रियाया इव गीतप्रक्रियाया अपि वराक्येव स्थितिः वरीवर्ति। इदानीन्तनाः कवयोपि वचनदारिद्र्यमिव गीतादारिद्र्यं व्यपोहितं समुत्सहन्ते समुद्यतन्ते च। अमुनैवलक्ष्येण गङ्गावतरण कथा पञ्चमात्रकेषु गीतेषु निबद्धा। प्रवाहस्यगतेः ध्वनेश्च वैविध्यं वैदग्ध्यं चाभिव्यञ्जयितुं प्रायोगमनार्थका ध्वन्यर्थकाश्च धातवः प्रयुक्ताः काव्येस्मिन्। नियतं सर्वाण्यपि गीतानि रचनासौन्दर्येण जीवन्ति। तथापि यथामति काव्येस्मिन् रसस्यभावस्य रीतेः नीतेः शब्दस्यार्थस्य च प्रामुख्यं सर्वधा परिकल्पितम्। प्रवाहगताविव अत्रापि क्वचिद् विलंबः क्वचिद् ऋजुता क्वचिद् विरल्यः क्वापि लघुता क्वापि गुरुता क्वाप्योजस्विता क्वापि प्रसन्नता चेत्यादयो गुणरीतयः समुदञ्चन्ति।

महाकवेरस्य छन्दसि रचनायां सामर्थे वर्तमानोपि संस्कृतभाषायां गीताप्रक्रिया न चिररूढा इति गेय काव्यरूपेण काव्यमिदं रचितमपि पाण्डित्यपूर्णं कृतम्। पाणिनीयधातुपाठे न्यूनाधिकाः द्विसहस्रं धातवो वर्तन्ते धातुपाठे दशविंशति धातून् गुच्छतया निबध्य सिद्धान्त कौमुद्यां गत्यर्थाः भाषार्थाः इति निरूपणं कृतम्। अपि च गेयस्य लयः जीवतुः एवं तालल्यान्वय अनुकूलान् धातून् मात्रा च्छन्दसि निबद्धान् एव गृहीत्वा एकत्र ग्रथनं सामान्यस्य कवेः असाध्यम्। विद्वानपि कवितानुकूलं तान् उपकर्तुं न शक्नोति।

अतः विजयसारथिः विद्वत्कविः। आन्ध्रदेशे आधुनिक महाकविषु जलप्रवाह वर्णनात्मकं किन्नेकसानि पाटलु इति खण्डकाव्यं विश्वनाथ सत्यनारायणेन कृतम्। किन्नेरसानी अद्यतन खम्मम् मण्डले गोदावर्यां मिलन्ती काचन

गिरिनदी। कविसाम्राट् विश्वनाथसत्यन्नारयणः तस्याः स्त्रीरूपं आपाद्य गृहिण्याः अवस्थां तस्यामारोप्य वर्णितवान्। श्वशुरगृहे भर्तुः मात्र कृतां पीडां असह्यमाना किन्त्रसानी भार्यायाः व्यथां दृष्ट्वापि मातरं किमपि वक्तुम् असमर्थः पतिं परित्यज्य श्वशुरगृहं त्यक्त्वा पितुः गृहं प्रस्थितवती। तस्याः गतिमपि विश्वनाथसत्यन्नारायणः लयानुकूलेन वर्णितवान्। काशीभट्ल सत्यन्नारायण महोदयेन कवेरस्य (विजयसारथेः) स्वर्णोत्सवसञ्चिकायां किन्नेरसानी गीतानि एव मन्दाकिनीं रचयितुं कवेः प्रेरणं अनुकुर्वन्तिति अभिप्रायं प्रकटीचक्रुः। तत्सादृश्यं द्योतयितुं काशीभट्ल महोदयः काव्यद्वयात् प्रत्येकं द्वित्राणि गीतानि उदाहृतवान्। आन्ध्रभाषा अनवगतापि तस्याः अर्थं अजानन्तोऽपि शब्दश्रवणमात्रेण तालल्या - नुकूलत्वे द्वयोः सादृश्यं ज्ञातुं प्रभवन्तीति एकमुदाहरणं दीयते।

కెరటాలలో నుర్యు
తెరచాలలో నీటి
పొరజాలులో కిన్నె
రటు కదలి యిటు కదలి
చిటి తరంగాలలో -- పోటి తరంగాలలో
నటనాలు మొదలెట్టేనే -- క్రొన్నీటి [1]
తుటుములాకదలాడేనే
పువు కన్నెలా జగా
రవ నవ్వులా వేల్పు
కవ చివ్వలా కన్నె
రటులూగి ఎటులూగి
చిరుగాలిలో నూగు [1]
కొఱవంపులో నీటు
తటి తీపులో కిన్నె
కటు లేచి ఇటు లేచి [1]

తదనుసారी मन्दाकिनी नृत्यं यथा
అతి మఞ్జుల పుష్ట
వన వఞ్జుల దీప్త
సుమకిఞ్జులాలో
కవదూర్మి పూరేణ------ కృతమఞ్జల పల్ల
విత జఞ్ఝల వీచి
మయ కంగుల పాస్త
గద జఞ్జుల

पराशक्तिस्वरूपिणी मन्दाकिनी गूढार्थरूपेण अत्र वर्णिता इति काशीभट्लमहोदयैः एव प्रतिपादितम्। अत्र तात्विकदृष्टिः प्रवाहेन सममुपवर्णिता ।

यथा --- **ओंकार गम्भीर मञ्जुलस्तानितेन**
विमलामृतासार पूरमा कल्पयति
कदम्बिनीव सा मन्दाकिनी
सा पराशक्तिरिवमन्दाकिनी।। 1

कदम्बनी मेघमाला तस्याः स्तनितमिव गङ्गाप्रवाह ध्वनिरपि भवति स च ध्वनिः प्रमवात्मक एव । तया स्वरूपा सा परत्वं परमात्मत्वं च भक्तस्य कल्पयति सा च पराशक्ति- रूपा कालत्रयामात्य क्रमस्वरूपा । एवं मन्दाकिनी महाशक्त्या अभिन्नां कविः ददर्श । राधाकृष्णयोः अद्वैतत्वे राधापि मन्दाकिन्येव

यथा --- **मुग्धमूलाधारमञ्जीरथा मञ्जु**
षड्वलादिष्ठानरशनयामणिपूर
वैजयन्त्याऽनहताब्ज
कन्धरामङ्गल सूत्रतायाब्ज
फलभूषणयेन्दु मिहिरताटंकया
कुण्डलीन्याख्यया श्रीपरशक्त्येव
विरहमाणः परन्धामा शिरोधाग्नि ।

मन्दाकिनी मूलधारं परिषिध्य पृथिवी तत्वं अमृतमयं कृत्वा अवगाढं अमृतं रकोति । एवं काशीभट्लमहोदयैः बहुधा मन्दाकिनी श्लाघिता । न. च रघुनाथाचार्यः अपि मन्दाकिनी परमात्मतत्वभूषिता इति प्रशशंसुः । कोराड सुब्रह्मण्य महोदयैः काव्यस्य अलङ्कारशैली बहुधा विवृता । अपि च पाणिनीय प्रयोगविशेषाः अपि बहवः अत्र दृश्यन्ते । यङन्ताः यङलुगन्ता क्यजन्ता क्यङन्ताः कृदन्ताश्च प्रयोगविशेषाः बहवः अस्मिन् काव्ये दृष्टाः । एतान् प्रयेगन् अर्थतः शब्दाश्च ज्ञातुं पाणिनीय प्रवीणाः नैरुक्ताः एव समर्थाः इति सुब्रह्मण्येन यदुदाहृतं तदक्षरशः सत्यम् ।

यथा ---- 1. विजहरीति ---- यङलुगन्तः
2. दंद्रम्ते , जंहम्यते ----- इति यङन्तौ
3. शब्दायते , वैरायते --- शब्दवैरकलहाभ्र कण्वर्महेभ्यः करणे (3-1-17) इति सूत्रं समर्थो प्रयोगौ । [1]
4. परिचाय्य - अग्नौ परिचाय्योपचाय्य समूह्याः (3-1-131) इति सूत्रम् । अनिविशेषे प्रयोगे निपातितः ।
5. सान्ताय्य - सान्नाय्यं हविर्विशेषः पाय्य सान्नाय्य निकाय्य

धाय्य - इति सूत्रेण (3-1-129) इत्यादयो ज्ञेयाः ।

एवं कवेः अभिनन्दनसञ्चिकायामेव काव्यमिदं

बहुभिः बहुधा प्रशस्तमिति अलं विस्तरेण ।

उपयुक्तग्रन्थसूची

1. बाणः, नैषधीयचरितम् , स –2, श्लो – 102
2. वाल्मीकिः, रामायणम् , बालकाण्डः, श्लो - 40
3. वाल्मीकिः, रामायणम, बालकाण्डः, श्लो- 29-30
4. श्री.भा.वि.सा, मन्दाकिनी, अवतारिका, पृ. सं - 15
5. श्री. भा.वि.सा, कृति, स्वर्णोत्सव संचिका, पृ .सं - 8
6. श्री.भा.वि.सा, कृति, स्वर्णोत्सव संचिका, पृ .सं - 8
7. श्री.भा.वि.सा , स्वर्णोत्सवसंचिका , पृ.सं - 11
8. श्री.भा.वि.सा, स्वर्णोत्सवसंचिका , पृ.सं - 11.

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प्रस्तावना

वेदवाङ्मयमिव पौराणिक साहित्यमपि भारतीय संस्कृतेः प्राच्यधर्म परम्परायाश्च ज्ञानाय परमोपदेयत्वमुपकल्पते । पुराणिना पुरातन विषयप्रतिपादकानि प्राच्य विधानाञ्च कोशरूपाणि । वेदानामात्मस्वरूपत्वेन अङ्गीक्रियते । आत्मा पुराणानां वेदानाम् इति वचनप्रामाण्यात् । यत्र पुराणेषु भूगोलेतिहासविधिविज्ञानादाय आधुनिक विषयाः सूक्ष्मतया दुरीदृश्यन्ते । तत्र कथाख्याधिका देव-माहात्म्य व्रतनियम धर्मशास्त्र तीर्थ माहात्म्य सृष्टि प्रलयराजवंश मन्वन्तर वंशानुचरितानि सुष्टुतया प्रतिपादितानि सन्ति । अपि च व्याकरणछन्दः काव्यनाट्यालङ्कारायुर्वेदानीति ज्योतिष कर्मकाण्ड विविध दर्शन विषयाः पुराणेषु प्रचुरतया वर्तन्ते । एतस्मादेव उक्तं यत् "पुराणविद् सर्वविद् भवति" इति । पुराणादृते न कश्चित् अन्यो विषयः प्रवर्तते । वेदज्ञानमपि पुराणज्ञानेन विना न पोपुष्यते । वेदार्थकस्य सम्यक् ज्ञानाय पौराणिक साहित्यं नितरांमपेक्ष्यते । वेदस्य गूढार्थपरिज्ञानं पुराणाध्ययनात् विना न भवितुमर्हति । "इतिहास पुराणाभ्यां वेदं समुपबृंह्येत् " इति कथनात् वेदोपबृंहणं पौराणिक साहित्येन विहितम् ।

पुराणशब्दस्य व्युत्पत्तिः

"पुरा भवं पुराणम् "इत्यर्थे पुरा इत्यव्ययशब्दात् "सार्वचिरं प्राह्वेप्रगेइव्ययेभ्यष्ट्युट् युला तुट् च" इति (अष्टाध्यायी 4—6—23) सूत्रात् ट्युप्रत्ययान्तरं इत् संज्ञस्य टकारस्य लोपे कृते युवोरजाकौ इति सूत्रेण योरनादेशे च विहिते तुडागमाच्च पुरातन इति शब्दो निष्पन्नो भवति ।

पुराणस्य लक्षणम् --- लोके केश्चित् पुराणानां वर्ण्यविषये यद्यपि पञ्चभिर्दशभिर्वा लक्षणानि उक्तानि । तथापि पुराणानां पञ्च लक्षणानि विद्वत्सु प्रथितानि सन्ति । एतस्मादेव कारणात् प्रायः सर्वेषु पुराणेषु पञ्च लक्षणमेव समुल्लिखितमस्ति ।

सर्गश्च प्रचिसर्गश्च वंशो मन्वन्तराणि च ।
वंशानुचरितं चैव पुराणं पञ्चलक्षणम् ।। [1]

अत्र सर्गशब्दस्यार्थः सृष्टिरिति विज्ञेयः । प्रतिसर्गेण सम्पूर्णस्य जगतः संहारः परिकीर्तितः । देवर्षिराज्ञां च उत्पत्तिपरम्परा जगतः वंश शब्दतो निर्दिष्टास्ति । सृष्ट्यादीनां कालव्यवस्था मन्वन्तरमित्यभिधीयते । तत्तजन्वयजातानां देवर्षिराज्ञां वृत्तान्तानि वंशानुचरितानि पदेन कथितानि वर्तन्ते ।

पुराणानां विभाजनम्

पुराणसाहित्यस्य प्रायः महापुराणोपपुराणभेदात् सर्वत्र विविध विभाजनमुपलभ्यते ।

मद्वयं भद्वयं चैव ब्रत्रयं वचतुष्टयम् ।
अनापलिङ्गकूस्कानि पुराणानि पृथक् पृथक् ।। [1]

अत्रास्य श्लोकस्याहभोहयमसि यत् मकाराद्याक्षरेण द्वे भकाराद्यक्षरेण द्वे, ब्रकाराद्याक्षरेण त्रीणि, वकाराद्यक्षरेण चत्वारि महापुराणानि बोधव्यानि तथा च अ -- ना --- प --- लि ---ग --- कू -- स्काद्याक्षरैः पृथक्त्वेनैनैकैके महापुराणं विजानीयात् ।

पुराणेषु विष्णुपुराणम् -

अष्टादश पुराणेषु विष्णुपुराणस्य स्थानं तृतीयम् । एतच्च भगवतो विष्णोः दक्षिणबाहुत्वेन स्वीक्रियते । मत्स्य पुराणानुकारं पुराणेञ्च श्लोकसंख्यामिमां समर्थयति । किन्तु तन्मते पुराणमिदं भागद्वयात्मकं पूर्वभागः उत्तरभागश्चेति । तत्र पूर्वभागे षडंशाः वर्तन्ते । ये तावत् शक्ति पुत्रेण पराशरेण मैत्रेयाय कथिताः सन्ति । उत्तरभागश्च तदन्तरं वर्तते । यत्र सूतः शौनकादिभिः प्रैरितः सन् विष्युधर्णोत्तरनामकाः नानाधर्मकथाः प्रोक्तवान् इति ।

विष्णुपुराणे -- प्रथमेऽशे --- 22 अध्यायाः ।
द्वितीयेऽशे --- 26 अध्यायाः ।
तृतीयेऽशे --- 27 अध्यायः ।
चतुर्थेऽशे --- 24 अध्यायाः ।
पञ्चमेऽशे ---- 36 अध्यायाः ।
षष्ठेंऽशे --- 07 अध्यायाः सन्ति ।

वर्ण्यविषयाः

अधुना उपलभ्यमानञ्च विष्णुपुराणस्य प्रथमेंऽशे विस्तरेण सृष्टेः वर्णनं वर्तते । द्वितीयेंऽशे भूगोलस्य वर्णनं तृतीये चतुर्णामाश्रमाणां चतुर्थे राजवंशवर्णनं पञ्चमे श्रीकृष्णस्य चरितं षष्ठे कलियुगस्वरूपम्, प्रायश्चित्त विधानम्, योगमायनिरूपणम्, ज्ञानेन सह भक्तेः सामञ्जस्य प्रतिपादनं चेति विषयाः सम्यक् वर्णिताः सन्ति । अत्र विष्णुपुराणे भगवतो विष्णोः सामप्रदायिक संकीर्णतां विस्मृत्य श्रद्धया सह पुराणखायास्य मतं स्वीकुर्वन्ति । अतः पुराणस्यास्य मुख्यवर्ण्यविषयोऽस्ति विष्णोः उपासना भक्तिर्वा । भक्तिद्वारा स्वाभाविकतया साधकस्य मनः भगवद्वुन्मुखं जायते । नवथा भक्तेः विस्तृत प्रति पादनं पुराणस्यास्य वैशिष्ट्यमस्ति । भगवतः विष्णोः नाम -- गुण -- लीलादि स्मरणस्य माहात्म्यं पराशरेण प्रदर्शितं यत् विष्णुस्मरणेन समस्तपापराशिः भस्मसाद्भवति । मोक्षश्च अनायासेन प्राप्यते ।

किन्तु स्वर्गप्राप्तिः तत्र विघ्नरूपानुमीयते ।
विष्णु संस्मरणा तद्यक्षीण समस्त केश संशयः ।
मुक्तिं प्रयाति स्वर्गाप्तिस्तस्य विघ्नोऽनुमीयते ।। [1]

न केवलं विष्णोः स्मरणं अपि तु तद्यशोगुणादीनां श्रवणं, कीर्तनं,

कृतस्य कर्मणः तत् समर्पणं च अत्यावश्यकमिति मत्वा पुराणेऽस्मिन् त्याग -- संयम - साधनादीनां महत्वपूर्णं चित्रणं कृतं वर्तते । एतेषां त्यागादीनां दृष्टान्तरूपेण ध्रुव प्रह्लाद -- सगर -- भगीरथ - नहुष -- ययाति -- जमदग्नि -- विश्वामित्र – केशिध्वजादीनां चरितं समुपन्यस्तमस्ति । सर्वेषु चरितेषु प्रह्लादचरितम् अत्र भक्तिकससवितततया समुपस्थापितं वर्तते । एतच्चरितमाध्यमेन भक्तेः सुमहात्म्यं प्रकाशितम् । भगवतश्च भक्तिप्रियत्वं प्रतिपादितमस्ति ।

कलिधर्मनिरूपणम्

विष्मुपुराणे षष्ठेऽशे प्रथमोध्याये मैत्रेयाय पराशर महर्णिणा कृत कलिधर्मनिरूपणं विद्रीयते -- कलियुगे मनुष्याणां प्रवृत्तिः न भवति। दाम्पत्यजीवनं यज्ञयागादि कर्मण्यापि च समीचीनरूपेण प्रचलन्ति ।

विवाह न कलौ धर्म्या न शिष्यगुरु संस्थितिः ।
न दाम्पत्यक्रमो नैव वहिनिदेवात्मकः क्रमः ।। [1]

कलियुगे यः बलवान् सैव नीचकुलोद्भवो सन्नपि सर्वेषां स्वामीरूपेण विराजते । कलियुगे यस्य कस्यापि जनस्य मुखात् यत्किमपि वाक्यं निस्सरति तदेव प्रमाणमित्यङ्गीक्रियन्ते । कलियुगे सर्वे क्षुद्रदेवाः प्रमुखदेवतारूपेण पूज्यन्ते । उपवासाः तीर्थाटनं धनत्यागः इत्यादि कार्याण्येव तपोरूपेण स्वीक्रियन्ते । स्वचित्तानुसारिण क्रियमाण-कार्यमेव धर्मः इति कथ्यन्ते । मदगर्वो जायते । स्वकेशानुगुणेनैव स्त्रियः स्वलावण्यालङ्कारं क्रियन्ते । निर्धनः पतिः सर्वथा त्याज्य एवा धनवानेव पतित्वेनाङ्गीक्रियन्ते । अधिकधनदानमात्रेणैव स्वामीति कथ्यन्ते । न तत्र कुलस्य विषयः । गृहकार्येष्वेव धनं सर्वं व्ययीक्रियन्ते । अपि तु सर्वदा धनार्जने एव रताः भवन्ति । न तु आत्मज्ञानार्जने । अतः अतिथिं सत्कारः प्रायः न स्यात् ।

गुहान्ता द्रव्यसङ्घाता द्रव्यान्ता च तथामतिः ।
अर्थाश्चात्मोप भोग्यान्ता भविष्यन्ति कलौयुगे ।। [1]

स्त्रियस्तु सुन्दरपुरुषकामुकाः स्वेच्छाचारिणीव भ्रमन्ति । पुरषाः अक्रममार्गे धनार्जने इच्छुकाः भवन्ति । शूद्राः अपि ब्राह्मणैस्सह सञ्चरन्ति । क्षीरसमाप्यनन्तरं गाः हनन्ति । अन्नावृष्टिकारणात् सर्वे जनाः भयभीताः सन्तः बुभुक्षया पीडितः तपस्विजनाः इव कन्दान् फलानि भक्ष्यन्ते । अनावृष्टिकारणात् आत्महत्यार्थं प्रयत्नन्ते । कलियुगे असमर्थाः सर्वदा आनन्देन जीवनं यापयन्ति । कलिसङ्क्रमण काले विना स्नानेन भोजनं कुर्वन्ति । अग्नि -- देवता अतिथीनां पूजां न कुर्वन्त्येव । पितृ-देवतेभ्यः श्राद्धादि कार्याण्यपि सम्यक् न निर्वहन्ति । गुरूणां सम्मानं प्रायः न भवत्येव । कलियुगे स्त्रियः क्षुद्रस्वभावयुक्ताः शारीरिक संस्कार रहिताः अपवित्राः असत्यवादिनः भवन्ति ।

स्वपोषणपराः क्षुद्राः देहसंस्कार वर्जिताः ।
परुषानृत भाषिण्यो भविष्यन्ति कलौस्त्रियः ।। [1]

ब्रह्मचारिणः वेदाध्ययनं विना कालं यापयन्ति । गृहस्थाः अपि विना अग्निहोत्रादिना सत्कर्मकार्याचरणेन सदाचारेण व्यर्थरूपेण कालं यापयन्ति । सन्यासिनः अपि ग्रामेष्वेव निवसन् मित्रैस्सह सत्सम्बन्धान् व्यवहरन्ति । कलियुगे राजानः प्रजा-रक्षणं विना करग्रहणे एव उत्सुकाः भवन्ति । धनवान् राजा इति दरिद्रः भृत्याः इति च विज्ञायते । कृषिकाराः वैश्यादि जनाः स्वकर्मणा विना अन्यकार्येषु निमग्नाः भवन्ति । शूद्रगणाः सन्यासिवेषं धृत्वा भिक्षा-वृत्तिमारभन्ति । जनानाम् आयुरपि न्यूनं भवति । बाल्यावस्थायामेव मरणं प्राप्यन्ते केचन द्वादश वर्षे केशाः फलिताः भवन्ति । विंशंति वर्षाणि एव आयुर्दायं भवति ।

पलितोद्भवश्च भविता तथा द्वादशवार्षिकाः ।
नातिजीवति वै कश्चित् कलौ वर्षाणि विंशतिः ।। [1]

कलियुगे जनाः मन्दबुद्धियुक्ताः व्यर्थचिह्नानि धृत्वा मरणात्यन्तिक समीपे निवसन्ति । महापुरुषाः यदि सन्दिग्धा भवन्ति तर्हि चिन्तनीयं कलियुगस्य आरम्भो जाता इति । कलिप्रभावेन मेघाः अत्यन्त न्यूनाः तथैव व्यवसाय क्षेत्राण्यपि अल्प फलदायिनः भवन्ति ।

कलि प्रभावेण धान्यमपि स्वल्पं भवति । दुग्धमपि न्यूनं भवेत् । श्वश्रूखशुरश्च गुरुरूपेण विराजते । शालाद्याः एव मित्रिणि रूपाणि आभान्ति । मातापितरौ त्यक्त्वा चित्तभ्रान्त्या सञ्चरन्ति । सर्वदा पाप-कर्मणि एव रताः भवन्ति । निर्लज्ज मनुष्याः सर्वदा दुःखगामिनः भवन्ति ।

वाङ्मनः कायजेंदोषरभिभूताः पुनः पुनः ।
नराः पापान्यनुदिन करिष्यन्त्यल्पमेथसः ।। 1

स्वाध्याय रहिते स्वाहारहिते संसारे धर्मः नश्यत्येव । परन्तु अत्र महान् विषयोऽयं भवति यत् किञ्चित् प्रयासेन एव बृहत्पुण्यं प्राप्तुं शक्यते कलियुगे ।

उपसंहारः

महापुराणेषु विष्णुपुराणम् अत्यन्तसुप्रसिद्धमस्ति । विष्णुमहापुराणे अनेके धार्मिक विषयाः उक्ताः तत्र कलिधर्मविषयाः महोन्नत रूपेण समुपवर्णिताः ।

उपयुक्तग्रन्थसूती

1. अष्टाध्यायी --------- पाणिनीमहर्षिः ।
2. कूर्मपुराणम् -------- वेदव्यासः ।
3. देवीभागवतम् ------ वेदव्यासः ।
4. विष्णुपुराणम् ----- वेदव्यासः ।

संधर्व सूची

1. कूर्मपुराणम्, श्लो --- 1, पृ --12
2. देवीभागवतम्, अध्या --- 1, श्लो ---3, पृ --2
3. विष्णुपुराणम्, 6—1--11
4. विष्णुपुराणम् --- 6—1--20
5. विष्णुपुराणम् --- 6 –1--30
6. विष्णुपुराणम् -- 6 –1—42
7. विष्णुपुराणम् --- 6 ---1---57

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प्रस्तावना

1. **गजः** - गजस्य मांसः न भोजनोपयोगि किन्तु गजवाहनेन उल्लाघः(रोगशक्तिः) त्रिदोषशआन्तिश्च भवति । गजक्षीरं च प्रमेहः, कुष्ठः,गुल्मः इति रोगान् निरयति ।

हस्तीद्विपोगजोनाग इभश्चद्विरदः करी ।
वारणः कुञ्जरोदन्ती मातङ्गष्षष्ठिहायनः ।।
दन्तावलो गजश्चैव पद्मीस्तम्भेरमोमतः ।
न एकलभः प्रोक्तः कुम्भीचकथितो बुधैः ।।
कुञ्जरारोहणं कुर्यास्त्रिदोषशमनं परम्
दुग्धं प्रमेहशमनं कुष्ठुगुल्महरंपरम् ।। [1]
धन्वन्तरिनिघअमटुः, सुवर्णादिवर्गः, पृ -- 254

2. **अश्वः** - अश्वस्यापि आरोहेण मूत्रव्याधिः नश्यति । व्याधेः नाम मूत्रकुच्छ्रः मूत्रस्य विसर्गे यः क्लेशः सनिवारितो भवतीति भावः। "स्यात् कष्टं कृच्छमाभीलं " इत्यमरः अपि च त्रिदोषाः अश्वयानेन नष्टाः भवन्ति । अश्ववेगेन शरीरस्य चलनाधिक्यत् एवं भवेत् । अश्व बऽबा क्षीरं तु चित्तभ्रम निवारकम् ।

घोटाश्वतुरगोवाजी हरिरुसस्तुवङ्गमः ।
शालिहोत्रोजवीसप्तिस्तुरङ्गस्सैन्धवोहयः ।।

अश्वीरोहणमात्रन्तु मूत्रकृच्छ्रविनाशयेत् ।
त्रिदोषशमनंबलं तद्दुग्धंभ्रमनाशकम् ।। [1]

3. **उष्ट्रः**- उष्ट्रस्य क्षीरं शीतनाशकं अर्शसः उपशमकं क्रिमिनाशकं महोरशमकं भवति ग्रहपीडां च निवारयति । वीर्य क्षणतां निरूध्य धातुपुष्ठिकरं भवति ।

उष्ट्रः क्रमेलकोधूम्रः करभोदीर्घमार्गगः ।
द्विककृच्छं कुनासश्च दीर्घग्रीवोघ म्रकः ।।
क्रमेलदुग्धं शीतघ्नमर्शः क्रियामबुदरपहम् ।
ग्रहभूतप्रशमनं नष्टेद्रियकरं परम् ।। [1]

4. **गर्दभः** - गर्दभस्य क्षीरं शरीरबलवर्धकं भ्रमं मूत्रकृच्छं निवारयति ।

बालानामपि रोगहारकम् ।
गर्दभश्शदुकर्णश्च बालोयोशसभःखरः ।
भारवाहीभूरिगामो धूसरोरेणु भूषितः ।।
खरदुग्धं भ्रमहरं कायसिद्धिकरं परम् ।
कामिलामूत्रकृच्छघ्नं बलरोगहरंपरम् ।। [2]

5. **छागलः** – छागलस्य मांसः गुरुः स्निग्धश्च भवति । श्वास रोगान् कासरोगान् वातरोगान् प्रदररोगान् शोषकि कफदोषं च जेतुं प्रभवति।

छागलोबर्कश्चागः चलपादस्तुबस्तकः ।
वोलकश्चैडकोजाच लम्बकर्णस्तथैवच ।।
अजमांसं गुरुस्निग्धं श्वासकासानिलापहम् ।
प्रदरंहन्ति गुरुस्निग्धं च पित्तघ्नं कफदोषजित् ।। [1]

6. **मेषः**- मोषमांसं मधुरं शीतवीर्यं गुरुं च भवति । त्रिदोषहारकं भवति ।
प्रतिश्चायं ज्वरं च निवारयति शरीरकान्तिं वृद्धिं करोति ।

तगरुर्मेद्रं मोषा च ह्युरभ्र उदणस्तथा अतिलोमस्तभोर्णाम् युरेडकोवृष्टिरेव च।
तगरोर्मधुरं मांसं शीतवीर्यन्त्रिदोषजित् प्रतिश्चायाज्वरहरं गुरुकान्ति प्रदंनृणाम् ।। [2]

7. **महिषः** - महिष वृषभयोः मांसेन गुणः यद्यपि धन्वरिनिघण्टौ ऐदाहृतः तथापि बहुभिःजनैःतन्मांसं उपयुज्यते । त्वचा च दूनि प्रयोजनानि प्राकृ निरूपितानि । आरण्यकस्य धेनोः गावयस्य मांसगुणं तु कोशे उदाहृतः । गवस्य मांसं वीर्यवृद्धिकरं बलकरं स्वदुः च उदाहृतम् ।

महिषः कासरः श्रृंगी श्यामः कृष्णोलूलायकः ।
विषाणीकलुषो दंशी पीनस्कन्धोरजस्वलाः ।।
महिषः सलिलप्रीतिस्वशक्तिर्भारवाहकः ।
अनड्वांस्तुबलीवर्धः ककुद्वान्वृषभोवृषः ।
उक्षा चे सौरभेच्च गोपतिश्शंतर प्रियः ।।
प्रातर्दशीं च वृषभं कल्याणं च फलप्रदम् ।
ऐश्वर्यं लभते नित्यं शंकरस्यानुवाहनम् ।।
तस्यपुत्रीभानुमती गौरत्रसुरभिर्वसु माहेयी बहुलाधेनुस्सौरभेयी च नन्दिनी ।
गावस्य तु मांसन्तु वृष्यंबल्यंरुचिप्रदम् धातुवृद्धिकरंचैव मुनिभिः परिकीर्तितम् ।।

8. **व्याघ्रः** - व्याघ्रस्य चर्मणा प्रयोजनमेव धन्वन्तरि निघण्ढौ दृष्टम् । व्याघ्रचर्म यौगिभः आसनार्थं उपयुज्यते । व्यघ्रनखधारणं ग्रहदोष निवारकं आयुः बुद्धिकरं देहवृद्धिम् च ।

व्याघ्रः पञ्चनखोव्यालश्शार्दूलश्च गुहाशयः
पुण्डरीकस्तीक्ष्णद्रष्ट्रस्तरस्वीघोरदर्शनः।।
सुखासनं व्याघ्रचर्म भिक्षुप्रीतिकरं तथा
तन्नखोग्रहदोषगघ्नस्तथायुर्देहवर्धनः ।। [1]

9. **वराहः** - वराह मांसं बहुजनप्रियम् । वातघ्नं बलवर्धकं कान्तिकरं च । वराहमांस धूपः ज्वरनाशकः भूतबादा निवारकः दुष्टव्रणहरश्च ।

सूकरोवज्रदंष्ट्राश्च बराहोमोमशः किटिः दंष्ट्री दंष्ट्रायुधः क्रोडो वीरस्कन्धो बहुप्रजः ।।
स्तब्धरोमाबलीकोलो घृष्टिः प्रोत्रीकिरस्मृतः स एव कथितो घोणी पृथुवाराहनामतः ।।
किटिमांसं वातकरं बलवर्णकरंपरम् तद्धमोज्वरभूतघ्नो दुष्टव्रणहरः परम् ।। [2]

10. **मृगः** - मृगमांसस्य जनप्रीतिकरत्वं प्रसिद्धमेव । मधुरगुणं लघुबलकरं वीर्यवृद्धिकरं त्रदोषहरं च ।

मृगोघहरिणोन्यङ्कः सारङ्गः पृषतोरुरुः ।
एणश्शबलपृष्ठश्च पिशङ्गश्चारुलोचनः ।।
न्यङ्कस्वादुर्लघुर्बल्योवृष्योदोषत्रयापहः ।। [1]

11. **शुनकः** - श्व मांसः यद्यपि मासाहारैः अपि नादृतः । तथापि वातनाशकरः उदाहृतः विपाकदशायां उष्णकरश्च ।

कुक्कुरः सारमेयश्च भीषणः शुनकस्तथा ।
भूस्तरोभषकरम्वास्यादलरको रात्रिजागरः ।।
कौलेयोमृगदंशश्च वान्तादः सुमुखस्तथा
सूकरारिः कृतज्ञश्च मृगयालुशलस्तथा ।। [2]

12. **बिडालः** - माराजालचर्मणा धूमेण ग्रहपीडा भूतपीडा ज्वरदोषः कुष्ठः कण्डूः दद्रः त्यादि रोगाः शान्ताः भवन्ति ।

बिडालोविषदंष्ट्रश्च मूषिकारिर्बिडालकः वृषदंशश्चमार्जालो मायावीदीप्तलोचनः ।।
मार्जालचर्मजोधूपो ग्रहघ्नो भूतनाशनः ज्वरदोषहरः कुष्ठ कण्डूदद्रुविनाशनः ।। [1]

13. **श्रृगालः** - श्रृगालमांसस्यापि गुणाः धन्वतिरिणा उदाहृताः । किन्तु श्रृगालमांसस्य विनियोगः न श्रूयते । तथापि औषधरूपेण तस्य गुणाः दर्शिताः । गुणेन कटुः समस्तवातरोगनिवारिकः क्षयरोगनिवारकश्च । धातूनां पुष्टिं शरीरपुष्टिं त करोति।

श्रुगालोजम्बुकः क्रोष्टा गोमायुः क्राष्टकः शिवा ।
वञ्चकोमृगधूर्तश्च भूरिमायोधभौरवः ।।
श्रुगालमांसं कटुकं गन्धर्वव्याधि नाशनम् ।
वृष्यं पुष्टिकरं बल्यं सर्ववातक्षयापहम् ।। [2]

14. **मर्कटः** - मर्कटस्य क्षीरगुणः एव दर्शितः । क्षीरं कटुकं भवति । सन्याज रोगस्य श्वासरोगस्य च निवारकं भवति ।

मर्कटो वानरः कीशो करिरशाखा मृगः कपिः ।
प्लवङ्गमो वनैकाश्च प्लवङ्गः प्लवगः प्लवः ।।
प्लवङ्गगं क्षीरं कटुकं कांससन्यास श्वासजित् ।। [1]

एवं सर्वेषां पशूनां लोकोपकारकत्वं अस्मिन् प्रकरणे निरूपितम् । जीवन्ता एताः अपि पशवः मानवजातिः बहुधा उपकारकाः इति स्पष्टम्। विशेषतः कृषिकर्माणि पशूनां विनियोगः मानवप्राणरक्षायाः जीवातुभूतः। गतां महिषां च क्षीरं मेषादीनां मांसं आहाररूपेण प्राणदायको भवत्येव । अपि च कैश्चित् औषध उपयोगोपि दृष्टः ।

एतान् उपकारान् अतीत्य पशूनां उत्सर्गैः अपि प्रयोजनं बहुधा दृष्टम् । मृगमलादयः पदार्थाः सुगन्धद्रव्यरूपेण औषधरूपेण उपयुज्यन्ते । एतेषां द्रव्याणां कृतं तत् घारकाः मृगाः हन्यन्ते च यथा-

चर्मणि द्वीपिनं हन्ति दन्तयो र्दन्ति कुञ्जरम् ।
बालेषु चमरीं हन्ति सीम्नि पुष्कलको हतः ।। [1]
सीमाः अण्डकोशः पुष्कलकः गन्धमृगविशेषः ।

गन्धमृगाश्च यथा ---

चित्ररेखाकरीधूर्णी मार्जालीगन्धमार्जनी ।
सुगन्धगात्रावसना कस्तूरीरोमशाल्यपि ।।
मुगन्ध पूतवृषणी बिडालीकाण्डचालिनी ।
तत्कोशंवृष्यमायुष्य सौरभंगात्रगन्धजित् ।। [2]

15. **कस्तूरी मृगः** - पुष्कलकः एव कस्तूरी मृगः नाम्ना प्रसिद्धः । प्रायशः हिमवत्पर्वत प्रान्तेषु एव एते चरन्ते । एतेषां सीम्नः कस्तूरी लभ्यते । शीतज्वरार्तानां औषधार्थं उपयुक्तः । अद्यापि सद्यः प्रसूताभ्यः गृहेषु प्रसूति वातनिवारणाय कस्तूरी दीयते । भगवत्सेवासु चन्दनेन मिश्रितोऽयं उपयुक्तः । "कस्तूरी तिलकं" - इत्यादि स्तोत्राणि प्रसिद्धान्येव । चन्दनेन तं मिश्रणात् यक्षकर्दमनामकः अनुलेपः क्रियते । एवं बहूपकारकाणां पशूनां प्रस्तावः संस्कृत वाङ्मये कीदृशो वर्तते इति विचारः एवं अत्र शोधनिबन्धे विवक्षितः ।

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COMPARISION OF CONTROL WITH CUSUM CHAARTS – A CASE STUDY

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ABSTRACT

The primary purpose of a CUSUM control chart is to detect small shifts from the process target. Over the years, latest publications have covered a number of different control charts. These charts are primarily Shewhart control charts, e.g., X-R, X-s, and X-mR control charts. These types of control charts are very useful in monitoring a process for statistical control or to see the impact of your process improvement efforts. You can also increase the sensitivity of these charts by using the zone tests and run tests in addition to the points beyond the control limits test for out of control conditions. The CUSUM control chart approach is different. The major aims of the CUSUM control chart are to keep the process on target. The "cumulative sum" in this type of chart is the sum of the deviations of individual sample results or subgroup averages from the target. The CUSUM control chart plots these cumulative deviations over time and will indicate when the process is "out of control" – or in this case, significantly off-target.

Key words: Control Charts, CUSUM, mR Chart

INTRODUCTION TO CUSUM CONTROL CHARTS

Like all control charts, a CUSUM control is a method of looking at your process variation. A CUCUSUM control chart monitors the deviations of indiIndividual samples results or subgroup averages from a target value. The target value is the same as your process aim – where you want the process to operate. The cumulative sum is the sum of

these deviations over time. To use a CUSUM control chart, you have to answer two basic questions.

1. What shift from the target do you want to be able to detect?

This shift is usually ½ or 1 standard deviation of the parameter being plotted. Thus, the dadata helps you set the shift to detect. This shift is often referred to as the "allowable slack" in th process.

2. Where do you want the action limits set?

The action limits are similar to control limits. The standard practice is to set these actions at four times the standard deviation. Again, the data helps set the action limits. It is clear you will need a method of estimating the standard deviation of the process. This is covered below.

A CUSUM control chart then tracks the deviations from the target. If a sample result is within the allowable slack, the process is considered to be operating on target. This results in a subtraction from the cumulative sum unless it is already zero. If a sample result is beyond the allowable slack, the process is considered to be operating off target. This results in an addition to the cumulative sum. If the cumulative sum exceeds the action limits, the process is assume to be off target.

There are two "one-side" cumulative sums. One for deviations that are positive (above the target) and one for deviations that are negative (below the target). Let's take a look at an example to see how this works. We will compare the results when using an individual's control chart to using the CUSUM control chart.

THE DATA SET

You take a sample every batch. The results for the last 25 batches are shown below You are producing a chemical that contains a small amount of component Y that is important in the use of the chemical. In addition, you know that the optimum concentration for that component is 0.16 wt. %. You want to control the manufacturing process as close to that as possible.

Table 1: The Data Set

Sample	Y wt.%	Sample	Y wt. %
1	0.175	14	0.164
2	0.152	15	0.141
3	0.150	16	0.186
4	0.207	17	0.127
5	0.136	18	0.149
6	0.212	19	0.155
7	0.166	20	0.210
8	0.141	21	0.197
9	0.157	22	0.191
10	0.197	23	0.211
11	0.172	24	0.158

12	0.183	25	0.201
13	0.166		

ANALYSIS USING SHEWHART CONTROL CHART

The data in Table 1 were analyzed using the X-mR chart. Fig. 1 shows the X- Chart Since our batch data represents one data point at a time, an individual's control chart (X-mR) can be used to monitor the results. This is the most common way the data would be analyzed. But remember, the purpose of a Shewhart control chart is primarily to keep the process in control and to see the impact of process improvement efforts. With a CUSUM control chart, you are trying to detect small shifts away from the process target.

This chart monitors the variation in the individual results over time. Fig. 2 shows the moving range (mR) chart. This chart monitors the variation in the range between consecutive samples.

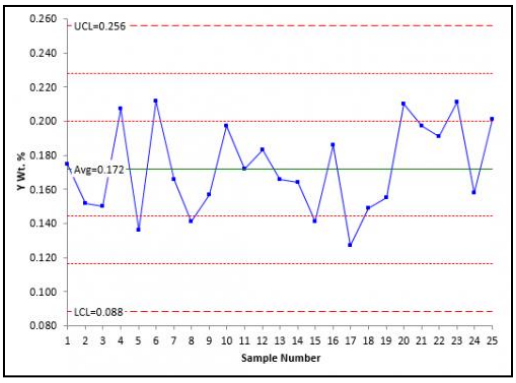

Fig. 1: X Chart for Y Wt. %

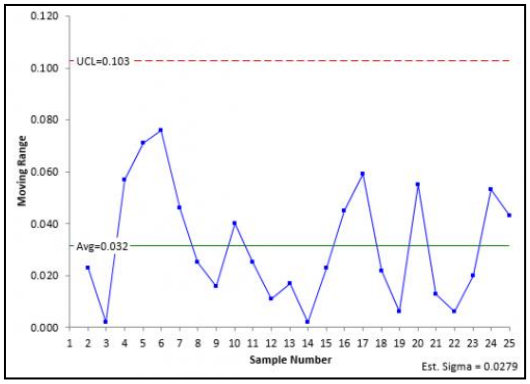

Figure 2: mR Chart for Y Wt. %

The X chart is in statistical control. There are no points beyond the limits and there are no issues with the zone tests or run tests. The process average is 0.172. This is different from the process target of 0.166. But not by much. Is it significantly different? The individual results can vary between the lower control (LCL) of 0.088 and the upper control limit (0.256). The process is consistent and predictable.

Note that 5 out of the last 6 points are above average. Is this a problem? Under the classical control chart approach, it is not a problem. There is not enough evidence of you to assume that a special cause of variation exists and spend time looking for that special cause. For more information on interpreting control charts, please see our April 2004 publication.

The mR chart is in statistical control as well. The average moving range between consecutive samples is 0.032. It can be as great as the UCL of 0.103. Since the range chart is in control, you can estimate the process standard deviation from the range chart. Remember above where we said you needed a standard deviation (σ). This is where you get it. For the X-mR chart, the standard deviation is given by the following:

σ =R /1.128 = 0.0279

ANALYSIS USING CUSUM CONTROL CHARTS

Let's take a look at the data using a CUSUM control chart. The first step is to determine the standard deviation of the data. This is used to set the allowable slack and the action limits. The best way to estimate the standard deviation is from a range control chart. For the individuals control chart, this is the moving range chart where sigma is estimated from the average range. As shown above, the estimate of the standard deviation is 0.0279.

The next step is to determine the allowable slack (k). It is common to set this value at ½ to 1 standard deviation. We will use ½. So,

k = 0.5(0.0279) = 0.01395

The next step is to determine the action limits (H). These are usually set as +/- 4 sigma. So,

H =+/- 4(0.0279) = +/- 0.1116

The next step is to calculate the cumulative sums on the high side (SH, above the target) and on the low side (SL, below the target). We know that our target is 0.16. The following are the definitions for SH and SL.

$SH(i)= Max[0, SH(i-1) + X_i – Target – k]$

$SL(i) = Min(0, SL(i-1) + X_i – Target + k]$

where

$i = i^{th}$ sample

$SH(i)$ is the high side cumulative sum for the ith point

$SH(i-1)$ is the high side cumulative sum for the $i^{th} – 1$ point

X_i is the result for the ith point

Target = target value

k = allowable slack

$SL(i)$ is the low side cumulative sum for the ith point

$SL(i-1)$ is the low side cumulative sum for the $i^{th} – 1$ point

Take a look at the equation for SH(i). It contains:

$X_i – Target – k = X_i – (Target + k)$

So, if $X_i >$ (Target + k), then the ith sample result is outside the allowable slack range on the high side.

Similarly, for SL(i):

X_i – Target + k = X_i – (Target – k)

So, if X_i < (Target – k), then the ith sample result is outside the allowable slack range on the low side. Figure 3 shows this.

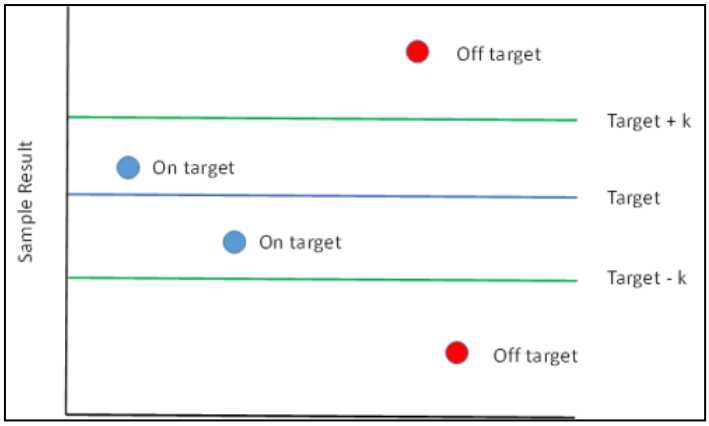

Figure 3: On or Off Target

Let's see how the calculations unfold. Note that the equations for SH and SL include the $i^{th}$and i^{th} -1 results for SH and SL. What about the first sample? There is no sample before the first sample. For the first sample, SH(0) = 0 and SL(0) = 0.

The first sample result is 0.175. Note that this result is above our aim of 0.16. If it is far enough away, then it should add to the high side value of SH.

SH(1)= Max[0, SH(0) + 0.175 – 0.16 – 0.01395] = Max[0, 0.001] = 0.001

SL(1) = Min(0, SL(0) + 0.175 – 0.16 + 0.01395] = Min[0, 0.029] = 0

The second sample result is 0.152. Note that this result is below our aim. It if is far enough away, then it should add to the low side value of SL.

SH(2)= Max[0, 0.001 + 0.152 – 0.16 – 0.01395] = Max[0, -0.021] = 0

SL(2) = Min(0, 0 + .152 – 0.16 + 0.01395] = Min[0, .00595] = 0

Table 2 shows the results of the calculations for all 25 samples. Note that both SH and SL return to zero several times.

Table 2: CUSUM Calculation Results

Sample	X	SH	SL
1	0.175	0.001	0.000
2	0.152	0.000	0.000
3	0.150	0.000	0.000
4	0.207	0.033	0.000
5	0.136	0.000	-0.010
6	0.212	0.038	0.000
7	0.166	0.030	0.000
8	0.141	0.000	-0.005
9	0.157	0.000	0.000

10	0.197	0.023	0.000
11	0.172	0.021	0.000
12	0.183	0.030	0.000
13	0.166	0.022	0.000
14	0.164	0.012	0.000
15	0.141	0.000	-0.005
16	0.186	0.012	0.000
17	0.127	0.000	-0.019
18	0.149	0.000	-0.016
19	0.155	0.000	-0.007
20	0.210	0.036	0.000
21	0.197	0.059	0.000
22	0.191	0.076	0.000
23	0.211	0.113	0.000
24	0.158	0.097	0.000
25	0.201	0.124	0.000

Remember that our action limits are +/- 0.1116. This value is exceeded by SH(23), our 23rd sample. This tells us that the process has moved significantly off target and needs to be adjusted to return the process to the target value.

You could simply use this tabular form for the cumulative sum. However, it is better to chart the results. Figure 4 is a plot of the CUSUM chart.

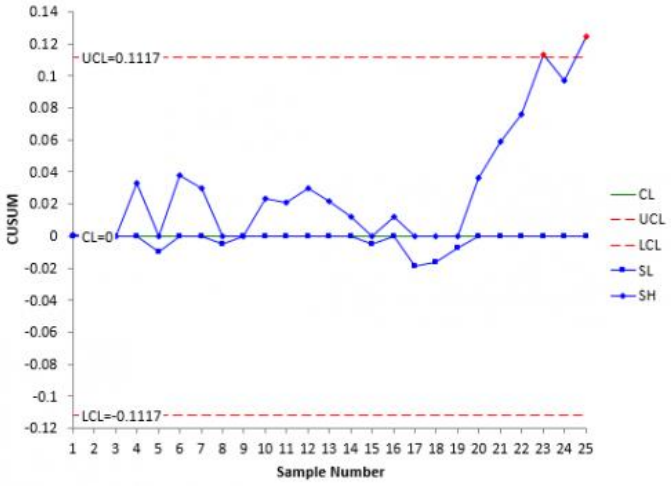

Figure 4: CUSUM Chart

You can see from this chart that both SH and SL are further away from 0 than in Figure 4. If the process is off target, then the CUSUM control chart will pick it up faster. If it is on target, the CUSUM values will trend back to zero as shown in the middle of the chart.

You can estimate the current process average when a point exceeds the action limits. In this example, that occurred at sample 23 and it occurred on the high side. The following can be used to estimate the current process average.

Current proces average = Target + k + (SH/NH)

where SH = value of SH when the action limit was exceeded and NH = the number of consecutive points above 0 when the action limits were exceeded. You can see in Figure 4 and Figure 5 that the 23rd result is the fourth point in a row above 0. So, NH = 4. With SH(23) = 0.113,

Current process average = .16 + 0.01395 + (0.113/4) = 0.202

If the action limit on the low size is exceeded, you find the current process average using:

Current process average = Target – k – (SL/NL)

Where SL is the value of SL when the action limit was exceeded and NL is the number of consecutive points below 0 when the action limit was exceeded.

COMPARING THE TWO APPROACHES

We examined the data using the individuals control chart and the CUSUM control chart. Did the two charts give us the same information? No, the individuals control chart indicated that the process was in statistical control. As of yet, there is no indication that anything has changed in the process. On the other hand, the CUSUM control chart indicated that the process had drifted off target on sample 23. So, it picked up the change quicker than the classical control chart.

But remember: the two types of control charts have different purposes. With the individuals control chart, the primary purposes are to keep the process in statistical control and to see the impact of process improvement efforts. All processes will have drift in the average. If small shifts in the average aren't critical to you, the individuals control chart is fine. It serves it purpose. However, if there is little room for error and it is more important that the process operate on target, then consider using the CUSUM control chart. Note that if there is a true shift in the process average, the individuals control chart will pick it up eventually.

One of the biggest disadvantages of the CUSUM control chart is that the underlying data is lost. You are not looking at actual values. The individuals control chart gives you a picture of what is happening in the process through the use of actual values. The CUSUM control chart is the sum of deviations – more difficult to see the underlying process behavior. And for the most part, if you use the additional tests for out of control (zone tests, etc.) with the individuals control chart, the ability of the individuals control chart to detect small shifts approaches that of the CUSUM control chart.

Note that there are other control schemes for the CUSUM control chart. These schemes are based on the ARL – the average run length. ARL is the average number of samples required before an out of control situation occurs. To minimize the number of false out of control points, the ARL should be large when the process is operating on target. It should be small if the process average has shifted off target. Different values of k and H create different ARL. Using k = 0.5 and H = 4 provides a CUSUM control chart that has good ARL for a shift in process average of one standard deviation. For more information, please see "Introduction to Statistical Quality Control" by Douglas C. Montgomery. You can also search online for "CUSUM ARL" to find more information. The ARL is based on a point beyond the action or control limits.

Conclusion

In this paper has introduced the one-sided CUSUM control chart. This type of control chart monitors the cumulative sum of deviations from the process target. It is designed to detect small shifts in the process average from the target value. It is called "one-sided" because there is a cumulative sum on the high side (above the target) and a cumulative sum on the low side (below the target). You can adjust the CUSUM ability to detect shifts by changing the values of k (the allowable slack) and H (the action limits). This type of chart will detect the process shift much quicker than waiting for a point beyond the control limits for the individuals control chart. However, applying the zones tests to the individuals control chart improves its ability to detect those small shifts.

REFERENCES

[1]. Arnold Jesse C., and Reynolds Marion R. Jr., "CUSUM Control with Sample Size and Sampling Intervals", Journal of Quality Technology, 33(1), pp. 66-81.(2001)

[2]. Jayaraman B., Valiathan G.M., Jayakumar K., Palaniyandi A., Thenumgal S.J., Ramanathan A., "Lack of mutation in p53 and H-ras genes in phenytoin induced gingival overgrowth suggests its non cancerous nature", Asian Pacific journal of cancer prevention : APJCP, ISSN : 13(11) (2012) PP. 5535-5538.

[3]. Box, and M. E. Mullerm "A Note on the Generation of Random Normal Deviates", Ann. Math. Statistics, 29, pp. 610-611. (1958)

[4]. Kalaiselvi V.S., Saikumar P., Prabhu K., Prashanth Krishna G., "The anti Mullerian hormone-a novel marker for assessing the ovarian reserve in women with regular menstrual cycles", Journal of Clinical and Diagnostic Research, ISSN : 0973 - 709X, 6(10) (2012) PP.1636-1639.

[5]. Costa A. F. B., "Chart with Variable Sample Size", Journal of Quality Technology, 26, pp.155-163. (1994)

[6]. Subhashree A.R., Shanthi B., Parameaswari P.J., "The Red Cell Distribution Width as a sensitive biomarker for assessing the pulmonary function in automobile welders-a cross sectional study", Journal of Clinical and Diagnostic Research, ISSN : 0973 - 709X, 7(1) (2013) PP. 89-92.

[7]. Lucas J. M., and Crosier R. B., "Fast Initial Response for CUSUM Quality Control Schemes", Technometrics, 24, (1982).

[8]. Gopalakrishnan K., Prem Jeya Kumar M., Sundeep Aanand J., Udayakumar R., "Analysis of static and dynamic load on hydrostatic bearing with variable viscosity and pressure", Indian Journal of Science and Technology, ISSN : 0974-6846, 6(S6) (2013) PP.4783-4788.

[9]. Montgomery D.C. Introduction to Statistical Quality Control, 5 th Edn. John Wiley and Sons, New York. 2005.

[10]. Srinivasan V., "Analysis of static and dynamic load on hydrostatic bearing with variable viscosity and pressure", Indian Journal of Science and Technology, ISSN : 0974-6846, 6(S6) (2013) PP.4777-4782.

[11]. Page. E.S "Cumulative Sum Control Charts," Technometrics, Vol 3, pp. 1-9. (1961)

[12]. Pandurangan A., "Some Applications of Markov Dependent Sampling Scheme in SQC", Unpublished Ph.D Thesis, Bharathidasan University, Tiruchirappalli, (2002).

[13]. Pandurangan A and R.Varadharajan, "Standardized CUSUM Control Chart". International Journal of Mathematics and Applied Statistics(IJMAS), Vol 1(2). pp. 99-110.(2010)

[14]. A. A. Olatunde. "On the Importance of Statistical Process Control in Health Care". *Research Journal of Medical Sciences*.3 (2): 87-90, 2009.

[15]. BPI Consulting, LLC Bulletin (2009); www.spcforexcel.com.

[16]. C. S. McArdle, D. Hole. "Impact of variability among surgeons on postoperative morbidity and mortality and ultimate.survival. *BMJ* 1991; 302: 1501-1505

[17]. D. C. Montgomery. "Introduction to Statistical Quality Control", *6th Edition. New York, NY: John Wiley & Sons*, 2009.

[18]. D. J. Wheeler, D. S. Chambers. "Understanding Statistical Process Control", *2nd Edition*. *Knoxville, TN: SPC Press*, 1992.

[19]. G. B. Wetherill. "Sampling Inspection and Quality Control". 2nd Edition. *New York: John Wiley & Sons, Inc.*, 1977.

[20]. J. C. Benneyan, R. C. Lloyd, P. E. "Statistical Process Control as a Tool for Research and Healthcare Improvement".*Quality & Safety Health Care* 2003; 12: 458-464.

[21]. M. A. Mohammed. "Using Statistical Process Control to Improve the Quality of Health Care". *Quality & Safety HealthCare* 2004; 13: 245-245

[22]. M. A. Mohammed, Worthington P, Woodall WH. "Plotting Basic Control Charts: Tutorial Notes for Healthcare Practitioners". *Quality & Safety Health Care* 2008; 17: 137-145

[23]. M. Diaz, D. Neuhauser. "Pasteur and Parachutes: When Statistical Process Control is Better than a Randomized Controlled Trial". *Quality & Safety Health Care* 2005; 14: 140-143.

[24]. M. Xie, T. N. Goh, V. Kuralmani. "Statistical Models and Control Charts for High Quality Processes". *Kluwer Academic Publishers*, 2002.

[25]. P. Adab, A. Rouse, M. A. Mohammed, "Performance League Tables: The NHS Deserves Better". *BMJ* 2002; 324:95-98.

MODERN REGRESSION WITH R PROGRAMMING

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ABSTRACT

Linear regression is used to predict the value of a continuous variable Y based on one or more input predictor variables X. The aim is to establish a mathematical formula between the the response variable (Y) and the predictor variables (Xs). You can use this formula to predict Y, when only X values are known. R is a programming language developed by Ross Ihaka and Robert Gentleman in 1993. R possesses an extensive catalog of statistical and graphical methods. It includes machine learning algorithm, linear regression, time series, statistical inference to name a few. Most of the R libraries are written in R, but for heavy computational task, C, C++ and Fortran codes are preferred. Our introduction to the R environment did not mention statistics, yet many people use R as a statistics system. We prefer to think of it of an environment within which many classical and modern statistical techniques have been implemented. The most convenient way to use R is at a graphics workstation running a windowing system.
This guide is aimed at users who have this facility. In particular we will occasionally refer to the use of R on an X window system although the vast bulk of what is said applies generally to any implementation of the R environment. To provide the statistics and p-values between percentages in the current and previous year we will perform an analysis of variance and a linear regression.
This tutorial presents a data analysis sequence which may be applied to environmental datasets, using a small but typical data set of multivariate point observations. It is aimed at students in geo-information application fields who have some experience with basic statistics, but not necessarily with statistical computing. Five aspects are emphasized:
1. Placing statistical analysis in the framework of research questions;
2. Moving from simple to complex methods: _rst exploration, then selection of promising modelling approaches;
3. Visualizing as well as computing;
4. Making correct inferences;
5. Statistical computation and visualization.
The analysis is carried out in the R environment for statistical computing and visualization
, which is an open-source dialect of the S statistical computing language.

Keywords: R, Regression, Least Square

1. INTRODUCTION

A Modern Approach to Regression with R is aimed, according to its preface, at _rst-year grad-uate students and senior undergraduates. R is a statistical programming language whose popularity is quickly overtaking SPSS and other "traditional" point-and-click software packages (Muenchen, 2015). But why would anyone use a programming language, instead of point-and-click applications, for data analysis? An important reason is that data analysis rarely consists of simply running a statistical test. Instead, many small steps, such as cleaning and visualizing data, are usually repeated many times, and computers are much faster at doing repetitive tasks than humans are. Using a point-and-click interface for these "data cleaning" operations is laborious and unnecessarily slow:

"[T]he process of tidying my data took me around 10 minutes per participant as I would do it all manually

through Excel. Even for a moderate sample size, this starts to take up a large chunk of time that could be spent doing other things like writing or having a beer" (Bartlett, 2016).

A programmed analysis would seamlessly apply the tidying steps to every participant in the blink of an eye, and would itself constitute an exact script of what operations were applied to the data, making it easier to repeat the steps later.

Learning to use a programming language for data analysis reduces human labor and saves time that could be better spent doing more important (or fun) things. In this post, I introduce the R programming language, and motivate its use in Psychological science. The introduction is aimed toward students and researchers with no programming experience, but is suitable for anyone with an interest in learning the basics of R.

R commands, case sensitivity:

Technically R is an expression language with a very simple syntax. It is case sensitive as are most UNIX based packages, so A and a are different symbols and would refer to different variables. The set of symbols which can be used in R names depends on the operating system and country within which R is being run (technically on the locale in use). Normally all alphanumeric symbols are allowed1 (and in some countries this includes accented letters) plus '.' and '_', with the restriction that a name must start with '.' or a letter, and if it starts with '.' the second character must not be a digit. Names are effectively unlimited in length. Elementary commands consist of either expressions or assignments. If an expression is given as a command, it is evaluated, printed (unless specifically made invisible), and the value is lost. An assignment also evaluates an expression and passes the value to a variable but the result is not automatically printed.

Commands are separated either by a semi-colon (';'), or by a newline. Elementary commands can be grouped together into one compound expression by braces ('{' and '}'). Comments can be put almost2 anywhere, starting with a hashmark ('#'), everything to the end of the line is a comment.

Recall and correction of previous commands

Under many versions of UNIX and on Windows, R provides a mechanism for recalling and reexecuting previous commands. The vertical arrow keys on the keyboard can be used to scroll forward and backward through a command history. Once a command is located in this way, the cursor can be moved within the command using the horizontal arrow keys, and characters can be removed with the DEL key or added with the other keys.

Building Valid Models

. We note that our graphics are formulated using the "character cell" unit. This is the pixel size that SAS provides for drawing a single character. It SAS works with three types of commands, or steps as the SAS literature calls them: data, proc, and option. Data steps read data into SAS and perform elementary operations on the data. Proc steps perform more sophisticated operations on the data, including statistical analysis. Option steps set attributes of the SAS session, such as the number of columns used for output and the default color of points in graphics. The data and proc steps are made up of component statements. These statements are similar to the one line commands in Stata. We may view the data and proc steps as batches of statements. The SAS literature refers to the options steps as statements as well, since their effects are atomic and straightforward.

In the Stata primer we focused on explaining individual statements. Due to the setup of the SAS language, we will discuss our programs in terms of data and proc steps and options statements. We should be consistently thorough, but some brevity may be required for readabilityis dependent on the pixel size of the SAS window. In rendering the graphics we generally have SAS maximized in a 1024 x 768 screen resolution environment.

In general, the size of elements of our graphics will vary depending on whether the window is maximized and what the overall screen resolution is. So depending on your desktop environment, your graphics may appear different. You can experiment with the "h=" option specification in the axis

statements and other size specifications to make the graphics visually clear in your environment.

2. LITERATURE REVIEW

Projections for US newsprint demand up to 2020 is studied . First, various specifications of the standard model used in forest product demand literature, called the classical model, are estimated using annual data from 1971–2000 (Hetemäki and Obersteiner, 2001). The results have indicated that structural change in the newsprint consumption pattern took place at the end of the 1980s. The classical model fails to explain and forecast the structural change. This finding has motivated the formulation of alternative models. Thus, a Bayesian model that allows industry experts prior knowledge about the future demand for newsprint to be included in the projections is estimated. Also, an ad hoc model, in which newsprint demand is a function of changes in newspaper circulation, is used to compute projections. Finally, the forecasts of these models are evaluated along with some of the existing projections. Besides providing an outlook for US newsprint demand, the study contributes to the existing literature of long-term forest product demand by raising some methodological questions and by applying new models to compute projections.

Tronto et al. (2006), have proposed a machine learning techniques such as neural networks, rule induction, genetic algorithm and case-based reasoning for application in a wide variety of fields such as computer vision, econometrics and medicine, where human abilities have proven to be superior to those of computers. Such techniques hold the promise of being able to make sense of a variety of inputs of different types in producing an output. Software effort modeling has always appeared to be a rather hit-or-miss business where statistical methods frequently result in low accuracy of prediction.

2.1 FORECASTING USING MULTIPLE LINEAR REGRESSION

The influence of economic and demographic variables on the annual electricity consumption in Italy has been investigated with the intention to develop a long-term consumption forecasting model. The time period considered for the historical data is from 1970 to 2007. Different regression models are developed, using historical electricity consumption, Gross Domestic Product (GDP), GDP per capita and population. A first part of the study by Bianco et al. (2009) has considered the estimation of GDP, price and GDP per capita elasticities of domestic and non-domestic electricity consumption. The domestic and non-domestic short run price elasticities are found to be both approximately equal to −0.06, while long run elasticities are equal to −0.24 and −0.09, respectively. On the contrary, the elasticities of GDP and GDP per capita present higher values. In the second part of the study, different regression models, based on co-integrated or stationary data, are presented. Different statistical tests are employed to check the validity of the proposed models. A comparison with national forecasts, based on complex econometric models, such as Markal-Time, has been performed and it has shown that the developed regressions are congruent with the official projections, with deviations of ±1% for the best case and ±11% for the worst. These deviations are to be considered acceptable in relation to the time span taken into account.

A formulation for predictive learning called multiple regression models and theoretical approach on construction of the regression model have been described. The key information of the article is the mathematical formulation for the forecast linear equation that estimates the multiple regression models. Calculation of the quantitative value of dependent variable forecast under influence of independent variables is done. The proposed study has presented the retail sales forecasting with multiple model estimation. It is one of the most important decisions a retailer can make with information obtained from the multiple regression. Checking model on the goodness of fit and statistical significance is explored. Finally, the quantitative value of retail sales forecast based on multiple regression models is calculated. Amral et al. (2007) have investigated the short term (up 24 hours) load forecasting of the demand for the South Sulewesi‘s (Sulewesi Island – Indonesia) Power System, using a Multiple Linear Regression (MLR) method. After a brief analytical discussion of the technique, the usage of polynomial terms and the steps to compose the MLR model is explained. Report on implementation of MLR algorithm using commercially available tool

such as Microsoft EXCELTM is also discussed. As a case study, historical data consisting of hourly load demand and temperatures of South Sulawesi electrical system is used, to forecast the short term load. The results are presented and analyzed; the potential for improvement using alternative methods is also discussed.

Stensrud, E., Foss et al. (2003) have investigated conclusion instability problem based on the evaluation criteria and the subset of the data used in the evaluation. With the use of non-parametric methods (the Mann-Whitney U test), it shows how to avoid conclusion instability. Menzies et al. (2006) have proposed a study that ranked 158 effort estimation methods via three different evaluation criteria and hundreds of different randomly selected subsets. The same four methods are ranked higher than the other 154 methods regardless of which evaluation criteria or data subset is applied. The recommended non-parametric evaluation is to evaluate and prune effort estimation methods. More specifically, while learning about the effort estimators from COCOMO-style data, it is found that the manual stratification defeats many complex algorithmic methods. To the best of their knowledge, this report is the first to offer stable conclusions regarding effort estimation across such a wide range of methods.

3. SYSTEM ANALYSIS

Formulation

The formulation of a problem is often more essential than its solution which may be merely a matter of mathematical or experimental skill. *Albert Einstein*

To formulate the problem correctly, you must

1. Understand the physical background. Statisticians often work in collaboration with others and need to understand something about the subject area. Regard this as an opportunity to learn something new rather than a chore.
2. Understand the objective. Again, often you will be working with a collaborator who may not be clear about what the objectives are. Beware of "fishing expeditions" - if you look hard enough, you'll almost always find something but that something may just be a coincidence.
3. Make sure you know what the client wants. Sometimes Statisticians perform an analysis far more complicated than the client really needed. You may find that simple descriptive statistics are all that are needed.
4. Put the problem into statistical terms. This is a challenging step and where irreparable errors are sometimes made. Once the problem is translated into the language of Statistics, the solution is often routine. Difficulties with this step explain why Artificial Intelligence techniques have yet to make much impact in application to Statistics. Defining the problem is hard to program.

That a statistical method can read in and process the data is not enough. The results may be totally meaningless.

Data Collection

It's important to understand how the data was collected.

- Are the data observational or experimental? Are the data a sample of convenience or were they obtained via a designed sample survey. How the data were collected has a crucial impact on what conclusions can be made.
- Is there non-response? The data you don't see may be just as important as the data you do see.
- Are there missing values? This is a common problem that is troublesome and time consuming to deal with.
- How are the data coded? In particular, how are the qualitative variables represented.
- What are the units of measurement? Sometimes data is collected or represented with far more digits than are necessary. Consider rounding if this will help with the interpretation or storage costs.
- Beware of data entry errors. This problem is all too common —almost a certainty in any real dataset of at least moderate size. Perform some data sanity checks.

Initial Data Analysis

This is a critical step that should always be performed. It looks simple but it is vital.

- Numerical summaries - means, sds, five-number summaries, correlations.

- Graphical summaries

– One variable - Box plots, histograms etc.
– Two variables - scatter plots.
– Many variables - interactive graphics.

Look for outliers, data-entry errors and skewed or unusual distributions. Are the data distributed as you expect? Getting data into a form suitable for analysis by cleaning out mistakes and aberrations is often time consuming. It often takes more time than the data analysis itself. In this course, all the data will be ready to analyze but you should realize that in practice this is rarely the case.

When to use Regression Analysis

Regression analysis is used for explaining or modeling the relationship between a single variable Y, called the response, output or dependent variable, and one or more predictor, input, independent or explanatory variables, X1……Xp
. When p = 1
1, it is called simple regression but when p > 1 it is called multiple regression or sometimes multivariate regression. When there is more than one Y, then it is called multivariate multiple regression which we won't be covering here.
The response must be a continuous variable but the explanatory variables can be continuous, discrete or categorical although we leave the handling of categorical explanatory variables to later in the course. Taking the example presented above, a regression of diastolic and bmi on diabetes would be a multiple regression involving only quantitative variables which we shall be tackling shortly. A regression of diastolic and bmi on test would involve one predictor which is quantitative which we will consider in later in the chapter on Analysis of Covariance. A regression of diastolic on just test would involve just qualitative predictors, a topic called Analysis of Variance or ANOVA although this would just be a simple two sample situation. A regression of test (the response) on diastolic and bmi (the predictors) would involve a qualitative response. A logistic regression could be used but this will not be covered in this book.

Regression analyses have several possible objectives including
1. Prediction of future observations.
2. Assessment of the effect of, or relationship between, explanatory variables on the response.
3. A general description of data structure.
Extensions exist to handle multivariate responses, binary responses (logistic regression analysis) and count responses (poisson regression).

4 DIAGNOSTICS AND TRANSFORMATIONS FOR SIMPLE LINEAR REGRESSION

Valid and Invalid Regression Models: Anscombe's Four Data Sets

In this chapter we will show how to use SAS to do simple linear regression diagnostics. We begin with the Anscombe datasets. We use a data step with the **infile** statement to do this.

```
data anscombe;
infile 'data/anscombe.txt' firstobs=2;
input case x1-x4 y1-y4;
run;
quit;
```

Now we draw figure 3.1. We use proc **gplot**, within a macro **%fourplots** that loops over the four datasets. Again, we use the & indexing notation.

```
%macro fourplots;
%do i = 1 %to 4;
goptions reset = all;
proc gplot data = anscombe;
axis1 label=(h=2 font=times angle=90 "y&i")
order=(2 to 14 by 2) value=(font=times h=1);
axis2 label=(h=2 font=times "x&i")
order=(0 to 20 by 5) value=(font=times h=1);
symbol1 value=circle interpol=r;
plot y&i*x&i / vaxis=axis1 haxis=axis2 vminor=0
hminor=0;
run;
quit;
%end;
%mend;
%fourplots;
```

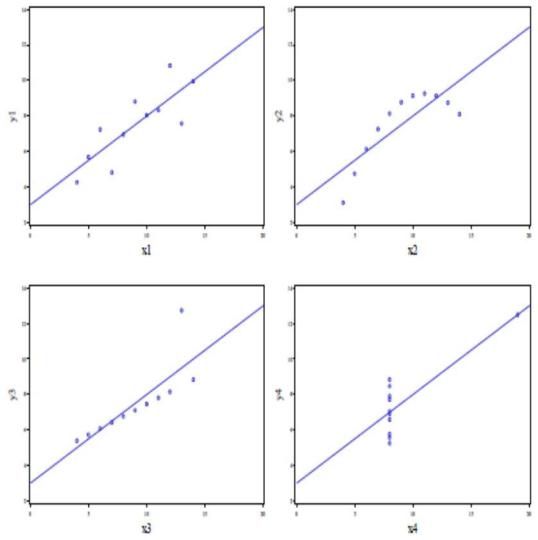

Fig. 4.1 Plots of Anscombe‟s four data sets

To produce the regressions for each dataset, we use another macro, **%reg**.

```
%macro reg;
proc reg data = anscombe;
%do i = 1 %to 4;
model y&i = x&i;
%end;
run;
quit;
%mend;
%reg;
```

To draw the residual plots, we again use a macro to loop over the four datasets. Within the macro **%regout**, we use the **out** statement within proc **reg** again. This time we specify the **r** argument so that SAS will store the residuals for regression i in variable ***resids*** of the dataset ***resdata&i***

```
%macro regout;
proc reg data = anscombe noprint;
%do i = 1 %to 4;
model y&i = x&i;
output out = resdata&i r=resids;
%end;
run;
quit;
%mend;
%regout;
```

Now we plot using the macro **%resplots**. This uses proc **gplot** to draw each individual plot.

```
%macro resplots;
%do i = 1 %to 4;
goptions reset = all;
proc gplot data = resdata&i;
title1 height=2 "Data Set &i" ;
axis1 label=(font=times h=2 angle=90 "Residuals")
order=(-2 to 2 by 1) value=(font=times h=1);
axis2 label=(font=times h=2 "x&i")
order=(0 to 20 by 5) value=(font=times h=1);
symbol1 value=circle;
plot resids*x&i / vaxis=axis1 haxis=axis2
vminor=0 hminor=0;
run;
quit;
%end;
%mend;
%resplots;
```

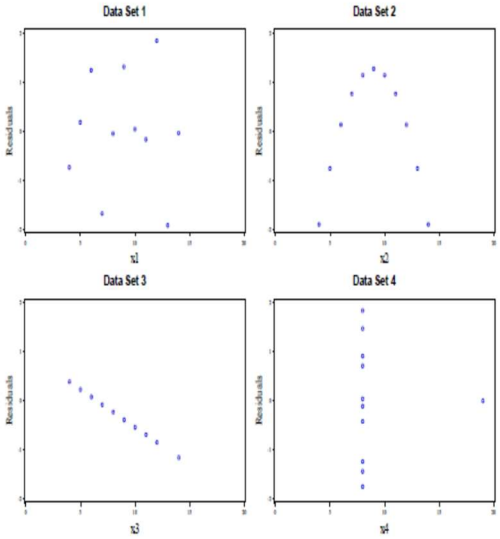

Fig.4.2 Residual plots for Anscombe‟s data sets

Features of R Programming

Now it’s time to discuss the features of R Programming:

- R is a comprehensive programming language that provides support for procedural programming involving functions as well as object-oriented programming with generic functions.
- There are more than 10,000 packages in the repository of R programming. With these packages, one can make use of functions to facilitate easier programming.
  - Being an interpreter based language, R produces a machine-independent code that is portable in nature. Furthermore, it facilitates easy debugging of errors in the code.
  - R facilitates complex operations with *vectors, arrays, data frames as well as other data objects* that have varying sizes.
  - R provides robust facilities for data handling and storage.

- As discussed in the above section, R has extensive community support that provides technical assistance, seminars and several boot camps to get you started with R.

4.1 Weighted Least Squares

4.1.1 Straight-Line Regression Based on Weighted Least Squares

We bring the cleaning data into SAS again. This time we use the special data that has been augmented with the standard deviations.

```
data inputit;
infile "data/cleaningwtd.txt" firstobs=2 expandtabs;
input case crews rooms stdev;
run;
quit;
```

Now we use a data step to add the inverse variance weight variable to the data.

```
data wtd;
set inputit;
wt = 1/(stdev**2);
run;
quit;
```

We perform the weighted regression using proc **reg**. The **weight** statement tells SAS to perform a weighted regression with the given variable as the weighting variable.

How R is better than Other Technologies

There are certain unique aspects of R programming which makes it better in comparison with other technologies:

- **Graphical Libraries –** R stays ahead of the curve through its aesthetic graphical libraries. Libraries like ggplot2, plotly facilitate appealing libraries for making well-defined plots.
- **Availability / Cost** – R is completely free to use which means widespread availability.
- **Advancement in Tool** – R supports various advanced tools and features that allow you to build robust statistical models.
- **Job Scenario** – As stated above, R is the primary tool for Data Science. With the immense growth in Data Science and rise in demand, R has become the most in-demand programming language of the world today.
- **Customer Service Support and Community** – With R, you can enjoy strong community support.

Applications of R Programming

- R is used in finance and banking sectors for *detecting fraud, reducing customer churn rate and for making future decisions.*
- R is also used by bioinformatics to analyse strands of genetic sequences, for performing drug discovery and also in computational neuroscience.
- R is used in social media analysis to discover potential customers in online advertising. Companies also use social media information to analyse customer sentiments for making their products better.
- E-Commerce companies make use of R to analyse the purchases made by the customers as well as their feedbacks.

CONCLUSION

R is free and open-source, making it possible for anyone to have access to world-class statistical analysis tools. It is used widely in academia and the private sector and is the most popular statistical analysis programming language today. we discussed about the R programming and its basic information. R has become a standard name in the world of programming. It is the most used tool in Data Science and many users are opting R due to its useful advantages and features. This is a basic regression class once you get a good idea on regression you can explore more in regression by going through the advance topics like adding the polynomial and interaction terms to your regression line , sometimes they work will charm. Adjusted R-squared is a good measure of training/in time sample error. We can't be sure about the final model performance based on this. We may have to perform cross-validation to get an idea on testing error.

REFERENCES

1. Leeb H and Potscher BM (2005) Model selection and inference: facts and fiction . *Econometric Theory* , 21 , 21 – 59 .

2. Li KC (1991) Sliced inverse regression (with discussion) . *Journal of the American Statistical Association* , 86 , 316 – 342 .

3. Li KC and Duan N (1989) Regression analysis under link violation . *Annals of Statistics* , 17 , 1009 – 1052

4. Loader C (1999) Local regression and likelihood. Springer, New York.

5. Mantel N (1970) Why stepdown procedures in variable selection ? *Technometrics* , 12 , 621 – 625 .

6. Maronna RA , Martin RD , and Yohai VJ (2006) Robust statistics: theory and methods . Wiley , New York .
7. Menard S (2000) Coefficients of determination for multiple logistic regression analysis . *American Statistician* , 54 , 17 – 24 .

8. Montgomery DC , Peck EA , and Vining GG (2001) Introduction to linear regression analysis (3rd edn.) . Wiley , New York .

9. David W. Letcher and Joao S. Neves (2010) Determinants of undergraduate business student satisfaction,

10. Estimated Simple Regression Equation, Retrieved from http://www.r-tutor.com/elementary-statistics/simple-linear-regression/estimated-simple-regression-equation.

11. Gupta S.P., and Gupta M.P. (2009). Business Statistics, Sultan Chand & Sons, New Delhi.

12. Juan-Carlos Ayala and Guadalupe Manzano (2014). The resilience of the entrepreneur. Influence on the success of the business. A longitudinal analysis. Journal of Economic Psychology Vol 42, pp 126:135.

13. Kothari C.R. and Garg Gaurav (2014). Research Methodology Methods and Techniques, New Age International Publishers, New Delhi.

14. . Multiple Linear Regression Analysis Retrieved from http://reliawiki.org/index.php/Multiple_Linear_Regression_Analysis

15. R.S. Rajput, Anjali Pant, and Santosh Kumar (2018). Development of Forecasting Model for Sugarcane Productivity using Multiple Linear Regression with Genetic Algorithm. Periodic Research, Vol 7.(2), PP 124:128.

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Global Epidemiology of Lung Cancer Prediction using Regression Analysis

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ABSTRACT

While lung cancer has been the leading cause of cancer-related deaths for many years in the Countries, incidence and mortality statistics – among other measures – vary widely worldwide. The aim of this study was to review the evidence on lung cancer epidemiology, including data of international scope with comparisons of tendency in economically, socially, and biologically different patient groups. In industrialized nations, evolving social and cultural smoking patterns have led to rising or death rates of lung cancer in women, lagging the long-declining smoking and cancer incidence rates in men. In contrast, emerging economies vary widely in smoking practices and cancer incidence but commonly also harbor risks from environmental exposures, particularly widespread air pollution. Recent research has also revealed clinical, radiologic, and pathologic correlates, leading to greater knowledge in molecular profiling and targeted therapeutics, as well as an emphasis on the rising incidence of adenocarcinoma histology. Furthermore, emergent evidence about the benefits of lung cancer screening has led to efforts to identify high-risk smokers and development of prediction tools. Varying trends in smoking largely dictate international patterns in lung cancer incidence and mortality. With declining smoking rates in developed countries and knowledge gains made through molecular profiling of tumors, the emergence of new risk factors and disease features will lead to changes in the landscape of lung cancer epidemiology.

Keywords: Lung Cancer, Regression, Prediction

INTRODUCTION

Lung cancer, also known as lung carcinoma, is a malignant lung tumor characterized by uncontrolled cell growth in tissues of the lung. This growth can spread beyond the lung by the process of metastasis into nearby tissue or other parts of the body.

The vast majority (85%) of cases of lung cancer are due to long-term tobacco smoking. About 10–15% of cases occur in people who have never smoked. These cases are often caused by a combination of genetic factors and exposure to radon gas, asbestos, second-hand smoke, or other forms of air pollution. Lung cancer may be seen on chest radiographs and computed tomography (CT) scans. The diagnosis is confirmed by biopsy which is usually performed by bronchoscopy or CT-guidance.

In Asia, China has high incidence and mortality rates from lung cancer, comparable to those of the US and Europe. Men have had a higher incidence of lung cancer than women since the 1970s and continue to comprise the majority of new lung cancer cases in China today, largely due to gender differences in smoking prevalence. Conversely, mortality rates in women are lower in China than in other industrialized nations, perhaps due to the higher incidence of adenocarcinomas with mutations responsive to targeted therapies.

Brazil, Russia, India, China, and US are recognized by their large and fast-growing economies. One of the few South American countries with a cancer registry is Brazil, where tobacco smoking peaked in the 1970s and lung cancer mortality in men peaked in 1993 and continues to rise among women. Likewise, in the Russian Federation, all-cause mortality in men is largely attributed to very high rates (60%) of both smoking and alcohol consumption, which are much lower in women. Accordingly, Russia has among the highest lung cancer mortality rate in men of all European countries but among the lowest in women. Mortality is now declining, after peaking in the early 1990s, but tobacco use remains a major barrier to effective cancer control. Additional risk factors in Russia include environmental pollution and workplace exposures in nuclear facilities and asbestos mines.

Comparatively, lung cancer incidence and mortality rates in India is somewhat better among the lowest in the world. The most common cancers in men are head and neck, gastric, and esophageal cancers, attributed to high usage of smokeless tobacco. One study in northern India noted that squamous cell lung cancer was the most common histology overall and among smokers. While cigarette smoking has a reported prevalence ranging from 28 to 57% among men, bidi smoking (hand-rolled tobacco) is the most commonly used (92%) tobacco product.

Burden of Disease

Internationally, lung cancer continues to be the leading cause of cancer-related deaths in men and women. A breakdown by level of economic development shows no differences in cancer deaths in men but a higher rate of lung cancer deaths in women in industrialized countries as compared with developing nations. Among females in developing countries, lung cancer deaths lag behind those due to breast cancer. Lung cancer incidence and mortality are tightly linked to cigarette smoking patterns. As smoking rates peak – generally first in men, followed by women – lung cancer incidence and

mortality rise in subsequent decades before declining following the initiation of comprehensive tobacco control programs. These trends have occurred earlier in industrialized countries as compared with the developing world. In the China, United States (US) and the United Kingdom (UK), lung cancer incidence and mortality rates have in fact been falling since the 1990s. In contrast, emerging nations – including China, Brazil, Russia,India, and South Africa (BRICS) – continue to have high rates of cigarette smoking in both men and women. They exhibit a lower incidence of cancer but a higher mortality burden compared with developed countries. Reasons for these patterns include unequal access to healthcare leading to delayed diagnosis and treatment, environmental contamination, and socio-cultural barriers.

Cigarette Smoking, Secondhand Smoking, and E-cigarettes

Cigarette smoking is the most recognized risk factor for developing lung cancer. Since the Surgeon General's report on smoking and health in 1964, at which time 52% of American men and 35% of American women were active smokers, the prevalence of cigarette smoking in US and consequently lung cancer has markedly decreased. While nicotine itself is not carcinogenic, there be as many as 55 substances in cigarette smoke that have been deemed carcinogenic by the International Agency for Research on Cancer including polycyclic aromatic hydrocarbons and 4-(methylnitrosamino)-1-(3-pyridyl)-1-butanone (NNK). Their activation leads to the formation of DNA adducts and subsequent gene methylation, DNA sequence changes, DNA segment amplification or deletion, or whole chromosome gains or losses. Relative risk of lung cancer in smokers as compared with smokers varies from 10- to 30-fold, and the degree of risk is dependent on number of cigarettes smoked daily and pack-years of smoking. Cigar and pipe tobacco smoking are also associated with increased odds of developing lung cancer.

Second hand smoke exposure also leads to a dose-dependent risk of lung cancer. Öberg and colleagues studied the effects of environmental tobacco exposure (ETS) in 192 countries on six continents and found that 40% of children and 33%–35% of non-smokers are exposed to secondhand smoke. The highest rates were in Europe, the western Pacific, and parts of Southeast Asia; the lowest rates were found in Africa. Over 600,000 deaths worldwide, most of them in women, were attributable to secondhand smoking in 2004. Similar to Öberg's finding of the relative risk for lung cancer in adult non-smokers exposed to secondhand smoke of 1.21 (95% confidence interval [CI]: 1.13–1.3), many regulatory bodies have reported an increase in lung cancer risk by 20 to 30% upon exposure to ETS. The largest numbers of estimated deaths in adults attributable to secondhand smoke, however, are not due to lung cancer but rather ischemic heart disease and asthma.

Electronic cigarettes have sparked much recent controversy over potential risks from long-term use, as well as their role in smoking initiation and potentially cessation. The 2012 National Youth Tobacco Survey found the prevalence of ever-use of e-cigarettes among middle and high school students in the US to be 6.8%, while prevalence among adults in the simultaneous National Adult Tobacco Survey was 1.9%. Although conventional cigarette use is far higher (18% among adults), the incidence of e-cigarette use is rising rapidly and has been associated with higher odds of cigarette smoking and lower odds of abstinence. Even more concerning, early research has shown that an e-cigarette vapor-conditioned media induced gene expression patterns in human bronchial epithelial cells concordant with that of cells exposed to a cigarette smoke-conditioned media. Lung cancer is the most commonly occurring cancer in men and the third most commonly occurring cancer in women. There were 2 million new cases in 2018. The top 25 countries with the highest incidence of lung cancer in 2018 are given in the tables below.

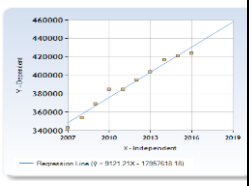
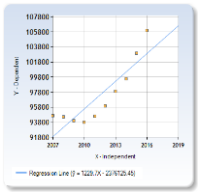
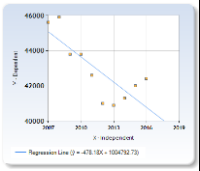
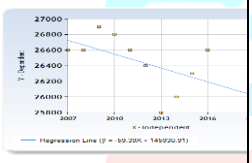
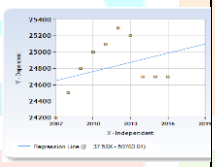
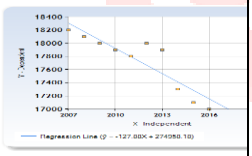
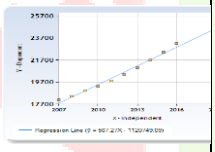
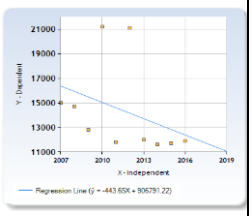
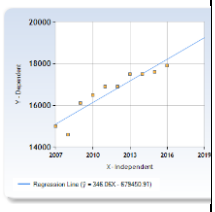
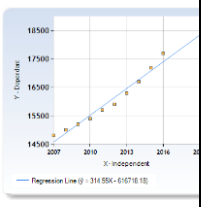
The Continuous Update Project Panel judged there was strong evidence that drinking water containing arsenic and taking high-dose beta-carotene supplements increase the risk of lung cancer. There is some evidence that consuming red meat, processed meat and alcoholic drinks might increase the risk of lung cancer. There is some evidence that the risk of lung cancer might be decreased by being physically active, consuming fruit and vegetables (current smokers), consuming foods containing retinol, beta-carotene or carotenoids, consuming foods containing vitamin C (current smokers), and consuming foods containing isoflavones (people who have never smoked). Read about all the evidence in our Third Expert Report. This is a summary measure of the rate of disease that a population would have if it had a standard age structure. Standardization is necessary when comparing populations that differ with respect to age because age has a powerful influence on the risk of dying from cancer. Tobacco smoking causes about two-thirds of all lung cancer deaths worldwide.

Problem Definition

In the following link r-directory > Reference Links > Free Data Sets, the lung cancer data of 188 countries of last 21years available. But I have choosen last 10 years data from 2007 to 2016 of top 15 countries i.e China, United States, Russia, Japan, India, Germany, Italy, France, United Kingdom, Poland, Vietnam and Spain for analysis purpose. Of 15 countries China Stands first in male cancer, United States stands in second place in male lung cancer, India takes 5th position and Brazil is in last position. The data showing table- 1 below. According to the data we observe that China is increases its Patients from 2007 to 2016 except 2012, India, Brazil and Vietnam also increased their patients from 2007 to 2016. The remaining countries are fluctuated from 2007 to 2016. Based on this information I developed the Linear Regression model and draw the figures. One of the major costs of cancer is cancer treatment. But lack of health insurance and other barriers to health care prevent many Countries from getting optimal health care. Lung cancer is a major source of morbidity, mortality, and health care costs in the developed and developing world. It is estimated that lung cancer is responsible for 20% of all cancer care costs

Table No - 1

Country	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
China	343000	354000	369000	385000	389000	395000	404000	417000	421000	424000
United States	94700	94500	94000	93800	94600	96000	97900	99600	103000	106000
Russia	45600	45900	43800	43800	42600	41000	40900	41300	42000	42400
Japan	48300	49300	49900	50800	51400	51600	51500	50700	50800	51000
India	35400	36600	38200	39900	41600	43400	45200	46700	48500	50400
Germany	30600	30700	30900	31000	31000	31200	31300	31200	31400	31700
Italy	26600	26600	26900	26800	26600	26400	25800	26000	26300	26600
France	24200	24500	24800	25000	25100	25300	25200	24700	24700	24700
United Kingdom	20600	20500	20300	20300	20400	20700	21000	21400	21700	22200
Poland	18200	18100	18000	17900	17800	18000	17900	17300	17100	17000
Vietnam	18100	18400	18900	19300	19800	20400	21000	21700	22400	23200
Spain	17700	17800	17800	17700	17700	17700	17300	17000	16900	16800
Ukraine	15000	14700	12800	12100	11800	12100	12000	11600	11700	11900
Turkey	15000	14600	16100	16500	16900	16900	17500	17500	17600	17900
Brazil	14800	15000	15200	15400	15700	15900	16300	16700	17200	17700

CHINA	**US**	**RUSSIA**
		
JAPAN	**INDIA**	**GERMANY**
		
ITALY	**FRANCE**	**UK**
		
POLAND	**VIETNAM**	**SPAIN**
		
UKRAIN	**TURKEY**	**BRAZIL**
		

Prediction

Next Prediction the data up to three more years' i.e 2020, 2025 and 2030, how to fluctuated the trend directions on countries in this year's whether its increasing order or decreasing order. In this case Linear Regression Model is the best prediction model to represent the data. Simple linear regression is a statistical method for obtaining a formula to predict values of one variable from another where there is a causal relationship between the two variables. The variables y and x are those whose relationship we are studying.

Consider the Model function

$$y = \alpha + \beta x$$

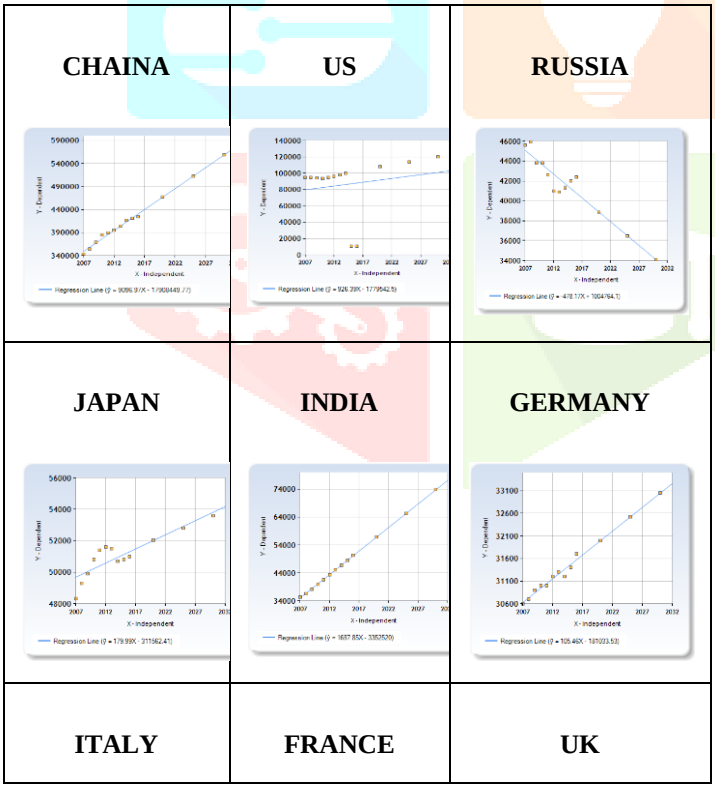
which describes a line with slope β and y-intercept α. In general such a relationship may not hold exactly for the largely unobserved population of values of the independent and dependent variables; we call the unobserved deviations from the above equation the errors. Suppose we observe n data pairs and call them $\{(x_i, y_i), i = 1, ..., n\}$. We can describe the underlying relationship between y_i and x_i involving this error term ε_i by

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

This relationship between the true (but unobserved) underlying parameters α and β and the data points is called a linear regression model.

Table - 2

Country	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2020	2025	2030
China	343000	354000	369000	385000	389000	395000	404000	417000	421000	424000	467424.242	512909.0909	558393.9
United States	94700	94500	94000	93800	94600	96000	97900	99600	103000	106000	107862.424	114010.9091	120159.394
Russia	45600	45900	43800	43800	42600	41000	40900	41300	42000	42400	38865.4545	36474.5455	34083.6364
Japan	48300	49300	49900	50800	51400	51600	51500	50700	50800	51000	52031.1111	52814.4444	53597.7778
India	35400	36600	38200	39900	41600	43400	45200	46700	48500	50400	56936.9697	65376.3636	73815.7576
Germany	30600	30700	30900	31000	31000	31200	31300	31200	31400	31700	31996.3636	32523.6364	33050.9091
Italy	26600	26600	26900	26800	26600	26400	25800	26000	26300	26600	25955.1515	25658.1818	25361.2121
France	24200	24500	24800	25000	25100	25300	25200	24700	24700	24700	25139.3939	25327.2727	25515.1515
United Kingdom	20600	20500	20300	20300	20400	20700	21000	21400	21700	22200	22491.5152	23421.8182	24352.1212
Poland	18200	18100	18000	17900	17800	18000	17900	17300	17100	17000	16643.0303	16003.6364	15364.2424
Vietnam	18100	18400	18900	19300	19800	20400	21000	21700	22400	23200	25141.8182	27978.1818	30814.5455
Spain	17700	17800	17800	17700	17700	17700	17300	17000	16900	16800	16430.303	15836.3636	15242.4242
Ukraine	15000	14700	12800	12100	11800	12100	12000	11600	11700	11900	9741.8182	8078.1818	6414.5455
Turkey	15000	14600	16100	16500	16900	16900	17500	17500	17600	17900	19591.5152	21321.8182	23052.1212
Brazil	14800	15000	15200	15400	15700	15900	16300	16700	17200	17700	18663.6364	20236.36	21809.0909





Results

The model we have built can be used for inference of how the different predictors influence the outcome. It is far from perfect. Their is still presence of non-linearity and non-constant variance of errors. Moreover, the outliers and leverage points should be analyzed to find a better model. In order to use it for prediction, more concrete measures should be taken for ensuring the accuracy of the model, like cross-validation. It still helps by providing good estimations of the significant relations between the

predictors and the outcome. These estimations can be used to summarize the data in a more useful and presentful way. By the Analysis we observed that the Countries like US, Italy, Poland, Spain and Ukraine are in decreasing order while the other Countries are in increasing order. In the Countries like China, Russia, Japan, India, Germany, France, UK, Vietnam, Turkey and Brazil are going to have impact on their economical, social and biological aspects compared to other Countries.

CONCLUSION

Lung cancer is the leading cause of death over the world and the only chance of cure for patients affected from this kind of cancer is surgical resection. This review has examined international trends in lung cancer epidemiology. Emerging economies and developing countries face many challenges in initiating tobacco cessation campaigns while also addressing environmental risk factors and cultural barriers. Over the past three to four decades, in contrast, industrialized nations have seen large declines in cigarette smoking and, consequently, lung cancer. However, despite knowledge gains in tumor biology that have led to targeted therapies, mortality from lung cancer remains high for most patients around the world. Future directions must include improvements in early detection and technological advances in genomics and genetics to achieve a more personalized approach to therapy and ultimately improve lung cancer survival.

References

1. Fitzmaurice C, Dicker D, Pain A, et al. The Global Burden of Cancer 2013. JAMA Oncology. 2015; 1(4): 505–527. DOI: 10.1001/jamaoncol.2015.
2. Torre LA, Bray F, Siegel RL, Ferlay J, Lortet-Tieulent J and Jemal A. 2012. Global cancer statistics, CA Cancer J Clin; 2015. DOI: 10.3322/caac.21262.
3. Thun MJ, Carter BD, Feskanich D, et al. 50-year trends in smoking-related mortality in the United States. The New England Journal of Medicine. 2013; 368(4): 351–364. DOI: 10.1056/NEJMsa1211127.
4. Youlden D, Cramb S and Baade P. . 2008; The international epidemiology of lung cancer: Geographic distribution and secular trends. Journal of Thoracic Oncology 3(8): 819–831. DOI: 10.1097/JTO.0b013e31818020eb.
5. Jemal A, Center MM, DeSantis C and Ward EM. 2010, Global patterns of cancer incidence and mortality rates and trends. Cancer Epidemiology Biomarkers & Prevention.; 19(8): 1893–1907. DOI: 10.1158/1055-9965.EPI-10-0437.
6. Goss PE, Strasser-Weippl K, Lee-Bychkovsky BL, et al. 2014; Challenges to effective cancer control in China, India, and Russia. The Lancet Oncology. 15(5): 489–538. DOI: 10.1016/S1470-2045(14)70029-4.
7. Siegel R, Ma J, Zou Z and Jemal A. Cancer statistics, 2014. CA: A Cancer Journal for Clinicians. 2014; 64(1): 9–29. DOI: 10.3322/caac.21208.
8. Freedman ND, Abnet CC, Caporaso NE, et al. 2015; Impact of changing US cigarette smoking patterns on incident cancer: Risks of 20 smoking-related cancers among the women and men of the NIH-AARP cohort. Int J Epidemiol;.
9. Meza R, Meernik C, Jeon J and Cote ML. 2015; Lung cancer incidence trends by gender, race and histology in the United States, 1973–2010. PLoS One. 10(3):

e0121323 DOI: 10.1371/journal.pone.0121323.

10. Rinh QD, Nguyen PL, Leow JJ, et al. 2015; Cancer-specific mortality of Asian Americans diagnosed with cancer: A nationwide population-based assessment. J Natl Cancer Inst. 107(6). DOI: 10.1093/jnci/djv054.
11. Bach PB, Cramer LD, Warren JL and Begg CB. 1999; Racial differences in the treatment of early-stage lung cancer. New England Journal of Medicine. 341(16): 1198–1205. DOI: 10.1056/NEJM199910143411606.
12. Wisnivesky JP, McGinn T, Henschke C, Hebert P, Iannuzzi MC and Halm EA. 2005; Ethnic disparities in the treatment of stage I non–small cell lung cancer. American Journal of Respiratory and Critical Care Medicine. 171(10): 1158–1163. DOI: 10.1164/rccm.200411-1475OC.
13. Ward E, Jemal A, Cokkinides V, et al. 2004; Cancer disparities by race/ethnicity and socioeconomic status. CA Cancer J Clin. 54(2): 78–93. DOI: 10.3322/canjclin.54.2.78.
14. UK CR. Lung cancer incidence projections to 2024: Future rates and numbers of new cases in Great Britain and the UK. Cancer Stats Cancer Projections Series; 2009.
15. Lortet-Tieulent J, Renteria E, Sharp L, et al. Convergence of decreasing male and increasing female incidence rates in major tobacco-related cancers in Europe in 1988–2010. European Journal of Cancer.
16. National Institutes of Health grant 1 RO1 CA163772.

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3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
2020-21									
S. No.	**Title of paper**	**Name of the author/s**	**Department of the teacher**	**Name of journal**	**Year of publication**	**ISSN number**	**Link of Journal**	**Link of Publication**	**Link to the recognition in UGC enlistment of the Journal**
1	*Method development and validation for simultaneous estimation of brinzolamide and brimonidine in pharmaceutical dosage form by using RP-HPLC*	Dr. M. Madhavi	Department of Botany	**International Journal of Chemical Research and Development**	**2-02-202**	**2664-6560**	https://www.chemicaljournal.in/	https://www.chemicaljournal.in/archives/2021.v3.i1.A.19/method-development-and-validation-for-simultaneous-estimation-of-brinzolamide-and-brimonidine-in-pharmaceutical-dosage-form-by-using-rp-hplc	NA
2	*Method development and validation for estimation of ticagrelor in pharmaceutical dosage form by using RP-HPLC*	Dr. M. Madhavi	Department of Botany	International Journal of Advanced Research and Development	**1-02-202**	2455-4030	https://www.advancedjournal.com/	https://www.advancedjournal.com/archives/2021/vol6/issue1/6-1-18	NA
3	*Optimization of Cultural Conditions to Enhance Bioactive Metabolite Production by Streptomyces Olivoverticillatus VLK – 15*	Dr. N. Krishna	Department of Botany	International Journal of Pharmaceutical Sciences and Research (IJPSR)	2021	2320-5148	https://ijpsr.com/	https://ijpsr.com/bft-article/optimization-of-cultural-conditions-to-enhance-bioactive-metabolite-production-by-streptomyces-olivoverticillatus-vlk-15/	NA

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3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
4	*Morphological and Anatomical Diversity of Bulbophyllum in India*	Dr. G. Ramesh Sri K.V.S. Durga Prasad	Department of Botany	Current Horticulture	2021	2347-7377	http://currenthorticulture.com/	https://www.indianjournals.com/ijor.aspx?target=ijor:chr&volume=9&issue=1&article=010	NA
5	*Stability and Validity Indicating UV Spectrophotometric Rapid Assay method for the Estimation of Promethazine Theoclate*	Dr. T. Umamaheswara Rao	Department of Chemistry	Science, Technology and Development	2021	0950-0707	https://journalstd.com/	https://drive.google.com/file/d/1xJtOCeFSSkrVKx9IoZ2P3wct7XXwZzrw/view	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
6	*Stability and Validity Indicating Assay Method for Estimation of Multidrug components of Etophylline and Theophylline Anhydrous –Forced Degradation Studies*	Dr. T. Umamaheswara Rao	Department of Chemistry	NOVYI MIR Research Journal	2021	0130-7673	https://novyimir.net/	https://novyimir.net/volume-5-issue-4-2020/	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
7	*Lung Cancer Detection Based on CT Images using Feature Extraction and Classification Techniques*	Sri TVN Hanumantha Rao	Department of Computer Science (PG)	Nat. Volatiles & Essent. Oils	2021		https://www.nveo.org/index.php/journal	https://www.nveo.org/index.php/journal/article/view/2535	NA
8	*Outcomes Bases Education : Planning, Teaching and Assessment Possibilities*	Smt. Ch. Aruna	Department of English	Dogo Rangsng Research Journal	2020	2347-7180	https://journal-dogorangsang.in/	https://journal-dogorangsang.in/no_1_Dec_20/14.pdf	https://ugccare.unipune.ac.in/Apps1/Content/Files/pdf/DogoRangsangResearchJournalOriginal.pdf

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DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
9	*Douglas Stewart's Ned Kelly A Unique Australian Phenomenon*	Smt. V. Beulah Rani	Department of English	INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY EDUCATIONAL RESEARCH	2020	2277-7881	http://www.ijmer.in/	http://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume9/volume9-issue11(7)-2020.pdf	NA
10	*Under Water Pollution Monitoring System to Prevent Fatalities of Aqua Ecosystem*	Dr. Y. Uday Kumar	Department of Mathematics	The Gazettee of India - Extraordinary [Part ii - Section 3(ii)]	2020	700301569	https://www.meity.gov.in/content/gazettes	https://www.quickcompany.in/patents/under-water-pollution-monitoring-system-to-prevent-fatalities-of-aqua-ecosystem	NA
11	*A New Family of Semicircular and Circular Arc Tan – Exponential Type Distributions*	Dr. S.V.S. Girija	Department of Mathematics	Thai Journal of Mathematics	2020	1686-0209	http://thaijmath.in.cmu.ac.th/index.php/thaijmath	http://thaijmath.in.cmu.ac.th/index.php/thaijmath/article/viewFile/1427/354354704	NA
12	*श्रीभाष्यं विजयसारथेः मन्दाकिनी काव्यस्य महैत्कृष्टत्वम्*	Dr. D.N. Deekshit	Department of Sanskrit	International Journal of Sanskrit Research	2020	2394-7519	https://www.anantaajournal.com/	https://www.anantaajournal.com/archives/?year=2020&vol=6&issue=5&part=F&ArticleId=1186	NA
13	*श्रीविष्णु महापुराणे कलिधर्म निरूपणम्*	Dr. D.N. Deekshit	Department of Sanskrit	International Journal of Sanskrit Research	2020	2394-7519	https://www.anantaajournal.com/	https://www.anantaajournal.com/archives/2020/vol6issue1/PartC/6-1-22-574.pdf	NA
14	*धन्वनतरिणा उक्तं पशुभिः प्रयोजनम्*	Dr. D.N. Deekshit	Department of Sanskrit	International Journal of Sanskrit Research	2020	2394-7519	https://www.anantaajournal.com/	https://www.anantaajournal.com/archives/2020/vol6issue1/PartC/6-1-18-696.pdf	NA

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S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
15	*Leadership of Women through Panchayat Governance (A Case Study of Elected Women Representatives of Prakasam District of Andhra Pradesh)*	Dr. K.Srilakshmamma	Department of Political Science	International Journal of Advanced Research (IJAR)	2020	2320-5407	https://www.journalijar.com/	https://www.journalijar.com/article/34314/leadership-of-women-through-panchayat-governance-(a-case-study-of-elected-women-representatives-of-prakasam-district-of-andhra-pradesh/	NA
16	*Challenges of Student – Centred Learning – A Note*	Dr. K.Srilakshmamma	Department of Political Science	Research Journal of English Language and Literature (RJELAL)	2020	2395-2636	http://www.rjelal.com/	http://www.rjelal.com/8.S1.2020/113-117.pdf	NA
17	*Women Political Participation in Local Bodies of Andhra Pradesh*	Dr. K.Srilakshmamma	Department of Political Science	Indian Journal of Research	2020	2250-1991	https://www.worldwidejournals.com/paripex/	https://www.worldwidejournals.com/paripex/recent_issues_pdf/2020/November/women-political-participation-in-local-bodies-of-andhra-pradesh_November_2020_3625904413_4107298.pdf	NA

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Method development and validation for simultaneous estimation of brinzolamide and brimonidine in pharmaceutical dosage form by using RP-HPLC

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Abstract
A high performance liquid chromatographic method was developed and validated for the simultaneous determination of brinzolamide and brimonidine by employing an isocratic RP-HPLC Kromasil C18 (4.6 x 250mm, 5µm) column resulted in an adequate separation for brinzolamide and brimonidine with retention time of 2.121 min and 2.801min respectively. Best resolution was achieved with the Perchloric acid (0.1%) and acetonitrile with ratio of (58:42) as mobile phase pumped at the flow rate of 1.0 ml/min with the detection wavelength of 251 nm. Regression coefficient for both brinzolamide and brimonidine was found to be 0.998 and 0.998 respectively indicating linearity within the concentration range. The validation parameters like linearity, precision, accuracy, limit of detection and limit of quantitation were also found to be suitable. The proposed method can hence be successfully applied to quantify brinzolamide and brimonidine during quality control of formulation.

Keywords: chromatographic, simultaneous determination brinzolamide, brimonidine

Introduction

Brinzolamide is a highly specific, non-competitive, reversible carbonic anhydrase inhibitor [1]. Carbonic anhydrase (CA) is an enzyme found in many tissues of the body including the eye. It catalyzes the reversible reaction involving the hydration of carbon dioxide and the dehydration of carbonic acid. In humans, carbonic anhydrase exists as a number of isoenzymes, the most active being carbonic anhydrase [2] II (CA-II). Inhibition of carbonic anhydrase in the ciliary processes of the eye decreases aqueous humor secretion, presumably by slowing the formation of bicarbonate ions with subsequent reduction in sodium and fluid transport. The result is a reduction in intraocular pressure. Brinzolamide is indicated in the treatment of elevated intraocular pressure[3] in patients with ocular hypertension or open-angle glaucoma. Structurally has (6R,7R)-7-[(2Z)-2-(2-amino-1,3-thiazol-4-yl)-2-[(Carboxymethoxy) imino]acetamido]-3-ethenyl-8-oxo-5-thia-1-azabicyclo[4.2.0]oct-2-ene-2-carboxylic acid and molecular weight is 453.45. It is soluble in water and DMSO (figure-1).

Brimonidine is an alpha-adrenergic [4] agonist and 2-imidazoline derivative that was first introduced in 1996. It is considered to be a third generation alpha-2 adrenergic receptor agonist, since it displays preferential binding at alpha-2 Adrenoreceptors over alpha-1 receptors. Chemically 5-Bromo-N-(4, 5-dihydro-1H-imidazol-2-yl)-6-quinoxalinamine tartrate and molecular weight 442.22. It displays a higher selectivity toward the alpha-2 adrenergic receptors than clonidine or apraclonidine, which are also alpha-2 adrenergic agonists. Early treatment and management of glaucoma, which predominantly involves the lowering of intraocular pressure, is critical since glaucoma is considered to be a common cause of blindness worldwide. Ophthalmically, brimonidine is used to lower intraocular pressure by reducing aqueous humor production and increasing uveoscleral outflow. Because it is oxidately stable, brimonidine is associated with fewer reports of ocular allergic reactions compared to other alpha-2 adrenergic agonists. Brimonidine mediates vasoconstrictive effects and it was shown to exhibit anti-inflammatory [5] properties in *ex vivo* human skin model and *in vivo* inflammation models. Brimonidine is reported to be metabolized in the cornea. Brimonidine that reaches the systemic circulation upon topical administration undergoes extensive hepatic metabolism mediated by hepatic aldehyde oxidases [6] and the systemic half-life was approximately 3 hours (figure-2).

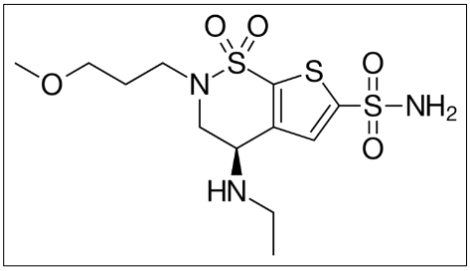

Fig 1: Chemical structure of Brinzolamide

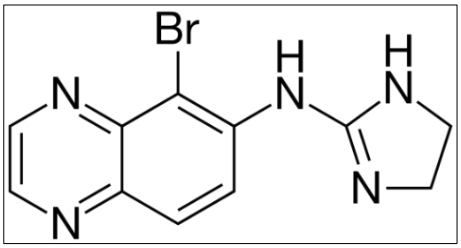

Fig 2: Chemical structure of Brimonidine

Literature survey revealed a few HPLC methods for the estimation of brinzolamide and brimonidine [7, 9]. Also some HPLC methods are available for the determination of brinzolamide and Timolol [10, 12]. Similarly some HPLC methods are reported for the estimation of brimonidine tartrate and timolol maleate [13, 14].Based on results only few works were done on brinzolamide and brimonidine and we plan to develop new simultaneous estimation of both drugs bulk and dosage forms by HPLC method.

Material and Methods

Quantitative simultaneous estimation of brinzolamide and brimonidine was done by using an isocratic Shimadzu HPLC instrument on Kromasil C18 (4.6 x 250mm, 5µm) column. The Instrument is equipped with binary pump and variable wavelength PDA detector. A 20µL Hamilton syringe was used for injecting the samples. Shimadzu UV-Visible spectrophotometer was used for spectral studies. Degassing of the mobile phase was done by using a Loba ultrasonic bath sonicator. A Shimadzu balance was used for weighing the materials. Brinzolamide and brimonidine pure drugs were provided by Rankem pharmaceuticals, Hyderabad. Optimised conditions: the mobile phase with 0.1% perchloric acid and Acetonitrile (58:42 %v/v) was employed in isocratic mode at a flow rate of 1.0 mL/min. The run time was 6 min and 20µL of the sample was injected for every run into the column at wavelength of 251nm.

Chromatographic Conditions

A mixture of 0.1% perchloric acid and Acetonitrile in the ratio of 58:42 %v/v was employed to be the most suitable mobile phase for identical chromatographic separation of Mucomelt Forte (Combination of Brinzolamide and Brimonidine). The solvent mixture was filtered through 0.45 µ membrane filter and sonicated to dissolve it completely. It was pumped through the column at a flow rate of 1.0 mL/min. Injection volume was 20µL and the column was maintained at ambient temperature and run time was set to be 6 min and detected of the drug was monitored at wavelength of 251nm and retention time was to be 2.121 and 2.801 min.

Preparation of standard stock solution

Accurately weighed 16mg of Brinzolamide, 3.2mg of Brimonidine and transferred to 25ml volumetric flask and3/4 thof diluents was added to these flask and sonicated for 10 minutes. Flask were made up with diluents and labelled as Standard stock solution. (640µg/ml of Brinzolamide and 128µg/ml of Brimonidine). 1ml from each stock solution was pipetted out and taken into a 10ml volumetric flask and made up with diluent. (64µg/mlBrinzolamide of and 12.8µg/ml of Brimonidine)

Preparation of sample solution

5 tablets were weighed and the average weight of each tablet was calculated, then the weight equivalent to 1 tablet was transferred into a 10)ml volumetric flask, 5ml of diluents was added and sonicated for 25 min, further the volume was made up with diluent and filtered by HPLC filters (640µg/ml of Brinzolamide and 128µg/ml of Brimonidine). 1ml from each stock solution was pipetted out and taken into a 10ml volumetric flask and made up with diluent. (64µg/mlBrinzolamide of and 12.8µg/ml of Brimonidine)

Linearity

The calibration graphs shows that the linear response was obtained over the range of concentrations used in the assay procedure. These data demonstrate that the methods have adequate sensitivity to the concentrations of the analyte. Several aliquots of standard solution of Ticagrelor was taken in different 10 mL volumetric flasks and diluted up to the mark with diluents such that the final concentration of Mucomelt Fortewas in the range 16-96 µg/mL for brinzolamide and 3.2-22.4 µg/mL for brimonidine. Evaluation of the drug was performed with UV detector at 251 nm; peak area was recorded for all the peaks. The correlation coefficient value of both brinzolamide and brimonidine was 0.9989. The results show that an excellent correlation exists between peak area and concentration of drug within the concentration range indicated. The data is tabulated in table 1 and Figure 3 & 4.

System suitability

System suitability parameters like retention time, theoretical plates and tailing factor were calculated and compared with standard values.

Accuracy

The recovery studies for the method were carried out by standard addition method. It was evaluated at three concentration levels (32, 64 and 96 µg/mL) and the percentage recoveries were calculated. The data is tabulated in table 2& 3 and figure 5, 6 &7.

Precision

The precision of the method was determined by intra and inter precision studies. This was evaluated by injecting three independent sample preparations of ticagrelor from a single formulation at three different concentration levels on the same day (Intraday) and on three different days (Inter day). The % RSD was then calculated. The data is represented in table 4 & 5 and figure 8 & 9.

Limit of detection and limit of quantification

LOD and LOQ were determined based on the standard deviation of the response and the slope of the calibration curve. The sensitivity of the method was established by the LOD and the LOQ values. Data is represented in table 6.

Robustness

Robustness was established by introducing small changes in the HPLC optimised conditions which include mobile phase, flow rate and temperature. Data is represented in table 7.

Results and Discussion

The proposed method was found to be simple. Six linear concentrations of Brinzolamide (16-96µg/ml) and Brimonidine (3.2-22.4µg/ml) were injected in a duplicate manner. Average areas were mentioned above and linearity equations obtained for brinzolamide was y =53463x + 61322 and of brimonidine was y = 62525x + 21764 and correlation coefficient obtained was 0.9989 for the two drugs. System suitability parameters indicates high column efficiency with large number of theoretical plates (>2000).The tailing factors were found to be 1.04 and 1.34 which does not exceed the critical value 2.The average retention time of brinzolamide and brimonidine was found to be 2.121 and 2.801

min. No interference was seen from any of the components of the pharmaceutical dosage form indicating the specificity of the method. The % RSD was found to be 0.3 and 0.3 for intraday and 0.2 and 0.7 for inter day precision studies. Thus the method was found to be accurate and precise as the %RSD was not more than 2%. The limit of detection of brinzolamide and brimonidine were 0.05 & 0.04, limit of quantification for brinzolamide and brimonidine were found to be 0.15 and 0.11µg/mL respectively. The RSD for the % assay of sample was calculated for each parameter in robustness and was found to be less than 2% confirming the robustness of the method.

Conclusion

A validated RP-HPLC method was developed for the determination of brinzolamide and brimonidine in tablet dosage form and bulk forms. As the proposed method is simple, rapid, accurate, precise and specific it can be employed for the routine analysis of brinzolamide and brimonidine in pharmaceutical dosage forms.

Tables and Figures

Table1: Linearity values of both brinzolamide and brimonidine

Brinzolamide		Brimonidine	
Concentration (µg/mL)	Peak area	Concentration (µg/mL)	Peak area
16	917929	3.2	234004
32	1766764	6.4	413515
48	2676790	9.6	632851
64	3391238	12.8	798128
80	4389813	16	1020948
96	5189115	22.4	1432899

Table 2: Accuracy data of brinzolamide

% Level	Amount Spiked (µg/mL)	Amount recovered (µg/mL)	% Recovery	Mean % Recovery
50%	32	31.752	99.22	99.16%
	32	31.581	98.69	
	32	31.844	99.51	
100%	64	64.088	100.14	
	64	64.028	100.04	
	64	63.822	99.72	
150%	96	94.220	98.15	
	96	94.365	98.30	
	96	94.701	98.65	

Table-3: Accuracy data of brimonidine

% Level	Amount Spiked (µg/mL)	Amount recovered (µg/mL)	% Recovery	Mean % Recovery
50%	6.4	6.453	100.82	99.43%
	6.4	6.419	100.30	
	6.4	6.350	99.22	
100%	12.8	12.58	98.25	
	12.8	12.75	99.59	
	12.8	12.55	98.05	
150%	22.4	22.21	99.14	
	22.4	22.23	99.24	
	22.4	22.46	100.26	

Table 4: Intraday precision (Repeatability) of brinzolamide

S. No	Area of Brinzolamide	Area of Brimonidine
1.	3286339	792187
2.	3294580	797745
3.	3265437	796380
4.	3280334	795969
5.	3274953	791564
6.	3290337	792160
Mean	3281997	794334
S.D	10708.5	2664.7
%RSD	0.3	0.3

Table 5: Intermediate precision of brimonidine

S. No	Area of Brinzolamide	Area of Brimonidine
1.	3309839	820216
2.	3308316	811974
3.	3299277	808454
4.	3313000	809483
5.	3322118	810069
6.	3301706	804178
Mean	3309043	810729
S.D	8209.5	5322.3
%RSD	0.2	0.7

Table 6: LOD & LOQ Values of Brinzolamide and Brimonidine

Molecule	LOD	LOQ
Brinzolamide	0.05	0.15
Brimonidine	0.04	0.11

Table 7: Robustness of Both Brinzolamide and Brimonidine

S. No.	Condition	%RSD of Brinzolamide	% RSD of Brimonidine
1	Flow rate (-) 1.1ml/min	0.0	1.0
2	Flow rate (+) 1.3ml/min	0.1	0.1
3	Mobile phase (-) 35B:65A	0.0	1.9
4	Mobile phase (+) 45B:55A	0.7	0.9
5	Temperature (-) 25°C	0.3	0.2
6	Temperature (+) 35°C	0.6	0.9

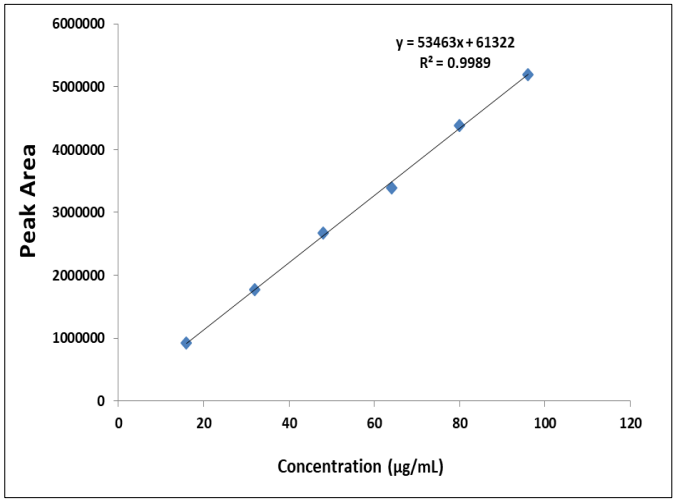

Fig 3: Calibration curve of Brinzolamide

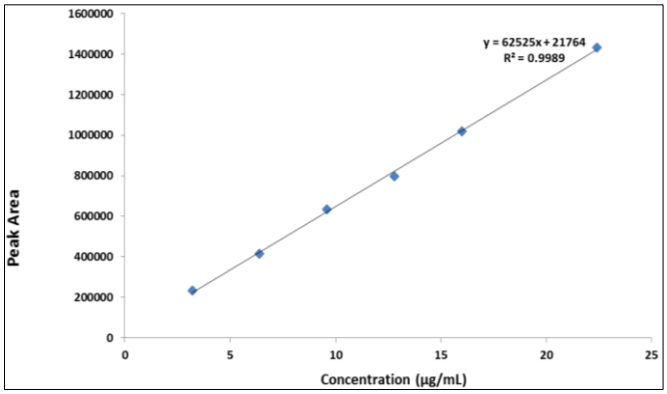

Fig 4: Calibration curve of Brimonidine

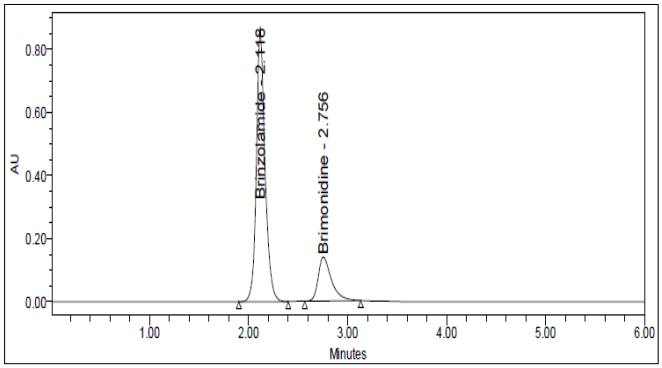

Fig 5: Accuracy 50% chromatogram of both brinzolamide and brimonidine

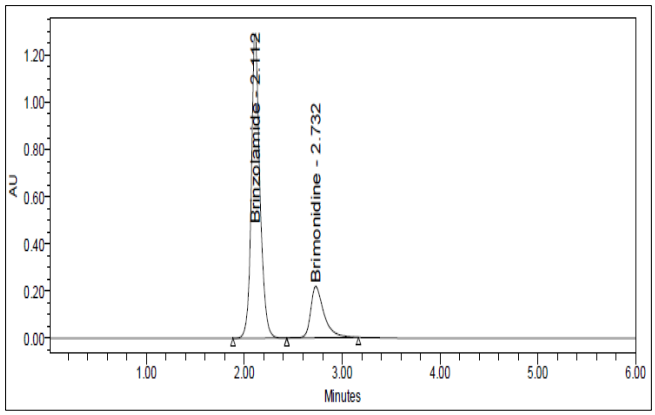

Fig 6: Accuracy 100% chromatogram of both brinzolamide and brimonidine

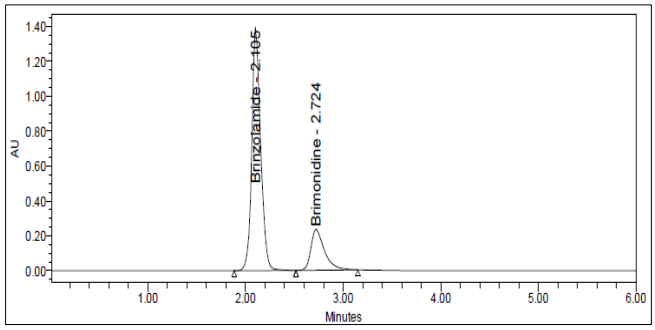

Fig 7: Accuracy 150% chromatogram of both brinzolamide and brimonidine

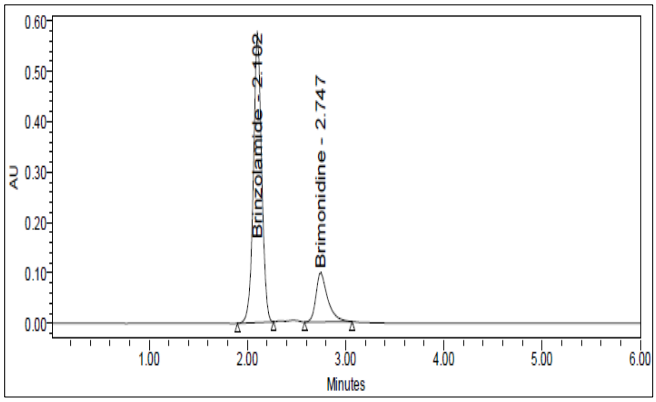

Fig 8: Repeatability of precision of both brinzolamide and brimonidine

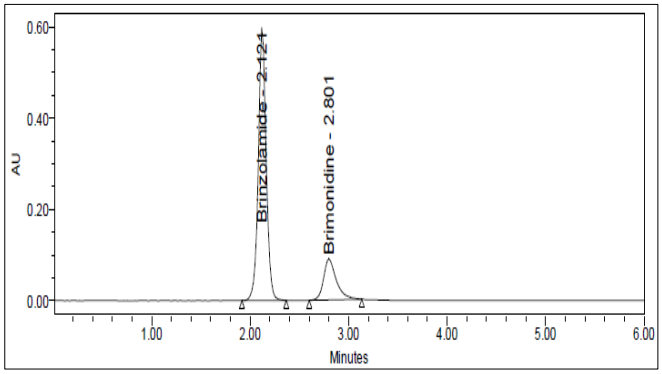

Fig 9: Intermediate Precision of Both Brinzolamide and Brimonidine

References

1. Vo A, Feng X, Patel D, Mohammad A, Patel M, Zheng J et al. Vitro physicochemical characterization and dissolution of brinzolamide ophthalmic suspensions with similar composition. International Journal of Pharmaceutics. 2020; 588:119761.
2. Scozzafava A, Supuran CT. Glaucoma and the applications of carbonic anhydrase inhibitors. Carbonic Anhydrase: Mechanism, Regulation, Links to Disease, and Industrial Applications, 2014, 349-59.
3. Kir E, Cakmak H, Dayanir V. Medical control of intraocular pressure with brinzolamide 1% after phacoemulsification. Canadian Journal of Ophthalmology. 2008; 43(5):559-62.
4. Cantor LB. The evolving pharmacotherapeutic profile of brimonidine, an 2-adrenergic agonist, after four years of continuous use. Expert opinion on pharmacotherapy. 2000; 1(4):815-34.
5. Piwnica D, Rosignoli C, de Ménonville ST, Alvarez T, Nollet MS, Roye O et al. Vasoconstriction and anti-inflammatory properties of the selective α-adrenergic receptor agonist brimonidine. Journal of dermatological science. 2014; 75(1):49-54.
6. Obach RS, Huynh P, Allen MC, Beedham C. Human liver aldehyde oxidase: inhibition by 239 drugs. The Journal of Clinical Pharmacology. 2004; 44(1):7-19.
7. Manoharan G, Al-Bratty M. Development and validation of ultra violet spectrophotometric and reversed-phase high performance liquid chromatography techniques for simultaneous estimation of brinzolamide and brimonidine

tartrate in ophthalmic suspension formulation. Orient. J. Chem. 2016; 32(2):1111-20.
8. Patel P, Darji VC, Patel BR. Analytical method development and validation of stability indicating RP-HPLC method for estimation of brinzolamide and brimonidine tartrate in an ophthalmic suspension. IJRAR. 2019; 6:150-6.
9. Agrawal VP, Desai SS, Jani GK. Development of RP-HPLC Method for Simultaneous Determination of Brimonidine Tartrate and Brinzolamide by QbD Approach and Its Validation.Eurasian Journal of Analytical Chemistry, 2016, 11(2).
10. Khatun R, Islam SA. Development and validation of analytical method for simultaneous estimation of brinzolamide and timolol by HPLC from ophthalmic preparation. International Journal of Pharmaceutical Sciences and Research. 2014; 5(3):1001.
11. Anusha B, Susmita AG, Rajitha G. Analytical Method Development and Validation of New RP-HPLC Method for Simultaneous Estimation of Brinzolamide and Timolol Maleate in Ophthalmic Solutions. Research Journal OF Pharmaceutical Biological and Chemical Sciences. 2016; 7(3):1290-8.
12. Elshanawane AA, Abdelaziz LM, Mohram MS, Hafez HM. Development and validation of HPLC method for simultaneous estimation of brimonidine tartrate and timolol maleate in bulk and pharmaceutical dosage form. Journal of Chromatography & Separation Techniques. 2014; 5(3):1.
13. Elshanawane AA, Abdelaziz LM, Mohram MS, Hafez HM. Development and validation of HPLC method for simultaneous estimation of brimonidine tartrate and timolol maleate in bulk and pharmaceutical dosage form. Journal of Chromatography & Separation Techniques. 2014; 5(3):1.
14. Büker E, Dinç E. A new UPLC method with chemometric design–optimization approach for the simultaneous quantitation of brimonidine tartrate and timolol maleate in an eye drop preparation. Journal of chromatographic science. 2017; 55(2):154-61.

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Method development and validation for estimation of ticagrelor in pharmaceutical dosage form by using RP-HPLC

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Abstract
The present study was conducted to develop a simple and precise analytical method for the estimation of ticagrelor in tablet formulation. Reverse Phase HPLC was used for method development and validation studies of ticagrelor. The optimum chromatographic conditions comprised of C18 column (Enable; 4.6 x 250mm, 5µm)) as the stationary phase and acetonitrile and 0.01M phosphate buffer in the ratio of 60:40 v/v as the mobile phase. The flow rate was 1 ml/min with detection at 230 nm and a run time of 10 min. The retention time of ticagrelor was 5.45 min. The linearity studies indicated that the range of the developed method was 50-150 µg/ml with a correlation coefficient of 0.999. The method was specific with a percent mean recovery was found to be 100.6 %. The % RSD in the precision studies was 0.069. The validated method was applied to conduct the assay of ticagrelor in tablets with a percent mean recovery of 98.98 %.Conclusion: The developed and validated RP-HPLC isocratic method was simple, accurate and precise as per the ICH guidelines. It was suitable for the analysis of ticagrelor in bulk and tablet formulation.

Keywords: ticagrelor, HPLC, anti-platelet

Introduction
Ticagrelor is a tablet dosage form which acts an antiplatelet [1] medicine, makes your blood flow through your veins more easily. It can help prevent blood clots and sold under the brand name Brilinta, is a medication used for the prevention of stroke, heart attack and others with acute coronary syndrome and coronary arteries. It is a CYP3A4 substrate [2] and weak CYP3A4 inhibitor. The drug was approved for use in the European Union by the European Commission on December 3, 2010. The drug was approved [3] by the US Food and Drug Administration on July 20, 2011. Chemically it is (1S,2S,3R,5S)-3-(7-{[(1R,2S)-2-(3,4-difluorophenyl)cyclopropyl] amino}-5-(propylsulfanyl)-3H-[1,2,3]triazolo[4,5-d]pyrimidin-3-yl)-5-(2-hydroxyethoxy) cyclopentane-1,2-diol and molecular weight is 522.56. The main metabolite [4] of this drug is AR-C124910XX is formed by CYP3A4 by dehydroxyethylation at position 5 of the cyclopentane ring in figure 1.

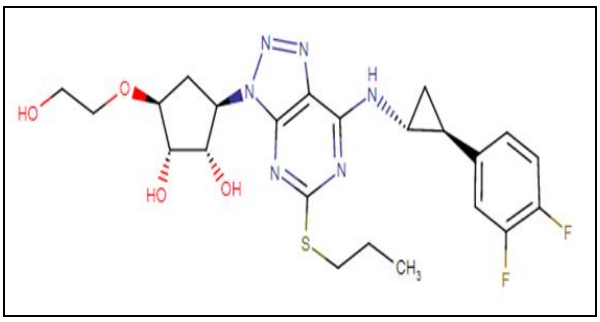

Fig 1: Chemical structure of Ticagrelor

Materials and Methods
Quantitative estimation of Ticagrelor was done by using an isocratic Shimadzu HPLC instrument on Enable C18 column (250 mm x 4.6 mm, 5µ). The Instrument is equipped with binary pump and variable wavelength PDA detector. A 20µL Hamilton syringe was used for injecting the samples. Data was analysed by using LC Solutions software. Shimadzu UV-Visible spectrophotometer was used for spectral studies. Degassing of the mobile phase was done by using a Loba ultrasonic bath sonicator. A Shimadzu balance was used for weighing the materials. Ticagrelor pure drug was provided by Aurobindo Pharma Hyderabad., Optimised conditions: the mobile phase with Acetonitrile: 0.01M phosphate buffer (60:40 %v/v) was employed in isocratic mode at a flow rate of 1.0 mL/min. The run time was 8 min and 20µL of the sample was injected for every run into the column at wavelength of 230nm.
Chromatographic conditions: A mixture of acetonitrile and 0.01M phosphate buffer in the ratio of 60:40 %v/v was employed to be the most suitable mobile phase for identical chromatographic separation of Ticagrelor. The solvent mixture was filtered through 0.45 µ membrane filter and sonicated to dissolve it completely. It was pumped through the column at a flow rate of 1.0 mL/min. Injection volume was 20µL and the column was maintained at ambient temperature and run time was set to be 10 min and detected of the drug was monitored at wavelength of 230nm and retention time was to be 5.45 min.

Preparation of standard stock solution
Accurately Weighed and transferred 90mg of ticagrelor working Standard into a 100ml clean dry volumetric flask, add 3/4th volume of HPLC graded mobile phase sonicated for 5 minutes and make up to the final volume with diluents to get concentrations like 0.9µg/ml of Ticagrelor.

Preparation of sample solution
Twenty tablets were accurately weighed, their mean weight was determined and they were mixed and finally powdered. Transfer the sample equivalent to 90mg of ticagrelor into a

100mL volumetric flask. Add about 85mL of diluents and sonicated to dissolve it completely and make volume up to the mark with diluents to concentration range 50-150 µg/mL. Mix well and filter through 0.45µ filter.

Linearity

The calibration graphs shows that the linear response was obtained over the range of concentrations used in the assay procedure. These data demonstrate that the methods have adequate sensitivity to the concentrations of the analyte. Several aliquots of standard solution of Ticagrelor was taken in different 10 mL volumetric flasks and diluted up to the mark with diluents such that the final concentrations of Ticagrelor were in the range 50 to 150 µg/mL. Evaluation of the drug was performed with UV detector at 230 nm; peak area was recorded for all the peaks. The correlation coefficient value of Ticagrelor was 0.9976. The results show that an excellent correlation exists between peak area and concentration of drug within the concentration range indicated. The data is tabulated in table 1& figure 2.

Table 1: Linearity values of Ticagrelor

Ticagrelor	
Concentration (µg/ml)	**Peak Area**
50	27287
70	46598
80	56931
100	76093
120	96186
130	106266
150	126359

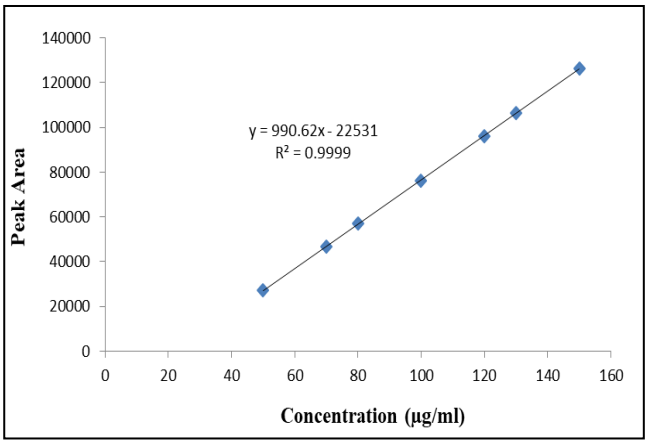

Fig 2: Calibration curve of Ticagrelor

System suitability

System suitability parameters like retention time, theoretical plates and tailing factor were calculated and compared with standard values.

Accuracy

The recovery studies for the method were carried out by standard addition method. It was evaluated at three concentration levels (80, 100 and 120 µg/ml) and the percentage recoveries were calculated. The data is tabulated in table 2& figure 3, 4 & 5.

Table 2: Accuracy data of Ticagrelor

% level	Amount added (µg/ml)	Amount recovered (µg/ml)	Percentage of recovery	Percentage of Mean recovery
80	80	80.0	100.0	100.09
	80	80.12	100.15	
	80	80.1	100.13	
100	100	100.12	100.12	100.17
	100	100.22	100.22	
	100	100.17	100.17	
120	120	121.06	100.88	101.56
	120	122.12	101.76	
	120	122.43	102.03	

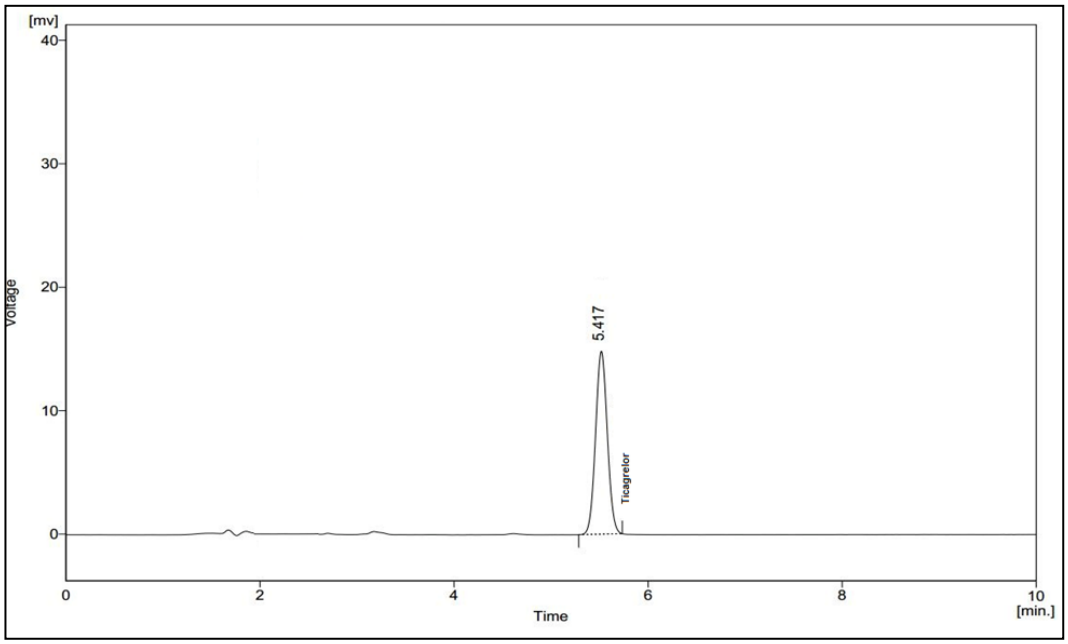

Fig 3: Accuracy 50% chromatogram of Ticagrelor

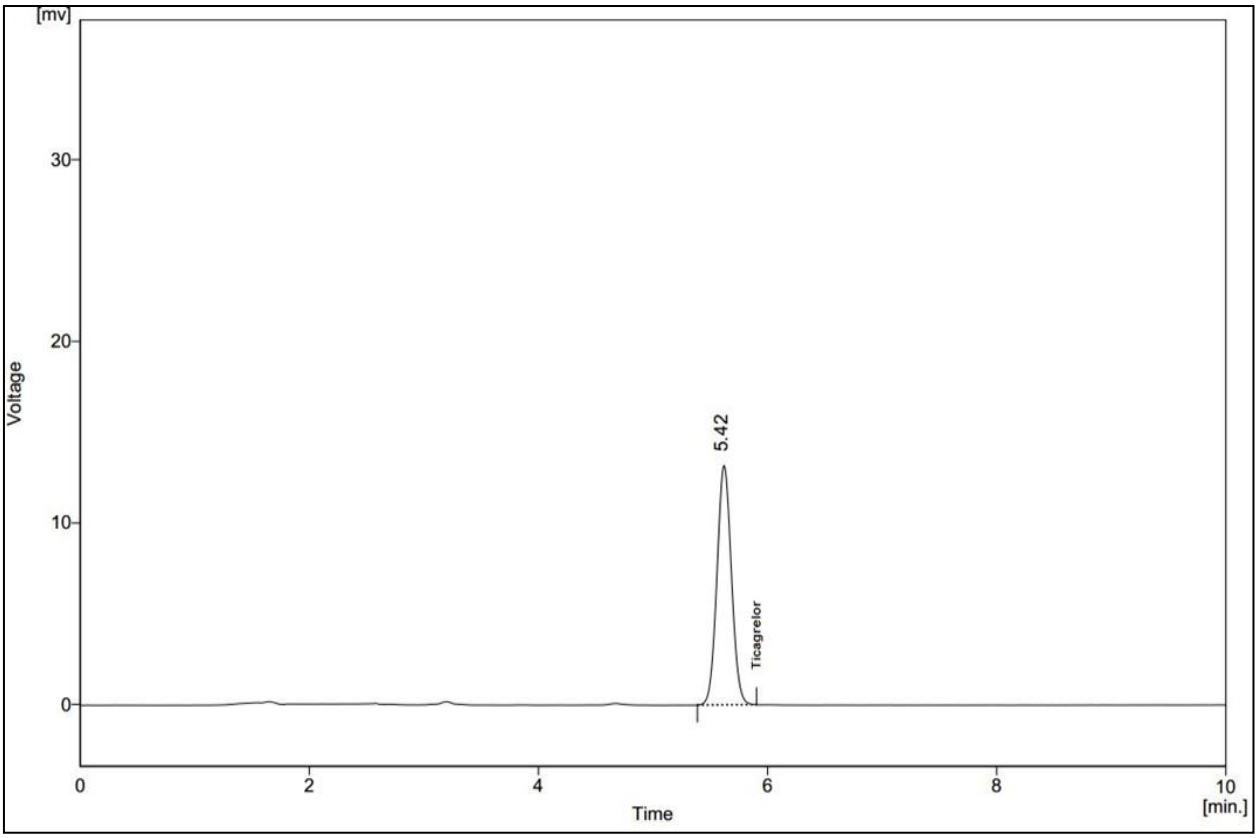

Fig 4: Accuracy 100% chromatogram of Ticagrelor

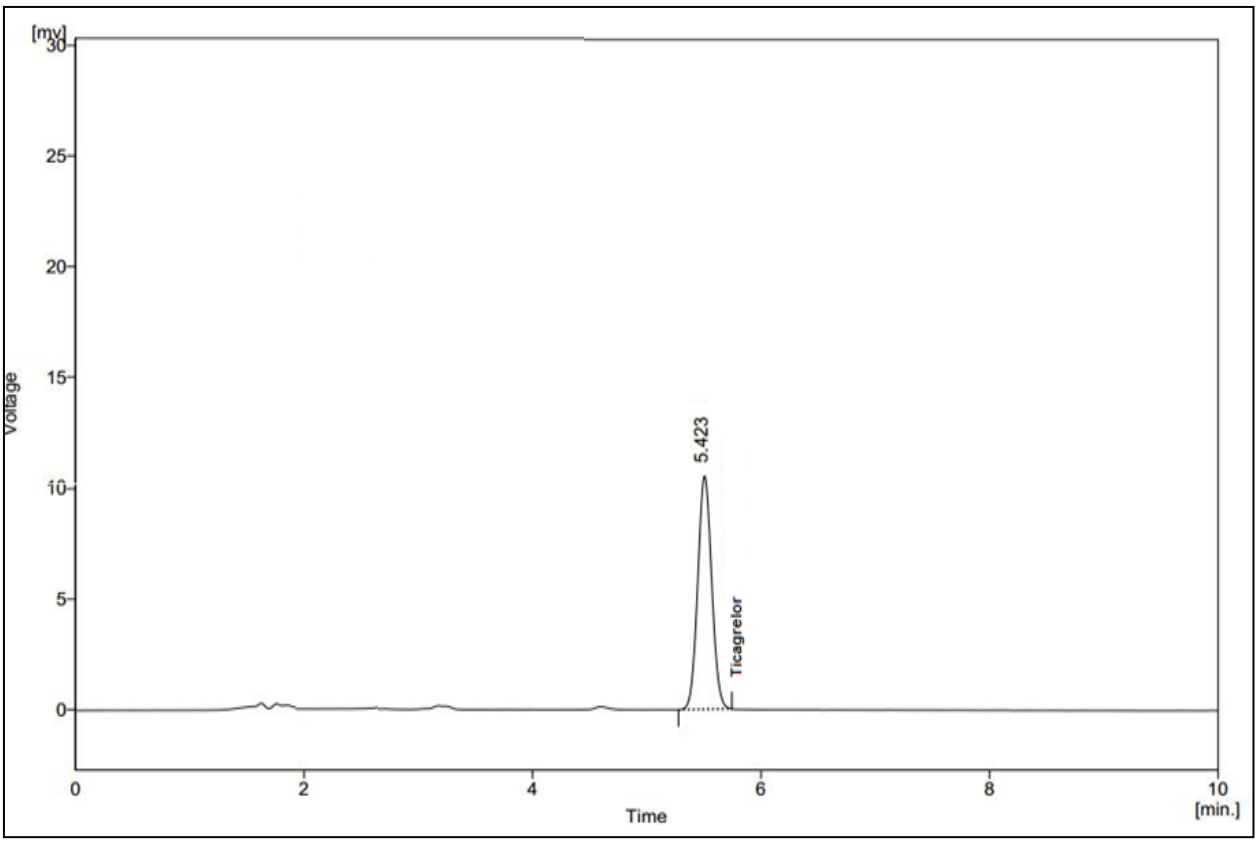

Fig 5: Accuracy 150% chromatogram of Ticagrelor

Precision

The precision of the method was determined by intra and inter precision studies. This was evaluated by injecting three independent sample preparations of ticagrelor from a single Formulation at three different concentration levels on the same day (Intraday) and on three different days (Inter day). The % RSD was then calculated. The data is represented in table 3 & 4 and figure 6 & 7.

Table 3: Precision (Intraday) values of Ticagrelor

Inj. Number	Area of Ticagrelor
1	75985
2	75036
3	75498
4	75268
5	75154
6	75167
Mean	75388
SD	347.08
%RSD	0.26

Table 4: Intermediate precision of Ticagrelor

Inj. Number	Area of Ticagrelor
1	75985
2	75036
3	75958
4	75268
5	75154
6	75280
Mean	75447
SD	415.99
%RSD	0.058

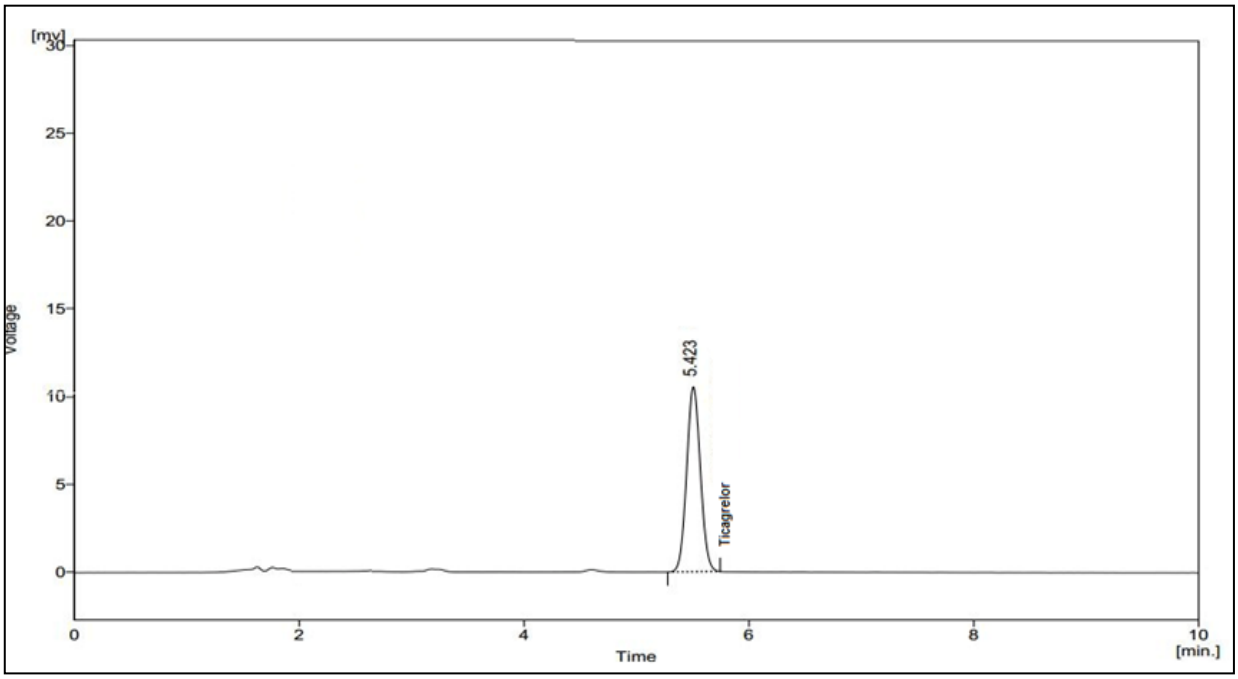

Fig 6: Repeatability of Ticagrelor

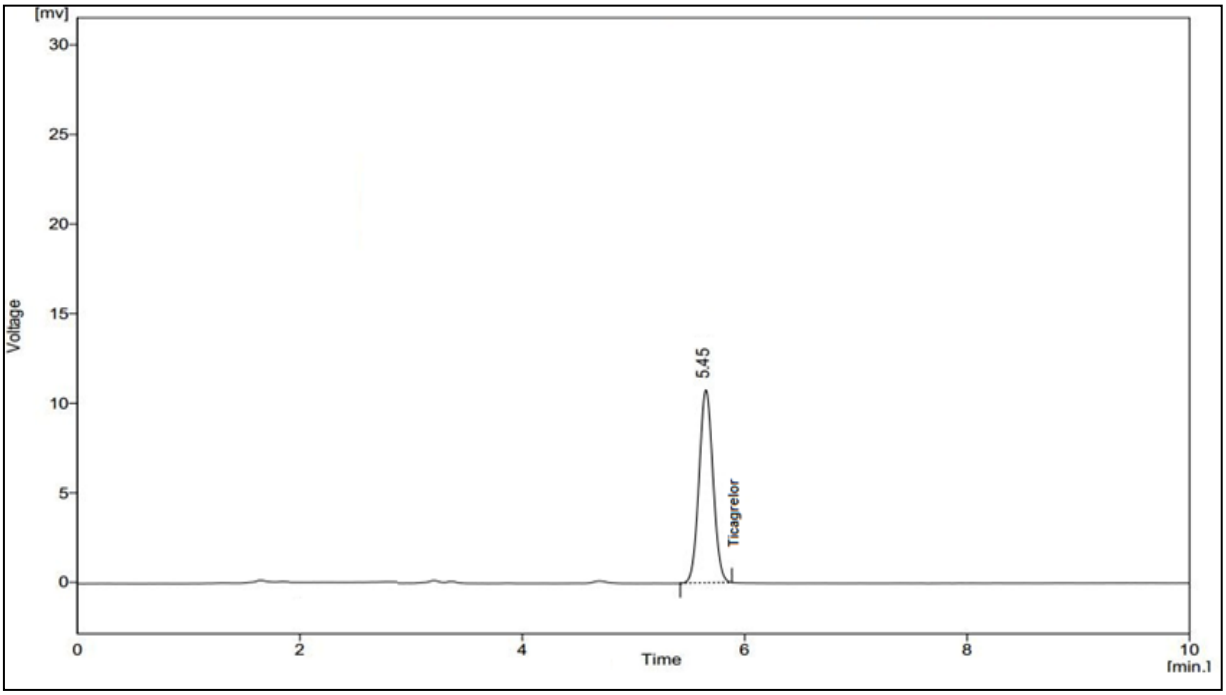

Fig 7: Intermediate precision of Ticagrelor

Limit of detection and limit of quantification

LOD and LOQ were determined based on the standard deviation of the response and the slope of the calibration curve. The sensitivity of the method was established by the LOD and the LOQ values. Data is represented in table 5.

Table 5: LOD & LOQ value of Ticagrelor

Drug name	LOD	LOQ
Ticagrelor	0.17	0.52

Robustness

Robustness was established by introducing small changes in the HPLC optimised conditions which include mobile phase, flow rate and pH. Data is represented in table 6.

Table 6: Robustness of Ticagrelor

S.N	Condition	Changed condition	Ticagrelor
1	Flow rate: 1ml/min	0.9ml/min-1.1ml/min	0.0-1.7
2	Mobile phase composition: 60:40	58:42-62:38	1.2-0.4
3	pH: 3	pH-2-p.H-4	0.1-0.0

Results and Discussion

The proposed method was found to be simple. Linearity was observed in the concentration range of 50-150 µg/mL with the regression equation y= 990.62x - 22531 and the correlation coefficient of 0.999. System suitability parameters indicates high column efficiency with large number of theoretical plates (>2000).The tailing factor was found to be 1.08 which does not exceed the critical value (2).The average retention time was found to be 5.45. No interference was seen from any of the components of the pharmaceutical dosage form indicating the specificity of the method. The % RSD was found to be 0.26 for intraday and 0.058 for inter day precision studies. Thus the method was found to be accurate and precise as the %RSD was not more than 2%. The limit of detection and limit of quantification for Ticagrelor were found to be 0.17 and 0.52 µg/mL respectively. The RSD for the % assay of sample was calculated for each parameter in robustness and was found to be less than 2% confirming the robustness of the method.

Conclusion

A validated RP-HPLC method was developed for the determination of Ticagrelor in tablet dosage form and bulk forms. As the proposed method is simple, rapid, accurate, precise and specific it can be employed for the routine analysis of Ticagrelor in pharmaceutical dosage forms.

References

1. Fuller R, Chavez B. Ticagrelor (brilinta), an antiplatelet drug for acute coronary syndrome. Pharmacy and Therapeutics. 2012; 37(10):562.
2. Na YG, Byeon JJ, Huh HW, Kim MK, Shin YG, Lee HK *et al*. Effect of ticagrelor, a cytochrome P450 3A4 inhibitor, on the pharmacokinetics of tadalafil in rats. Pharmaceutics. 2019; 11(7):354.
3. https://www.alembicpharmaceuticals.com/wp-content/uploads/2018/07/Press-Release-USFDA-Tentative-Approval-Ticagrelor-Tablets-90mg-July-2018-1.pdf
4. Adamski P, Buszko K, Sikora J, Niezgoda P, Barańska M, Ostrowska M *et al*. Metabolism of ticagrelor in patients with acute coronary syndromes. Scientific reports. 2018; 8(1):1-8.

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OPTIMIZATION OF CULTURAL CONDITIONS TO ENHANCE BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15

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Keywords:

Streptomyces olivoverticillatus, Mangrove ecosystem, Bioactive compounds, Optimization

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ABSTRACT: The study was designed to optimize cultural conditions for enhanced production of bioactive compounds by *Streptomyces olivoverticillatus* VLK-15 isolated from mangrove ecosystem of South Coast of Andhra Pradesh, India. The soil dilution plate technique was employed for the isolation of actinobacteria using starch-casein agar (ISP-6) medium. The potent strain was identified as *Streptomyces olivoverticillatus* VLK-15 based on polyphasic taxonomy. The productivity of the strain was enhanced by amending the medium (ISP-2) with mannitol (0.5%), peptone (0.5%), and K_2HPO_4 (0.05%) adjusted to pH 7.0 and incubated with the culture at 30ºC for ninety-six hours. The secondary metabolites produced under optimized conditions exhibited high antimicrobial activity against Gram-positive and Gram-negative bacteria as well as fungi. Hence, it could be a possible source for novel and potential antimicrobial compounds.

INTRODUCTION: The mangrove ecosystem provides a unique environment to support the growth of different microorganisms which are involved in the production of potential secondary metabolites possessing pharmaceutical properties. Mangrove ecosystems are found in tropical and subtropical intertidal regions around the world [1]. Only 5% of microbes, including bacteria, fungi, and actinobacteria isolated from these ecosystems have been studied [2]. Among the microorganisms, marine actinobacteria produce unique and novel secondary metabolites, which play an extensive role in the pharmaceutical and medical industry due to their capacity to produce secondary metabolites.

QUICK RESPONSE CODE

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More than 10,000 bioactive compounds have been reported from actinobacteria [3], of which the major share (70%) is contributed by the genus *Streptomyces* alone [4]. There is an urgent need to access new actinobacterial species for the exploration of novel bioactive compounds to hit multi-drug-resistant pathogens. Hence, we focused on the isolation and identification of novel species as well as optimization of cultural conditions to enhance the bioactive compounds from rare actinobacterial strains from the mangrove ecosystem of the south coast of Andhra Pradesh.

MATERIALS AND METHODS:

Microorganism: The strain VLK-15 was isolated from the mangrove ecosystem of the south coastal region of Andhra Pradesh, India, and identified as *Streptomyces olivoverticillatus* VLK-15 by using polyphasic taxonomy including molecular analysis (16S r RNA). The gene sequence of the strain VLK-15 is deposited in the GenBank database of NCBI with the accession number MG309759 [5].

Antimicrobial Activity: The antimicrobial activity of strain VLK-15 was tested against bacteria such as *Bacillus megaterium* (NCIM2187), *Staphylococcus aureus* (MTCC 3160), *Pseudomonas aeruginosa* (ATCC 9027), *Escherichia coli* (ATCC 35218) and *Candida albicans* (ATCC 10231) by agar well diffusion assay [6]. Nutrient agar and Czapek-Dox agar media were used for culturing the test bacteria and fungi, respectively. The culture filtrate was extracted with ethyl acetate, and the solvent extract (50 µL) was added to the well made on agar media seeded with test microbe. Wells with only ethyl acetate served as control. The diameter of the inhibition zone was measured after 24 h of incubation at 30°C for bacteria and 24-72 h for yeast.

Optimization of Cultural Conditions for Improved Production of Bioactive Compounds: An attempt was made to optimize the culture conditions such as pH, temperature, carbon sources, nitrogen sources and minerals to enhance the bioactive metabolite production by *Streptomyces olivoverticillatus* VLK-15. The secondary metabolite production by the strain was studied at regular intervals up to eight days. Fermentation was carried out in 250-mL Erlenmeyer flasks with constant shaking at 120 rpm.

The strain was cultured on ten different media such as ISP-1 to ISP-7 and Czapek-Dox, nutrient broth and yeast-starch broth to determine the ideal conditions for the maximum production of bioactive compounds. The bioactive metabolite production in each medium is evaluated [6].

The effect of initial pH on the bioactive compounds was determined by altering pH of the production medium from 4 to 10. Similarly, the optimal temperature for the production of bioactive compounds was evaluated by inoculating the strain at temperatures ranging from 25 to 45°C, while maintaining all other conditions at optimum levels [7].

The influence of carbon sources on bioactive compound production was determined by adding different carbon sources to the fermentation medium, such as dextrose, galactose, maltose, sucrose, mannitol, lactose, starch, cellulose, fructose, and xylose, each at a concentration of 0.4% (w/v) [8]. Impact of different concentrations of the best carbon source (0.1-1% w/v) on bioactive compound production was also studied.

The effect of various nitrogen sources such as ammonium nitrate, peptone, tryptophan, L-proline, tyrosine, urea, yeast extract, histidine, cysteine, and alanine were studied by supplementing nitrogen source (0.4%) to the medium. Further, the best nitrogen source (0.1–1% w/v) for improved production of bioactive metabolites was also recorded [9]. The optimized medium was supplemented with mineral salts such as KH_2PO_4, K_2HPO_4, $FeSO_4$, $ZnSO_4$ and $MgSO_4$ to measure the impact of minerals on the production of bioactive metabolites [10]. The metabolites produced by the strain grown under optimal conditions were tested for antimicrobial activity.

Statistical Analysis: Data obtained on antimicrobial activity under different culture conditions are statistically analyzed with one-way analysis of variance (ANOVA).

RESULTS AND DISCUSSION:

Influence of Incubation Time on Biomass and Antimicrobial Activity: The growth pattern and bioactive compound production by the strain VLK-15 were studied at regular intervals up to eight days. The stationary phase of *Streptomyces olivoverticillatus* VLK-15 continued from 72 h to 120 h of incubation **Fig. 1**. The bioactive compounds obtained from the four-day-old culture exhibited good antimicrobial activity against the test microorganisms. This is in conformity with the earlier reports that the extracts of 4-day old cultures of *Streptomyces lavendulocolor* VHB-9 [11], *S. cheonanensis* VUK-A [12], and *S. tendae* TK-VL_333 [13] exhibited high antimicrobial activity against the test microorganisms.

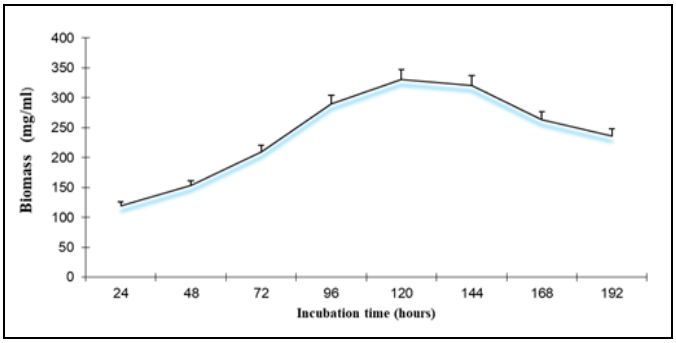

FIG. 1: GROWTH CURVE OF *STREPTOMYCES OLIVO-VERTICILLATUS* VLK-15. DATA ARE STATISTICALLY ANALYZED AND FOUND TO BE SIGNIFICANT AT 5%

Effect of Culture Media on Antimicrobial Compounds: Bioactive compound production by the strain VLK-15 was studied in different culture media **Fig. 2**. Among the ten media tested, YMD (ISP-2) supported high levels of bioactive metabolites followed by starch-casein (ISP-6) and nutrient broth.YMD broth for *Streptomyces violaceoruber* VLK-4 [6] and Czapek- Dox broth for *Streptomyces* sp. MNK-7 [14] supported high rates of antibiotic production.

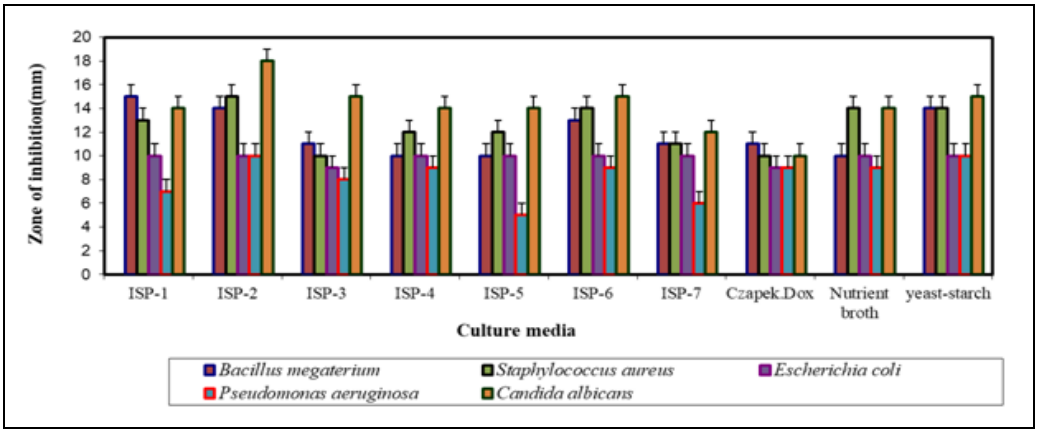

FIG. 2: EFFECT OF DIFFERENT CULTURE MEDIA ON BIOACTIVE COMPOUNDS PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

Impact of pH and Temperature on Bioactive Compounds: Maximum antimicrobial production by the strain VLK-15 was found at pH 7 **Fig. 3**. Bioactive metabolites obtained from *Streptomyces lavendulocolor* VHB-9 [11], *S. cheonanensis* VUK-A [12], *S. violaceoruber* VLK-4 [6,] and *Streptomyces* sp. VITSVK9 [7] exhibited good antimicrobial activity when grown at pH 7.

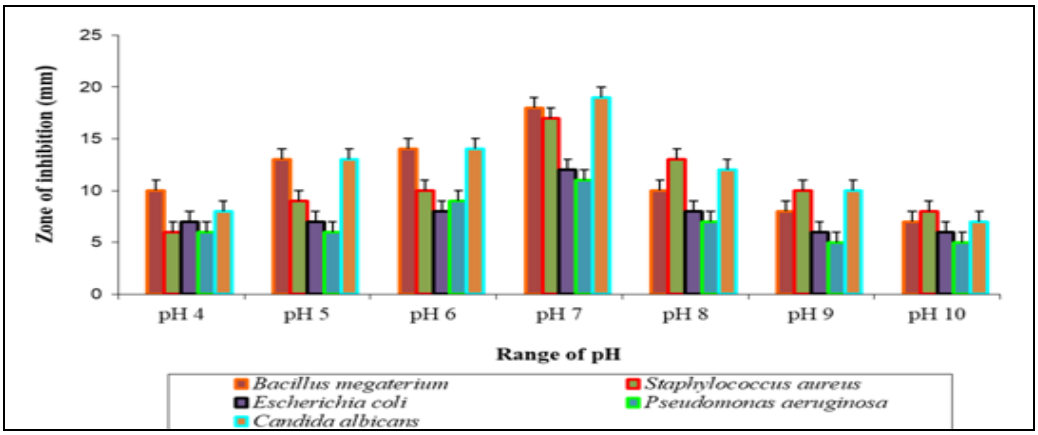

FIG. 3: INFLUENCE OF pH ON SECONDARY METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

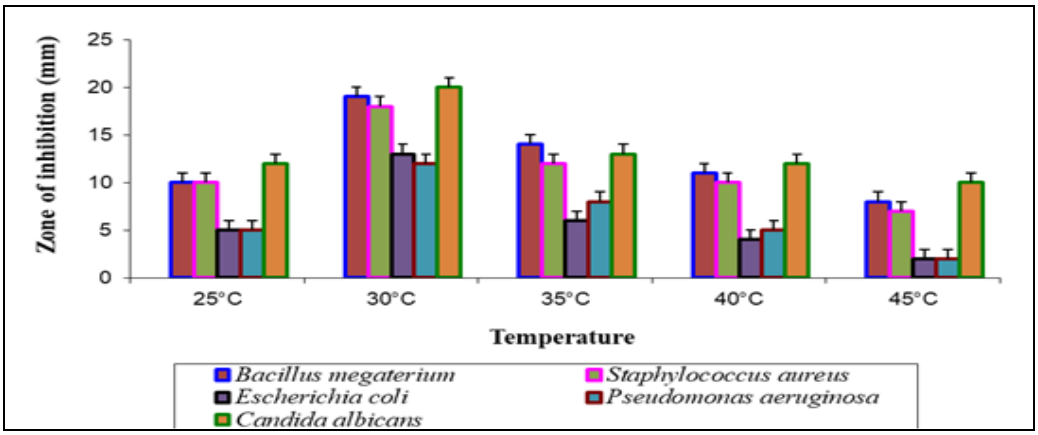

FIG. 4: EFFECT OF TEMPERATURE ON BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

The production of antimicrobial compounds by the strain VLK-15 increased with rise in the incubation temperature from 25 -30°C **Fig. 4**. Further increase in temperature (above 30°C) resulted in the decline in the production of bioactive metabolites. Earlier reports on *Streptomyces* species also indicated that culturing the strains at 30°C could lead to high levels of bioactive metabolites [11, 12, 15, 16].

Impact of Carbon and Nitrogen Sources on Antimicrobial Activity: The effect of carbon sources on production of secondary metabolites by the strain VLK-15 is shown in **Fig. 5**. Significant production of bioactive compounds was found in mannitol amended medium (ISP-2) followed by dextrose, lactose and starch. As mannitol appeared as the most favored carbon source for secondary metabolite production, different concentrations of mannitol (0.1-1%) were added to find the optimal concentration. Mannitol @ 0.5% exhibited high yields of bioactive compounds **Fig. 6**. Mannitol was reported as a best carbon source to enhance the bioactive metabolite production by *Streptomyces violaceoruber* VLK-4 [6], whereas *S. lavendulocolor* VHB-9 [11] and *Streptomyces cheonanensis* VUK-A [12] utilized lactose as carbon source for secondary metabolite production.

Different nitrogen sources were tested for their impact on bioactive metabolite production by the strain VLK-15. Among the nitrogen sources used, peptone followed by yeast extract and tryptophan was found to be the best nitrogen source for the production of bioactive compounds. As Peptone improved the bioactive compound production by the strain VLK-15, the influence of various concentrations of peptone was assessed, and 0.5% was found to be good for the production of bioactive metabolites **Fig. 8**. These results are comparable with *Streptomyces lavendulocolor* VHB-9 [11] and *Streptomyces cheonanensis* VUK-A [12].

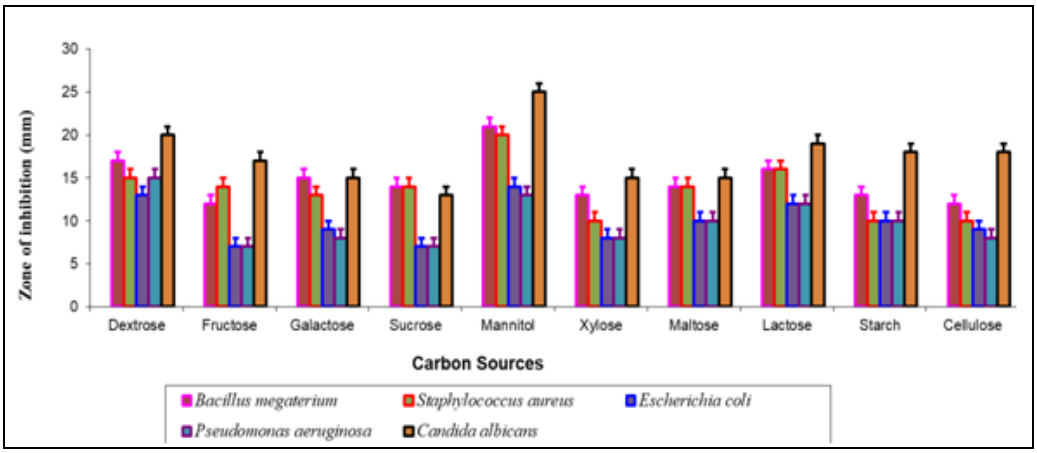

FIG. 5: EFFECT OF DIFFERENT CARBON SOURCES ON BIOACTIVE METABOLITES PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

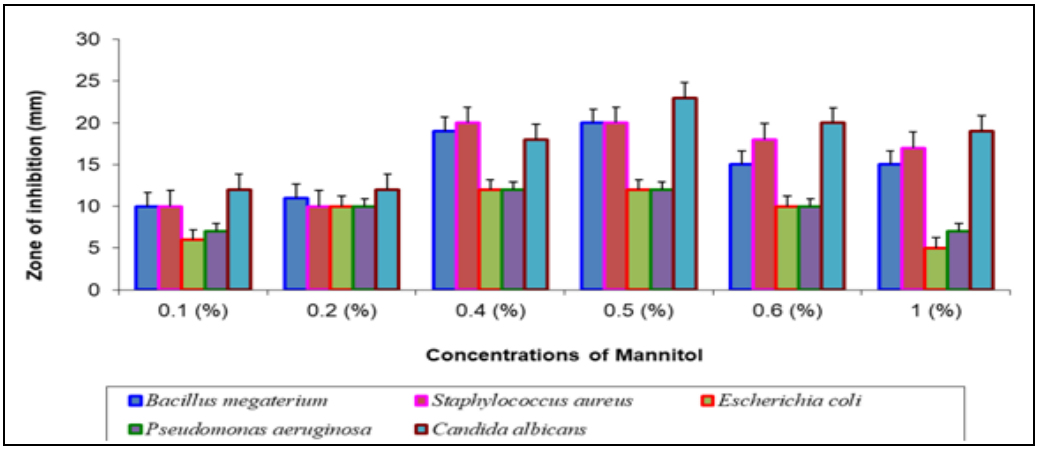

FIG. 6: IMPACT OF DIFFERENT CONCENTRATIONS OF MANNITOL ON BIOACTIVE METABOLITES PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

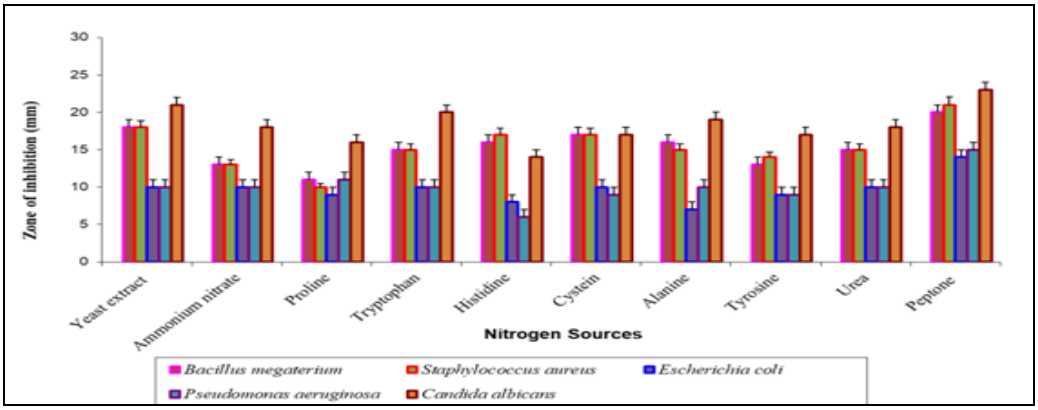

FIG. 7: EFFECT OF NITROGEN SOURCES ON SECONDARY METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

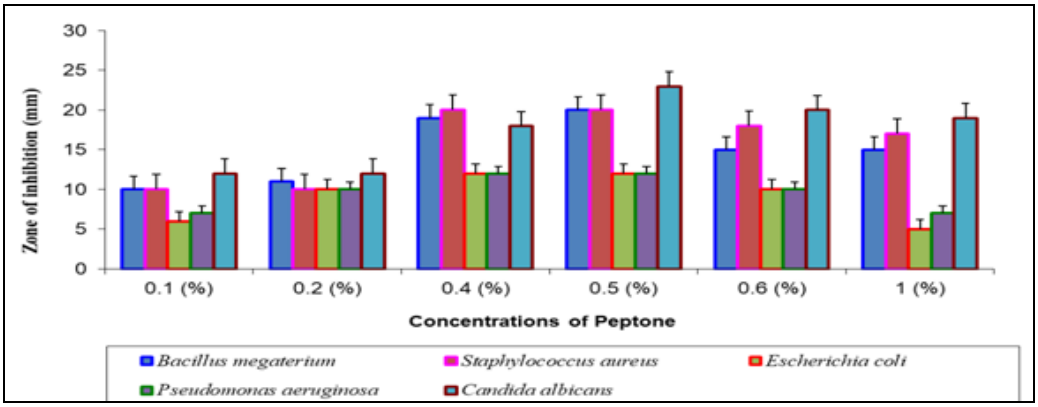

FIG. 8: INFLUENCE OF DIFFERENT CONCENTRATIONS OF PEPTONE ON BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

Influence of Minerals on Production of Antimicrobial Compounds: The effect of minerals on secondary metabolite production by the strain VLK-15 is shown in **Fig. 9**. Among the minerals tested, K_2HPO_4 followed by $FeSO_4$, KH_2PO_4 and $ZnSO_4$ supported a high yield of bioactive compound production. Similar results were found for *Streptomyces lavendulocolor* VHB-9 [11], *S. violaceoruber* VLK-4 [6,] and *Streptomyces cheonanensis* VUK-A [12].

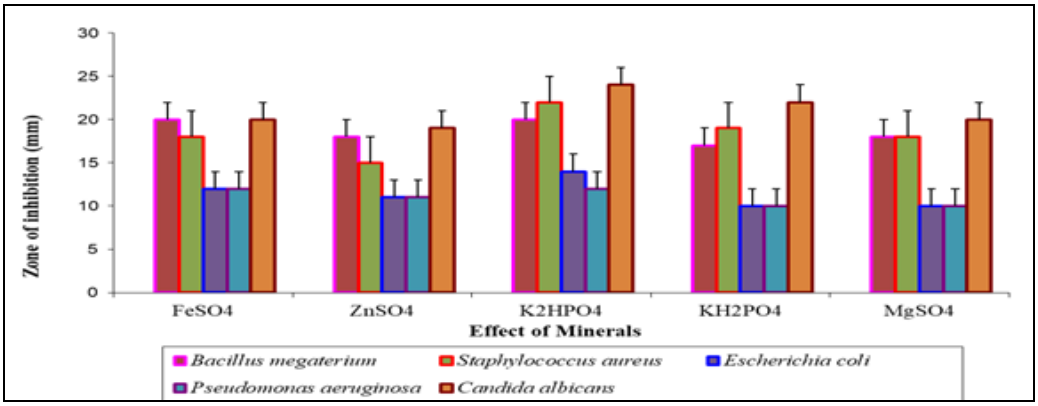

FIG. 9: IMPACT OF DIFFERENT MINERALS ON BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

The bioactive compounds obtained from optimized fermentation broth exhibited high antimicrobial activity against test bacteria and fungi **Table 1**. Among the bacteria tested, *Bacillus megaterium* was highly sensitive to the compounds produced by the strain VLK-15 followed by *Staphylococcus aureus*, *Pseudomonas aeruginosa* and *E. coli*. *Candida albicans* showed high sensitivity to the metabolites.

TABLE 1: ANTIMICROBIAL ACTIVITY OF ETHYL ACETATE EXTRACT OF *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15 UNDER OPTIMIZED CONDITIONS

Test microorganisms	Zone of inhibition (mm)
Bacillus megaterium	32
Staphylococcus aureus	30
Pseudomonas aeruginosa	27
Escherichia coli	26
Candida albicans	30

CONCLUSION: In the present study a potent strain *Streptomyces olivoverticillatus* VLK-15 isolated from the mangrove ecosystem of the south coast of Andhra Pradesh, India, possessing antagonistic activity, revealed its potential to inhibit several pathogenic bacteria and fungi. Bioactive metabolite production was often influenced by various environmental factors, including nutrients (nitrogen and carbon source), temperature and pH. Maximum bioactive compound production by the strain was found when cultured on ISP-2 medium amended with mannitol @ 0.5% and peptone @ 0.5% adjusted to pH 7 for 96 hours at 30ºC. Further studies on purification, characterization, and identification of bioactive metabolites of *Streptomyces olivoverticillatus* VLK-15 are in progress. This is the first report on the optimization of culture conditions to enhance the bioactive metabolites by the strain *Streptomyces olivoverticillatus* VLK-15.

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REFERENCES:

1. Ottoni CA, Amini MS, Alam I, Alzubaidy H, Mokhtar N, Archer JAC and Bajic VB: Soil and rhizosphere associated fungi in gray Mangroves (*Avicennia marina*) from the Red Sea-a metagenomic approach. Genom Prot Bioinform 2015; 13: 310-20.
2. Xu J: Bioactive natural products derived from mangrove-associated microbes. RSC Adv 2015; 5: 841-92.
3. Azman AS, Othman I, Fang CM, Chan KG, Goh BH and Lee LH: Antibacterial, anticancer and neuroprotective activities of rare Actinobacteria from mangrove forest soils. Indian J Microbiol 2017; 57: 177-87.
4. Zengler K, Parakar A and Keller M: New methods to access microbial diversity for small molecule discover," in Natural Products therapeutic Medicine, L. Zhang and A. L. Demain, Eds., 2005; 275–294. Humana Press, Totowa, NJ, USA.
5. Naragani K, Indupalli MD and Muvva V: Phylogenetic characterization of potential bioactive metabolites producing actinomycetes from mangrove sediments. Int J Pharm Sci & Res 2020; 11(12): 1000-05.
6. Krishna N, Munaganti RK and Muvva V: Optimization studies for enhanced bioactive metabolite production by *Streptomyces Violaceoruber* VLK-4 isolated from the south coast of Andhra Pradesh, India. Int J Pharm Sci Res 2014; 5: 4760-68.
7. Saurav K and Kannabiran K: Diversity and optimization of process parameters for the growth of Streptomyces VITSVK 9 sp. Isolation from Bay of Bengal. India J Nat Environ Sci 2010; 1: 56-65.
8. Elliah P, Srinivasulu B and Adinarayana K: Optimization studies on neomycin production by a mutant strain of *Streptomyces marinensis* in solid state fermentation process. Biochemistry 2000; 39: 529-34.
9. Kathiresan K, Balagurunathan R and Selvam MM: Fungicidal activity of marine actinomycetes against phytopathogenic fungi. Ind J Biotechnol 2005; 4: 271-6.
10. Kumar MR, Vijayalakshmi M, Saidulu K, Krishna N, kiranmayi MU, Reddy DK and Akkewar DM: Antimicrobial profile of *Arthrobacter kerguelensis* VL-RK_09 isolated from Mango Orchards. Brazilian Journal of Microbiology 2016; 47: 1030-38.
11. Bindu-Hima BSSN, Kumar MR, Vijayalakshmi M, Krishna N and Deepa IM: Optimization, isolation and characterization of bioactive compounds from *S. lavendulocolor* VHB-9. AJPClin Res 2018; 11: 361-68.
12. Kiranmayi MU, Sudhakar P, Krishna N and Vijayalakshmi M: Influence of cultural conditions for improved production of bioactive metabolites by *Streptomyces cheonanensis* VUK-A Isolated from Coringa Mangrove Ecosystem. Curr Tren Biotechnol Phar 2012; 6: 99-111.
13. Kavitha A and Vijayalakshmi M: Optimization and purification of L-asparaginase produced by *Streptomyces tendae* TK-VL_ 333. Z Naturforsch 2010; 65: 528-31.
14. Saha MR, Rifa FA, Islam MZ and Khondkar P: Optimization of conditions and in vitro antibacterial activity of secondary metabolite isolated from *Streptomyces* sp. MNK 7. J Appl Sci Res 2010; 6: 453-59.
15. Atta HM, Bahobail AS and El-Sehrawi MH: Studies on isolation, classification and phylogenetic characterization of antifungal substance produced by *Streptomyces albidoflavus*- 143. New York Science J 2011; 4: 40-53.
16. Ghosh U and Prasad B: Optimization of carbon, nitrogen sources and temperature for hyper growth of antibiotic producing strain *Streptomyces kanamyceticus* MTCC 324. The Bioscan 2010; 5: 157-58.

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Morphological and Anatomical Diversity of *Bulbophyllum* in India

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ABSTRACT

The Orchidaceae is one of the largest families of flowering plants comprising about 40 per cent of the Monocotyledons. India, due to its tropical location, physiological variation associated with favourable climatic conditions, has a moderately rich orchid flora of about 1350 species in 186 genera. The structure of leaf and leaf basis are taken primary importance in this paper. On this investigation, the anatomy of leaf of *Bulbophyllum* species is taken for the detailed study, to find out the ecological and taxonomic significance.

KEY WORDS: *Bulbophyllum,* Leaf, Anatomy, Ecology.

The Orchidaceae is one of the largest families of flowering plants comprising about 40 per cent of the Monocotyledons (Ramesh and Khasim 2019). It comprises about 779 genera and 22,500 species (Mabberley, 2008). In India, orchids are concentrated mostly in Eastern Himalaya (Hooker, 1895), Western Himalaya and also Western Ghats. Orchids with their attractive range of varied flowers form excellent ornamental plants and they are rich source of medicines and aesthetic pleasure. In present work, 7 species of *Bulbophyllum* collected from different geographical areas were slected for study (Table 1a). The anatomy of leaf of all the samples are taken on the morphological structure is well defined from all the samples. *Bulbophyllum affine* (Arunachal Pradesh), *B. bisetum* (Darjeeling), *B. careyanum* (Sikkim), *B. cauliforum* (Darjeeling), *B. cornutum* (Darjeeling) and *B. crassipes* (Darjeeling), *B. Fischerii* (Darjeeling).

MATERIALS AND METHODS

Plant materials were collected from Arunachal Pradesh, Darjeeling and Sikkim Himalayas and at various altitudes over the period of 2 years (Table 1a). Vegetative organs such as leaves, stems, pseudobulbs and roots were fixed in FAA (5 cc formalin + 5 cc acetic acid + 90 cc 70 per cent ethanol) for 24 hours and then they were transferred to 70 per cent alcohol and stored in it for laboratory studies. Sections were stained with safranin and fast green. For leaf epidermal peelings, small bits of leaf were put in 10% potassium hydroxide solution and then boiled until the epidermis was loosened from the mesophyll and veins. These peelings were mounted in 50 per cent glycerine. For microtome sections, soft parts of the plant were dehydrated in alcohol and xylene series, infiltrated and embedded in paraffin wax (melting point 60-62°C) and sectioned with a rotary microtome at a thickness of 15-20 µm; double staining was done by safranin-fast green combination and sections were mounted in DPX mountant (Khasim, 2002).

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RESULTS AND DISCUSSION

The *Bulbophyllum* is probably the largest pantropical genus with approximately 2400 species (Sieder *et al.,* 2007). The Paleotropical region is the richest of this genus, with hundreds occurring in Asia, followed by Africa and the Neotropics. In India, it is represented by 87 species (Manilal and Kumar, 2004).

B. affine: Rhizome stout, long with dense roots all along; pseudobulbs cylindrical with slightly thickened bases; leaf oblong, obtuse with narrow base. In out line, leaf is angular at the midrib region (Fig. 1A). Cuticle is well developed on both surfaces. Adaxial epidermal cells are larger in their size than the abaxial epidermal cells. The guard cells are clearly seen with

cuticular ledges. The length and width of the stoma is 0.017 µm and 0.023 µm respectively (Table 1b).

B. bisetum: Creeping rhizome with roots; pseudobulbs conical-ovoid, to 2 cm long; leaves thick, fleshy to 15 cm long, lanceolate, obtuse. Inflorescence pendent, densely flowered; flowers olive green and brown, sepals dull-purple, petals purple, lip purple with yellow tip. Epidermal cells are rectangular in shape. Stomatal apparatus with 4 subsidiary cells (tetracytic) confined to abaxial surface only. The length and width of stoma is 0.019 and 0.021 µm respectively. Leaf is thick, fleshy. Adaxial epidermal cells are comparatively larger than abaxial ones (Fig. 1a).

B. careyanum: Pseudobulbs 8 cm long, smooth and 3 cm in diameter; leaf oblong - linear, 20 cm long and 2 cm broad, thick, leathery, tongue shaped. Epidermal cells are polygonal in shape. Stomata are paracytic and confined to abaxial surface only. In out line, leaf is almost flat, slightly angular groove in the midrib region (Fig. 1C). Abaxial side of mid-rib region broadly conical. Epidermis - Cuticle is well developed on both surfaces. Adaxial epidermal cells are comparatively larger than abaxial ones (Fig. 1B). The thickness of the cuticle is 0.009 µm (Table 1b). Guard cells with cuticular ledges (stomatal ledges) are observed (Ramesh and Khasim, 2017). The mid-rib vascular bundle is 0.084 m and the laminar vascular bundle is 0.051 µm (Table 1b). All vascular bundles are associated with sclerenchyma.

B. cauliflorum: B. cauliflorum is known as "Stem-Flowering *Bulbophyllum*". Stomatal apparatus with 2 or 4 subsidiary cells is present on the abaxial surface only. Two-celled absorbing trichomes are confined to abaxial surface only. Two-celled absorbing trichomes are found in abaxial epidermis. Assimilatory cells are thin-walled parenchymatous with chloroplast. Majority of mesophyll cells are hyaline, larger, pleated and function as water storage cells (Fig. 1D).

B. cornutum: A small sized, warm growing epiphyte with pseudobulbs enveloped in long stiff bristles and carry a single, apical, thick leaf. Inflorescence single-flowered. The epidermal cells are polygonal in shape. Absorbing trichomes are absent. Stomata with 4-6 subsidiary cells (cyclocytic) are confined to abaxial surface only. Adaxial epidermal cells are comparatively larger than abaxial ones. The adaxial epidermal cells at the midrib region are slightly elongated. Stomata are confined to abaxial surface only (hypostomatic distribution), 3-celled absorbing trichomes (Fig. 1E). Vascular bundles are in a single series. Larger midrib vascular bundle is in the centre and, small and large laminar vascular bundles on either side of it (Fig. 1E).

B. crassipe: Epidermal cells are isodiametric to polygonal in shape. Stomata with 4 subsidiary cells

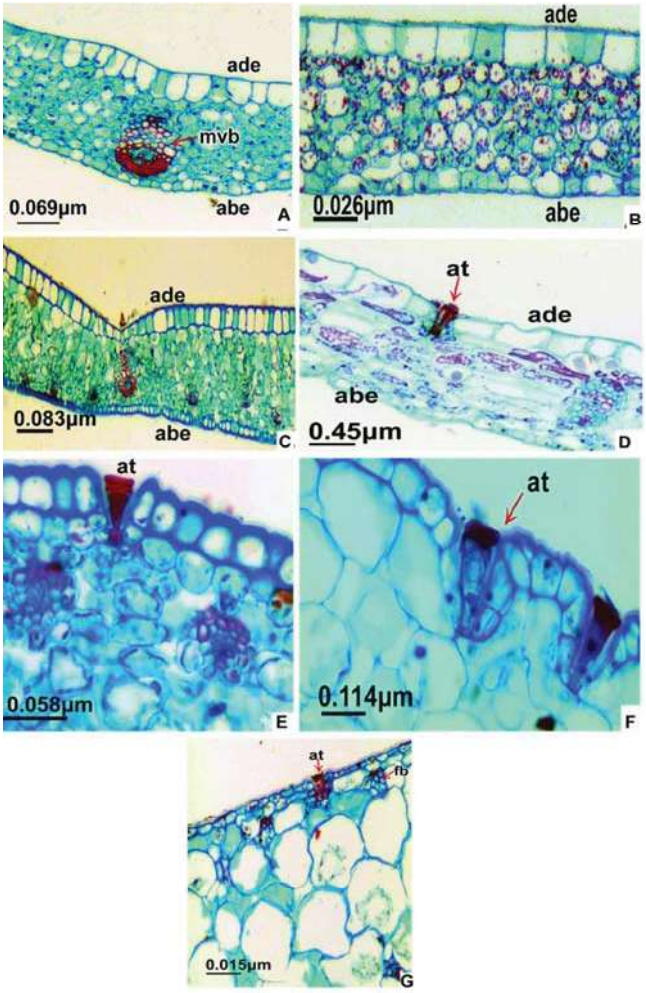

Fig. 1(A-G): Leaf Anatomy of Bulbophyllum

A. *Bulbophyllum affine,* Leaf cross section sowing larger adaxial epidermal cells and midrib vascular bundle.
B. *B. bisetum,* Leaf cross section showing adaxial and abaxial epidermis.
C. *B. careyanum.* Leaf cross section showing larger adaxial epidermal cells and midrib vascular bundle.
D. *B. cauliflorum.* Leaf cross section showing absorbing trichome towards adaxial epidermis
E. *B. cornutum.* Leaf cross section indicating elongated 3-celled absorbing trichome on adaxial epidermis.
F. *B. crassipes.* Leaf cross section showing absorbing trichomes towards adaxial epidermis.
G. *B. fischerii.* Leaf cross section indicating the absorbing trichome and fibre bundles toward adaxial epidermis.

(Table 1) are confirmed to abaxial surface only (hypostomatic distribution). Leaf is angular at the midrib region. Epidermis is well developed with turgid barrel-shaped cells which have thick cuticle on both the surfaces. The adaxial epidermal cells are two times larger in their size than the abaxial epidermis, prominent absorbing trichomes are seen (Fig. 1F).

Table 1a. Morphological and Anatomical Features of Ecological Interest

Sl. No.	Taxa	Habitat	External features	Ade cells size; Stomata, distribution; ssc	Absorbing trichomes
1	*Bulbophyllum affine*	E	Thick leaves, fleshy pseudobulbs	**ade** cells comparatively larger; with 2 or 4 subsidiary cells, prominent stomatal ledges, **h**; well-developed **ssc**	--
2	*B. bisetum*	E	Thick leaves, fleshy Pseudobulb	**ade** cells comparatively larger; with 4 subsidiary cells, h; ssc present	+
3	*B. careyanum*	E	Fleshy psueudobulb, leathery leaves	**ade** cells comparatively larger; with 2 subsidary cells (paracytic), stomatal ledges present **h**; well- developed **ssc**	--
4	*B. cauliflorium*	E	Long-sheathed rhizome, fleshy pseudobulbs	**ade** cells comparatively larger; with 2 or 4 subsidiary cells, **h**; small **ssc**	+
5	*B. cornutum*	E	Thick leaves, fleshy Pseudobulb	**ade** cells are comparatively larger; with 4-6 subsidiary cells (mostly cyclocytic, **h**; **ssc** present	--
6	*B. crassipes*	E	Leathery leaves, fleshy Pseudobulb	**ade** cells are comparatively larger; with 4 subsidiary cells (tetracytic), **h**; ssc present	--
7	*B. fischerii*	E	Leathery leaves, fleshy Pseudobulb	**ade** cells are comparatively larger; with 4 subsidiary cells (tetracytic), **h**; **ssc** present	--

B. fischerii: B. fischerii has small ovoid, 2- leaved pseudobulbs scattered on a slender creeping rhizome; leafless during flowering recemes and inclined. Epidermal cells are rectangular to polygonal in shape. Stomata with 4-5 subsidiary cells are present on the abaxial epidermis only. Epidermis cells are rectangular to squarish. Thick cuticle is covered on both the surfaces. Adaxial epidermal cells are two times larger in size than the abaxial epidermal cells shows fibre bundles towards adaxial epidermis. Vascular bundles are arranged in a single series. Large, midrib vascular bundle is in the centre and, other small and large laminar vascular bundles on either side it. All vascular bundles are associated with sclerenchyma (Fig. 1G).

LEAF - Epidermis : Epidermal cells possess smooth and thin walls in almost all investigated taxa belonging to tribe Dendrobieae. In most of the presently studied taxa, the size of the adaxial epidermal cells is comparatively larger than abaxial ones (Table 1a). Khasim (1996) reported adaxial epidermal cells that are three times larger than abaxial ones in *Paphiopedilum fairrieanum.*

Stomata : The stomata are hypostomatic in distribution, restricted to abaxial surface of leaf. Similarly, hypostomatic distribution is found in other groups of Orchidaceae (Möbius, 1887). Interestingly Vij *et al.* (1991) observed the hypostomatic leaves in mesophytic orchids. Rasmussen (1987) opined that hypostomaty is more frequent in mesophytic orchids, whereas amphistomaty dominates in those of dry and humid habitats.

Absorbing trichomes : The trichomes known to be absorbing in function, are 2 or 3-celled structures with dome-shaped apical cell and basal stalk cell. It preferred to call them as 'Handle cells'. However, in the present investigation, these are observed in some species such as *B. careyianum* and *B. fischerii.*

Hypodermis : In the presently investigated taxa, hypodermis is almost absent. However, fibre bundles at hypodermal position are appeared in *D. anceps.* In *B. careyanum* of present study, a single layer of hypodermis (distinguished from epidermis) without thickenings is reported.

Table 1b. Leaf: Anatomical features in *Bulbophyllum* (in μm)

Sl. No.	Accs. No. Anat. Feat	1	2	3	4	5	6	7
1	Absorbing trichome	-	-	+	-	+	+	+
2	Cuticle thickness	0.008	0.015	0.011	0.009	0.007	0.004	0.012
3	Stomatal width	0.023	0.018	0.021	0.019	0.016	0.024	0.028
4	Stomatal length	0.017	0.012	0019	0.015	0.019	0.011	0.021
5	Midrib vb. Size	0.089	0.075	0.062	0.084	0.058	0.078	0.071
6	Laminar vb. Size	0.041	0.047	0.052	0.051	0.044	0.049	0.057

Mesophyll : In all the investigated taxa, mesophyll is homogeneous, not differentiated into palisade and spongy parenchyma. Mesophyll tissue is tightly packed in some cases, which favours the fixation of carbon through C_4 pathway. Various tracheoidal elements including water storage cells with cellulosic thickenings and without thickenings are observed in the presently studied taxa.

Vascular bundles : In general, vascular bundles are arranged in a single series in all the presently investigated taxa. In all vascular bundles of leaf, phloem is situated towards abaxial side, and xylem towards adaxial side (Ramesh, 2018). The phloem and xylem ends possess with some amount of sclerenchyma (sclerotic sheath). Tracheids with helical thickenings and vessel-like tracheids are abundant in leaf and also other parts of plant body.

CONCLUSION

These data suggest that all the 7 species studied are xeromorphic in nature and can tolerate long periods of drought. *Bulbophyllum cauliflorum,* however, showed maximum xeromorphic features. The leaves of this orchid are very thick and has the thickest adaxial cuticle with extremely small and sunken stomata. Thus, it can be concluded that all the species studies are able to tolerate long periods of drought and efficient in water-use and these traits can be useful for conservation of these species under green house conditions also. The sps. *B. cauliflorum* is showing close affinity with *B. careyanum* which is a taxonomic significance, helps in classifying the large group of monocotyledons.

REFERENCES

Ramesh G and S M Khasim. 2017. Genetic diversity in some Indian Bulbophyllinae (Orchidaceae) with reference to Ecological adaptability and Phylogenetic significance. Acta Horticulture Arditti, J. 1992. *Fundamentals of Orchid Biology*. John Wiley, New York, USA.

Bose T K and Bhattacharjee S K. 1980. *Orchids of India*. Naya Prakash, Calcutta, India.

Brühl P. 1926. *A Guide to the Orchids of Sikkim*. Thaker, Spink and Co., Calcutta, India.

Curtis K M. 1917. The anatomy of six epiphytic species of the New Zealand Orchidaceae. *Ann. Bot.* **31**: 133-149.

Holttum R E 1955. Growth habits of monocotyledons: variations on a theme. *Phytomorphology,* **5**: 399-413.

Hooker J D. 1890. *Hooker's Icones Plantarum*, Vols. **I** and **II**. Williams and New Gate, London, U.K.

Khasim S M and Mohana Rao P R. 1986. Anatomical studies in relation to habitat tolerance in some epiphytic orchids. In: S.P. Vij (ed.), *The Biology, Conservation and Culture of Orchids*. East-West Press, New Delhi. pp. 49-57.

Khasim S M. 1996. Ecological anatomy and conservation of *Paphiopedilum fairieanum* Pfitz. (Orchidaceae) - an endangered species from India. *Newsletter of Himalayan Botany,* **20**: 11-13.

Khasim S M. 2002. *Botanical Microtechnique: Principles and Practice*. Capital Publishing Company, New Delhi.

Khasim S M. 1986. *Anatomical Studies in some Indian Orchidaceae*. Doctoral thesis, Nagarjuna University, India.

Mehra P N and Vij S P. 1974. Some observations on the ecological adaptations and distribution pattern of the East Himalayan Orchids. *Amer. Orchid Soc. Bull.* **43**: 301-315.

Möbius M. 1887. Über den anatomischen Bau der orchideenblätter und dessen Bedeutang fur das system dieser Familie. *Jahrb. Wiss. Bot.* **18**: 530-607.

Mohana Rao P R and Khasim S M. 1986. Leaf anatomy in relation to ecological adaptability of some Himalayan Orchids. *J. Swamy Bot. Cl.* **3**: 9-13.

Pijl L Van Der and Dodson CH. 1966. Orchid Flowers: Their Pollination and Evolution.

Miami Univ. Press, Florida, USA.

Pridgeon A M. 1986. Anatomical adaptations in Orchidaceae. *Lindleyana* **1**: 90-101.

Ramesh G and SM Khasim, 2017. Genetic diversity in some Indian Bulbophyllinae (Orchidaceae) with reference to Ecological adaptability and Phylogenetic significance. Acta Horticulture Arditti, J. 1992. *Fundamentals of Orchid Biology.* John Wiley, New York, USA.

Ramesh G and Khasim S M. 2018. Morpho-anatomical and molecular characters of *Bulbophyllum* and *Dendrobium* spp. Found in Southern Ghats of India. *Current Horticulture*. **6**(1). 50-54.

Ramesh G and Khasim S M. 2019. The Orchid Root: Anatomical and Ecological Adaptability with Special Reference to Bulbophyllum and Dendrobium in the Book of BMPO 2018 Proceedings: Orchid Biology: Recent Trends and Biology, Springer International in 2019. 2 Chapter entitled "Orchid. The diversity of stomatal development in Orchidaceae, subfamily: Orchidoideae. *Bot. J. Linn. Soc.* **82**: 381-393.

Rasmussen H. 1987. Orchid stomata--structure, differentiation, function and phylogeny. *In:* J. Arditti (ed.), *The Orchid Biology: Reviews and Perspectives"*, Vol. **IV.** Cornell University Press, Ithaca, USA. pp. 107-138.

Sieder A, Rainer H. and Kiehn M. 2007. CITES checklist for *Bulbophyllum* and allied taxa (Orchidaceae). Botanical Garden of Vienna. http://www.cites.org/ common/com/NC/ tax_ref/Bulbophyllum.

Stern W L and Morris M W. 1992. Vegetative anatomy of *Stanhopea* (Orchidaceae) with reference to pseudobulb water-storage cells. *Lindleyana,* **7**: 34-53.

Stern W L, Morris M W and Judd W S. 1994. Anatomy of the thick leaves in Dendrobium section *Rhizobium* (Orchidaceae). *Int. J. Pl. Sciences,* **155**: 716-729.

Vij S P, Kaushal P S and Kaur P. 1991. Observations of leaf epidermal features in some Indian orchids: taxonomic and ecological implications. *J. Orchid Soc. India,* **11**: 93-97.

Williams N H. 1976. Subsidiary cell development in the *Catasetinae* (Orchidaceae) and related groups. *Bot. J. Linn. Soc.* **72**: 299-309.

Stability and Validity Indicating UV Spectrophotometric Rapid Assay Method for The Estimation of Promethazine Theoclate

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Abstract

Asimple, selective, linear, precise and accurate UPLC method was developed and validated for rapid assay of Promethazine Theoclatein Bulk and Pharmaceutical tablet Formulation. The chromatographic analysis was performed on a Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell at ambient temperature. The UV detection wavelength was observed at 252nm and injected sample is 20µl. Isocratic elution at a flow rate of 1.0ml/min was employed. The Mobile phase consisting of 0.05M KH_2PO_4 (to pH 3 with Phosphoric acid) 95V/V and Acetonitrile 5V/VMethanol and waterin the ratios of 50v/v and 50v/v. The Rt for Etophyllinewas ± 12 minutes approximately is identified. The Average percentage recovery of the method was in the range of 99-102%. The method was validated as per the ICH guidelines. The method was successfully applied for routine quality control analysis of pharmaceutical formulation.

Key Words: Promethazine Theoclate, UV Spectroscopy, Suitability, Recovery, Precise

1. Introduction

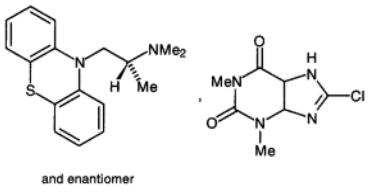

Fig.- 1: Structure of Promethazine Theoclate

A white or almost white powder, very soluble in water, sparingly soluble in ethanol (96%), practically insoluble in ether. The molecular formulae is $C_{17}H_{20}N_2S$,$C_7H_7ClN_4O_2$. It is a generic medication under many brand names globally.[1]Promethazine, a phenothiazine derivative, is structurally different from neuroleptic phenothiazines, with similar effects.[2]Promethazine is a first-generation antihistamine.[3] It is taken by mouthor by injection into muscle.[3]It is in the phenothiazine family of medications.[3]It is used to treat allergies, trouble sleeping, and nausea. It may also be used for sedating people who are agitated or anxious.[4,5] It is unclear if use during pregnancy or breastfeeding is safe for the baby.[3,4] Use by injection into a vein is not recommended due to potential skin damage.[4] Promethazine made in 1940s by a team of scientists from Rhône-Poulenc laboratories.[6] The cost by mouth formulation is less than 0.20 USD per dose as of 2018.[7] Another notable use of promethazine is a local anesthetic, by blockade of sodium channels.[8]It is a chiral compound, occurring as mixture of enantiomers.[9]It was first synthesizedin the early 1940s.[10] The team was seeking to improve on diphenhydramine; the same line on medical chemistry led to the creation of chlorpromazine.[11]Thumma S et.al.,[12] proposed PMZ in hot-melt extruded. Separation was achieved on a 150 mm x 4.6 mm i.d., 3 microm particle size, C8(2) column by acetonitrile-25mM phosphate buffer (pH 7.0), 50:50 (v/v) as mobile phase at a flow rate of 1 mL min^{-1}. UV detection is 249 nm. Jagdish Kakadiya et.al.,[13] proposed PMZ and PCM. The separation was carried out on Hyperchrom ODS-BP (4.6 mm X 250 mm, 5 µm) column using Methanol: Water with 1% TEA in the ratio of 30:70 v/v as eluent. The flow rate was 1ml/min and effluent was detected at 250 nm. Rt were 3.10 and 1.72min.Linear range is 2-10 µg/ml for PMZ and 50-250 µg/ml PCM. % recoveries were 99.00-100.33% and 99.56-101.28%. V. D. N. Kumar Abbaraju et.al.,[14] proposed Flucloxacillin Sodium in tablet Formulation. Flow rate is 1.0ml/min was employed on symmetry Bondpeak C18 10 µm (3.9 x 300 mm). The mobile phase consisted of Acetonitrile and 0.0.1M Phosphate buffer in the ratio of 35:65/v/v. Wavelengthis 273nm and 10µl sample is injected. The Rt is 1.5min. % RSD for precision and accuracy of method is 0.1%.

2. Experimental

2.1 Instrumentation,

Peak HPLC containing LC 20AT pump and variable wavelength programmable UV-Visible detector and Rheodyne injector was employed for investigation. The chromatographic analysis

was performed on a Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell. Degassing of the mobile phase was done using a Loba ultrasonic bath sonicator. A Denwar analytical balance was used for weighing the materials.

2.2 Chemicals and solvents

The reference powdered sample Content of Theophen Compound tablets Anhydrous is collected. The Formulation was procured from the local market. KH2PO4, Phosphoricacid, Acetonitrile, water and buffer used were of HPLC grade and purchased from Merck Specialities Private Limited, Mumbai, India.

2.3 The mobile phase

The Mobile phase consisting of 0.05M KH_2PO_4 (pH 3 with Phosphoric acid) 95V/V and Acetonitrile 5V/V Methanol and water in the ratios of 50v/v and 50v/v.

2.4 Preparation of solutions

Standard Solution is prepared byweighing 50 mg of Promithazine Theoclate standard into a 100 ml standard flask. Added 5 ml water and 1 ml concentrated ammonia to the flask, mix, allow to stand for 5 minutes and make up to volume with absolute ethanol. Diluted 5 ml to 50 ml with absolute ethanol in an amber standard flask. Diluted a further 5 ml to 50 ml in standard flask and marked up to the mark by absolute ethanol. Sample solution is prepared by weighing 20 tablets, determined average mass and grind to a fine powder. Accurately weighed 500 mg of sample into a 100 ml standard flask. Added 5 ml water and 1 ml concentrated ammonia to the flask, mixed, allowed to stand for 5 minutes. Added 30 ml of absolute ethanol, sonicated for 5 minutes and allowed to cool. Maked up to volume with ethanol and filtered. Diluted 5 ml of this solution to 50 ml by absolute ethanol in 50 ml standard flask. Further diluted 5 ml to 50 ml in standard flask and make up to mark by absolute ethanol. Blank solution is prepared by diluting 0.05 ml concentrated ammonia to 50 ml standard flask with absolute ethanol. Further diluted 5 ml of this solution to 50 ml standard flask with absolute ethanol.

3. Method Development

A systematic study of the effect of various factors was undertaken by varying one parameter at a time and keeping all other conditions constant. Method development consists of selecting the appropriate wave length and choosing stationary and mobile phases. The following studies were conducted for this purpose. The spectrum of 10 ppm solution of Theophen Compound Elixirwas recorded separately on UV spectrophotometer. The peak of maximum absorbance wavelength 252nm was observed. Finally the expected separation and peak shapes were obtained on Jasco Model: V-630,

with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell. Flow rates of the mobile phase were changed from 0.5-1.5 ml/min for optimum separation. It was found from experiments that 1.0ml/min flow rate was ideal for elution of analyte. The injection volume is estimated at 20 µl. Approx. Rt. for Theophylline Anhydrous is+ 12 minutes

4. Validation of the proposed method

4.1 Specificity

The Blank and placebo solution spectra must not show any absorbance corresponding in wavelength to that of the active compound. The spectra obtained from the drug active and sample must correspond. The blank and placebo does not to have any absorbance at specified wavelength corresponding to Promithazine Theoclate and drug active and product have corresponding absorbencies at about 203 nm, 255 nm and 276 nm. Therefore it is considered spectrally pure.The solvent and placebo solutions must contain no components, which co-elute with the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous. The peak purity results from the photo diode-array analysis must show that the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous peak is pure – i.e. the purity angle must be less than the threshold angle. The solutions listed below were injected using the conditions specified in the method of analysis. Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous stable are under UV light exposure. No components are seen to co-elute with the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous peaks, and the peak purity results indicate that the Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous peaks can therefore be considered spectrally pure. The chromatograms are shown from figure 2 to 8.

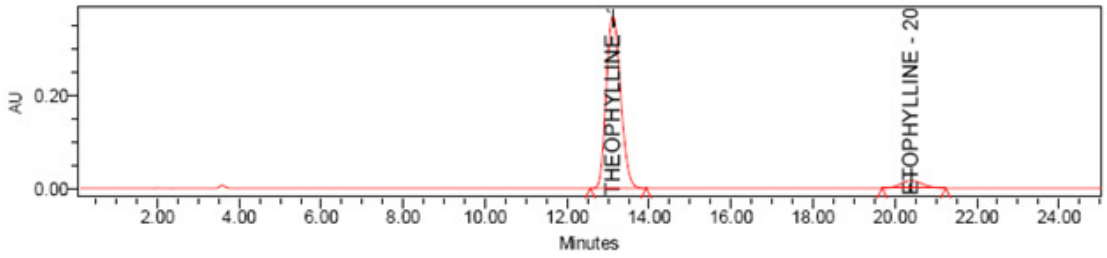

Fig.- 2:Chromatogram 1: Drug active – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

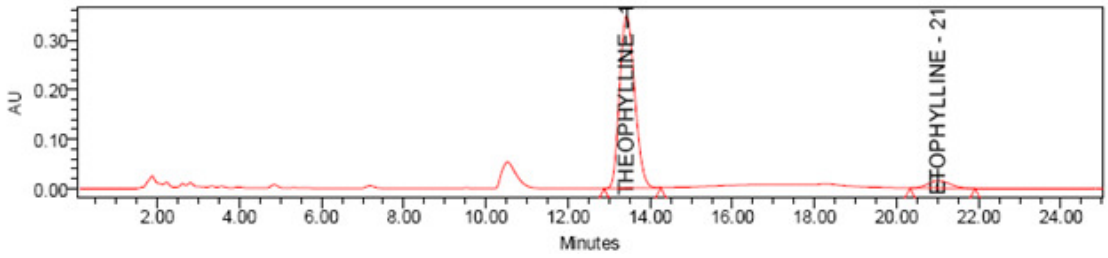

Fig.- 3:Chromatogram 2: Product – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

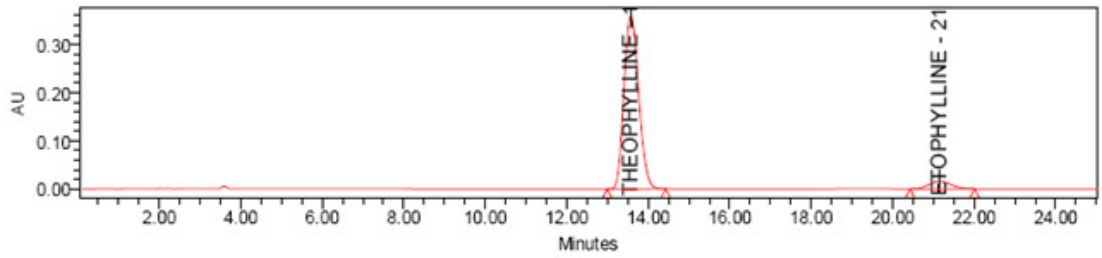

Fig.- 4:Chromatogram 3:Active – UV stress: Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

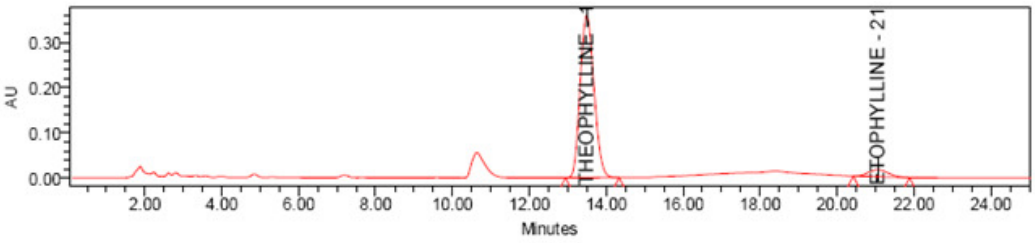

Fig.- 5:Chromatogram 4: Product – UV stress: Peak due Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

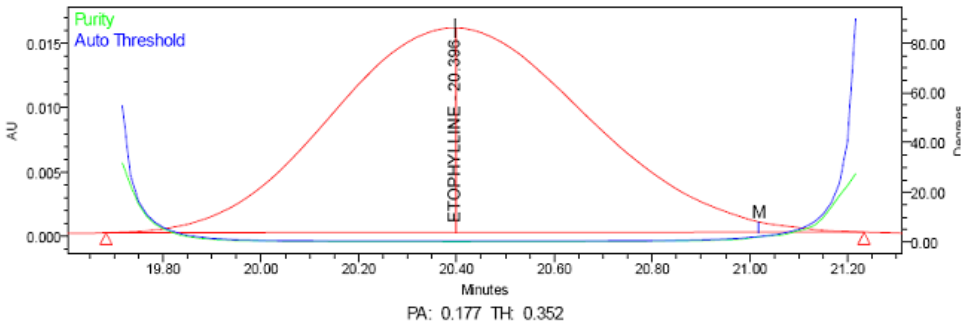

Fig.- 6: Peak Purity 1: Drug Active Etophylline (β Hydroxyethyl Theophylline)Purity Angle <Threshold: 0.177 < 0.352

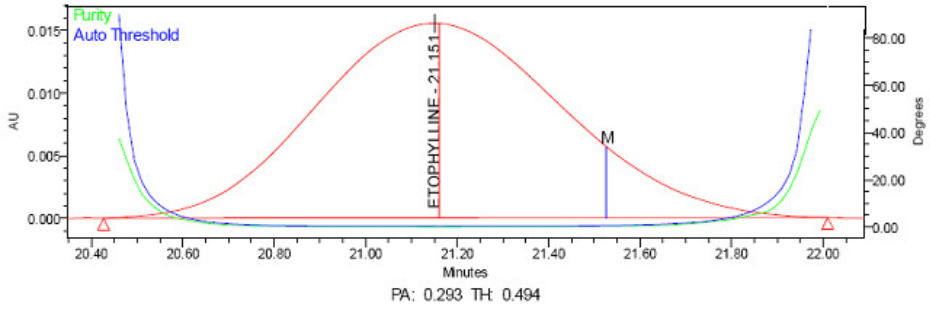

Fig.- 7. Peak Purity 2: Drug Active Etophylline (β Hydroxyethyl Theophylline)Purity Angle <Threshold: 0.293 < 0.494

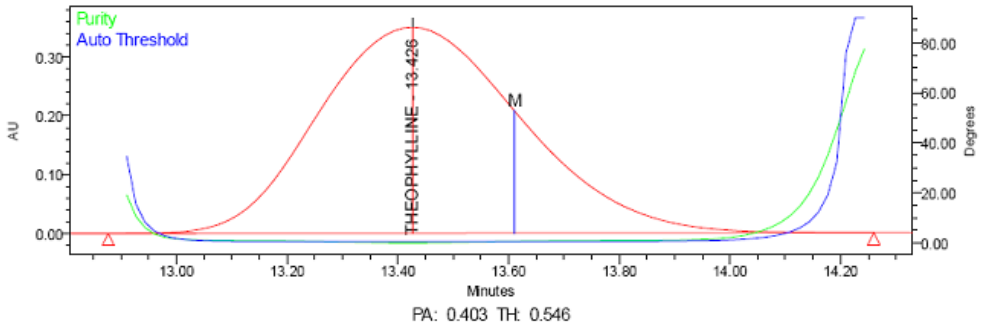

Fig.- 8: Peak Purity 3: Theophylline Peak Purity Angle <Threshold: 0.403 < 0.546

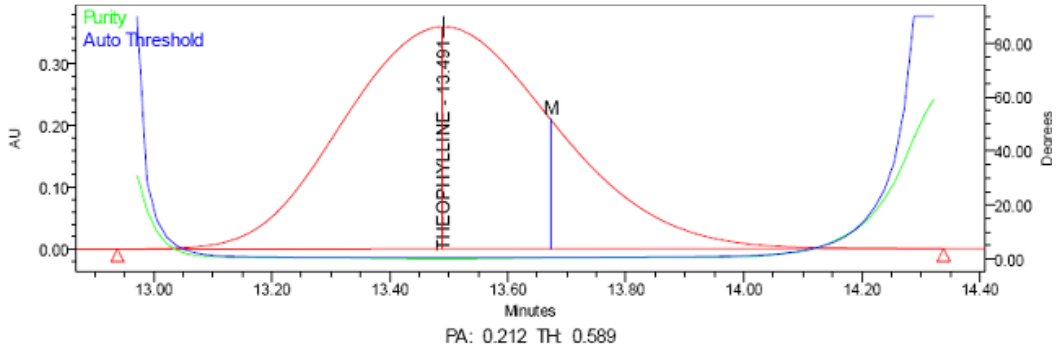

Fig.-9: Peak Purity 4: Theophylline UV Stressed Drug Product

4.2 System Suitability

The % RSD of the peak responses due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous for the six replicate injections for each must be less than or equal to 2.0 %. Six replicate injections of working standard solution were injected according to the method of analysis. The % RSD for the peak responses was determined. The analytical system

complies with the requirements specified by the system suitability. The results are tabulated in the table 1.

Sample	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline
1	583050	8322581
2	584320	8333629
3	584644	8339267
4	585002	8334340
5	585018	8329352
6	584723	8350223
Mean	584460	8334899
% RSD	0.1	0.1

Table1. System Suitability results

4.3 Linearity

The correlation coefficient of the regression line for Promethazine Theoclate should be greater than or equal to 0.99. The Y-intercept of the line should not be significantly different from zero, i.e. the assessment value (z) falls within the specified limits only when +5 > z > -5. Five solutions containing 50, 75, 100, 125, and 150 % of Promithazine Theoclate, relative to the working concentrations, were prepared and read according to the method of analysis. A linear regression curve was constructed, and the R^2 and assessment values calculated. The R^2 for Promithazine Theoclate is 1.00. The plot is a straight line, and the assessment value (z) is 1 for Promithazine Theoclate. Linearity Curve was shown in Figure 10 and results are in table 2.

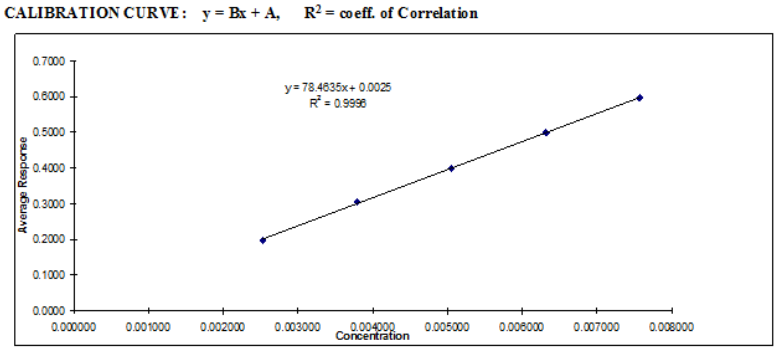

Fig.- 10. Linearity curve

Sample Number	Concentration	Response 1	Response 2	Average Response
50%	0.002528	0.1970	0.1970	0.1970
75%	0.003791	0.3050	0.3050	0.3050
100%	0.005055	0.3990	0.3990	0.3990
125%	0.006319	0.4990	0.4990	0.4990
150%	0.007573	0.5950	0.5950	0.5950

Table 2. Linearity results

4.4 Accuracy

The percentage recovery of the active compounds for each solution prepared are within 95.0 – 105.0 % of the actual amount. Sample solutions were spiked with known concentrations of Promethazine Theoclate to result in concentrations representing 50, 75, 100, 125, and 150 % of Promethazine Theoclate relative to the working concentration. The absorbance of the above samples was read in duplicate according to the method of analysis. From the accuracy results above, the percentage recovery values for Promethazine Theoclate satisfy the acceptance criteria for accuracy across the range of 50 - 150 %. Results are tabulated in table 3.

Sample	Theoretical Weight in mg	Response(Abs)	Average Response(Abs)	Average Actual Weight in mg	Average % Recovery
50%	24.31	0.189	0.189	23.90	98.3
50%		0.189			
75%	36.47	0.283	0.283	35.79	98.1
75%		0.283			
100%	48.61	0.379	0.379	47.93	98.6
100%		0.379			
125%	60.77	0.474	0.474	59.82	98.4
125%		0.473			
150%	72.92	0.563	0.563	71.20	97.6
150%		0.563			

Table 3. Accuracy Results

4.5 Method precision

4.5.1Repeatability

The % RSD due to Promethazine Theoclate concentration for the six samples must be less than or equal to 2.0 %. Six separate sample preparations of batch 241645 were analyzed according to the method of analysis. The % RSD due to Promethazine Theoclate concentration for the assay meets the requirements. Results are tabulated in table 4.

Sample number	Results (mg/tab)
	Promethazine Theoclate content
1	25.15
2	25.22
3	25.36
4	24.96
5	25.08
6	25.14
Mean	25.15
% RSD	0.5

Table 4. Repeatability results

4.5.2Intermediate Precision

The % RSD due to Promethazine Theoclate concentration for the six samples must be less than or equal to 2.0 %. The mean results obtained in the repeatability, and the intermediate precision must not differ by more than 3.0 %. Six separate sample preparations of batch 241645 were analysed according to the method of analysis. The % RSD for intermediate precision is 0.4 %.The intermediate precision and repeatability comply as they differ by 0.1 %. Results are tabulated in table 5.

Sample	Results (mg/tab)
	Promethazine Theoclate
1	24.99
2	25.05
3	25.09
4	25.18
5	25.16
6	25.24
Mean	25.12

% RSD	0.4
Sample	Mean Results (mg/tab)
	Promethazine Theoclate
Repeatability	25.15
Intermediate Precision	25.12
Mean	25.14
% RSD	0.1

Table 5: Intermediate precision results

4.5.3 Stability Of Solution

The assay results showed that no loss of active in the sample preparation while standing for time period of 24 hours. The % RSD due to the β-Hydroxyethyl Theophylline and Theophylline Anhydrous content for the samples is 1.5 %. The stability of solution mean of β-Hydroxyethyl Theophylline and Theophylline Anhydrous deviates from the obtained intermediate repeatability mean by 0.50 %.

4.5.4Range

Based on the accuracy results, the range for the assay of Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous is 5.0 – 15.0 mg/30ml and 50.0 – 150.0 mg/30 ml respectively, which represents 50 % to 150 % of the working concentration.

5. Resultsand Discussion

Several mobile phase compositions were tried. The chromatographic analysis was performed on a Jasco Model: V-630, with a fixed bandwidth 2 nm and a pair of 1cm matched quartz cell at ambient temperature. A satisfactory separation and good peak symmetry was found in a mixture of 0.05M KH_2PO_4 (to pH 3 with Phosphoric acid) 95V/V and Acetonitrile5v/v are used as a mobile phase. Flow rate proved to be better than the other mixtures in terms of resolution and peak shape. The wavelength for detection was set at 254nm at which much better detector responses for drug was obtained. The Rtfor β-Hydroxyethyl Theophylline and Theophylline Anhydrous is ± 12 minutes. A system suitability test was applied to representative chromatograms for various parameters. The results obtained were within acceptable limits. Therefore the system meets suitable criteria. The calibration curve was obtained for a series of

concentration in the range of 0.6-1.4µg/ml and it was found to be linear. Five points graphs was constructed and identified that the correlation coefficient (R^2) for β-Hydroxyethyl Theophylline and Theophylline Anhydrous is 0.0007.The plot is a straight line and the assessment value (z) is 0.6 for β-Hydroxyethyl Theophylline and Theophylline Anhydrous. All these factors lead to the conclusion that the proposed method is accurate, precise, simple, sensitive and rapid and can be applied successfully. The Cleaning Validation method is proven to be valid and the validation test results show that the method complies with the validation requirements.The method is therefore acceptable as valid.

6. Conclusion

The method for the assay of Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous in Theophen Compound elixir complies with the requirements for linearity, specificity, system suitability, method precision and accuracy across the range of 50% to 150% of the working concentration. The method therefore indicates stability and is acceptable and valid.The developed method is also found to be precise and robust for the simultaneous determination of Promethazine hydrochloride and Theophylline Anhydrous in liquid dosage forms.

8. REFERENCES

1. "Promethazine international brands". Drugs.com. Retrieved 17 July 2017.
2. Strenkoski-Nix LC, Ermer J, DeCleene S, Cevallos W, Mayer PR. *American Journal of Health-System Pharmacy*,57,16 (2000)
3. "Promethazine Hydrochloride Monograph for Professionals". Drugs.com. American Society of Health-System Pharmacists. Retrieved 24 October2018.
4. British national formulary, British Medical Association,BNF 74 (74 ed.). p. 276. (2017).
5. Malamed, Stanley F. Sedation, A Guide to Patient Management. Elsevier Health Sciences, p. 113(2009).
6. Li, Jie Jack. Laughing Gas, Viagra, and Lipitor: The Human Stories behind the Drugs We Use,Oxford University Press,United Kingdom, p. 146(2006).
7. "NADAC as of 2018-10-24". Centers for Medicare and Medicaid Services. Retrieved 24 October 2018.

8. Jagadish Prasad, P. Conceptual Pharmacology. Universities Press, pp. 295,303,598(2010).
9. "RxList: Promethazine Description". 06, 21(2007)
10. Ban, TA, Dialogues in clinical neuroscience. 8, 3(2006)
11. "Paul Charpentier, Henri-Marie Laborit, Simone Courvoisier, Jean Delay, and Pierre Deniker". Science History Institute. 6, 2015.
12. Thumma S, Zhang SQ, Repka MA, *Pharmazie*, 63, 8 (2008)
13. Jagdish Kakadiya, Naiya Parmar , Nehal Shah, *Asian Journal of Research in Biological and Pharmaceutical Sciences*, 2, 1(2014)
14. V. D. N. Kumar Abbaraju & V. Sreeram, *International journal of AppliedPharmaceutical and Biological Research,* 2, 3 (2017)

Stability and Validity Indicating Assay Method for Estimation of Multi drug components of Etophylline and Theophylline Anhydrous – Forced Degradation Studies

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Abstract

A very simple, more selective, high linear, most precise along with accurate HPLC process is advanced also validated to rapid assay of Etophylline and theophylline which are in Bulk and in pharma oral system formulation. Chromatographic investigation is subjected by utilization of Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column at ambient temperature. UV detection wavelength is noted at 254nm, sample passed into system as 20µl. Rate of flow is 1.0ml/min. Iin this process utilized mobile phase as 0.05M KH_2PO_4 whose pH is adjusted to 3 by phosphoric acid as 95V/V along with acetonitrile as 5V/V. Rt to Etophylline is ± 12 minutes, run time is 19 min. Average percentage recovery is in the range of 99-102%. The method is favorably applied to periodic quality control analysis of pharma formulation.

***Key Words*:** Etophylline, Theophylline, UV Spectroscopy, Suitability, Recovery, Precise.

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1. Introduction

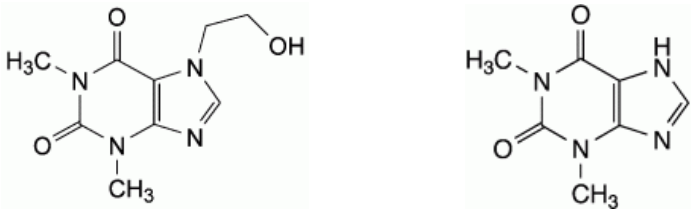

Fig:1 Structure of Etophylline and Theophylline Anhydrous

Etophylline also represented as β- Hydroxyethyl Theophylline which is stored in very tight, light antagonistic container. Proposed drug is white, more crystalline powder. This drug is more soluble in water, in rare cases low soluble in alcohol. Theophylline anhydrous is white also highly crystalline powder. This drug is a little soluble in water, faintly soluble in ethanol. It subjects to dissolution in various solutions like alkali hydroxides, ammonia along with various mineral acids. Tablets includes Etophylline, theophylline are more effective ingredients. For this chemical name is 7-(2-hydroxyethyl)-1, 3-dimethyl-3, 7dihydro-1H-purine-2, 6-dione[1]. THEO indicated to chronic obstructive problems of air ways, COPD bronchial asthma, child apnea[2]. Various processes[3-6]are characterized to determine ETO individually also in combination by different drugs. Procedure include serum RP-HPLC[7],TLC methods[8-9]along with derivative spectrophotometric process[10]. Devang.N.A et.al.,[11] proposed both Etophylline and Theophylline in injectable forms. Quantification is borne by utilizing Silica gel column, which is packed by octadecylsilane, 2.1 mm × 100 mm, i.d, 1.7µm particle size. In this process proposed segregation is carried in isocratic mode which consists a mixture of 0.05 M Sodium Acetate along with acetonitrile as 90:10 composition, pH is noted as 4.5 flow rate is fixed at 0.3 ml / min. Detected wavelength as 270nm. Retention time for Theophylline is 2.368 min and for Etophylline is 3.129 min. Recovery to proposed drugs are in the range of 99.84 - 100.89%. S.Nagajyothi et.al.,[12] proposed Etophylline as ETO ,Theophylline as THEO in bulk along with solid dosage forms. Wavelengths to both selected drugs are 271nm for ETO, 273nm THEO. V. D. N. Kumar Abbaraju et.al.,[13] proposed Pholcodine at rate of flow as 1.0ml/min column utilized Symetry or Luna C8 5µm (4.6 x 150 mm). Mobile phase consists of methanol as 70v/v, water as 20v/v along with acetonitrile as 10v/v. For this process wavelength noted as 220nm, sample passed in to column as 100µl. Run time as 10minutes, flow rate is 1.5ml. % R.S.D for this drug is 0.4. LOD is identified as 15µg/ml and LOQ is noted at 25µg/ml. Mean percentage recovery is 80%. Finally by studying above literature developed process is simpler, superior to other methods reported in the literature.

2. EXPERIMENTAL

2.1 Instrumentation

For this investigation peak HPLC containing LC 20AT pump along with changeable wavelength programmable UV-Visible detector, Rheodyne injector is engaged for through check. chromatographic examination and determination is accomplished over Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column. Degassing of mobile phase is done utilizing Loba ultrasonic bath sonicator. Denwar analytical balance is operated inorder to weigh materials.

2.2 Chemicals and solvents

To this analysis grinded sample content of Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous is poised. KH_2PO_4, Phosphoric acid, Acetonitrile, water along with buffer utilized are HPLC grade those are procured from Merck, India.

2.3 Mobile phase

A mixture of 0.05M KH_2PO_4 which pH is adjusted to 3 by phosphoric acid taken as 95v/v and acetonitrile 5v/v is utilized.

2.4 Ground work for solutions

Mobile phase preparation

Exactly 6.8045g of KH_2PO_4 is taken in 500ml beaker which contains water, after that adjusted pH to 3 by phosphoric acid. Transferred same solution in 1000ml standard flask. After that subjected dilution to suitable volume by water. Latter mixed with Methanol in the ratio which is provided to dilute fragments. Filtered and run over line in ratio likely by methanol.

Standard Solution of Etophylline and Theophylline

Exactly 67.00 mg β-Hydroxyethyl Theophylline RS standard is taken in 100ml standard flask this is noted as solution A. After that subjected to dissolution latter diluted to suitable capacity by methanol. In order to prepare theophylline, exactly 67.00mg Theophylline Anhydrous taken in 250ml flask. To this about 10ml of solution A is pipetted and transferred in 250ml. After that subjected to dissolution and diluted to suitable volume by mobile phase. Finally, filtered through 0.22 µm filter by separating first 5ml filtrate.

Preparation of Specimen

Exactly 10.00gm fragment is taken in 100ml standard flask. To this added 50.00ml mobile phase and subjected to sonication for mixing. After that diluted to volume by above said mobile phase. Subjected to filtration by 0.45µm filter. Finally discard first 5ml of filtrate.

3. Development of Method

Various optimized circumstances are followed to conclude in bulk samples along with its combined assorted oral tablet formulations. Obtained chromatograms of both standard and sample are recorded. Spectrum to 10ppm solution of Theophen compound elixir is documented separately over spectrophotometer. Finally peak of maximum absorbance wavelength is 254nm is recognized. Expected separation along with shapes of peak is recognized over Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column. To identify rate of flow various mobile phase are alternated as 0.5-1.5 ml/min to get excellent disengagement. Finally ideal rate of flow is founded as 1.0ml/min inorder to elute analyte. Volume injected as 20 µl. Approx. Rt. to Theophylline Anhydrous is± 12 minutes.

4. Validation Of Proposed Method

Different parameters designed to validation are specificity, linearity, precision, accuracy, robustness, system suitability, limit of detection, limit of quantification and solution stability.

4.1 Specificity

Both solvent along with inactive drug solutions won't accommodate any constituents, those co-elute by Etophylline also anhydrous of Theophylline. Purity in peak values by photo diode-array investigation showed that both peaks are pure indicates that angle of purity is inferior to angle of threshold. Solutions like Etophylline and Theophylline anhydrous passed by utilizing required circumstances which is proposed in analysis method are stable under UV light vulnerability. There are no other components observed for co-elute by Etophylline along with Theophylline anhydrous peaks. Finally results of purity peak demonstrate both Etophylline and Theophylline anhydrous deliberated as spectrally pure. Finally from observations the chromatograms obtained are shown in the figure 2 to 5, peak purity results are shown in figure 6 to 9.

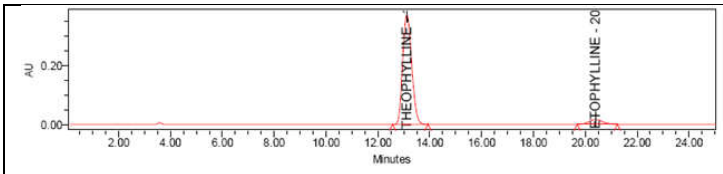

Fig: 2 Chromatogram 1: Drug active – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

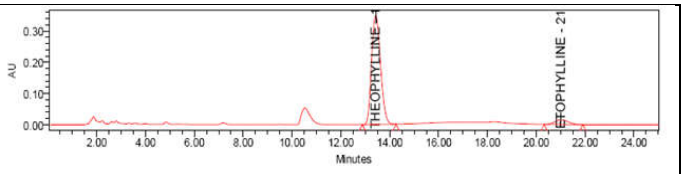

Fig: 3 Chromatogram 2: Product – Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

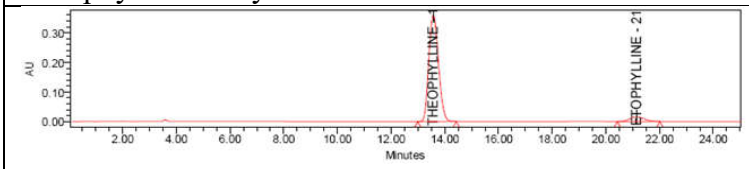

Fig: 4 Chromatogram 3: Active – UV stress: Peak due to Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

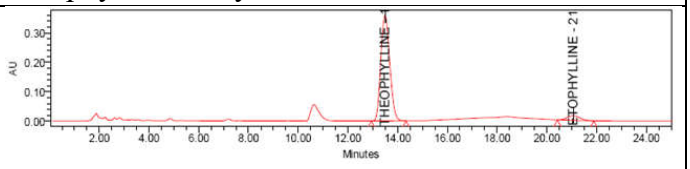

Fig: 5 Chromatogram 4: Product – UV stress: Peak due Etophylline (β-Hydroxyethyl Theophylline) and Theophylline Anhydrous

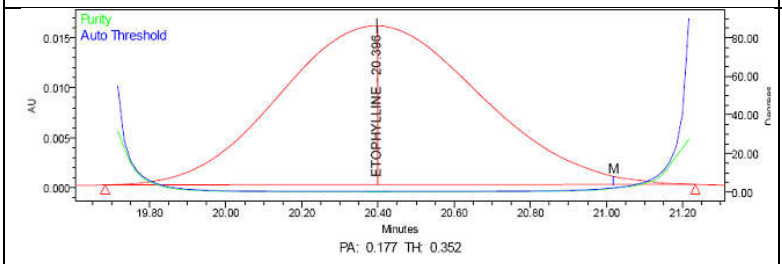

Fig: 6 Peak Purity 1: Drug Active Etophylline (β Hydroxyethyl Theophylline) Purity Angle <Threshold: 0.177 < 0.352

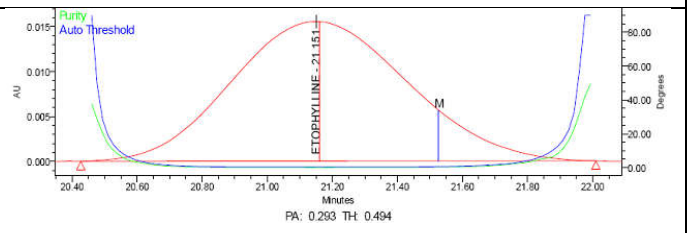

Fig: 7: Peak Purity 2: Drug Active Etophylline (β Hydroxyethyl Theophylline) Purity Angle <Threshold: 0.293 < 0.494

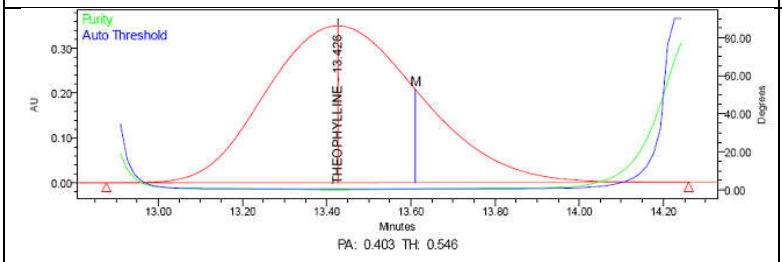

Fig: 8: Peak Purity 3: _Theophylline Peak Purity Angle <Threshold: 0.403 < 0.546

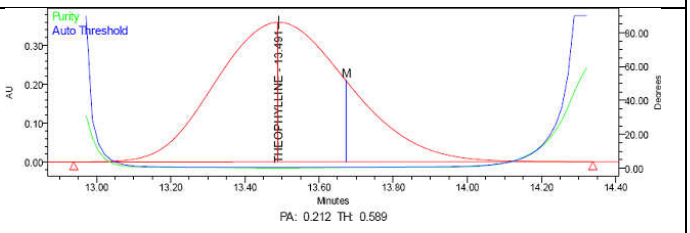

Fig: 9: Peak Purity 4: Theophylline UV Stressed Drug Product

Forced Degradation Studies:

These studies are performed by considering various sample preparations which includes specificity by forced degradation, preparation of sample, construction of placebo, 1.0N Hcl stressed sample, sample of 1.0N NaOH stressed, sample of 3.0% w/v Hydrogen Peroxide stressed, stressed

sample of neutral, sample which is exposed to UV light, sunlight along with sample which is also stressed by thermal denoted as dry heat. Finally these studies are concluded as total degradation products were taken out from Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peak also each other. Peak purity angle is minimum to purity threshold by waters empower-2 software. Finally it is concluded samples are undergoing degrading in both circumstances. Also found that in sunlight condition Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peaks are subjected to degradation normally, for remaining no degradation is identified. Anyhow total degradation products are separated very well by Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peak along with each other. Etophylline (β-Hydroxyethyl Theophylline), product Theophylline anhydrous peak and impurities peaks are more pure. Due to this reason related substances method is considered more specific & highly stability indicating.

4.2 System Suitability

% RSD of response in peak is mainly as a result of Etophylline along with Theophylline anhydrous to total six simulate injections to each and every prerequisite be minimum as 2.0 %. Total six replicate injections of working standard sample solution are passed by considering analysis of method. % RSD to peak responses are measured. Investigative system accumulate by prerequisite which is particularized by this parameter. Finally results obtained are tabulated in the table: 1.

Sample	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline
1	583050	8322581
2	584320	8333629
3	584644	8339267
4	585002	8334340
5	585018	8329352
6	584723	8350223
Mean	584460	8334899
% RSD	0.1	0.1

Table:1 System Suitability results

4.3 Linearity

Correlation coefficient for regression line to β-Hydroxyethyl Theophylline along with Theophylline anhydrous are very nearer to value 0.99. Y-intercept for line to assess z lowering along specified range only when + 5 > z > - 5. Total five solutions which contains 50, 75, 100, 125, and 150 % of β-Hydroxyethyl Theophylline along with Theophylline anhydrous relative for working strength, those are designed after that injected as per procedure of analysis. By values obtained a linear regression curve is assembled, after that both R^2 along with computation values are determined. R^2 to both β-Hydroxyethyl

Theophylline and Thephylline anhydrous are equal to 1.0. Both plots constructed are appeared as straight lines finally value of z for both were 0.49 and 1.99. Therefore, from total results obtained concluded that analysis for Etophylline and Theophylline in Theophen Compound Elixir are noted as linear. Results obtained are tabulated in table.2 and linearity curve is shown in the fig 10.

Sample Number	Concentration	Response 1	Response 2	Average Response
50%	0.134000	4261261.0000	4264119.0000	4262690.0000
75%	0.201200	6293041.0000	6296606.0000	6294823.5000
100%	0.268000	8422748.0000	8422146.0000	8422447.0000
125%	0.335200	10447659.0000	10467901.0000	10457780.0000
150%	0.402000	12438737.0000	12435040.0000	12436888.5000

Table:2 Linearity results

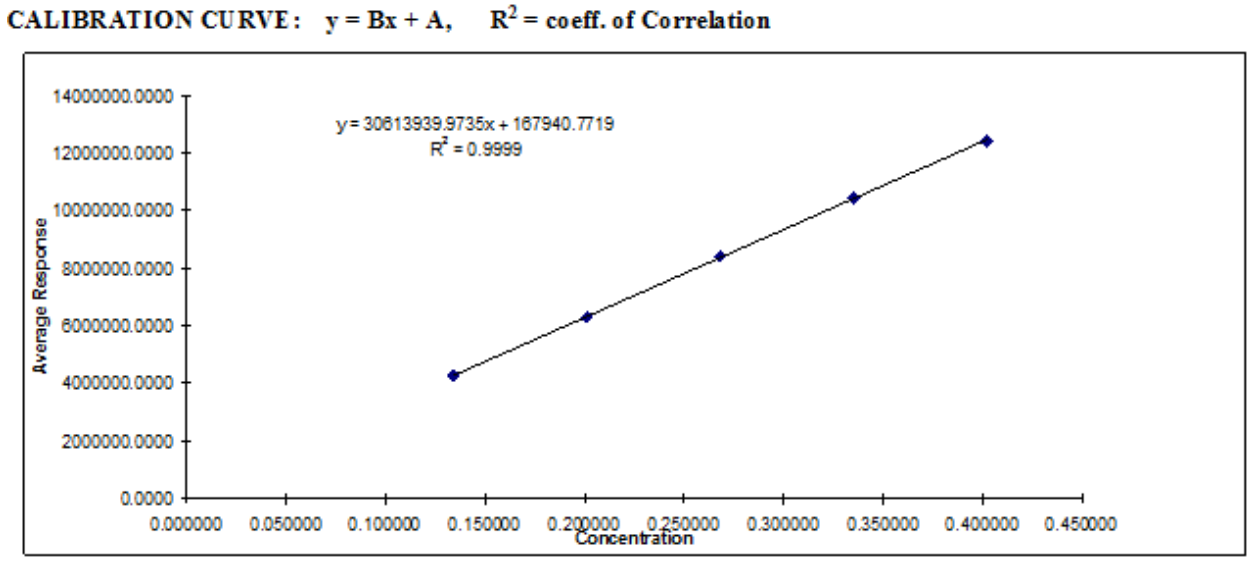

Fig: 10 Linearity curve

4.4 Accuracy

To assess this parameter percentage recovery of active substance to each solution constructed is in the range of 95.0 – 105.0 % to actual amount. collected samples are subjected to weighing in various 100ml standard flasks those consists of strengths β-Hydroxyethyl Theophylline after that Theophylline anhydrous are 50, 75, 100, 125 and 150 % of β-Hydroxyethyl Theophylline also Theophylline anhydrous those are analogues to working stregnths. Represented samples are passed in duplicate as

per analysis process. By values of accuracy obtained, percentage recovery values to both β-Hydroxyethyl Theophylline along with Theophylline anhydrous satisfies acceptance criteria to this parameter over range of 50 % - 150 %. Finally, obtained values are noted in table 3 and 4.

Sample %	Sample weighed (g)	Theoretical % Active	Actual % Active	% Recovery	Average % Recovery
50	5.0005	4.95	4.9	99.0	99.0
			4.9	99.0	
75	7.5002	7.43	7.3	98.3	98.3
			7.3	98.3	
100	10.0005	9.85	9.8	99.5	100.0
			9.9	100.5	
125	12.5004	12.38	12.3	99.4	99.8
			12.4	100.2	
150	15.0003	14.85	14.8	99.7	99.7
			14.8	99.7	

Table 3: Etophylline (β-Hydroxyethyl Theophylline) - Percentage recovery results

Sample %	Sample weighed (g)	Theoretical % Active	Actual % Active	% Recovery	Average % Recovery
50	5.0005	50.8	51.6	101.6	101.7
			51.7	101.8	
75	7.5002	76.2	76.2	100.0	99.9
			76.0	99.7	
100	10.0005	101.6	101.4	99.8	100.0
			101.8	100.2	
125	12.5004	127.0	126.3	99.4	99.5
			126.5	99.6	
150	15.0003	152.4	150.2	98.6	98.6
			150.2	98.6	

Table 4: Theophylline Anhydrous - Percentage recovery results

4.5 Method precision

4.5.1 Repeatability

For this parameter % RSD is due to β-Hydroxyethyl Theophylline and Theophylline anhydrous strength to total six samples is found that 2.0 %. % RSD due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous strength assay meets necessitates to repeatability founded as 0.9 % and 1.0 % for both drugs. Results obtained are noted in table 5.

Sample number	Results (mg/30ml)	
	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline Anhydrous
1	9.8	100.2
2	9.8	99.9
3	9.8	99.7
4	9.6	97.6
5	9.8	99.8
6	9.7	99.1
Mean	9.7	99.4
% RSD	0.9	1.0

Table 5: Repeatability results

4.5.2 Intermediate Precision

By utilizing proposed parameter % RSD is mainly due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous strength to total six samples is corresponding to 2.0 %. Finally ungenerous values achieved over both repeatability along with intermediate precision should not vary to value 3.0 %. Total six various sample preparations are assayed as per protocol of analysis of method. % RSD is mainly due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous strength assay meets possible concern to Intemediate Precision at 0.6 % and 1.0 %. % difference of mean to both drugs in the intermediate precision and repeatability both comply as they varied as 1.0 % and 1.4 % respectively. Finally values are tabulated in table

Sample number	Results (mg/30ml)	
	Etophylline (β-Hydroxyethyl Theophylline)	Theophylline Anhydrous
1	9.7	97.5
2	9.7	98.5
3	9.8	98.3
4	9.9	98.5
5	9.8	97.3
6	9.9	98.1
Mean	9.8	98.0
% RSD	0.6	1.0

Table 6: Intermediate precision results

5. Stability Of Solution

This aspect provided values showed that there no loss of active in testing sample preparation while standing at a period of 24 hours. % RSD is mainly due to β-Hydroxyethyl Theophylline along with Theophylline anhydrous content to samples is noted as 1.5 %. Stability to solution mean of β-Hydroxyethyl Theophylline and theophylline anhydrous varies by attain intermediate repeatability mean by 0.50 %.

4.6 RANGE

This parameter is derived as per values obtained by accuracy, range to assay of β-Hydroxyethyl Theophylline along with Theophylline anhydrous is 5.0 – 15.0 mg per 30ml and 50.0 – 150.0 mg per 30 ml respectively, this denotes 50 % to 150 % to working strength.

4.7 Limit of Detection and Limit of Quantification

These parameters to proposed drugs are measured by utilizing standard equation proposed by ICH guidelines. For this parameter results obtained to proposed drug β-Hydroxyethyl Theophylline is 2.3, 7.2 and for Theophylline anhydrous is 1.98, 6.0

5. Results And Discussion

To optimize the parameters, several mobile phase compositions were tried and concluded with Cosmosil C18 5 µm (4.6 x 150 mm) or equivalent column. A satisfactory separation and good peak symmetry was found in a mixture of 0.05M KH_2PO_4 whose pH is adjusted to 3 by phosphoric acid as 95V/V along with acetonitrile as 5V/V. Flow rate is identified as 1.0ml/min. proved to be better than the other mixtures in terms of resolution and peak shape. Wavelength for detection is 254nm. Rt values for both

proposed drugs is ± 12 minutes. To various parameters we are performed system suitability test after that applied to representative chromatograms for various parameters. Results obtained were acceptable limits. Therefore, system meets suitable criteria. Calibration curve was obtained to series of strengths those are in the range of 0.6-1.4µg/ml, it was founded as linear. Total five points graph is constructed after that identified R^2 to both β-Hydroxyethyl Theophylline and Theophylline anhydrous as 0.0007. Plot is straight line and z is 0.6 for both drugs. Forced degradation studies are also studied by preparing different solutions noted in the article. All these factors lead to the conclusion that the proposed method is more accurate, most precise, very simple, sensitive along with rapid also applied more successfully to estimate both β-Hydroxyethyl Theophylline and Theophylline anhydrous in bulk and in pharmaceutical formulations as oral dosage without interference and with good sensitivity.

6. Declaration On Validity Of The Method

The method for the assay of β-Hydroxyethyl Theophylline and Theophylline Anhydrous comply with the requirements for linearity, method precision and accuracy for β-Hydroxyethyl Theophylline and Theophylline Anhydrous across the range of 50 % to 150 %. The Cleaning Validation method is proven to be valid and the validation test results show that the method complies with the validation requirements. Therefore, the method is acceptable as valid.

7. Acknowledgement

Authors are thankful to management of GITAM Deemed to be University for providing research facilities to perform the proposed work.

8. References

[1] *United States of pharmacopoeia United Kingdom, 1,* ***(1988)****,pp. 238.*

[2] *Indian pharmacopoeia government of India, 1996; 2,* ***(1966)****,pp. 752.*

[3] *Ritschel WA, Thompson GA and Lucker PW. "Biopharmaceutic evaluation of etofylline clofibrate and its drug formulation", Arznejmittelforschung, 30,* ***(1980)****, 2020.*

[4] *Bernd S and Josef G, "Assessment of psychoactivity and pharmacodynamics of a cerebral vasodilating hexobendine-combination by quantitative electroencephalographic and psychometric analyses", J. Progress in neuro-psychopharmacology, 1978; vol.2,****(1978)****, pp.543.*

[5] *Vajner L, Konradova V and Uhlik, "The effects of intravenously administered methylxanthines on the proportion of goblet cells containing fucosylated glycoconjugates in rabbit tracheal epithelium", J. Acta Histochem., vol.106,* ***(2004)****, pp.47.*

[6] *Erking W, Lucker PW and Niehch G, "Human pharmacokinetics of theophylline and etofylline from different formulations of a cardiotonic", Arznejmittelforschung. vol.31,**(1981)**, pp. 371.*

[7] *Nirogi RV, Kandikere VN and Shukla M, "A Simple and rapid HPLC/UV Method for the simultaneous quantification of theophyllin and etofyllin in human plasma", Journal of chromatogr B Analyt Technol Biomed Life Sci., 2007; vol. 848,**(2007)**, pp. 271.*

[8] *Schlemmer W, "Quantitative thin-layer chromatography: Assay of drug mixtures by scanning of remission peaks", Journal of chromatography, vol.63, **(1971)**, pp. 121.*

[9] *Schlemmer W, "Application of quantitative thin-layer chromatography in drug assay and stability testing: determination of codeine phosphate, noscapine, diphenhydramine hydrochloride, phenylephrine hydrochloride, caffeine, etofyllin, Phenobarbital and thiamine hydrochloride by in situ reflectance spectroscopy', Journal of chromatography A, vol.82, **(1973)**, pp.143.*

[10] *Dave HN, Mashru RC and Thakkar AR, "Simultaneous determination of salbutamol sulphate, bromhexine hydrochloride and etofylline in pharmaceutical formulations with the use of four rapid derivative spectrophotometric methods", Analytica chimica Acta, vol.597, **(2007)**, pp. 113.*

[11] *Devang. N. Wadia and Hemant. T. Desai, "Ultra Performance Liquid Chromatography (Uplc) Method Development And Validation For The Simultaneous Estimation Of Etophylline And Theophylline In Pharmaceutical Dosage Form", International journal of life sciences and pharma research; 2012; vol.2, no.3, **(2012)**, pp.19-24.*

[12] *S.Nagajyothi, A. Elphine Prabahar, P.V.Suresh and RamaRao Nadendla,"Simultaneous Estimation of Etophylline and Theophylline in Bulk and solid Dosage Form by Chemometric Assisted Spectrophotometric Methods",IOSR Journal of Research & Method in Education;; vol.7, no.3, **(2017)**, pp. 23-29.*

[13] *V.D.N.Kumar. Abbaraju, V.Sreeram and M.Venkateswara Rao,"Development and Validation of Pholcodine In Pharmaceutical Formulation – A New Rp-HPLC Method", International Journal for Research in Engineering Application & Management, IMC, 18601, **(2017)**,pp.179-182.*

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Lung Cancer Detection Based On Ct Images Using Feature Extraction And Classification Techniques

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Abstract

The image detection in medical field is a difficult task due to small lesions which are not identifiable in an early stage. The early medical diagnosis is important because it increases the mortality rate. Therefore prediction of lung cancer in its early stage is important. Here Linear Discriminant Analysis (LDA) classification is used to predict lung nodules and normal cells on MRI images. Dual Tree M-Band Wavelet Transform is used for feature extraction and LDA concentrates on the image classification. The proposed method undergoes three important stages includes in lung cancer detection such as pre-processing, feature extraction and classification. The accuracy results up to 95% of the images have been identified precisely.

Keywords: GLCM Feature Extraction, M-band wavelet co-efficient, Linear Determinant Analysis, Lung cancer detection.

1. Introduction

Abnormal growth of cells in any part of the body occupies additional space and leads to a disease called cancer. Therefore the growth of abnormal cells in lungs causes lung cancer [1]. Lung cancer might treated as a non-curable disease in the past period and this disease is considered to be 5th cause of death compared to all type of cancers that exists in the world [2]. It is the most common type of cancer and the death rate results nearly 1.59 million people per anum. Among all cancers the lung cancer presents the highest incidence for new cases of count 222,500 in both men and women with the death count of 155,870 in US in the year of 2017 [3]. Lung cancer be the second cause of death among men and women, it is the kind of prostate cancer for men and breast cancer for women [4].

Many lives can be saved through the earlier diagnosis of cancerous cells [5]. But accuracy detection of nodule cells by this earlier detection technique is difficult because of variations of morphological patterns in the lung tissues. Therefore classification method for lung nodule detection [6] was proposed to classify the lung nodules as benign or malicious. In correspondence with Lung Image Database Consortium (LIDC) the contour maps are extracted on basis of excel files [11]. The proposed cancer detection techniques are based on radiation method and every technique has own pros and cons. If radiation is too high then the image quality will get reduced also affects the patients health, hence an alternative method was developed through Computer Aided Diagnosis (CAD) in concern with human healthcare system. The main classification methods used nowadays are Artificial Neural Network (ANN) [7] and Support Vector Machine (SVM) [8]. The accuracy of the CAD system is improved by training the dataset and testing the dataset on basis of the system requirement. Results obtained for SVM classifier is 97% accuracy when compared with ANN classifier including significant reduction in false rate densities [9].

An appropriate method called Computed Tomography (CT) screening is considered to identify the lung cancerous cells [10] in the earlier stages. Lung carcinoma or lung nodule is defined as the malignant lung tumor caused in lung tissues because of hysterical cell growths. These cells can grow beyond the lungs into neighbour tissues and nearby parts of the body or other various parts of the body. This process is said to be metastasis. Benign nodules are referred to normal patients and malignant nodules are represented for the patients who suffering with cancerous cells [12].

2. Proposed Method

In this proposed method the lung nodule identification is carried out in three stages such as image pre-processing using median filter, feature extraction using GLCM features and classification of lung nodules using linear discriminant analysis classifier. Figure 1 gives the overall process of the proposed lung nodule classification method.



Figure 1: Nodule classification using LDA

The CT lung images collected from the dataset are usually consists of noise with low contrast because of image acquisition process. Hence the input image should undergo the process of image preprocessing in order to remove the impulse noise as well as to get better scan image contrast value. The preprocessing method not only improves the precision but also increases the nodule detection accuracy value. Here median filter is applied for removing the noise and this provides good smoothing consequences when compared with other noise removing filters.

The next stage is extracting the features and this feature extraction process plays a significant role in detection of true positive nodules precisely. Extraction of good features can efficiently differentiate the true nodules from the acquired candidate nodules. Intensity based features is used to extract the Dual Tree M-Band Wavelet (DTMBW) sub-band coefficients. The features such as skewness and kurtosis are used to decompose the DTMBW features. By taking the intensity values presented in each region of pixels the average intensity values for all the pixels in the region are calculated. The intensity value in the wide range of pixels is identified and interchanges with the overlapping of cells in an image. The pulmonary nodules are detected to be circular in shape and the blood vessels are

identified with slender like structures. Skewness and variance features are defined in following equations 1 and 2.

$$Skewness_{Sk} = \frac{1}{\sigma_r^3}\sqrt{\sum_{r=0}^{R-1}(r-\bar{r})^3 s(r)} \quad (1)$$

$$Variance\ \sigma_r^2 = \sum_{r=0}^{R-1}(r-\bar{r})^2 s(r) \quad (2)$$

The skewness, variance, kurtosis and contrast measure are used to extract the lower and higher frequency components of DTMBT. These extracted features are stored in the feature database and used for the classification.

Once the intensity based features for each and every nodule candidate is calculated then the extracted features are analyzed with the suitable classifier. Here LDA classifier method is applied for the classification of benign (normal) cells and malignant (cancerous) lung nodules. The feature values are evaluated with the LDA classifier to discriminate between the malignant nodule and the benign cells, the extracted features is combined with the LDA. The function of linear discrimination analysis g(x) is defined using equation 3.

$$g(x) = W(X) \oplus \omega_0 \quad (3)$$

The term W(X) denotes the product weight vector and product feature vector. The term ω_o indicates the bias that is used to determine or locate the position of the decision surface.

3. Results and Discussions

The implementation process is done using the tool called MATLAB r2018 and the consequences are shown further. 130 images are taken for training process and 40 images are used for testing process. The size of the input image taken here is 256X256 and the classified output of nodules and non-cancerous cells and its intensity level is measured through accuracy calculation. The sample input image that taken in the dataset is given in figure 2.



Figure 2: Sample Input Image

Mainly 2 features are extracted from this feature extraction process which is carried out by applying GLCM feature reduction technique and fed as input to the LDA classifier. The image size after the feature extraction process reduced to 15X19.

Accuracy measurement is defined as the number of precise predictions made and it is termed as a different label classification. To evaluate performance of classification confusion matrix is used for evaluation. The confusion matrix in general comprises of four tables with the combination of predicted values and actual values. But the output consists of two or more classes based on the requirement. The accuracy is measured using the equation 4.

$$Accuracy\,(\%) = \frac{(TP + TN)}{(TP + TN + FP + FN)} \qquad (4)$$

Here P represents Positive i.e. resulted as positive, N represents Negative where the observation is resulted as not positive. TP stands for True Positive which implies the observation as positive therefore results in positive %. FN stands for False Negative that denotes the observation as positive and it results in negative %. TN stands for True Negative and observation seems to be negative as well as results in negative. FP stands for False Positive and observation is considered as negative and results as predicted positive. Histogram oriented gradient image representation for non-cancerous cells are shown in figure 3.

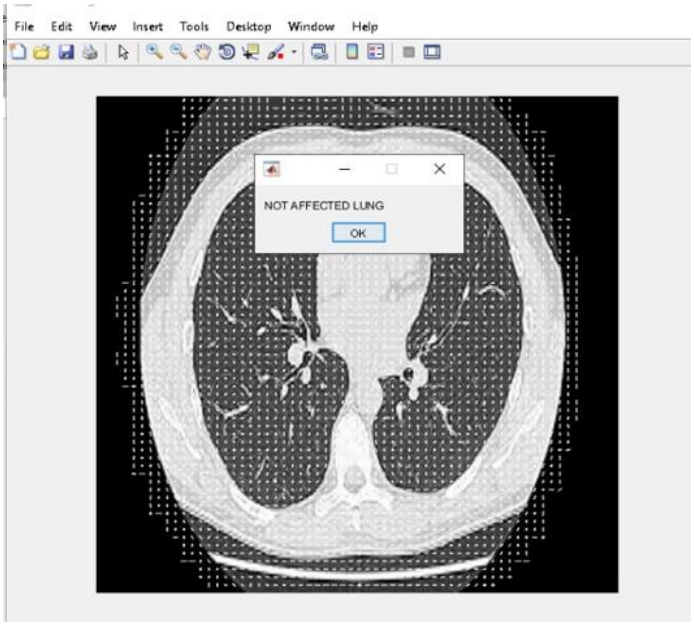

Figure 3: Histogram representation of non-cancerous lung



Figure 4: LDA classifier output

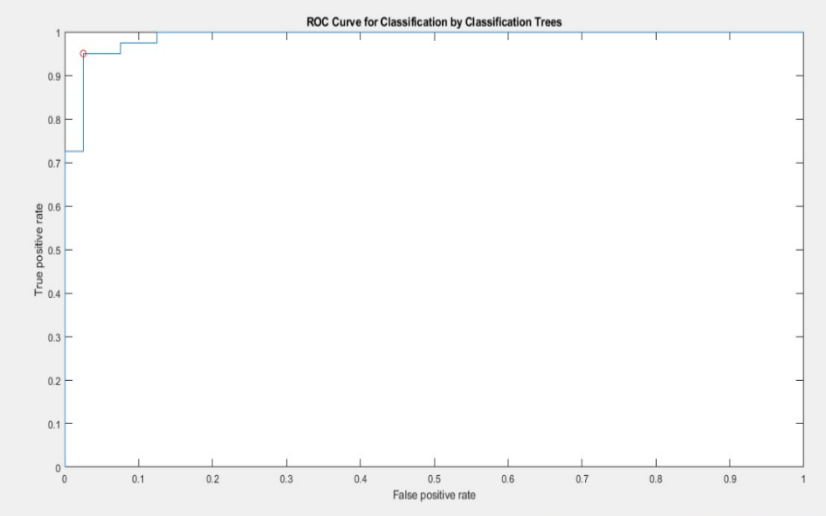

Figure 5: ROC curve for LDA Classifier

The confusion matrix result and the Region of Curve (ROC) which is applied for the calculation of accuracy of the LDA classifier are shown in figures 4 and 5 respectively. Therefore for our proposed model provides better accuracy of 99% compared to the conventional classifiers.

4. Conclusion

Detection of lung cancer in its early stage is significant since this leads to fatality. Therefore several digital imaging techniques had been proposed for the detection of cancerous cells but the precision accuracy is still being a challenging one. Hence a novel and precise lung cancer detection method is proposed. The proposed method undergoes three important stages includes in lung cancer detection such as pre-processing, feature extraction and classification. This method uses median filter for pre-processing the image, DTMBW is applied for feature extraction and LDA classifier is used for classification purpose to predict lung nodules and normal cells on MRI images. The accuracy results up to 99% of the images have been identified precisely.

References

1. Gajdhane, V. A., & Deshpande, L. M. (2014). Detection of lung cancer stages on CT scan images by using various image processing techniques. IOSR Journal of Computer Engineering (IOSR-JCE), 16(5), 28-35.

2. Vardell, E. (2020). Global Health Observatory Data Repository. Medical Reference Services Quarterly, 39(1), 67-74.

3. Siegel, R. L., Miller, K. D., Fedewa, S. A., Ahnen, D. J., Meester, R. G., Barzi, A., & Jemal, A. (2017). Colorectal cancer statistics, 2017. CA: a cancer journal for clinicians, 67(3), 177-193.

4. Moradi, P., & Jamzad, M. (2019). Detecting Lung Cancer Lesions in CT Images using 3D Convolutional Neural Networks. In 2019 4th International Conference on Pattern Recognition and Image Analysis (IPRIA) (pp. 114-118). IEEE.

5. Ravindranath, K., & Somashekar, K. (2017). Early detection of lung cancer by nodule extraction—A survey. In 2017 International Conference on Electrical, Electronics, Communication, Computer, and Optimization Techniques (ICEECCOT) (pp. 1-4). IEEE.

6. Tariq, A., Akram, M. U., & Javed, M. Y. (2013). Lung nodule detection in CT images using neuro fuzzy classifier. In 2013 Fourth International Workshop on Computational Intelligence in Medical Imaging (CIMI) (pp. 49-53). IEEE.

7. Nasser, I. M., & Abu-Naser, S. S. (2019). Lung cancer detection using artificial neural network. International Journal of Engineering and Information Systems (IJEAIS), 3(3), 17-23.

8. Asuntha, A., Brindha, A., Indirani, S., & Srinivasan, A. (2016). Lung cancer detection using SVM algorithm and optimization techniques. J. Chem. Pharm. Sci, 9(4), 3198-3203.

9. Taher, F., Werghi, N., & Al-Ahmad, H. (2015). Computer aided diagnosis system for early lung cancer detection. Algorithms, 8(4), 1088-1110.

10. Gopi, K., & Selvakumar, J. (2017). Lung tumor area recognition and classification using EK-mean clustering and SVM. In 2017 International Conference on Nextgen Electronic Technologies: Silicon to Software (ICNETS2) (pp. 97-100). IEEE.

11. Gupta, A., Märtens, O., Le Moullec, Y., & Saar, T. (2015). A tool for lung nodules analysis based on segmentation and morphological operation. In 2015 IEEE 9th International Symposium on Intelligent Signal Processing (WISP) Proceedings (pp. 1-5). IEEE.

12. Setio, A. A., Jacobs, C., Gelderblom, J., & van Ginneken, B. (2015). Automatic detection of large pulmonary solid nodules in thoracic CT images. Medical physics, 42(10), 5642-5653.

Outcomes-Based Education: Planning, Teaching and Assessment Possibilities

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Abstract:

This paper discusses some of the basic principles of outcomes-based education, and addresses them in Indian Higher Education planning, teaching and evaluation. It will help teachers understand that OBE's theory and principles can be converted into concrete steps in the preparation, teaching and assessment of education for students. The paper acknowledges the fact that OBE has both opponents and supporters and quickly discusses some of the common concerns about OBE. In several ways, the paper is deliberately provocative and encouraging educators to challenge their existing teaching practices and explore ways of integrating those concepts in results-oriented learning. The article is not meant as a detailed OBE treatise. This presents several concepts that are central to OBE and suggests means of discussing these theories more closely.

Keywords: *Teachers' instructional planning, teaching and assessment of student learning.*

Introduction:

"A good archer is not known by his arrows but by his aim."
-Thomas Fuller

"A windmill is eternally at work to accomplish one end,
although it shifts with every variation of the weathercock,
and assumes ten different positions in a day."
-Charles C. Colton

Outcome-based education is easy to conceptualize but difficult to define. This is an education approach in which curriculum decisions are driven by the results that students must show at the end of the course. Education based on results can be summarized like' results-oriented thinking' and is the opposite of' input-based education,' which emphasizes the educational process and accepts whatsoever. The results agreed in outcomes-based education on what is being taught and assessed for the curriculum guide.

In the context of the learning outcomes it is clear what the education results will be and how they will be organized, the educational strategy, the teaching methods, the assessment procedures and the educational setting. At least three viewpoints are available to judge the quality of the educational system. The system inputs, what happens inside the system and the system outputs. Anyone willing to receive information should focus their attention mainly on money, capital, services, etc. and may base their opinion on the performance and usefulness of the process by using economic rationalism.

Those who are interested in results will focus mainly on products or educational outcomes. All aspects of education can be argued that are important and that quality is not to be assessed from a narrow perspective. However, there have been increased calls for more attention in western society in recent years to the outcomes of education in order to assess the return on educational investments (especially public education). The growing demand for accountability is one reason why diverse

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forms of outcome education spread rapidly over the 1980s and 1990s, in countries like the United States and the United Kingdom.

Each education system has two basic types of outcomes. The first type includes results such as results of tests, completion rates, employment after-course , etc. The second type of result is less measurable and generally shows the skill, the ability or the essence of the training of the learners. This is the second kind of result.

In Spady's words:

> " Outcome-Based Education means clearly focusing and organizing everything in an educational system around what is essential for all students to be able to do successfully at the end of their learning experiences." "This means starting with a clear picture of what is important for students to be able to do, then organizing the curriculum, instruction, and assessment to make sure this learning ultimately happens "
>
> (Spady, 1994:1).

Spady's main idea is that the OBE approach to planning, delivery and evaluation requires that administrators, teachers and students focus their attention and effort on the desired outcomes of education— the outcomes that are expressed by student learning in terms of individuals. One approach highlights the students ' mastery of traditional academic results (usually focused on the subject) and certain cross-disciplinary outcomes (e.g. the ability to solve problems or work collaboratively). The second approach focuses on long-term cross-curricular results directly related to the future roles of students (e.g. productive workers or responsible citizens or parents). These two approaches correspond to the traditional and transitional OBE and transformational OBE that Spady (1994) calls.

Spady clearly favours a revolutionary approach to OBE where "high quality, culminating demonstrations of significant learning in context" (Spady, 1994:18). For Spady, learning is not significant unless the results reflect the complexities of real life and promote the life roles faced by the students after their formal education has been finished.

OBE is guided by three basic assumptions in addition to the belief that the findings must reflect meaningful long-term learning:

- Every student can learn and excel, but not all at the same time.
- Successful learning encourages much more learning.
- Schools (and teachers) monitor the circumstances under which students will or will not be successful in school.

From his three premises, Spady developed four essential principles of OBE.

1. clarity of focus
2. designing back
3. high expectations
4. expanded opportunities

The first principle is clarity of focus: this means that all teachers must clearly focus on what they want students to do. In planning and teaching teachers should therefore focus on helping students to develop the knowledge, competences and arrangements that ultimately will allow them to

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achieve significant, clearly articulated results. This principle obliges teachers to make clear for the students at all stages of the teaching process their short-and long-term purpose for learning. It also needs educators to rely on clearly defined significant results for every student review.

The second principle is often described as a backrest and is inseparably bound up with the first principle. That means that a consistent description of the essential education that students should accomplish by the completion of their formal training must be made the starting point for all course development. All educational decisions are then based on the "desired final result" and the "building blocks" of learning that students must achieve so that the longer-term outcomes can eventually be achieved. Killen and Spady (1999) describes a systematic framework to develop curricula for higher education in this way, and Collier (2000) illustrates its application.

The third fundamental principle of OBE is that teachers should expect all students very much. In literature, there is ample evidence that teachers need to set the high and challenge level for performance to encourage students to get deeply involved in the issues they are learning from (e.g. Queenland School Reform Longitudinal Study 1999). The idea that successful learning promotes successful learning is very closely related to helping students achieve high standards (Spady 1994). As learners excel, they build their confidence, increase their faith, and embark on further learning challenges. One of the main reasons for using OBE is that it can motivate all students to do hard things properly.

Intellectual quality is not something for a few pupils: something that all pupils should expect, which is linked with the fourth principle: that pupils should endeavour to offer all pupils more opportunities. The belief that not all students learn the same way at the same time is based on this notion (Spady, 1994). However, most students can achieve high standards by offering adequate opportunities— the truth is that the students learn important things; they do not learn them in a specific manner or by a certain arbitrary means.

For example, we cannot conveniently ignore the design principle or the expanded opportunities principle and continue to say that we have an OBE system. One of the attractions of results-based education is the ability for managers to control the outcomes of education while at the same time offering teachers a great deal of freedom to select the content and methodologies, they use to help their students achieve the results. Both of these issues of control and freedom can create tension; teachers can struggle with authority and teachers can not like how teachers use their choice. Alternatively, it will try and show that the teachers can operate within a results-based system and have the ability to deal with a lot of social, moral and racial issues of teaching and learning at the same time. The paper will not settle the controversy because that challenge is tough.

Instructional Planning:

The OBE system has three main elements: deciding what students are to achieve, how students are to help them achieve these results (i.e., decide content and teaching strategy) and how students are to be determined when the students have reached the results (i.e., decide assessment and reporting procedure). The system includes three main steps in educational planning. Such decisions are made as a curriculum expert (e.g. a secondary science teacher) in the view of most students. However, if students are to achieve broader results–for example the key skills–they have to organize learning programs in an integrated fashion that draws on elements from all areas of learning.

Curriculum documents are broadly written in any State or in any national education system; therefore, they do not address the particular needs of individual schools or particular groups of students directly. Teachers need therefore sufficient detail for their daily work in order to be able to

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translate whatever course guidelines they receive into specific teaching curricula. The programs are therefore planning which guide the selection of lesson results, content, teaching strategies, resources and assessment procedures by each teacher. The programs may be for large work areas (e.g. four-year courses) or small work groups (e.g. a subject area). The nature of these programs will be very different, but they can be similar in structure. The objective of each program should be to clarify (to explain why it has been implemented), goal (to explain the successes of the program) outcomes statements, quality statements (to show the specific scope of the program), instructional method statements (to illustrate how the training experiences should be organized), and assessments. Every programming form addresses these questions at some point, but three basic styles of programming can be used by stressing different key elements.

1. Content-based programming:

The selection of content precedes the consideration of results or teaching strategies in content programming; content-based programming is the approach most teachers are familiar with. This stress "covers the course" almost entirely by implying that teachers must teach specified material over each period of time (lesson, word, year, etc.). In many cases, the content being taught is closely linked to a subject-based textbook. This approach takes little account of how much each student will learn in the time available and causes teachers to believe that it is acceptable and appropriate to learn different amounts of each individual student. However, we can find out that different differences are based on their learning ability. Yet teachers have not forgotten learner's outcomes.

2. *Activities-based* programming:

The choice of learning experiences takes precedence over other considerations in the activity-based programming; experience-based curriculum stresses student interactions by requiring students to think about what they could learn from these activities. It is very difficult to justify content or experience-based programming on a non-administrative basis.

3. *Outcomes-based* programming:

The first choice in outcomes-based programming is what students will learn about and be able to do when the course is done. "Scheduling results means organizing education to achieve predetermined results." It begins by specifying clearly what the students need to know, what they can do and what attitudes or values are desirable at the end of the programme.

> "In outcomes-based education . . . you develop the curriculum *from* the outcomes you want students to demonstrate, rather than writing objectives *for* the curriculum you already have."

(Spady, 1988:6).

The program is built to offer all students equal opportunities for each outcome, using these findings as a guide. The most important element of results-based education is that' all students are expected to be successful. There are many different means to approach results-based programming and evaluation.

In order for all the students to learn well and to achieve particular achievements, teachers must follow certain instructional processes and each has consequences for the plan and curriculum of their teachers.

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- Teachers need to properly prepare their students to succeed. This requires teachers to understand exactly what students want to know, to anticipate difficulties and to minimize those difficulties.
- Teachers need to create a positive learning environment that helps students, no matter how easy or difficult the learning process is to find. In large part, your relationship with students depends on this positive environment, but it also depends on your effort to help the physical environment learn.
- Teachers need to help students understand why they need to learn (including what it will be of use to them in future) and how they will know when they learn it. Do not assume that students will notice the importance of what you teach because you know why. And never teach anything that is not useful to you.
- Teachers need to use a variety of teaching methods to help each student learn. We should not presume that all students are similarly capable of learning from one single education approach, and that no specific education method is a good way of helping students to achieve these learning outcomes. If you have taken the findings into account that students want to achieve, the content you are going to use to help students achieve these results, the characteristics of the students and the resources available, you have to select the best strategy. The so-called "student-centred" approaches should not always be considered to be the correct OBE technique to use. These are often adequate but more specific instructional approaches sometimes are acceptable (Killen, 1998).
- Teachers need to offer students sufficient opportunities to practice with their new knowledge and skills, so that they can explore, experiment, concrete errors and adapt their thinking under the guidance of the teacher. It is important to help students apply their new knowledge and skills instead of just building up new knowledge and skills.
- Teachers must help every student to personally each study episode (lesson or class of lessons) so that they are aware of what they have learned and where they are led. Don't assume that without your guidance students can do this.

TEACHING STRATEGIES

Teaching is only teaching when students learn. Therefore, "it remains the responsibility of educators to construct meaningful learning experiences that lead to the mastery of outcomes" (Cockburn, 1997:7). Teachers have to make informed decisions about teaching strategies to build meaningful learning experiences. There are often two basic teaching approaches: teacher-centred and student-centred.

1. ***Teacher-centred approaches:*** These are also referred to as explicit, deductive and expository instructional — examples of which are lectures or demonstrations. Through these teaching methods, the teacher determines what is to be understood and how the knowledge to be learned is delivered.
2. ***Student-centred approaches:*** These also referred to as learning through discovery, inductive learning, or learning from inquiries emphasize much more the role of the learners in the learning process, such as cooperative learning and research projects for students. You still have the education agenda when you use instructional methods focused on students, but you do have much less influence over what and how students learn. You are no longer a filter that needs to pass through all information before you reach the learners.

Based on how learning is organized, teaching strategies often include labels such as reading, class discussion, group work, cooperative learning, troubleshooting, student research, etc. Killen (1998) explains how these and several other techniques can be used in a broad range of instructional

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scenarios and points out that no technique is an appropriate method in which students can produce all learning results.

Whatever approach to teaching you use; it is important to keep the following points in mind:

- • LEARNING rather than teaching should be the main focus.
- • Impossible for students to learn unless they THINK.
- • The PROCESSS you use to include students in material and the CONTENT itself will promote and enable the analysis.
- • There is no distance on your subject – you need to support students to make LINKS in other topics.

• It is your job to help LEARN HOW TO LEARN students.

In the OBE system, you cannot assume that all students learn from a strategy such as a small group discussion equally well, and you cannot assume that every student in any fixed time will learn the same thing. If you are to assist all students in achieving the results of what you teach, you have to be flexible in the way you teach and the expectations you have at any time for each student. You have to accept that students will be in different stages in most courses and therefore work towards various short-term outcomes at the same time. You'll have to be creative, and you'll probably notice that you won't use full class teaching very often to support each pupil in your curriculum (under the restrictions of a traditional school system).

ASSESSMENT

It is only natural to think about how far individual students achieve these results when we focus our planning and instruction on the results of education, so that we can see to what extent our intentions have been met if you wish. So, we must think about the evaluation and this can also be a matter for certain teachers in an OBE system. Nonetheless, in theory good assessment practices in OBE do not vary from good evaluation practices in any other education system.

The four concepts that characterize OBEs have many significant consequences for the assessment of student learning. If the concepts of emphasis and development are followed, all assessments must be related to the essential long-term outcomes to be accomplished by students, or to the short-term effects that can emerge from these long-term outcomes. Focusing evaluation only on specific results that neglect the long-term intent of the program of research is clearly not appropriate. Of example, it is important to periodically review the progress of the students towards the accomplishment of core competences.

To be useful in an OBE system, assessment should conform to the following principles:

- The evaluation methods should be valid — in reality, they should evaluate what you intend to evaluate.
- Reliable testing methods – reliable findings should be produced.
- Assessment processes should be realistic — no insignificant considerations such as the cultural background of the applicant can affect them.
- Testing must demonstrate the knowledge and skills that students need to learn more.
- Testing should suggest something that educators and individual students do not learn. This means that students should be able to apply their knowledge and to extend them to the limit of their understanding.

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- A thorough and clear assessment should be carried out.
- Assessment will promote the ability for every student to know important things;
- The evaluation should allow this individuality to be demonstrated because students are individuals.

At OBE, assessment should always help improve the education of students. For evaluation should help research, appraisal activities will actually provide students with opportunities to show what they have achieved and recognize what they still need to learn. Since learning is not just an addition of prior knowledge, it should help student's connective what they learn to their prior knowledge. Because this process is continuous restructuring.

The development of equity evaluations would require careful analysis not only of our assessments and the way we conduct them, but also of how the evaluation processes we use respond to different individuals and groups. The challenge is to develop evaluation tasks flexibly enough to give students a sense of achievement, challenge the upper reaches of understanding of the student and provide a window in the thinking of each student. In this way, students may need to allow several points of entry and exit for evaluation activities and to respond in ways that reflect different levels of knowledge and sophistication. Equity involves every student being able to learn the significant knowledge and qualifications that are assessed and the content that they have not been given an opportunity to learn cannot be assessed fairly.

Evaluations can help students learn important things only if they are based on standards that reflect high expectations of all students; assessment equity cannot be achieved as long as excellence is not required of all. If we want excellence, the expectation must be high enough to allow each student to learn important knowledge and skills with effort and good instruction. This realism is important, as Gardner (1991) indicates, if teachers know what students learn.

Learner's Responsibility for Learning:

In an OBE model, "learners are responsible for their own learning and success" are often proposed (Cockburn, 1997, p.6). The issue was probably caused by a misunderstanding of the theory behind the concept. This is of interest for teachers and parents. The theory accepts that education is essentially an individual and private activity, no matter what the educators do. For its students the teacher cannot learn; the teacher can make learning easier only. In this respect, OBE stresses the teacher's role to clearly defining the goals and helping students achieve the results. It also stresses the responsibility of the learner to try and achieve the results.

One of the challenges that students are accountable for their own education is that they can find it very difficult to know if they are learning or not. They may be able to see that they make mistakes or answer questions wrongly, but this doesn't necessarily mean they are aware that they don't learn. Missing performance may be equated rather than incomprehension to lack of effort. Many people have trouble understanding why they don't understand even when they know what it is, they must remember (Killen, Meade, Yli-Renko & Fraser, 1996). This gives the teachers a new responsibility to help students to diagnose and evaluate their approaches to learning. One of the drawbacks of outcomes-based education is that it helps students understand, be aware of what they really know and be aware of their own control over their own learnings.

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Conclusion:

The principles of the OBE and the philosophies discussed above in this article are subject to four simple questions: What do we want students to learn? Why do we want students to learn these things? How can we best help students to learn these things? and, how will we know when students have learned? Such concepts can be seen as an interesting mix of philosophical positions, but are naturally most strongly rooted in logical empiricism. Spady's view on OBE is that we should not allow schooling (or other aspects of education and training) to be driven by an

> "Edu-centric paradigm—a paradigm defined by what the system is and (always) has been rather than by what it should and could be if student learning and future success in the Information Age were its true purpose and priority"
>
> (Spady, 1998:10).

In the assumption however, that OBE offers a text as a way to achieve things that are appropriate for all school contexts, OBE becomes ideologically fixed. Successful implementation of OBE would allow educators to contextualize the OBE concepts correctly. It should be clear from these principles that OBE is not an' event,' but a comprehensive approach to education. It is not a matter of "fitting the calendar" but of a set of ideas that affect the overall curriculum of the college. One of the reasons for successful student learning is that results-based education encourages teachers to be well-trained.

My answer to OBE's critics is generally to say: first understand, then try, then criticize. No learning system is perfect, and no program can "act," except for the dedication of educators. Clearly, OBE has not been the spectacular achievement its supporters have hoped would have been in certain other nations. Teachers should encourage students in their efforts to achieve significant and meaningful results to follow this approach to learning. Teachers will know that if all students succeed, and nobody engaged in the education should be satisfied by their efforts, they achieve their goals.

If teachers want to succeed with outcomes-based education, they need to adopt the position that "there is no such thing as failure, only feedback and results . . . success depends on how well we process the feedback we get regarding our efforts" (Alessi, 1991, p.14).

References:

Alessi, F. V. (1991). ODDM: The gentle bulldozer. *Quality Outcomes-Driven Education*, April, 11-18.

Ames, R. & Ames, C. (1991). Motivation and effective teaching. In L. Idol and B. F. Jones (Eds.). *Educational values and cognitive instruction: Implications for reform*. (pp.247- 271).

Ames, C. & Archer, J. (1988). Achievement goals in the classroom: Student learning strategies and achievement motivation. *Journal of Educational Psychology*, 76, 535-556.

Biggs, J. B. & Collis, K. F. (1982). *Evaluating the quality of learning: The SOLO taxonomy (Structure of the Observed Learning Outcome)*. New York: Academic Press.

Block, J. H. (Ed.). (1971). *Mastery learning: Theory and practice*. New York: Holt, Rinehardt and Winston.

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Bloom, B. S. (1973). *Every kid can: Learning for mastery*. Washington, DC: College/University Press.

Brady, L. (1992). *Curriculum development* (4th ed.). Sydney: Prentice Hall.

Brophy, J. (1986). *Socialising student motivation to learn*. East Lansing, MI: Institute for Research on Teaching. (ERIC Document Reproduction Service, ED269384).

Brown, A. S. (1988). Outcome-based education: A success story. *Educational Leadership*, 46(2), 12.

Burns, R. (1987). *Models of instructional organization: A casebook on mastery learning and outcome-based education*. San Francisco, CA: Far West Laboratory for Educational Research and Development. (ERIC Document Reproduction Service, ED318117).

Burns, R. & Squires, D. (1987). Curriculum organization in outcome-based education. *The OBE Bulletin*, 3, 1-9. (ERIC Document Reproduction Service, ED294313).

Cockburn, P. (1997). *Building a brighter future: Curriculum 2005*. Pretoria: Department of Education.

Collier, D. (2000). *Assessment based on outcomes of significance*. A paper to be presented at the Annual Conference of the Association for the Study of Evaluation in Education in Southern Africa, Port Elizabeth, South Africa. 26-29 September.

Eltis, K. (1995). *Focusing on learning: Report of the review of outcomes and profiles in New South Wales Schooling*. Sydney: University of Sydney, Faculty of Education.

Fitzpatrick, K. A. (1991). Restructuring to achieve outcomes of significance for all students: A progress report from Township High School District 214. *Outcomes*, 9(4), 14-22.

Franc, L. H. (1978). *Toward improving patterns of instruction*. Durham, NH: New England Teacher Corps Network.

Fritz, M. (1994). *Why OBE and the traditionalists are both wrong*. Educational Leadership, 51(6), 79-81.

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Douglas Stewart's Ned Kelly
A unique Australian phenomenon

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Abstract

Australian literature is shaped by environment than by Nationality. Douglas Stewart's writings are more concerned with environment, than growth and culture. Ned Kelly is unique and defines Australian personality through it. It is a sequence of adventure s coming to a head in a triumph over the concept of death of a dauntless spirit that flies back into its primordial home. Ned finds in death not a negation but a strong fulfilment. Right from the beginning of Ned's dramatic career, the reader is aware of the strong personality beneath the finished literary character. Ned's last moments are the most glorious. we see free nobility and heroism. The play shows us the quest of the hero for his identity, Ned Kelly is symbolic of everyman's desire for the impossible and it is because he stands for the impossible that he is admired.

Key Words

Australian literature-national hero-conflicts and affinities - society's rejection- chivalry -death -strongfulfilment-self-confidence - conviction-nobility and heroism-voyage of self - discovery

Douglas Stewart is an Australian writer. As in most Commonwealth countries, in Australia to the literature is shaped more by environment than by nationality. The Australian writers are more concerned with their own environment, their growth and culture.

One striking feature of Australian literature is that its colouring is strictly local, and the writer has given voice to the majestic

scenery of the land. The character of the land itself is a major factor which goes into the making of the Australian literature. A bushfire, a drought and the moisture all constitute a remarkable feature of the Australian landscape. The lack of harmony between man and nature in a recurring theme in Australian literature.

The play 'Ned Kelly' was first produced as a radio play on the ABC, 21st June 1942. It became a stage play presented by the Sydney University dramatic society, 14th October 1942.

Ned Kelly is a bushranger (a nomadic outlaw). He is an outlaw, yet possess excessive individualism, humanity and fair play. He also possesses deep strain of humanism. The common Australian trend to recognize in Ned all that they held most sacred: Defiance of law and authority, kindness to the common man and chivalry. So, Ned Kelly easily evolves into a pro type of the Australian – a national hero accompanied by the glory of the Bush and romance.

Stewart born in New Zealand and adopting Australia sees Australia with romantic eyes. No Australian before Steward had written anything so arrestingly and characteristically Australian. He is making Australian drama real and relevant in Australian context. Douglas Stewart has taken up Ned Kelly theme for a dramatic treatment and made Australian drama unique and defines Australian personality through it. Ned Kelly reflects his concern with the conflicts and affinities between human being who are Australian products. He projects the clash between the self and the social facts. The Bush- ballad has become the rich backdrop for the play.

Douglas Stewart has given us a Noble and live portrait of a bushman in the person of Ned Kelly. Ned's life is a hymn to Australia himself and to the ideals of equality. It is not where he began his career, but it matters where he ended it. He tries to heighten the heroic potentialities of man into limelight with a new meaning and significance.

In the first Act we see the aggressive side of Ned. In the second Act we see the freshness of his response to nature and a lively mind, a vivid personality of distinct charm. In the 3rd and 4th acts we see him as an embodiment of social, physical and spiritual freedom. He strikes at the very centre of a society which has rejected him. He is able to tear the painted veil of life and bring to our view the underlying shame and hypocrisy of the sophisticated society. Towards the end of the play, we find that his chivalry in his undoing. His sympathetic treatment of Curnow leads to his betrayal by Curnow and the traps track him down from the final encounter. But his final configuration with death is seen as a process of synthesis with nature. Death holds no terror for him. It is only a challenge he says," I will fight but not surrender." And we also notice that a life of painful separation from society and a life an endless longing for adventure and romance culminates in a union with nature through his death.

The play is a sequence of adventures coming to a head in a triumph over the concept of death of dauntless spirit that flies back into its primordial home, -The Australian Bush ranger Ned thus finds in death not a negation but a strong fulfilment. It is not a gloomy, inevitable surrender to death, but a consequence of a faith that looks through death and remains indomitable and optimistic of its ultimate reunion with nature. Thus, in Ultimately merged himself into nature with all his unshaken faith and undimmed hope. It is this which makes Ned Kelly **a unique Australian phenomenon.** It is this which makes him a typical Australian legendary figure.

In the poem of Judith wright titled "Legend"

He ran like a have

He climbed like a fox

He caught it in his hands

The colours and the cold-

Like a bar of ice, like a column of a fountain,

Like a ring of gold.

And again

"He went home as early as could be with the singing and his shoulder".

This Australian altitude is in sharp contrast with the western tendency to master nature.

The Australian seeks a total identification with landscape.

The spell of the play is also different from others. At the beginning of the play Ned appears as an unabashed, arrogant and flamboyant self.

He says to living, "Maybe you know there is power in the name of Kelly as there was in clank's, Ben Halls and grandness; Later he refers to his own dark, desperate character",

But his regard for the glory of the self is gravely objective without a touch of self-love or of self-interest. Right from the beginning of Ned's dramatic career, the reader is aware of the strong personality beneath the finished literary character. It is negative independent and radical in the Australian manner. With remarkable self-confidence and conviction, he welcomes death at the end of the play.

We are reminded of the famous words of Julius Caesar...

"cowards die many times before their death

The valiant never taste of death but once."

Ned's last moments are the most glorious. We see true nobility and heroism. It is evident in his last words...

"come out from behind the trees if you can stop me, you are thirty to one. Isn't it how you like it?

Stand up and meet me then."

Here he rises to his fullest stature, the very incarnation of energy and resolve.

Thus, the tragic angle of the play shows us the quest of the hero for his identity. He embarks on a voyage of self-discovery and in the end, he may have died, but his tragic struggle has brought sufficient illumination to the reader.

Ned Kelly is symbolic of every man's desire for the impossible and it is because he stands for the impossible that he is admired.

He also stands as a prototype of Australian sensibility and values and presents of phase in the emergent Australian conscience.

Bibliography:

1.Douglas Stewart-Angus and Robertson Limited 1943 – Australia-206 Pages.

2. Companion to Douglas Stewart; Ned Kelly Maze Lawson-Angus and Robertson,1965-56 Pages.

3. Bushrangers; Australia's Greatest self-made Heroes – Evan MC Hugh Published 2011 by penguin.

4. Wild Colonial Boys- Tall Tales & True Australian Bushrangers-Geoff Hocking – First Published 2015 By Boolarong Press.

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FORM 2
THE PATENT ACT 1970
(39 of 1970)
&
The Patents Rules, 2003
PROVISIONAL/COMPLETE SPECIFICATION
(See Section 10 and Rule 13)

COMPLETE

UNDER WATER POLLUTION MONITORING SYSTEM TO PREVENT FATALITIES OF AQUA ECOSYSTEM

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Summary of Invention

The crucial aim of the research is towards track the consistency of the water samples by developing smart water quality monitoring system capable of detecting four different physical parameters: pH, temperatures, humidity, water conductivity and evaluating the derived importance of the listed parameters using the correct machine learning method. With the assistance of Arduino dependent sensors, various water samples are checked, and their values are obtained from different measurements. A quick forest binary classifier provides more considerable efficiency testing to verify the precision and efficacy of the method when forecasting the water quality. A system device consumes demonstrated its value using delivering reliable results in detecting water quality dependent on physical parameters.3 This device could be applied for real-time water monitoring elucidation. This could detect the chemical water parameters.

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We Claim

What is claim is

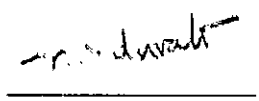
1) Proposed intelligent system consist a modules namely underground water quality monitoring. In water quality monitoring various sensors are involved to check the quality of water in terms of Hydrogen ions (pH), humidity, Temperature, and water conductivity. In quality water distribution management system it used to monitor

2) The water quality monitoring module as claimed in claim 1, receives all the real time information from various sensors to check the quality of water in terms of suitable to living begin or not. All the sensor information's are automatically processed for efficient observation, monitoring and controlling thought hardware platform.

3) The quality water distribution management system as claimed in claim 1, receives only the quality water from the claim 1 and distribute it to the consumer in proper way. Providing accurate water consumption levels, smart meter billing, reducing wastage of water in water-pipeline and finding the capacity of stored water level are monitored by hardware platform.

4) As claimed in claim 2 and claim 3, the observed real time sensor values are monitored and controlled by secure mobile application for the use of remote accessing.

5) As claimed in claim 2 and claim 3, the observed real time sensor values are also stored system for data storage and data processing for present and future need.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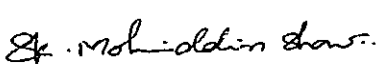
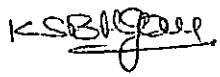

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FIELD OF INVENTION

This invention is a device for tracking the water contamination that is used to measure water constantly, based on the four physical parameters, such as temperature, Humidity, pH, and Pressure properties. The ARDUINO-UNO is mounted on the four sensors which detect water parameters. Extracted sensory data were passed to the WHO (World Health Organization) software package according to the normal values. The proposed method of machine innovation would expertly measure water limitations using the fast wood binary classification to decide whether or not the measured water sample will drink based on the determined results.

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BACKGROUND OF INVENTION

The impact of water is beyond any human thing's description. Water conservation is a major concern with the exponential growth of the worldwide population, especially in production, agriculture and other industries. Most people around the world struggled behind drinking water. Per year, many people die of all lethal water-borne diseases. A research found that nearly five million deaths are mainly caused by unregulated water use.

Analysis of the WHO shows that almost 1.4 million child deaths are stopped by providing them with drinking water. The key purpose of this project remains the introduction of an intelligent water quality management network to help us monitor many physical parameters of drinking water rather than manual processes. Many computational studies have been conducted in recent years to create smart systems for water parameter specification and monitoring.

It is suggested to track the quality and delivery of water in-road control network based on sensor nodes. Their conceptual design, optical sensors are based on the lightweight, low-cost, electro-chemical pipe frames and sensors for this design. This approach is suitable for broad scale water consumers, water generators and colonial water plan classifications. For water protection control of both publisher and customer, a broker less software framework is built. The pH, temperature and oxygen measured from water samples are analyzed, leading to a proportional reversal to the relationship between them.

There are four basic sensors, such as ph, temperature and humidity, based on an industrial water quality control system. The goal of the research remains the development by the wireless framework of an intelligent water quality monitoring system. Four physical constraints are determined using four independent Arduino Uno equipped sensors: temperature, humidity and *pH of various water samples.*

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Object of Invention

- Microcontroller (Arduino)
- Humidity Sensor
- Pressure sensor
- Chemical sensor
- Temperature sensor
- RF Transmitter

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Summary of Invention

The crucial aim of the research is towards track the consistency of the water samples by developing smart water quality monitoring system capable of detecting four different physical parameters: pH, temperatures, humidity, water conductivity and evaluating the derived importance of the listed parameters using the correct machine learning method. With the assistance of Arduino dependent sensors, various water samples are checked, and their values are obtained from different measurements. A quick forest binary classifier provides more considerable efficiency testing to verify the precision and efficacy of the method when forecasting the water quality. A system device consumes demonstrated its value using delivering reliable results in detecting water quality dependent on physical parameters.3 This device could be applied for real-time water monitoring elucidation. This could detect the chemical water parameters.

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Detailed Description of Invention

The detailed description of various exemplary embodiments of the disclosure is described herein with reference to the accompanying drawings. It should be noted that the embodiments are described herein in such details as to clearly communicate the disclosure. However, the amount of details provided herein is not intended to limit the anticipated variations 10 of embodiments; on the contrary, the intention is to cover all modifications, equivalents, and alternatives falling within the spirit and scope of the present disclosure as defined by the appended claims.

It is also to be understood that various arrangements may be devised that, although not explicitly described or shown herein, embody the principles of the present disclosure. Moreover, all statements herein reciting principles, aspects, and embodiments of the present disclosure, as well as specific examples, are intended to encompass equivalents thereof

A real-time remote water quality observing device is invented to monitor the quality of water of a static water reservoir or a water distribution system of an urban or sub-urban area. The water quality data can be saved locally in the device and also the water quality data can be visualizing locally by a display unit that is attached with the device. As well as the data can be visualizing from any location through central server using any remote monitoring device through remote login.

In one embodiment of the present invention, a water quality monitoring unit is illustrated in the figure 1. The real-time water quality monitoring unit comprises a sensing unit for measuring a physical condition parameter of water producing measured data of said monitoring physical parameters. The measuring components are the sensors which are used for sensing 30 water quality parameters (pH, pressure, temperature, humidity) of underground water. The sensing unit includes a pH sensor, a humidity sensor, a pressure sensor, a temperature sensor. The said sensors are fixed the controller. Said measured physical condition parameter of said water can be pH, humidity, pressure, temperature of underground water being used for producing said monitoring data accordingly. All the data coming from the sensors like Temperature, Pressure, pH and humidity data are sent to the main processor. After collecting the data by the

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Main Processor Unit, it saved the data locally in local storage unit in CSV format as a backup, displayed the measured value and data in attached Display Unit and send the data to a central server (201) via transmitting unit. Furthermore, the local monitoring unit comprises all the sensor data coming from different sensors, a main processor unit, display, local storage unit and a transmitting unit. The water quality data can be seen from any remote monitoring device through remote login. This key water quality misbrands are best in use for classifying/defining water quality as well as governing a distributed water supply network/pipeline network/ static reservoir, and could simply be measured by respective monitoring units. Moreover, said monitoring unit can be a sensor, especially a (online) membrane sensor, further more specifically monitoring real-time water quality of said water online

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Detailed Description of Drawings

(1) Figure (i) shows the Block Diagram

BATTERY/POWER SUPPLY:

A power supply unit (or PSU) converts mains AC to low-voltage regulated DC power for the internal components.

(2) Figure (ii) shows Micro Controller

A microcontroller is a compact integrated circuit designed to govern a specific operation in an embedded system. A typical microcontroller includes a processor, memory and input/output (I/O) peripherals on a single chip.

(3) Figure (iii) shows the Temperature Sensor

An electronic device for the temperature sensor measures the temperature of your environment and converts your input data into electronic data to record, track or signal temperature changes.

(4) Figure (iv) shows the Humidity Sensor

A sensor is a moisture device for water vapour detection and measurement. There was a mistake. These humidity sensors accurately measure the dew point and absolute humidity based on our robust, capacitive technology.

(5) Figure (v) shows the Pressure Sensor

By converting pressure to an analogue electrical signal, a pressure sensor works. During the steam age, demand for pressure measurement instruments increased. There was a misunderstanding. We now use pressure transducers and pressure switches to electronically measure pressure.

(6) Figure (vi) shows the Chemical Sensor

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Chemical sensors are measuring instruments which convert chemical or physical properties of a particular analyte into a measurable signal, the magnitude of which is usually proportional to the analyser 's concentration

(7) Figure (vii) shows the RF Transmitter

The transmitter is the generator of the radiofrequency current that is supplied to the transmitter spool. The transmitter is RF. This creates a signal that excites protons in the field of photography. There was a misunderstanding. This energy is transmitted in the form of a quick, intense RF explosion at the resonant frequency of hydrogen.

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Different Embodiment of Invention

Economical

This system is economical as compared to many of the existing manual solutions. It saves a lot of money and time of the government.

Real time monitoring

This system works in real time to monitor the quality of underground water. The system enables to a real time view of data using wireless data transmission and its store the monitor data's for future use.

Requires no human supervision: -

This system requires almost no human supervision. All of these sensors are continually monitor the underground water quality and send to the monitor data to server using RF transmitter.

Effective

Monitoring offers reliable data needed for sound water quality management decisions now and in the future. Water quality control is used to warn us about established, potential and developing problems; to determine whether or not drinking water requirements are met with and to safeguard other useful water uses. so this system is very important and effective.

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Applications of Invention

ADVANTAGES

1. Due to automation, the parameter management time is reduced.
2. For average citizens this is economically available.
3. Low maintenance.
4. Prevention of water diseases.

APPLICATIONS

1. This system is used in domestic and industrial applications.
2. Water supply agencies.
3. To determine the cause for water diseases in the health department.

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Drawings

BLOCK DIAGRAM

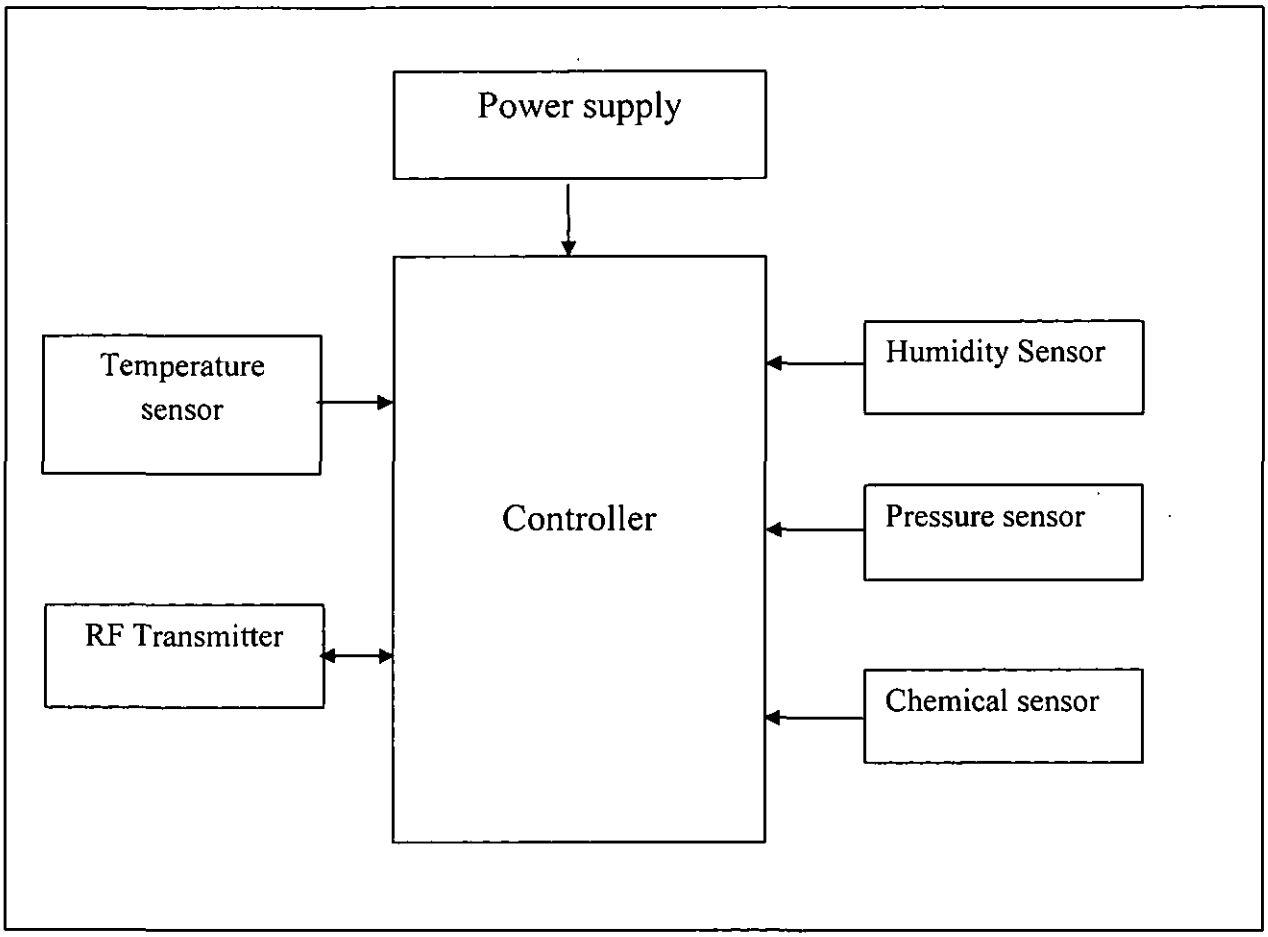

Figure (i) shows the Block Diagram

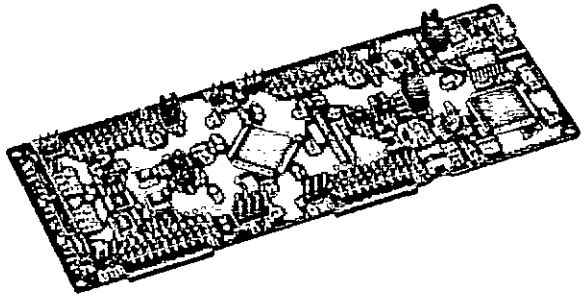

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Figure (ii) shows Micro Controller

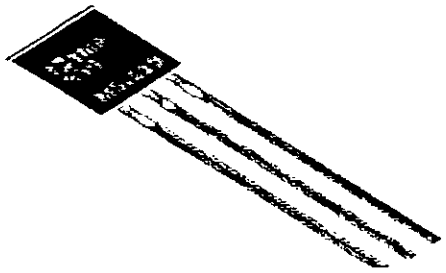

Figure (iii) shows the Temperature Sensor

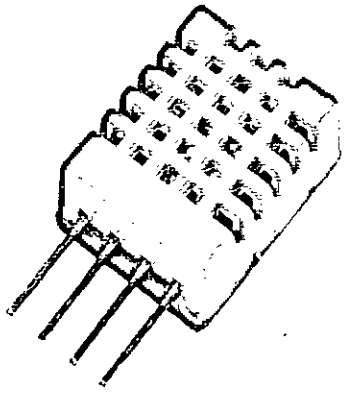

Figure (iv) shows the Humidity Sensor

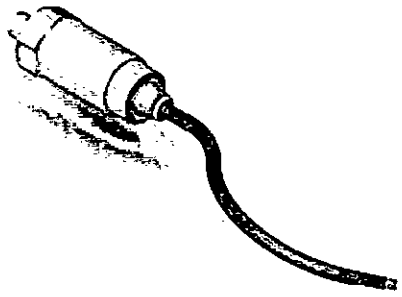

Figure (v) shows the Pressure Sensor

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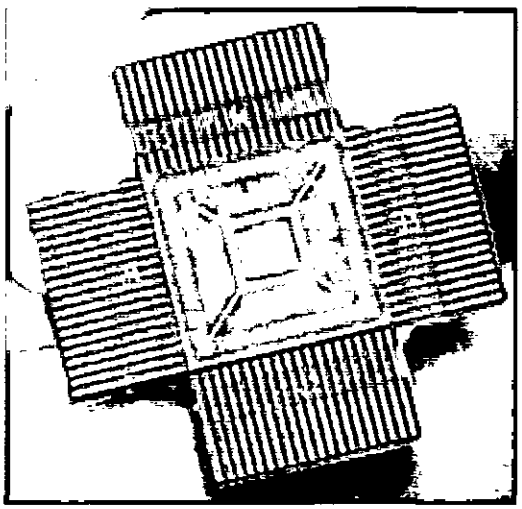

Figure (vi) shows the Chemical Sensor

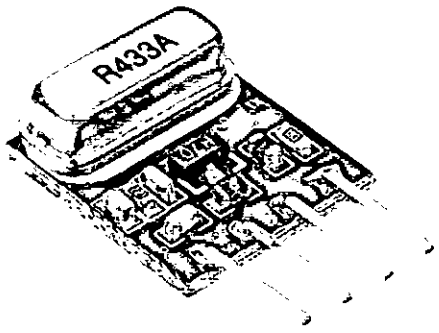

Figure (vii) shows the RF Transmitter

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NewAppl. (D) 89136-F-9
P.V. 12/10/20

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"FORM 1 THE PATENTS ACT 1970 (39 of 1970) and THE PATENTS RULES, 2003 **APPLICATION FOR GRANT OF PATENT** (See section 7, 54 and 135 and sub-rule (1) of rule 20)	(FOR OFFICE USE ONLY) 700301568

	Application No.	202041044264
	Filing date:	12/10/2020
	Amount of Fee paid:	1750/-
	CBR No:	34169
	Signature:	P.V. 12/10/20
1. APPLICANT'S REFERENCE / IDENTIFICATION NO. (AS ALLOTTED BY OFFICE)		

2. TYPE OF APPLICATION [Please tick (✓) at the appropriate category]

Ordinary (✓)		Convention ( )		PCT-NP ( )	
Divisional ( )	Patent of Addition (✓)	Divisional ( )	Patent of Addition ( )	Divisional ( )	Patent of Addition ( )

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			State	Andhra Pradesh
			Country	India
			Pin code	522019
Name in Full	Nationality	Country of Residence	Address of the Applicant	
Dr. J L RAMA PRASAD	INDIAN	INDIA	House No.	P B Siddhartha College of Arts & Science
			Street	
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			State	ANDHRA PRADESH
			Country	India

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PATENT OFFICE CHENNAI 13/10/2020 12:36

			Pin code	522 010
Name in Full	Nationality	Country of Residence	Address of the Applicant	
Dr. G. DHARMAIAH	INDIAN	INDIA	House No.	Narasaraopeta Engineering College,
			Street	Yellamanda, Kotappakonda Road
			City	Narasaraopet.
			State	Andhra Pradesh
			Country	India
			Pin code	522 601
Name in Full	Nationality	Country of Residence	Address of the Applicant	
SREELATHA SANKURU	INDIAN	INDIA	House No.	SINGARENI COLLIERIES WOMEN'S DEGREE COLLEGE
			Street	
			City	KOTHAGUDEM
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Name in Full	Nationality	Country of Residence	Address of the Applicant	
NADIMPALLI UDAYA BHASKARA VARMA	INDIAN	INDIA	House No.	DNR COLLEGE OF ENGINEERING & TECHNOLOGY
			Street	BALUSUMUDI, WEST GODAVARI DISTRICT
			City	BHIMAVARAM
			State	ANDHRA PRADESH
			Country	INDIA
			Pin code	534 202
Name in Full	Nationality	Country of Residence	Address of the Applicant	
KRANTHI KUMAR KOTHA	INDIAN	INDIA	House No.	Gudlavalleru Engineering college
			Street	Shesadri Rao Knowledge village
			City	Gudlavalleru
			State	Andhra Pradesh

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ATENT OFFICE CHENNAI 13/10/2020 12:36

			Country	India
			Pin code	521 356

3B. CATEGORY OF APPLICANT [Please tick (✓) at the appropriate category]

Natural Person ( )	Other than Natural Person		
	Small Entity ( )	Startup ()	Others ( )

4. INVENTOR(S) [Please tick (✓) at the appropriate category]

Are all the inventor(s) same as the applicant(s) named above?	Yes (✓)	No ( )

If "No", furnish the details of the inventor(s)

Name in Full	Nationality	Country of Residence	Address of the Applicant	
Dr. Ch. BABY RANI	INDIAN	INDIA	House No.	V.R. Siddhartha Engineering College, Kanuru, 520007
			Street	Krishna District,
			City	Vijayawada
			State	Andhra Pradesh
			Country	India
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Name in Full	Nationality	Country of Residence	Address of the Applicant	
DR. NALLAPATI VEDAVATHI	INDIAN	INDIA	House No.	Koneru Lakshmaiah Education Foundation Deemed to be University
			Street	Vaddeswaram
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			State	Andhra Pradesh
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			Pin code	522 502

Name in Full	Nationality	Country of Residence	Address of the Applicant	
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			Street	Market Centre

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Dr. Y. UDAYA KUMAR	INDIAN	INDIA	City	Guntur
			State	Andhra Pradesh
			Country	India
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Name in Full	Nationality	Country of Residence	Address of the Applicant	
DR. SHAIK MOHIDDIN SHAW	INDIAN	INDIA	House No.	Narasaraopeta Engineering College (Autonomous)
			Street	
			City	Narasaraopet
			State	Andhra Pradesh
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Name in Full	Nationality	Country of Residence	Address of the Applicant	
Dr. K.S. BALAMURUGAN	INDIAN	INDIA	House No.	RVR & JC College of Engineering
			Street	
			City	Guntur
			State	Andhra Pradesh
			Country	India
			Pin code	522019
Name in Full	Nationality	Country of Residence	Address of the Applicant	
Dr. J L RAMA PRASAD	INDIAN	INDIA	House No.	P B Siddhartha College of Arts & Science
			Street	
			City	VIJAYAWADA
			State	ANDHRA PRADESH
			Country	India
			Pin code	522 010
Name in Full	Nationality	Country of Residence	Address of the Applicant	
Dr. G. DHARMAIAH	INDIAN	INDIA	House No.	Narasaraopeta Engineering College,
			Street	Yellamanda, Kotappakonda Road
			City	Narasaraopet.

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

			State	Andhra Pradesh
			Country	India
			Pin code	522 601
Name in Full	Nationality	Country of Residence	Address of the Applicant	
SREELATHA SANKURU	INDIAN	INDIA	House No.	SINGARENI COLLIERIES WOMEN'S DEGREE COLLEGE
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Name in Full	Nationality	Country of Residence	Address of the Applicant	
NADIMPALLI UDAYA BHASKARA VARMA	INDIAN	INDIA	House No.	DNR COLLEGE OF ENGINEERING & TECHNOLOGY
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			City	BHIMAVARAM
			State	ANDHRA PRADESH
			Country	INDIA
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Name in Full	Nationality	Country of Residence	Address of the Applicant	
KRANTHI KUMAR KOTHA	INDIAN	INDIA	House No.	Gudlavalleru Engineering college
			Street	Shesadri Rao Knowledge village
			City	Gudlavalleru
			State	Andhra Pradesh
			Country	India
			Pin code	521 356

5. TITLE OF THE INVENTION: UNDER WATER POLLUTION MONITORING SYSTEM TO PREVENT FATALITIES OF AQUA ECOSYSTEM.

~~6. AUTHORISED REGISTERED PATENT AGENT(S)~~	~~IN/PA-No.~~	
	~~Name~~	
	~~Mobile No.~~	

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

7. ADDRESS FOR SERVICE OF APPLICANT IN INDIA	Name	Dr. Ch. BABY RANI, Associate Professor, V.R. Siddhartha Engineering College,
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	Mobile No.	9959503380
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	Name	Dr. NALLAPATI VEDAVATHI, Koneru Lakshmaiah Education Foundation Deemed to be University
	Postal Address	Vaddeswaram, Guntur-522 502, Andhra Pradesh
	Telephone No.	
	Mobile No.	9849067889
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	Name	Dr. Y. UDAYA KUMAR, Associate Professor, Mathematics, Hindu college, Market Centre
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	Postal Address	Narasaraopet – 522 601, Andhra Pradesh
	Telephone No.	

12-Oct-2020/89136/202041044264/Form 1

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Mobile No.	9440644473
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Fax No.	
E-mail ID	dharma.g2007@gmail.com
Name	SREELATHA SANKURU, LECTURER, MATHEMATICS,

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

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	Mobile No.	8500406256
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	E-mail ID	ssreelathamsc@gmail.com
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	Mobile No.	9866157035
	Fax No.	
	E-mail ID	Varma9866157035@gmail.com
	Name	KRANTHI KUMAR KOTHA, Assistant professor Basic sciences and Humanities, Gudlavalleru Engineering college
	Postal Address	Shesadri Rao Knowledge village, Gudlavalleru-521 356, Andhra Pradesh
	Telephone No.	
	Mobile No.	9848117580
	Fax No.	
	E-mail ID	kotha_kranthi78@yahoo.co.in

~~8. IN CASE OF APPLICATION CLAIMING PRIORITY OF APPLICATION FILED IN CONVENTION COUNTRY, PARTICULARS OF CONVENTION APPLICATION~~

~~Country~~	~~Application Number~~	~~Filing date~~	~~Name of the Applicant~~	~~Title of the invention~~	~~IPC (as classified in the convention country)~~

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PATENT OFFICE CHENNAI 13/10/2020 12:36

~~9. IN CASE OF PCT NATIONAL PHASE APPLICATION, PARTICULARS OF INTERNATIONAL APPLICATION FILED UNDER PATENT CO-OPERATION TREATY (PCT)~~	
~~International application number~~	~~International filing date~~
~~10. IN CASE OF DIVISIONAL APPLICATION FILED UNDER SECTION 16, PARTICULARS OF ORIGINAL (FIRST) APPLICATION~~	
~~Original (first) application No.~~	~~Date of filing of original (first) application~~
~~11. IN CASE OF PATENT OF ADDITION FILED UNDER SECTION 54, PARTICULARS OFMAIN APPLICATION OR PATENT~~	
~~Main application/patent No.~~	~~Date of filing of main application~~

12. DECLARATIONS

(i) Declaration by the inventors

(In case the applicant is an assignee: the inventors may sign herein below or the applicant may upload the assignment or enclose the assignment with this application for patent or send the assignment by post/electronic transmission duly authenticated within the prescribed period).

We, the above-named inventors are the true & first inventors for this Invention and declare that the applicants herein are our assignee or legal representative.

(a) Date: 10.10.2020

(b) Signature: 

(c) Name: Dr. Ch. BABY RANI

(a) Date:10.10.2020

(b) Signature: _____

(c)Name: Dr. NALLAPATI VEDAVATHI

(a) Date: 10.10.2020

(b) Signature: 

(c) Name: Dr. Y. UDAYA KUMAR

(a) Date: 10.10.2020

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

(b) Signature:

(c) Name: Dr. SHAIK MOHIDDIN SHAW

(a) Date: 10.10.2020

(b) Signature:

(c) Name: Dr. K.S. BALAMURUGAN

(a) Date: 10.10.2020

(b) Signature:

(c) Name: Dr. J L RAMA PRASAD

(a) Date: 10.10.2020

(b) Signature:

(c) Name: Dr. G. DHARMAIAH

(a) Date: 10.10.2020

(b) Signature:

(c) Name: SREELATHA SANKURU

(a) Date: 10.10.2020

(b) Signature:

(c) Name: NADIMPALLI UDAYA BHASKARA VARMA

(a) Date: 10.10.2020

(b) Signature:

(c) Name: KRANTHI KUMAR KOTHA

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

(ii) Declaration by the applicant(s) in the convention country

(In case the applicant in India is different than the applicant in the convention country: the applicant in the convention country may sign herein below or applicant in India may upload the assignment from the applicant in the convention country or enclose the said assignment with this application for patent or send the assignment by post/electronic transmission duly authenticated within the prescribed period)

I/We, the applicant(s) in the convention country declare that the applicant(s) herein is/are my/our assignee or legal representative.

(a) Date

(b) Signature(s)

(c) Name(s) of the signatory

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

(iii) Declaration by the applicants

We the applicants hereby declares that: -

We are in possession of the above-mentioned invention.

- ☐ The provisional/complete specification relating to the invention is filed with this application - NA
- ☐ The invention as disclosed in the specification uses the biological material from India and the necessary permission from the competent authority shall be submitted by us before the grant of patent to us.
- ☐ There is no lawful ground of objection(s) to the grant of the Patent to us - NA
- ☐ We are the true &first inventors.
- ☐ We are the assignee or legal representative of true & first inventors
- ☐ The application or each of the applications, particulars of which are given inParagraph-8, was the first application in convention country/countries in respect of my/our invention(s) - NA
- ☐ We claim the priority from the above-mentioned application(s) filed in convention country/countries and state that no application for protection in respect of the invention had been made in a convention country before that date by me/us or by any person from which We derive the title.
- ☐ Our application in India is based on international application under Patent Cooperation Treaty (PCT) as mentioned inParagraph-9 - NA
- ☐ The application is divided out of Our application particulars of which is given in Paragraph-10 and pray that this application may be treated as deemed to have been filed on DD/MM/YYYY under section 16 of the Act.
- ☐ The said invention is an improvement in or modification of the invention particulars of which are given inParagraph-11 – NA

13. FOLLOWING ARE THE ATTACHMENTS WITH THE APPLICATION

(a) Form 2

Item	Details	Fee	Remarks
Complete/ provisional specification) #	No. of pages One		
No. of Claim(s)	No. of claims Five and No. of pages - Two		
Abstract	No. of pages - One		
No. of Drawing(s)	No. of drawings and No. of pages – Seven and Five		

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PATENT OFFICE CHENNAI 13/10/2020 12:36

In case of a complete specification, if the applicant desires to adopt the drawings filed with his provisionalspecificationasthedrawingsorpartofthedrawingsforthecompletespecification underrule13(4), the number of such pages filed with the provisional specification are required to be mentioned here.

(b) Complete specification (in conformation with the international application)/as amended before the International Preliminary Examination Authority (IPEA), as applicable(2copies).

(c) Sequence listing in electronic form

(d) Drawings (in conformation with the international application)/as amended before the International Preliminary Examination Authority (IPEA), as applicable (2copies).

(e) Priority document(s) or a request to retrieve the priority document(s) from DAS (Digital Access Service) if the applicant had already requested the office of first filing to make the priority document(s) available to DAS.

(f) Translation of priority document/Specification/International Search Report/International Preliminary Report on Patentability.

(g) Statement and Undertaking on Form3

(h) Declaration of Inventorship on Form5

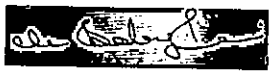
(i) Power of Authority

(j).....

Total fee ₹4,500.00 in Demand Draft No. 211703 Date 07.10.2020 Drawn on Axis Bank, Nanganallur Branch.

We hereby declare that to the best of our knowledge, information and belief the fact and matters slated herein are correct and We request that a patent may be granted to us for the said invention.

Dated this Tuesday of 10th October.2020

(a) Signature: 

(b) Name: Dr. Ch. BABY RANI

(a) Signature: _____

(b) Name: Dr. NALLAPATI VEDAVATHI

(a) Signature: 

(b) Name: Dr. Y. UDAYA KUMAR

(a) Signature: Sk. Mohiddin Shaw.

(b) Name: Dr. SHAIK MOHIDDIN SHAW

(a) Signature:

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

(b) Name: Dr. K.S. BALAMURUGAN

(a) Signature:

(b) Name: Dr. J L RAMA PRASAD

(a) Signature:

(b) Name: Dr. G. DHARMAIAH

(a) Signature:

(b) Name: SREELATHA SANKURU

(a) Signature:

(b) Name: NADIMPALLI UDAYA BHASKARA VARMA

(a) Signature:

(b) Name: KRANTHI KUMAR KOTHA

To,

The Controller of Patents

The Patent Office, at Guindy, Chennai – 600 032.

Note: -

* Repeat boxes in case of more than one entry.
* To be signed by the applicant(s) or by authorized registered patent agent otherwise where mentioned.
* Tick (✓)/cross (x) whichever is applicable/not applicable in declaration inparagraph-12.
* Name of the inventor and applicant should be given in full, family name in the beginning.
* Strike out the portion which is/are not applicable.
* For fee: See First Schedule";

12-Oct-2020/89136/202041044264/Form 1

PATENT OFFICE CHENNAI 13/10/2020 12:36

700301569

FORM 2
THE PATENT ACT 1970
(39 of 1970)
&
The Patents Rules, 2003
PROVISIONAL/COMPLETE SPECIFICATION
(See Section 10 and Rule 13)

COMPLETE

UNDER WATER POLLUTION MONITORING SYSTEM TO PREVENT FATALITIES OF AQUA ECOSYSTEM

1. APPLICANTS

(a) NAME: Dr. Ch. BABY RANI
(b) NATIONALITY: INDIAN
(c) ADDRESS: V.R. Siddhartha Engineering College, Kanuru, Krishna District, Vijayawada- 520 007

(a) NAME: Dr. NALLAPATI VEDAVATHI
(b) NATIONALITY: INDIAN
(c) ADDRESS: Koneru Lakshmaiah Education Foundation Deemed to be University Vaddeswaram, Guntur- 522 502

(a) NAME: Dr. Y. UDAYA KUMAR
(b) NATIONALITY: INDIAN
(c) ADDRESS: Hindu college, Market Centre, Guntur - 522003, Andhra Pradesh,

(a) NAME: Dr. SHAIK MOHIDDIN SHAW
(b) NATIONALITY: INDIAN
(c) ADDRESS: Narasaraopeta Engineering College (Autonomous), Narasaraopet- 522 601, Andhra Pradesh

(a) NAME: Dr. K.S. BALAMURUGAN
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12-Oct-2020/89136/202041044264/Form 2(Title Page)

PATENT OFFICE CHENNAI 13/10/2020 12:36

(a) NAME: Dr. J L RAMA PRASAD
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Andhra Pradesh

(a) NAME: Dr. G. DHARMAIAH
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(a) NAME: SREELATHA SANKURU
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(a) NAME: KRANTHI KUMAR KOTHA
(b) NATIONALITY: INDIAN
(c) ADDRESS: Gudlavalleru Engineering college, Shesadri Rao Knowledge village,
Gudlavalleru-521 356, Andhra Pradesh

The following specification particularly describes the invention

and the manner in which it is to be performed

12-Oct-2020/89136/202041044264/Form 2(Title Page)

PATENT OFFICE CHENNAI 13/10/2020 12:36

FORM 9
THE PATENT ACT, 1970
(39 of 1970)
&
The Patents Rules, 2003
REQUEST FOR PUBLICATION
[See Section 11A[2]; rule 24A]

CBR : 34169
DATE : 12/10/2020
AMT : 2750/-
P.V
12/10/20

1. Name, Address and Nationality of the Applicants

1. APPLICANTS

(a) NAME: Dr. Ch. BABY RANI
(b) NATIONALITY: INDIAN
(c) ADDRESS: V.R. Siddhartha Engineering College, Kanuru, Krishna District, Vijayawada- 520 007

(a) NAME: Dr. NALLAPATI VEDAVATHI
(b) NATIONALITY: INDIAN
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(a) NAME: Dr. Y. UDAYA KUMAR
(b) NATIONALITY: INDIAN
(c) ADDRESS: Hindu college, Market Centre, Guntur - 522003, Andhra Pradesh,

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700301570

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(a) NAME: Dr. G. DHARMAIAH
(b) NATIONALITY: INDIAN

12-Oct-2020/89136/202041044264/Form 9

PATENT OFFICE CHENNAI 13/10/2020 12:37

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(b) **NATIONALITY:** INDIAN
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(a) **NAME: KRANTHI KUMAR KOTHA**
(b) **NATIONALITY:** INDIAN
(c) **ADDRESS: Gudlavalleru Engineering college, Shesadri Rao Knowledge village, Gudlavalleru-521 356, Andhra Pradesh**

2. To be Signed by the Applicant Or his authorized registered Patent Agent	hereby request for early publication of our application for Patent No.....dated ..... under section 11A(2) of the Act. Dated this .....day of .....20.

2. Name of the Natural Persons Has signed

(a) Signature: 

(b) Name: Dr. Ch. BABY RANI

(a) Signature: _____

(b) Name: Dr. NALLAPATI VEDAVATHI

(a) Signature: 

(b) Name: Dr. Y. UDAYA KUMAR

(a) Signature:

(b) Name: Dr. SHAIK MOHIDDIN SHAW

(a) Signature:

(b) Name: Dr. K.S. BALAMURUGAN

12-Oct-2020/89136/2020410442 64/Form 9

PATENT OFFICE CHENNAI 13/10/2020 12:37

(c) Signature:

(d) Name: Dr. J L RAMA PRASAD

(a) Signature:

(b) Name: Dr. G. DHARMAIAH

(a) Signature:

(b) Name: SREELATHA SANKURU

(a) Signature:

(b) Name: NADIMPALLI UDAYA BHASKARA VARMA

(a) Signature:

(b) Name: KRANTHI KUMAR KOTHA

To

The Controller of Patents,
The Patent Office,
At Guindy, Chennai 600 032.

Note:- For Fee: See First Schedule

12-Oct-2020/89136/202041044264/Form 9

PATENT OFFICE CHENNAI 13/10/2020 12:37

Thai Journal of **Math**ematics
Volume 18 Number 2 (2020)
Pages 775–781

http://thaijmath.in.cmu.ac.th

ISSN 1686-0209

A New Family of Semicircular and Circular Arc Tan-Exponential Type Distributions

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Abstract In this paper, an attempt is made to construct a new family of two parameter semicircular model, we call this as Semicircular Arc Tan-Exponential Type distribution, by applying simple projection on Arc Tan-Exponential Type distribution ([Y. Phani, S.V.S. Girija, A.V.D. Rao, Arc Tan-Exponential Type distribution induced by stereographic projection/ bilinear transformation on modified wrapped exponential distribution, Journal of Applied Mathematics, Statistics and Informatics (JAMSI) 9 (1) (2013) 69–74]) for modeling semicircular data, probability density and cumulative distribution functions of said model are presented and their graphs are plotted for various values of parameters. It is extended to the l-axial Arc Tan-Exponential Type distribution by simple transformation for modeling any arc of arbitrary length.

MSC: 60E05; 62H11
Keywords: circular models; characteristic function; probability distribution; projection

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1. Introduction

Angular / circular data are very common in the areas of Biology, Geology, Meteorology, Earth Science, Political Science, Economics, Computer Science, etc. Full circular models are prevalent in most of the text books (Fisher, 1993 [1]; Jammalamadaka and Sen Gupta, 2001 [2]; Mardia and Jupp, 2000 [3]). The authors like Gradshteyn and Ryzhik (2007) [4], Ahn and Kim (2008) [5] and Yedlapalli et al. (2013) [6] noticed that for some cases angular data do not require full circular models for modeling. For example, when sea turtles emerge from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random

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variable is sufficient for such angular data. A few more examples of semicircular data are available in Ugai et al. (1977) [7].

Guardiola (2004) [4] obtained the semicircular normal distribution by using a simple projection and Byoung et al. (2008) [5] developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Phani et al. (2013) [6] constructed some semicircular distributions by applying Inverse Stereographic projection, Dattatreya Rao et al. (2016) [8] developed a circular logistic distribution by applying inverse stereographic projection. In this paper we developed a family of Semicircular Arc Tan-Exponential Type distribution (**SCATETD**) by projecting Arc Tan-Exponential Type distribution (Phani et al., 2013 [9]) over a semicircular segment. We plotted the graphs of the density function and distribution function for various values of parameters. We derive the characteristic function for some of the members of the proposed family, and also we extend this family for l-axial also.

2. Arc Tan-Exponential Type Distributions

Definition 2.1. Let X be a continuous random variable and $\alpha > 0$ and $\lambda > 0$ be real numbers. Then X is said to have an *Arc Tan-Exponential Type distribution* with parameters α and λ, if the probability density and cumulative distribution functions are given respectively by

$$f_X(x;\alpha,\lambda) = \frac{\alpha\lambda \operatorname{cosech}(\pi\lambda)\exp\left(-2\lambda\tan^{-1}\left(\frac{x}{\alpha}\right)\right)}{(x^2+\alpha^2)} \tag{2.1}$$

and

$$F_X(x;\alpha,\lambda) = \frac{\operatorname{cosech}(\pi\lambda)\left[\exp(\pi\lambda) - \exp\left(-2\tan^{-1}\left(\frac{x}{\alpha}\right)\right)\right]}{2} \tag{2.2}$$

where $-\infty < x < \infty$, $\alpha > 0$ and $\lambda > 0$.

By applying simple projection defined by $x = \nu\tan(\theta)$, $\nu > 0$, which leads to a Semicircular Arc Tan-Exponential Type distribution, whose probability density and cumulative distribution functions are given by

$$g(\theta) = \frac{\sigma\lambda \operatorname{cosech}(\lambda\pi)\sec^2(\theta)}{(\sigma^2+\tan^2(\theta))}\exp\left(-2\lambda\tan^{-1}\left(\frac{\tan(\theta)}{\sigma}\right)\right) \tag{2.3}$$

and

$$G(\theta) = \frac{\operatorname{cosech}(\pi\lambda)}{2}\left[\exp(\pi\lambda) - \exp\left(-2\lambda\tan^{-1}\left(\frac{\tan(\theta)}{\sigma}\right)\right)\right] \tag{2.4}$$

where $-\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$, $\lambda > 0$ and $\sigma = \frac{\alpha}{\nu} > 0$ and it is denoted by $\mathbf{SCATETD}(\lambda,\sigma)$.

Case(1) When $\sigma = 1$, i.e., $\nu \to \alpha$, we get

$$g(\theta) = \lambda \operatorname{cosech}(\pi\lambda)\exp(-2\lambda\theta), \text{ where } -\frac{\pi}{2} < \theta \leq \frac{\pi}{2},$$

we call it as Semicircular Exponential-Type distribution.

Case(2) When $\sigma = 1$, i.e., $\nu \to \alpha$ and $\lambda \to 0^+$, we get

$$g(\theta) = \frac{1}{\pi}, \text{ where } -\frac{\pi}{2} < \theta \leq \frac{\pi}{2},$$

which is a Semicircular uniform distribution.

2.1. Semicircular Exponential Type Distribution

Definition 2.2. A random variable X_{SC} on the Semicircle is said to have the *Semicircular Exponential-Type distribution* with parameter $\lambda > 0$ denoted by **SCETD**(λ), if the probability density, the cumulative distribution and the characteristic functions are respectively given by

(i)

$$g(\theta) = \lambda \text{cosech}(\pi\lambda) \exp(-2\lambda\theta) \tag{2.5}$$

where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $\lambda > 0$,

(ii)

$$G(\theta) = \frac{\lambda}{2} \text{cosech}(\pi\lambda) [\exp(\pi\lambda) - \exp(-2\lambda\theta)] \tag{2.6}$$

where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $\lambda > 0$ and

(iii)

$$\varphi_p = \alpha_p + i\beta_p, \ p = 0, \pm1, \pm2, \pm3, ...,$$

where

$$\alpha_p = \left(\frac{4\lambda^2}{4\lambda^2+p^2}\right) \cos\left(\frac{p\pi}{2}\right) + \left(\frac{2\lambda p}{4\lambda^2+p^2}\right) \sin\left(\frac{p\pi}{2}\right) \coth \pi\lambda$$

$$\beta_p = \left(\frac{2\lambda p}{4\lambda^2+p^2}\right) \cos\left(\frac{p\pi}{2}\right) - \left(\frac{4\lambda^2}{4\lambda^2+p^2}\right) \sin\left(\frac{p\pi}{2}\right) \coth \pi\lambda$$

i.e.

$$\alpha_p = \begin{cases} (-1)^{\left(\frac{p-1}{2}\right)} \left(\frac{2\lambda p}{4\lambda^2+p^2}\right) \coth \pi\lambda, & \text{if p is odd} \\ (-1)^{\left(\frac{p}{2}\right)} \left(\frac{4\lambda^2}{4\lambda^2+p^2}\right), & \text{if p is even} \end{cases}$$

$$\beta_p = \begin{cases} (-1)^{\left(\frac{p+1}{2}\right)} \left(\frac{4\lambda^2}{4\lambda^2+p^2}\right) \coth \pi\lambda, & \text{if p is odd} \\ (-1)^{\left(\frac{p}{2}\right)} \left(\frac{2\lambda p}{4\lambda^2+p^2}\right), & \text{if p is even.} \end{cases} \tag{2.7}$$

Graphs of pdf and cdf of Semicircular Exponential Type Distribution for Various Values of the Parameter

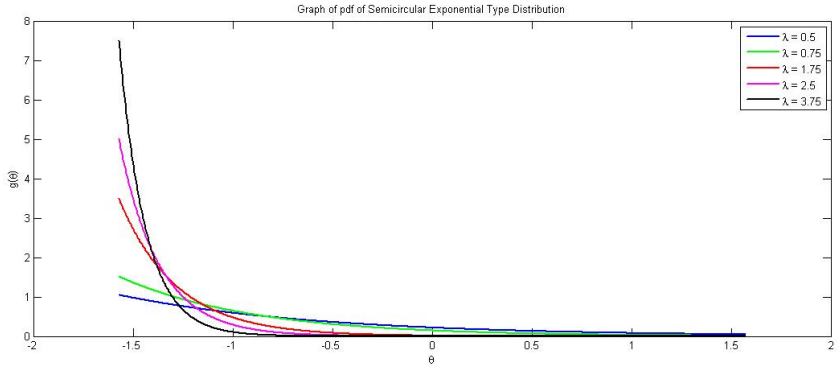

FIGURE 1. Graph of pdf of Semicircular Exponential Type Distribution

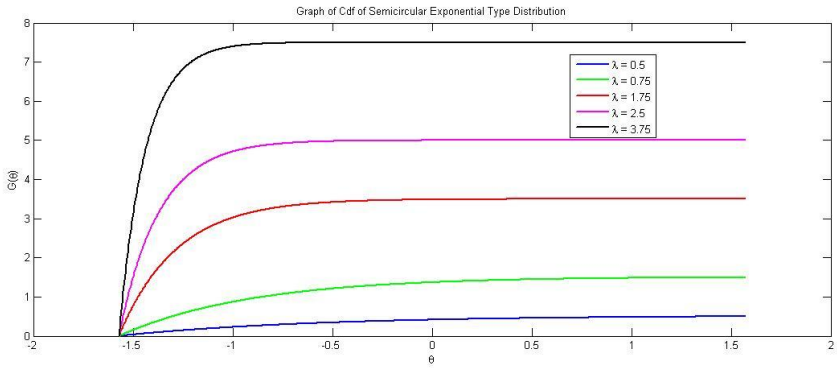

FIGURE 2. Graph of cdf of Semicircular Exponential Type Distribution

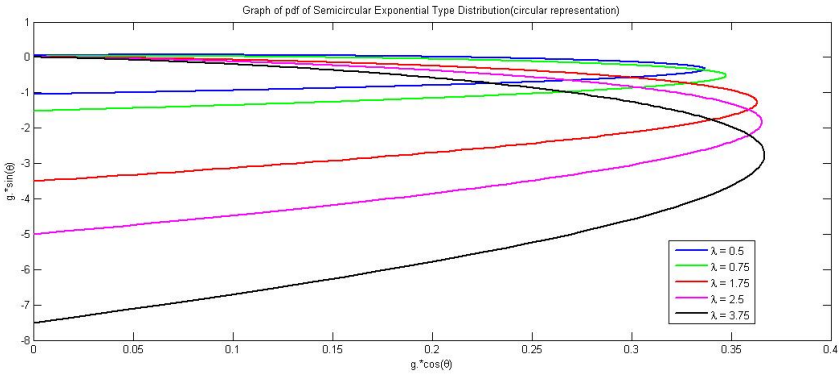

FIGURE 3. Graph of pdf of Semicircular Exponential Type Distribution (Circular Representation)

3. Extension to l-Axial Distribution

We extend the proposed family to the l -axial distribution, which is applicable to any arc of arbitrary length say $\frac{2\pi}{l}$ for $l = 1, 2, 3, ...$ To construct the l -axial Arc Tan-Exponential Type distribution, we consider the density function of Semicircular Arc Tan-Exponential Type distribution and use the transformation $\phi = \frac{2\pi}{l}$, $l = 1, 2, 3, ...$ The probability density function of ϕ is given by

$$g(\phi) = \frac{l\sigma\lambda \text{cosech}(\lambda\pi)\sec^2\left(\frac{l\phi}{2}\right)}{2\left(\sigma^2 + \tan^2\left(\frac{l\phi}{2}\right)\right)} \exp\left(-2\lambda\tan^{-1}\left(\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}\right)\right), \; -\frac{\pi}{l} < \phi < \frac{\pi}{l}. \tag{3.1}$$

Case (1): When $l = 1$ in (3.1), we get a density function

$$g(\phi) = \frac{\sigma\lambda \text{cosech}(\lambda\pi)\sec^2\left(\frac{\phi}{2}\right)}{2\left(\sigma^2 + \tan^2\left(\frac{\phi}{2}\right)\right)} \exp\left(-2\lambda\tan^{-1}\left(\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}\right)\right), \; -\pi < \phi < \pi. \tag{3.2}$$

We call it as Circular Arc Tan-Exponential Type distribution.

Case (2): When $l = 1$ and $\sigma = 1$ in (3.1), we get a density function

$$g(\phi) = \frac{\lambda \text{cosech}(\lambda\pi)}{2} \exp(-\lambda\phi), \quad -\pi < \phi < \pi. \tag{3.3}$$

We call it as Circular Exponential Type distribution.

Case (3): When $l = 2$ in (3.1), we get a density function

$$g(\phi) = \frac{\sigma\lambda \text{cosech}(\lambda\pi)\sec^2(\phi)}{\sigma^2 + \tan^2(\phi)} \exp\left(-2\lambda\tan^{-1}\left(\frac{\tan(\phi)}{\sigma}\right)\right), \; -\frac{\pi}{2} < \phi < \frac{\pi}{2}. \tag{3.4}$$

Which is same as Semicircular Arc Tan-Exponential Type distribution.

3.1. Circular Exponential Type Distribution

Definition 3.1. A random variable X_s on the circle is said to have the *Circular Exponential Type distribution* with parameter $\lambda > 0$ denoted by **CETD**(λ), if the probability density, the cumulative distribution and the characteristic function are respectively given by

$$\text{i)} \quad g(\theta) = \frac{\lambda \text{cosech}(\lambda\pi)}{2} \exp(-\lambda\theta), \quad -\pi < \theta < \pi, \tag{3.5}$$

$$\text{ii)} \quad G(\theta) = \frac{\lambda}{2}\text{cosech}(\lambda\pi)\left[\exp(\pi\lambda) - \exp(-2\lambda\theta)\right], \quad -\pi < \theta < \pi \tag{3.6}$$

and

$$\text{iii)} \quad \varphi_p = \alpha_p + i\beta_p, \; p = 0, \pm1, \pm2, \pm3, ... \tag{3.7}$$

where

$$\alpha_p = \left(\frac{4\lambda^2}{4\lambda^2+p^2}\right)\cos\left(\frac{p\pi}{2}\right) + \left(\frac{2\lambda p}{4\lambda^2+p^2}\right)\sin\left(\frac{p\pi}{2}\right)\coth(\pi\lambda),$$

$$\beta_p = \left(\frac{2\lambda p}{4\lambda^2+p^2}\right)\cos\left(\frac{p\pi}{2}\right) - \left(\frac{4\lambda^2}{4\lambda^2+p^2}\right)\sin\left(\frac{p\pi}{2}\right)\coth(\pi\lambda).$$

Characteristics of Semicircular Exponential Type Distribution

Parameter(s)	**Value(s)**
Trigonometric moments	$\alpha_p = \begin{cases} (-1)^{\left(\frac{p-1}{2}\right)}\left(\frac{2\lambda p}{4\lambda^2+p^2}\right)\coth(\pi\lambda) \text{ if p is odd} \\ (-1)^{\left(\frac{p}{2}\right)}\left(\frac{4\lambda^2}{4\lambda^2+p^2}\right) \text{ if p is even} \end{cases}$ $\beta_p = \begin{cases} (-1)^{\left(\frac{p+1}{2}\right)}\left(\frac{4\lambda^2}{4\lambda^2+p^2}\right)\coth(\pi\lambda) \text{ if p is odd} \\ (-1)^{\left(\frac{p}{2}\right)}\left(\frac{2\lambda p}{4\lambda^2+p^2}\right) \text{ if p is even} \end{cases}$
ρ_p and μ_p^0	$\rho_p = \frac{2\lambda}{\sqrt{(4\lambda^2+p^2)}}\left[\sqrt{\cos^2\left(\frac{p\pi}{2}\right)+\sin^2\left(\frac{p\pi}{2}\right)\coth^2(\pi\lambda)}\right]$ $\mu_p^0 = \tan^{-1}\left(\frac{p\cos\left(\frac{p\pi}{2}\right)-2\lambda\sin\left(\frac{p\pi}{2}\right)\coth(\pi\lambda)}{2\lambda\cos\left(\frac{p\pi}{2}\right)+p\sin\left(\frac{p\pi}{2}\right)\coth(\pi\lambda)}\right)$
Resultant length	$\rho = \frac{2\lambda\coth(\pi\lambda)}{\sqrt{4\lambda^2+1}}$
Mean direction	$\mu_0 = 2\pi + \tan^{-1}(2\lambda)$
Circular variance	$V_0 = \frac{\sqrt{4\lambda^2+1}-2\lambda\coth(\pi\lambda)}{\sqrt{4\lambda^2+1}}$
Circular standard deviation	$\sigma_0 = \sqrt{-2\ln(1-V_0)}$ $= \sqrt{\ln(4\lambda^2+1)-2\ln(2\lambda\coth(\pi\lambda))}$
Central trigonometric moments	$\alpha_p^* = \rho_p\cos\left(\mu_p^0 - p\mu_0\right)$ $\beta_p^* = \rho_p\sin\left(\mu_p^0 - p\mu_0\right)$
Skewness	$\gamma_1^0 = \frac{\beta_2^*}{V_0^{3/2}}$ where $\beta_2^* = \left(\frac{\lambda}{1+\lambda^2}\right)\sin\left(\tan^{-1}\left(\frac{1}{\lambda}\right) - 4\pi - 2\tan^{-1}(2\lambda)\right)$
Kurtosis	$\gamma_2^0 = \frac{\alpha_2^*-(1-V_0)^4}{V_0^2}$ where $\alpha_2^* = \left(\frac{\lambda}{1+\lambda^2}\right)\cos\left(\tan^{-1}\left(\frac{1}{\lambda}\right) - 4\pi - 2\tan^{-1}(2\lambda)\right)$

4. Conclusion

In this paper, we investigated a family of semicircular and circular Arc Tan-Exponential Type distributions, which follows from inducing simple projection on Arc Tan-Exponential Type distribution. The density and distribution functions of this distribution admit explicit forms, as do characteristic function. As this distribution is asymmetric, it is hoped that it offers a better option for modeling skew circular as well as l-axial data.

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References

[1] N.I. Fisher, Statistical Analysis of Circular Data, Cambridge University University Press, Cambridge, 1993.

[2] S.R. Jammalamadaka, A. Sen Gupta, Topics in Circular Statistics, World Scientific Press, Singapore, 2001.

[3] K.V. Mardia, P.E. Jupp, Directional Statistics, John Wiley, Chichester, 2000.

[4] I.S. Gradshteyn, I.M. Ryzhik, Table of Integrals, Series and Products (7th edition), Academic Press, 2007.

[5] B.J. Ahn, H.M. Kim, A new family of semicircular models: the semicircular Laplace distributions, Communications of the Korean Statistical Society 15 (2008) 775–781.

[6] P. Yedlapalli, S.V.S. Girija, A.V.D. Rao, On construction of stereographic semi circular models, Journal of Applied Probability and Statistics 8 (1) (2013) 75–90.

[7] S.K. Ugai, M. Nishijima, T. Kan, Characteristics of raindrop size and raindrop shape, Open Symposium URSI Commission F (1977) 225–230.

[8] A.V.D. Rao, S.V.S. Girija, Y. Phani, Stereographic logistic model - application to noisy scrub birds data, Chilean Journal of Statistics 7 (2) (2016) 69–79.

[9] Y. Phani, S.V.S. Girija, A.V.D. Rao, Arc Tan-Exponential Type distribution induced by stereographic projection/ bilinear transformation on modified wrapped exponential distribution, Journal of Applied Mathematics, Statistics and Informatics (JAMSI) 9 (1) (2013) 69–74.

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RESEARCH ARTICLE

LEADERSHIP OF WOMEN THROUGH PANCHAYAT GOVERNANCE (A CASE STUDY OF ELECTED WOMEN REPRESENTATIVES OF PRAKASAM DISTRICT OF ANDHRA PRADESH)

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Leadership, RuralGovernance, Awareness, Women Participation

Abstract

Women's participation in politics has been an issue of discussion in recent years. The dominance of caste, culture and patriarchy has resulted in ignorance among women about public life. The process of womens participation in local governance is in the transitory stage and needs to be carefully scurried to bring about an improvement in the better participation of women in rural politics. The leadership of women at grassroots democracy level still requires validation for effective participation. The goal of enlightening rural women will be achieved only if women are brought up to a level where they can participate in all walks of life. The goal of political equality can be achieved through women's involvement in the process of empowerment. The objective of this study is to examine the political status of women in the local institution of Grama Panchayat in the Prakasam district of Andhra Pradesh in India. 73rd Amendment of the Indian Constitution has been given a space to women to enable them to participate in the village development through gross root governance. It is seen in recent local body elections that the huge number of women is being elected to panchayat institutions. However, it is observed that lack of leadership is being created various types of problems while performing their legitimate roles in the public sphere. This paper deals the poor leadership faced by the women elected representatives from the coastal area of Prakasam district of Andhra Pradesh. The findings revealed that the political participation of women in the functioning and the electoral process of panchayats are not satisfactory at the grassroots democracy. A parochial society and low level of education are responsible for their inactive participation in the rural local democracy.

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Introduction:-

Women's leadership is crucial in the effective implementation of any development project. Majority of schemes and projects fail due to lack of women's involvement in its planning and implementation.

Gender discrimination is an ongoing debate worldwide. Women's participation in politics has been an issue of discussion in recent years. The dominance of caste, culture and patriarchy has resulted in ignorance among women

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about public life. The process of women's participation in local governance is in the transitory stage and needs to be carefully scurried to bring about an improvement in the better participation of women in rural politics. The leadership of women at village level still requires validation for effective participation. The concept of women's participation in grassroots political institution remains a formal due to the absence of education, exposure and opportunities. The goal of enlightening rural women will be achieved only if women are brought up to a level where they can participate in all walks of life. The goal of political equality can be achieved through women's involvement in the process of empowerment at grass-root democracy.

Women need decision-making capacity and ability to take part in the community and national activities. Women leadership in panchayat governance is crucial not only for the development of women themselves but also for the availability of their creative potential for the development of society. The 73rd Constitutional Amendment Act 1992 has given a positive response to their involvement and participation to the realization of women's leadership and encouraged women to participate in the democratic process at the lower tire of political institution. The low education, ignorance, casteism and rigid patriarchal norms are subjected to exploitation at the hands of officials and the society dominated by male.

In the above context, it is important to analyze the nature and extent of women's leadership in the rural democratic process. The present paper is an attempt to measure the magnitude of women leadership and changing gender composition after the introduction of reservation for women in Gram Panchayat governance. In democratic set up, people need to wider participation irrespective caste, colour, creed and gender. The system of decentralization through Panchayati Raj Institutions (PRIs) was introduced by the 73rd Amendment Act of 1992. As women consist of half the total population, their participation in local governance is essential to make democracy successful at the lower tier of the panchayat. The question of their participation arises due to their long-standing neglected social status in Male-dominated Indian society. (Alok Pandey, 2009) [1]

A developing democracy needs the wider participation of women in grassroots politics is essential to make democratic governance successful. (Artatrana Gochhaayat, 2013) [2]. Leadership assumes significance and wider proportions in a democratic system, particularly at the lower level of governance. In the context of the rural situation, particularly in backward societies, the problem of leadership has acquired much importance in terms of the introduction of community projects, land reforms, decentralized local administration and selection of beneficiaries etc., which aim at the reconstruction of rural society into a large extent. It is believed in the constitution may well depend upon the type of leadership available at the grassroots level. (Yashoda T. 2007) [3]. Women's entry into the rural political system would ensure a change in the political system of the village, the perceptions of women and develop grassroots leadership among women. (Snehalata Panda, 1995) [4]. To facilitate people's participation in governance, new leadership also needs to be facilitated. This includes marginalized leadership sections of Dalits, tribals and youth. The social, economic, cultural and psychological aspects prevent the emergence of leadership in these groups. Strengthening women's leadership in the social-economic and political sphere becomes essential for removing gender inequality and facilitating women's empowerment (Alok Pandey, 2009) [5].

Significance of the study:

The question of the political leadership of women in rural India has assumed a considerable significance after introducing the 73rd Constitutional Amendment. This historic Amendment resulted in one-third reservation for women to take part in the democratic political process at the local body institutions. Women's leadership in political bodies at various levels is the need of the hour. The leadership of women in public life will be able to gain equal access to control over the political process. Research studies on the subject of the leadership of elected women's role in political bodies must provide an insight into the nature of their participation as well as the obstacles to their effective participation.

A silent revolution takes place in the rural Andhra Pradesh after 73rd enactment as it becomes significant to study leadership of women role in the panchayat governance in the Prakasam district of Andhra Pradesh state. The government of Andhra Pradesh has taken a historical step to enhance the percentage of reservation for women from

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one-third to at least 50 per cent in elections held in 2013 in the 3-tire rural local bodies. The study intends to bring out issues around this milestone step and all those major hurdles for the leadership of women in the public sphere. It is expected that the finding will help policy-making agencies to take suitable measures to streamline the process of women leadership and to promote women participation with an active and energetic role in the development of overall panchayat governance.

Objectives of the study:-

1. To study the socio-economic status of rural elected female leaders.
2. To examine the comparative profile of a different sample of women leaders in the Grama Panchayat.
3. To explore the awareness, leadership, decision making power and their involvement in the democratic process.
4. To explore the problems that discourages the participation of female leaders among all categories of women.

Methodology of the study:-

In Andhra Pradesh women constitute of little more than 50% of the total population. Their socio-economic status has been relatively low, and they are discrimination in all walks of life. Women have been restricted to their home and being suppressed and deprived to get fair treatment from the male-dominated society. The present study attempted to learn the extent of the political leadership of women in the functioning as well as the decision-making process of panchayats and their problems. The study was undertaken in February 2018 using a sample of women respondents from 19 Grama Panchayats of Markapuram Division in Prakasam district of Andhra Pradesh

This research paper is based on primary and secondary sources of data. The main objective of the study is to do an analysis of the leadership of the women in Markapuram Revenue Division of Prakasam District of Andhra Pradesh in India. Of the three divisions in Prakasam District Markapuram, Revenue Division is considered a backward area. (Chief Planning Officer2016-2017). Markapuram Revenue Division comprises 76-gram panchayats. In those 36 grama panchayats are headed women sarpanches. There are 380 ward members in the 76 Grama Panchayats. The primary source of data was collected by conducting personal Interviews. The secondary source data related to Panchayat representative's data was collected from the district panchayat office (DPO) and journals and publications.

In the present study, an attempt has been made to understand the social background features of the women leaders in the Grama Panchayat of Prakasam district the state of Andhra Pradesh. This empirical work is based on the Primary data collected from women representatives elected in 2013 in the western part of Prakasam District of Andhra Pradesh. The data were collected during 2018, February while conducting thesis work on women empowerment through village panchayat in the Prakasam district. This study focused on understanding the role of elected women leaders at the Grama Panchayat level. There are around 350 women representatives in the Gram Panchayat spread over three mandalas under the Markapuram Revenue Division of Prakasam district. The researcher collected primary data from 76 women representatives, including 19 presidents 14 Vice-Presidents, and 43 members were selected for an in-depth study on the basis of multi-stage stratified random sampling. After the collection of data from sample respondents, it has been systematically processed, classified and converted into Tables in the sample and accordingly analyzed. It should be noted that the data have been entirely processed on the basis of the community (caste group) of the women representatives. The researcher has drawn 20 women samples from OC, 30 from the BC, 22 from the SC and 4 from ST category as sample respondents with an intension to measure their leadership and other qualities in the extension of their empowerment.

Here, it would be necessary to deal with a brief profile of Prakasam district location, which is the area of the present investigation. Prakasam District formed on 02-02-1970 with Ongole as a headquarter by carving out portions of Ongole Revenue division from Guntur, Kandukur Revenue division from Nellore district and Markapur Revenue division from Kurnool. The district was renamed as Prakasam in 1972 in memory of the great freedom fighter late Sri Tanguturi Prakasam Panthulu also called as Andhra Kesari, who was chief minister of the composite Madras State and the first chief minister of Andhra Pradesh State.

The district was bounded by the East: Bay of Bengal, West: Kurnool district, North: party by Guntur and Mahaboobnagar Districts, South: party by Nellore and Cuddapah districts. The district is located in a tropical region between 14-57'-00" to 16-17'-00" Northern Latitude and 78-43-00' to 80-25'-00" Eastern Longitude. The central

part of the District comprises large tracts of low shrubs and Jungle diversified with rocky hills and stony plains and is a unique feature of the district. The Markapuram Division, which is a western portion of the district, is purely an upland area.

Area, population and other related particulars

The District covered an area of 17,626 sq.kms with a density of 193 persons per sq.km. It is much more in size in the area when compared to other districts of Andhra Pradesh. The district has covered a coastline of 102kms. Spread over in 10 Mandals. According to 2011 census, there are 1093 villages in the district, of which 1058 are Revenue Villages, and the rest of 35 are forest villages. In those 1093 villages, 1002 villages are inhabited, and the remaining 91 is un-inhabited villages. The Mandals in the district are not homogeneous in terms of the physical characteristics, natural resources and potentialities. The total population of the district is 33, 97,448 as per the 2011 census: It stands for 6.88 of the total population of the state and is placed 9th position in the size of the population. The district has 16, 82,684 female population, and this constituted 49.53 per cent and 6.83 per cent of the state female population.

The rural population of the district is 27, 33,866 and it comprises 80.44 per cent of the district population and 7.86 per cent to that of state rural population. Similarly, the urban population of the district spread over in 9 towns is 6, 64,582 comprising 19.56 per cent of the district population and 4.55 per cent of the state urban population.

According to the 2011 Census community-wise population, the Scheduled Caste population of the district is 7, 87,861 which 23.19 per cent of the district and 9.33 of the state total Scheduled Caste population. Similarly, the Scheduled Tribe population of the district is 1,51,145, and it stands for 4.45percent of the district and 5.04percent of the state Scheduled Tribe population. The decennial growth of population in the district is seen from 2001 census to 2011 census was 11.05percent. As regards the density of population is 193 per sq.km. Whereas it is recorded 308 per sq. Km for the Andhra Pradesh State. The district literacy rate is 63.08 per cent, which was lower than the state literacy rate of 67.41 per cent. The sex ratio of the district is 981 females per 1000 males as against 996 of the states as per the 2011 census with regards main workers are 14, 63,508 constituting 43.08 per cent of a total population of the district and 7.61% of the state population.

Administrative units of the District

The District has divided three Revenue Divisions Viz, Ongole Kandukur and Markapur. There are 56 Mandals which were formed on 25.05.1985 with the purpose of bringing the administration to access the people. There are four municipalities in the district viz, Ongole, Kandukur, Chirala and Markapuram. Besides another 4 Nagara Panchayats were also formed i.e., Addanki, Chimakurthi, Kanigiri and Giddalur. Ongole Revenue Division comprises of 20 Mandals, Kandukur Division constitutes with 24 Mandals and Markapuram Division formed with 24 Mandals. So far, the district has 56 Mandals. There are 1002 inhabited Revenue Villages are existing in Prakasam District. They were constituted into 1043 Grama Panchayats duly classifying them into 62 and 981 Grama Panchayats as notified and non-notified respectively.

According to the latest census, the district has a total population of 33.97 lakhs, consisting of 17.15 lakhs males and 16.82 females. The district rural population is accounted for 27.33 lakh persons, and the urban population is 6.64 lakhs persons. The total S.C population is 7.88 out of which 3.97 lakhs are males and 3.91 females. The district consists of 1.51 lakhs S.T population, out of which 0.77 lakhs are males and 0.74 females. It has a population density of 193. The sex ratio is 981 females for every 1000 males. The district literacy rate is 63.08, while 72.92 per cent in men and 53.11percent women.[6]

Table -1:- Sample framed.

Sl. No	Name of the Mandal	GP's Selected	No. of Members	No.of Sample Respondents Taken
1	Markapuram	8	42	32
2	P.Arthavedu	5	26	20
3	Cumbum	6	34	24
	Total	19	102	76

Source: District Panchayat office, Ongole

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Table 2:- Distribution of Sample Respondents by Position Held.

Sl. No	Position Held	Social status				
		O.C	B.C	S.C	S.T	Total
1	1	2	3	4	5	6
2	Member	7 (16.27) (35.00)	19 (44.19) (63.33)	14 (32.56) (63.63)	3 (6.98) (75.00)	43 (100%) (56.58)
3	President	5 (26.32) (25.00)	8 (42.10) (26.67)	6 (31.58) (27.27)	0 (0) (0)	19 (100%) (25.00)
4	Vice-president	8 (57.14) 40.00	3 (21.42) (10.00)	2 (14.28) (9.09)	1 (7.14) (25.00)	14 (100%) (18.42)
	Total	20 (26.31) (100%)	30 (39.47) (100%)	22 (28.95) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Respondents by Position Held

Table 2 presents information about different positions held by the women candidates in the local body institution of Grama Panchayat in the sample study area. Of the total sample, 43 respondents (56.58 per cent) were different categories of Grama panchayat Members. Of the total four respondents from the ST community interviewed for this study, 3 are members and one Vice–President. A total of 76 respondents each 4 from one panchayat has been chosen as a sample in the study area (Panchayat term, 2013 - 2018) Care was taken to include all four categories of female sample respondents of *Open Category (OC), Backward Caste (BC), Schedule Caste((SC), and Schedule Tribe (ST).*

Table 3:- Distribution of Sample Respondents by Age Group.

Sl No	Age-group (in years)	Social status				
		O.C	B.C	S.C	S. T	Total
1	1	2	3	4	5	6
2	21-35	9 (31.02) (42.86)	12 (41.38) (40.00)	7 (24.14) (31.82)	1 (3.44) (25.00)	29 (100%) (38.16)
3	36-45	8 (25.00) (38.10)	13 (40.63) (43.33)	9 (28.12) (40.90)	2 (6.25) (50.00)	32 (100%) (42.10)
4	50 Above	3 (20.00) (19.05)	5 (33.33) (16.67)	6 (40.00) (27.28)	1 (6.67) (25.00)	15 (100%) (19.74)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%))	76 (100%) (100%)

Age of Respondents:

Age is considered as a very important variable to be in public life. The data on women representatives' age has been presented in Table 3. On the whole, the majority (42.10 per cent) of women candidates have come from the middle age group of 36-45.

Table 4:- Distribution of Sample Respondents by Educational Level.

Sl. No	Educational Status	Social status				
		O.C	B.C	S.C	S.T	Total
1	1	2	3	4	5	6
2	Illiterate	5 (15.62) (25.00)	12 (37.5) (40.00)	13 (40.63) (59.09)	2 (6.25) (50.00)	32 (100%) (42.11)
3	Primary	6 (28.58) (30.00)	10 (47.61) (33.33)	3 (14.29) (13.64)	2 (9.52) (50.00)	21 (100%) (27.63)
4	High school	7 (41.17) (35.00)	5 (29.41) (16.67)	5 (29.42) (22.73)	0 (0) (0)	17 (100%) (22.37)
5	College	2 (33.33) (10.00)	3 (50.00) (10.00)	1 (16.67) (4.54)	0 (0) (0)	6 (100%) (7.89)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

A higher percentage of women leaders found from BC category in the young age group of 21-35 (40.00%) and 36-45 (43.33%). A considerable percentage (19.74) of women leaders belonged to older people found in the sample study. It is found that the women of most active age category, i.e., young age and the middle age groups have come forward to participate in public life. It is observed from the study that the local body provides new blood with a chance to utilize their capacities in the rural governance

Educational Status

Education is a very significant element in many ways, as it facilitates to aware knowledge about their powers and functions of governance. An educated woman will have a better vision and helpful to understand the Panchayat Raj System. It is a widely accepted that the overall literacy rate of women is very low in rural Andhra Pradesh. This reflects their educational backwardness of women Sarpanches and Ward members in the present study area. Now a day's educational status has a strong bearing on leadership performance. The field data in Table 4 shows that a majority (42.11%) 32 of the women candidates were illiterates from all caste groups in the sample study area. Next heist per cent (27.63) of women respondents had primary education. With regard to no education among marginalized is concerned, a higher percentage is found between SC and ST categories of women leaders, i.e., (59.09 and 50.00). As per as high school and a college education is concerned, none contributed from ST community. The study clearly reveals that only six respondents constitute nearly 8 per cent studied up to college level. Regarding post-graduation education, no respondent was found in the study area. As per the Bhargava B.S and Vidya K.C (1992)[7] study, the women representatives of PRIs on the whole lag behind men representatives in their educational level. Lack of education manifests itself in their relatively low degree of involvement in the decision-making process.

Sl. No	Occupational Status	Social status				
		O.C	B.C	S.C	S. T	Total
1	1	2	3	4	5	6
2	Housekeeper	8 (25.81) (40.00)	12 (38.72) (40.00)	10 (32.25) (45.45)	1 (3.22) (25.00)	31 (100%) (40.25)
3	Labor	5 (15.15) (25.00)	13 (39.39) (43.33)	12 (36.36) (54.55)	3 (9.10) (75.00)	33 (100%) (42.86)

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4	Business	7 (58.33) (35.00)	5 (41.67) (16.66)	00 (00) (00)	00 (00) (00)	12 (100%) (16.88)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Table 5: Distribution of Sample Respondents by Occupational Status.

Occupational Status

Occupational status is one of the most important variables in determining the socio-economic profile of the elected women representatives in the grassroots political institution. Financially, rural women have always been depended on her family. Table 3 provides data on the occupation level of the women leaders. The respondents were categorized into three groups as per their occupation.

The data shows that most of the women candidates (42.86 per cent) have been involved in agriculture-based labour work with their family. As per a house- maker is concerned, a good percentage of (40.25%) respondents reported in the sample study area. It is interesting to note that, no SC and ST women leaders found under business activity in the sample population. A considerable percentage (35.00%) of sample respondents have been found doing business in the open category. They were assisting their family in business activity. It has observed thefrom the table that most of the women leaders were found meager economic activity.

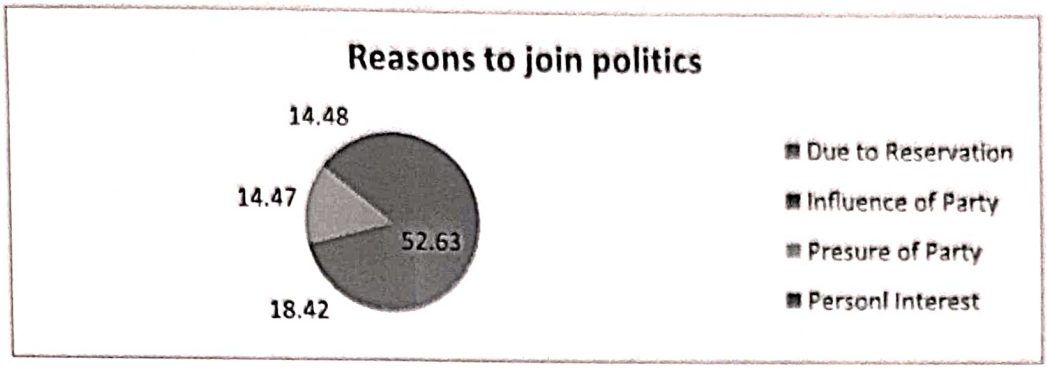
Table 6:- Distribution of Sample Respondents by Their Reasons to Join Politics.

Sl No	Reasons to Join Politics	Social status				
		O.C	B.C	S.C	S. T	Total
1	1	2	3	4	5	6
2	Due to Reservation	11 (27.50) (55.00)	16 (40.00) (53.33)	10 (25.00) (45.45)	3 (7.50) (75.00)	40 (100%) (52.63)
3	Influence of Family	7 (50.00) (35.00)	2 (14.28) (6.67)	4 (28.58) (18.18)	1 (7.14) (25.00)	14 (100%) (18.42)
4	Pressure of Party	1 (9.09) (5.00)	6 (54.55) (20.00)	4 (36.36) (18.18)	0 (00) (00)	11 (100%) (14.47)
5	Personal Interest	1 (9.09) (5.00)	6 (54.54) (20.00)	4 (36.37) (18.18)	0 (00) (00)	11 (100%) (14.48)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Respondents Reasons to Join Politics

The women in rural areas still confined mostly to domestic chores. The decision of women candidates to participate in local politics and push them into the election has been influenced by various factors. Elected women representative's responses on factors influenced to enter Grama Panchayat governance is reported in Table 6. A good percentage of the women candidates 40 (52.63%) have been entered into as new entrants in politics by reservation policy for women provided by the 73rd Constitutional Amendment which was earlier totally a male domain.

Figure-1: Respondents Reasons to Join Politics



With regard to the influence of family is concerned 14 (18.42%) respondents from all categories reported. As per a personal interest variable is concerned, it is interesting to note that of the total sample, a small percentage (14.48) of all types of respondents were contested on their own. Of the total ST community category sample, a major chunk of respondents 3 (75.00%) are elected on reservation provision. The data reveals that legislation regarding 50 per cent mandatory reservation policy has paved the way to enter the large size of women particularly marginalized women.

A considerable (14) respondent (18.42 per cent) felt that due to getting into public life by the support extended by family members. 55.00 for OC, 53.33 for BC, and 45.45 for SC women candidates opine that reservation policy is very much benefited. Majority of women candidates are in the political sphere because of reservation policy and support of their family. It is evident from the table that many of the women leaders contested local election due to other reasons than self-interest. Kumar Das (2009) [8]study also reveals that non-high caste women leaders combined dominated the grassroots politics

7. Attending Panchayat Meetings

Women respondents were asked about their punctuality in attending Gram Panchayat meetings have been reported in Table 7. On the basis of their responses, their contribution is grouped as regularly, sometimes, rarely and never. It is a positive step as the majority of the respondents were attending panchayat meetings. It is evident from the data that of the total sample respondents 19 (25.00 per cent) women candidates have been attending panchayat meetings on a regular basis. As per as never attending respondents are concerned 12 out of 76 reported in the sample study area. These respondents do not attend the meetings because the chairpersons do not inform about the meetings in time, dislike to attend and heavy domestic work. This practice is seen in mostly opposition groups and party. It is observed from the study that non-attendance of a meeting by women candidates are attributed to the traditions of orthodox rural society.

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Table 7:- Distribution of Sample Respondents by Nature of Attending Panchayat Meetings.

Sl. No	Attendance to meetings	Social status				
		O.C	B.C	S.C	S.T	Total
1	1	2	3	4	5	6
2	Regularly	7 (36.85) (38.89)	8 (42.10) (33.33)	4 (21.05) (22.22)	0 (0) (0)	19 (100%) (25.00)
3	Sometimes	5 (19.23) (27.78)	10 (38.47) (41.67)	9 (34.61) (50.00)	2 (7.69) (50.00)	26 (100%) (34.21)
4	Rarely	6 (31.58) (33.33)	6 (31.58) (25)	5 (26.31) (27.78)	2 (10.53) (50.00)	19 (100%) (25.00)
5	Never	2 (23.08) (14.29)	6 (46.15) (20.00)	4 (30.78) (18.18)	0 (0) (0)	12 (100%) (15.79)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Knowledge on GP Affairs

The data presented in Table 8 reveals that women candidates' level of knowledge about Panchayat activities. As per knowledge about panchayat affairs is concerned, most of the sample respondents have not been aware. It is said that there is a poor understanding of about fifty per cent reservation among all categories of women. Only one had knowledge about the reservation of seats for women. Nearly 13 per cent of women candidates were knowledge relating to functions, source of income. It is evident from the Table that a large majority of them do not have knowledge issues relating the local governance.

Vikas Nandal, (2013) [9] in his study found that representatives have very less awareness and have very little knowledge about the Panchayati Raj Institutions. They have very little knowledge about the Panchayati Raj Act 73rd Amendment, powers and works of the gram panchayats, sources of income of gram panchayats.

Figure -2:- Knowledge on GP.

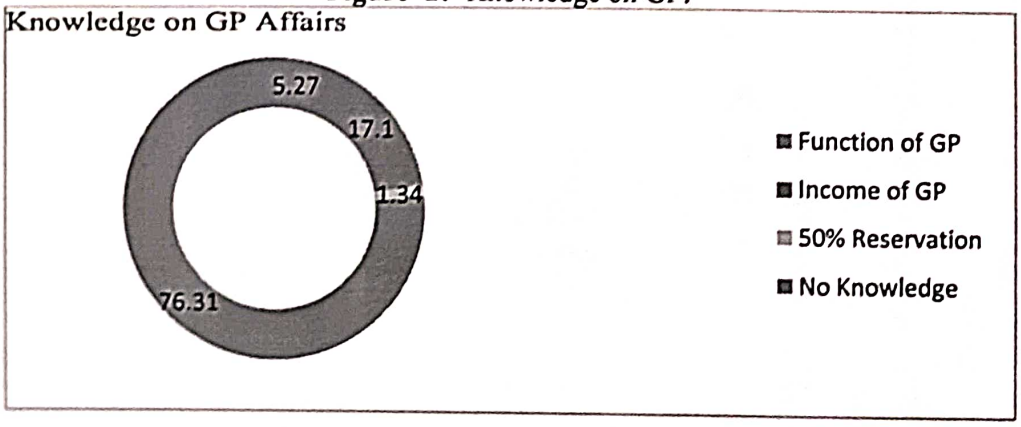


Table 8:- Distribution of Sample Respondents by Knowledge on Panchayat.

Sl. No	Respondents Knowledge on Panchayat	Social status				
		O.C	B.C	S.C	S.T	Total
1	1	2	3	4	5	6
2	Aware of Functions of GP	2 (50.00) (10.00)	1 (25.00) (3.33)	1 (25.00) (4.55)	0 (0) (0)	4 (100%) (5.27)
3	Aware of the Source of Income of GP	5 (35.72) (25.00)	6 (50.00) (20.00)	2 (14.28) (9.09)	0 (0) (0)	13 (100%) (17.10)
4	Aware of 50 per cent Reservation	0 0 0	01 100 (3.33)	0 0 0	0 0 0	01 (100%) (1.34)
5	Not at all	13 (22.41) (65.00)	22 (37.94) (73.34)	19 (32.76) (86.36)	4 (6.89) (100)	58 (100%) (76.31)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Table 9:- Participation in Decision- making.

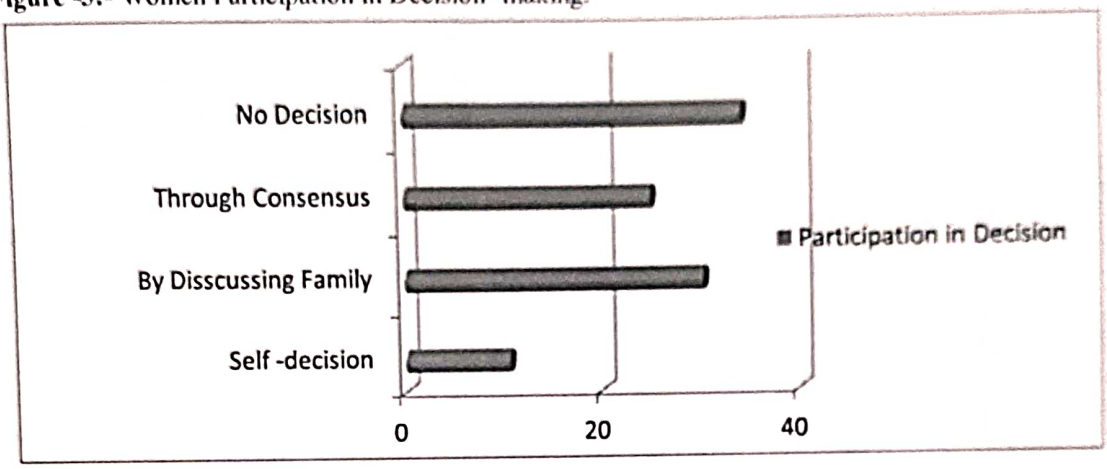
Sl. No	Respondents Decision	Social status				
		O.C	B.C	S.C	S.T	Total
1	1	2	3	4	5	6
2	Self- decision	2 (25.00) (10.00)	2 (25.00) (6.66)	3 (37.5) (13.63)	1 (12.5) (25.00)	8 (100%) (10.52)
3	By discussing Family	4 (17.39) (20.00)	9 (39.13) (30.00)	9 (39.13) (40.90)	1 (4.35) (25.00)	23 (100%) (30.27)
4	Through Consensus	6 (31.58) (30.00)	9 (47.36) (30.00)	3 (15.78) (13.63)	1 (5.28) (25.00)	19 (100%) (25.00)
5	No decision	8 (30.77) 40.00	10 (38.47) (33.33)	7 (26.92) (31.81)	1 (3.84) (25.00)	26 (100%) (34.21)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Participation in Decision- making

Participation in rural governance through decisions is considered significant for women empowerment. Respondents were asked to respond on the decisions taking while exercising their legitimate role in Grama Panchayat aspects in the Table 9. It shows from the above data that a majority of elected women leaders 26 (34.21%) from all categories are not acted in decision- making process relating to panchayat affairs. This is the heist percentage as per as the no decision is concerned. With regard to self-decision is concerned, only a very little (10.52 per cent) of women respondents have been reported. The women leaders act on their own without depending on others is found more in SC category (37.5 per cent) and take independent decisions. In the category of women leaders of BC and SC (39.13%, 39.13%) who were taking decisions with the support extended of their spouses.

A good number of respondents (23) constitute 30.27 per cent were taking decisions in relation to their role as representatives after discussing with on their spouses and family members. Of the total sample from open category 40.00 per cent of women leaders did not participate in any Grama Panchayat activities. It is also found that women candidates who are attending panchayat meetings also admit that they are not involved in the process of decision – making process.

Figure -3:- Women Participation in Decision- making.



It is seen from the opposition group women, particularly other party affiliated members. A similar finding is also seen in the study of Jilly Philippa MS., and S Manimekalai N. (2011) [10] women were able to occupy leadership positions, but they were mostly confined to women's wing not general and mainstream politics and strategically important positions .h e voices of and participation in decision making space in the parties have been very limited, and often women are only observed as passive participants.

Table 10:- Distribution of Sample Respondents by Their Nature of Leadership in Political Party.

Sl. No	Respondents Leadership	Social Status				
		O.C	B.C	S.C	S. T	Total
1	1	2	3	4	5	6
2	Primary Member	19 (25.68) (95.00)	29 (39.18) (96.66)	22 (29.73) (100)	4 (5.41) (100)	74 (100%) (97.37)
3	Village level Leader	01 (50.00) (5.00)	01 (50.00) (3.34)	0 (0) (0)	0 (0) (0)	02 (100%) (2.63)
4	Mandal level Leader	0 (0) (0)	0 (0) (0)	0 (0) (0)	0 (0) (0)	0 (0) (0)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

Nature of Leadership in Political Party.

As per as leadership at the village level is concerned, a low percentage of women candidates contributed from all categories. Majority of the respondents were taking political party primary membership after electing Gram Panchayat. It is observed that the male members are associating membership of the political party in the Andhra Pradesh rural society. The notable aspect of this study is that only two respondents had a status village-level leader, one each from the OC and BC categories. It is interesting to note that no respondent found from Mandal level leadership in the sample study area. It is evident from the table that poor leadership is seen among women candidates in studying area.

Political Participation

A question was asked to sample respondents how their political participation was relating to the Grama Panchayat affairs is presented in Table 9. Seeking information about women candidate's responses relating activities like the involvement of political meeting, mobilization of party cadre and campaign for elections and non-participation in local government activities.

Table .11:- Distribution of Sample Respondents by Participation in Party Activities.

Sl. No	Respondents Political Participation.	Social status				
		O.C	B.C	S.C	S.T	Total
1	1	2	3	4	5	6
2	Attending Political Meeting	7 (30.43) (35.00)	7 (30.43) (23.33)	8 (34.79) (36.37)	1 (4.35) (25.00)	23 (100%) (30.26)
3	Mobilization of Party Cadres	2 (22.22) (10.00)	6 (66.67) (20.00)	1 (11.11) (4.54)	0 (0) (0)	9 (100%) (11.85)
4	Campaigning for Election	0 (0) (0)	1 (100) (3.33)	0 (0) (0)	0 (0) (0)	1 (100%) (1.31)
5	No Participation	11 (25.59) (55.00)	16 (37.20) (53.33)	13 (30.23) (59.09)	3 (6.98) (75.00)	43 (100%) (56.58)
	Total	20 (26.31) (100%)	30 (39.48) (100%)	22 (28.94) (100%)	4 (5.27) (100%)	76 (100%) (100%)

As regards to political involvement, of the total sample, a comfortable majority of the women leaders (56.58 per cent) in all four categories have not been participating in any political activity. Women candidates admit that they did not come out of their houses to involve political participation. For a significant 23 respondents (30.26%) of the total sample, are involved in political meeting, while 1 respondent participated in canvassing local elections. The data shows that women leaders were reluctant to political participation due to socio-cultural constraints prevailed in rural society. It is evident from the table that women leaders depended on their family members and political party for their actions in Grama Panchayat. It is seen much higher in ST sample women leaders. It is also observed that due lack of communication skills and has less exposure to political experience.

Challenges to women leadership

A variable like age, education, occupation etc. examine whether these could have any impact on the emerging women leadership in Prakasam district. The study observed that majority of the women representatives are young and middle age and the women between the age group of 35-46 years have dominated grama panchayat politics. Earlier, older women have been found much higher in rural public life. Observing the caste factor, all categories of women combined dominated the grass-root governance, to enhance their socio-economic status due to fifty per cent of reservation introduced in the Panchayat Raj System. The overall educational status of women leaders in Prakasam district is not impressive. Thus, the study observed that women have come to the leadership position without proper education. Agriculture based labour work exclusively dominate the occupation of women leaders. The female leaders were spent time in searching daily wage labour required to work to support their families in agriculture. House-maker occupation is also found to be concentrated by the female leaders. The elected women depended on their husbands on economic aspects can be deterred the way on their acquiring leadership position. The study observed that women leaders are not economically sound, which may have affected their leadership position.

Most of the elected women leaders from all categories are first-time entrants to public life. They came to rural governance without any political background, accessibility in fact provided by the 73rd Constitutional Amendment Act. It is observed that current women, particularly those, belong to the marginalized sections with large scale, would not have entered rural political sphere without reservation of seats. They considered it as the turning point in the process of empowerment. Women candidates are backed by family members, elites of the village and political

party to contest Gram Panchayat institution. Women leaders have been loyalty them, as they are taking responsibility of selection and nomination for a rural grassroots political institution. It is found in more elected women leaders of marginalized sections of society like SC and ST.

The study reveals that women leaders are not shown interested in attending panchayat meetings. Those women who were attended accompanied to the office by male guardians such as husband, son and other family members and party leaders of the respective village. The women usually accompany male members whenever they go for Mandal and district offices to meet higher officials in relations to Panchayat affairs. Local bureaucracy is also comfortable with men to manage the affairs of panchayat rather than women. And they do not allow the women to play a key role in the grassroots political arena. It is found from field data that some of the female leaders are never attending panchayat meetings due to non-information, non-invitation as they belong to an opposition party. Some were not attending due to unattractive in the affairs of the panchayat and busy with daily domestic work and find it difficult to allot necessary time. The non-attendance of panchayat meetings has also been observed due to the traditions of the orthodox rural society. Their husbands and party leaders represent them in the meetings leading to proxy representation found in all categories of women leaders.

The study reveals that female leaders have very little knowledge about their rights and duties relating to the governance of panchayat, even completing four years of tenure as representatives. They have poor knowledge about Constitutional provisions of reservation of seats for women in Grama panchayats. Due to women leaders lack political knowledge men of their family interfere in matters relating to grama panchayat. This is because of their lack of consciousness and low levels of education. Dependence on their spouse, low level of education, lack of interest in electoral politics, and inadequate training appear to be the cause of women's low level of knowledge. In other words, they are mere instruments in the hands of their spouse and political parties relating to panchayat governance.

Participation of women in the affairs of panchayat is not a desired extent and discouraging. It is observed from the study that a number of reasons are responsible for their apathy and indifference to participate in the political institution. Socio-cultural constraints, political isolation, passive and subordinates to male counterparts stand in the way of effective participation of women in the grama panchayat politics. The study also found that patriarchal family set up and social-cultural values prevailing in the rural society such as low education, economic dependence and lack of confidence, low level of knowledge and awareness, low exposure to the public domain are responsible for low participation in panchayat institution. A majority of the respondents were taking political party membership after electing Gram Panchayat. It is observed that the male members are having membership of political party in the Andhra Pradesh rural society.

The study indicates that the domination of male members of the family, particularly their husband and local political party, have taken decisions in the Grama panchayat aspects. It is found that elected women candidates were prevented from discharging their functions as members of panchayat. The hostile environment, muscle power and expensive nature of electoral politics deprive women to enter into public life. It is said that gender disparity was found to be the major reasons and hesitant to present their views in front of the male members of panchayat. Male guardian's dominance has been prevailing unabated in the grama panchayat level, while only women members remained passive leaders. It is observed that the leaders who are not attending panchayat meetings also responded that they are not involved in the decision-making process. It is also observed that those women candidates of educated and having knowledge of panchayat affairs were more involved in the decision-making process. The data show that majority of the different respondents did not play any role in the decision- making process.

Suggestions:-

1. The marginalized elected women representatives who are engaged in livelihood activities like daily wage labour do not show interest in attending panchayat meetings regularly. So, it is required to compensate for their loss. Female leaders made more economically self-reliance.
2. The role of the family, political party and community are crucial to getting changed in the outlook of men towards facilitating women participation in the public domain. The immediate requirement is the conduction of gender-sensitive programs for all those persons involving panchayat governance.
3. Government institutions or voluntary organizations must come forward to organize women and make them active participants in the grass-root political institution. Their contribution is required to get empower them and

to make a strong and independent leader. The Self-Help Groups role is also crucial to streamline women success at the grassroots democracy.

4. The present training is inadequate for first-time elected female leaders. Leadership education required, particularly in running panchayat activities, is need of the hour. It is necessary to have such working knowledge about the matters of Panchayat. It is, therefore, highly essential to provide systematic training to emerge as powerful women leaders to understand their rights, duties, and also to tide over crisis situations when they arise for smooth conduct of Gram Panchayat. It will enhance their capacity and boost their morale in dealing with administration and handle village problems effectively.
5. To further improve the female leadership in the lower tire democratic process, proxy attendance should be strictly curbed, and it should be made mandatory for every women representative to be presented in the meetings and other affairs of local governance.
6. Political party leaders and family members of women representatives usually intervene in the functioning of Panchayats at all levels and process. The government should take strict action in such intervention of party leaders and male family members and provide a comfortable working environment for them. It is required consistent effort towards strengthening female leadership for better participation at the grass-root governance.

Conclusion:-

To sum up, in recent years, the provision of reservation for women to ensure numeric strength and their role in the local governance system has assumed considerable importance. On the contrary, undermining the capacities of women, this opportunity is in the majority of cases being utilized by their male family members and counterparts and affiliated political party, not by them. So far, the participation of women in the activities of panchayat affairs is not satisfactory. The female leaders have still not capable to utilize their full potential as they encountered many challenges such as patriarchy, low self-confidence and ignorance in Grama Panchayat governance arena. Their participation is confined to their as political representatives only. The study observed that economic independence, support from family and community is required to exercise their legitimate role in rural governance and to overcome challenges. Despite all odds imposed against them, the post-73rd Constitutional Amendment has shown an unprecedented huge number and the emerging women leadership at the rural governance in Prakasam district.

References:-

1. Alok Pandey (2009) Learning Leadership in Rural India: Empowerment of Elected Women Representatives in Local Governance, Women Empowerment in Political Institutions, An Indian perspective. Kanishka Publishers, New Delhi. p 147
2. Artatrana Gochhayat (2013), Political participation of women in Grama Panchayat Elections in Odisha: A Case Study of Hindol Block in Dhenkanal District. International Journal of Rural Studies, vol.20no.2 Oct. P.1
3. Yashoda T. (2007) Women Leadership In Grama Panchayats: A Field Analysis From Hassan District. ISDA Journal January –March & April –June, p.73
4. Snehalata Panda (1995) Women in Rural Local Government, Grass Roots Democracy: Challenges Ahead, Kurukshetra. April 1995.p.103
5. Alok Pandey (2009) Ibid.147
6. Handbook of Statistics (2015), Prakasam District, Chief Planning Officer-Ongole.
7. Bhargava and Vidya (1992). "Position of Women in Political Institutions" (*Journal of rural development.* Vol.11 (5), pp.619, 620, 621
8. Jogendra Kumar Das (2009), "Women in the Panchayati Raj Institutions: An Empirical Study in Kamrup District of Assam state" Sundaram (Eds) (Women Empowerment in Political Institutions: An Indian perspective), Kanishka Publishers, Distributors, New Delhi, pp.131.
9. Vikas Nandal,(2013). "Participation of women in panchayati raj Institutions: A Sociological study study" of Haryana, India. *International research journal of social sciences* Vol.2 (12), PP 47-50
10. Jilly Philippa and Manimekalai (2011), "Political Participation of women in Southern Region of Tamil Nadu: Challenges Ahead", *Women's Link*, Vol.17, No.2, April-June.pp.26.

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References:-

1. Alok Pandey (2009) Learning Leadership in Rural India: Empowerment of Elected Women Representatives in Local Governance, Women Empowerment in Political Institutions, An Indian perspective. Kanishka Publishers, New Delhi. p 147
2. Artatrana Gochhayat (2013), Political participation of women in Grama Panchayat Elections in Odisha: A Case Study of Hindol Block in Dhenkanal District. International Journal of Rural Studies, vol.20no.2 Oct. P.1
3. Yashoda T. (2007) Women Leadership In Grama Panchayats: A Field Analysis From Hassan District. ISDA Journal January –March & April –June, p.73
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5. Alok Pandey (2009) Ibid.147
6. Handbook of Statistics (2015), Prakasam District, Chief Planning Officer-Ongole.
7. Bhargava and Vidya (1992). "Position of Women in Political Institutions" (*Journal of rural development.* Vol.11 (5), pp.619, 620, 621
8. Jogendra Kumar Das (2009), "Women in the Panchayati Raj Institutions: An Empirical Study in Kamrup District of Assam state" Sundaram (Eds) (Women Empowerment in Political Institutions: An Indian perspective), Kanishka Publishers, Distributors, New Delhi, pp.131.
9. Vikas Nandal,(2013). "Participation of women in panchayati raj Institutions: A Sociological study study" of Haryana, India, *International research journal of social sciences* Vol.2 (12), PP 47-50
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Women Entrepreneurship:

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POLITICAL PARTICIPATION OF WOMEN IN INDIA - A NOTE

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Introduction

Women in India are marginalized in all aspects of life; social-economic, Political and cultural. India is a country of villages where rural women are in a most disadvantaged position. Around 80 per cent of female population of the country lives in rural India. Despite constitutional provisions of equal rights and privileges, women still marginalized. India has the most adverse sex ratio for women in the world. Life expectance is also lower than men. Education and health levels have been lower. They have been involved in lows killed low paid jobs and they received lower wages and lower income than men and hardily own and or control property and means of production. Therefore gender equity and empowerment of women has become issue of debate of in the developmental programmes of India especially in the last three decades.

Empowerment of Women

Women empowerment is ongoing debate and mostly used and discussed today. Empowerment has wide spread uses and is the key focus of development programmes. Women is integral part of society and be given equal status in all spheres of life. Women constitute half of the population and get equal opportunities to live, to grow, to participate in all decisions. Gender equality and women's empowerment have been considered for the development and well-being human society. Most countires of the world women have been discriminated, marginalized, disempowered in all aspects of life. Women have little role in the formulation of social, economic, legal political which govern their lives and keep them subjugated. The issue of women empowerment has gained ground after a movement for their oppressed status in 1980.

Empowerment is multidimensional process which enable people to assert their control over the resources which influence their lives. It is a process of helping the hands of women who have affecting from various disabilities, inequalities and gender discrimination. It is the expansion of granting human rights and social justice to marginalize and disadvantage sections of people. Women Empowerment is, participation of women in decision making process of social, economic and political aspects.

Political Empowerment

Political participation means exercising the right to vote, power [illegible] membership of political parties, electoral campaigning, attending party [illegible] holding party positions, contesting elections, co-decision making, [illegible] making at all levels of governance of the state. Simply put, it is a process [illegible] which individuals plays a role in political life deciding the common goals [illegible] society and determining the best way to achieve it.

The root of political participation of women can be traced back [illegible] century reform movements. Social reformers like Rajaram Mohan Roy, [illegible] Chandra Vidyasagar, Mahadeva Govinda Ranade, Behramji Malabari [illegible] that social evils could be eradicated by raising consciousness and [illegible] women. The Constitution of India guaranteed justice-social, economic [illegible] political, liberty of thought, and equality to all citizens. Constitution provides [illegible] equality of women and called State to take measures to neutralize the [illegible] economic, educational and political disadvantage faced by women.

Empowerment of women in al spheres and in particular the political sphere is crucial for their advancement and for the foundation of gender [illegible] society. It is central to the goals of equality, development and peace. The [illegible] democracy which is more than seven decades old but a large mass of women [illegible] kept out of political arena still at higher decision making bodies. There can be [illegible] true democracy, or no true people's participation in governance and development without equal participation of men and women at different levels of decision-making. Participation of women in political life is integral to the advancement of women.

Even after seven decades of women's suffrage, the number of women in high positions of political power and influence is still small. The number of women in political institutions is very low. The participation of women in political and social decision-making has been increasing rapidly but it is not satisfactory. Women participation in the Parliament has never been [illegible] higher than 14 per cent. They are excluded from political and administrative positions which important policy decisions affecting them. They have little say in the formation of social, economic, legal political rules which govern their lives and keep them subjugated. Several factors are responsible for women's low participation. The most important are literacy, traditionalism, prejudice, economic dependence and the unfavorable political structure. They need to be involved in all the decision making process and allow them to raise their voice.

Political participation includes the right to vote, right to contest and right to candidacy, for this women must accepted as campaigners as well as members of parties and they need to be involved in all the decision-making process. Reservation of seats is one of the [illegible] for women as it guarantee [illegible] Kuldeep Fadia, [illegible]

fundamental and non-negotiable principles: (a) the equality between women and men; (b) Women's right to the full development of their potentials; (c) women right to self-representation and self-determination.

Women Political status in India

The indicates some of the prominent women leaders in Indian politics are furnished in Table – 1.

Table - 1

Women political status in India – 2019

Name	Political Party	Belonging state	Positions occupied
Pratibha Patil	INC	MH	Ex-president of India
Sonia Gandhi	INC	UP	Ex-president of INC
Meera Kumar	INC	UP	First woman speaker of lok sabha
Mamata Banerjee	TMC	WB	CM, West Bengal
Mayavathi	BSP	UP	National president of BSP, EX-CM, UP
Menaka Gandhi	BJP	UP	Ex Minister for women & child development
Nirmala Sitharaman	BJP	TN	Ministry of Finance and ministry of corporate affairs
Anandiben patel	BJP	GJ	Ex-chief minister, Gujarat
Brinda Karat	CPI(M)	WB	Member, rajya sabha
Sumitra Mahajan	BJP	MP	Ex-speaker of lok sabha
Uma Bharti	BJP	UP	Ex-Minister of drinking water & Sanitation
Sushma Swaraj	BJP	MP	Ex-Minister of external affairs
Sheila Dikshit	AICC	PB	Ex-CM, Delhi and Ex-governor, kerala
Ambikasoni	AICC	PB	Member, rajya sabha
Kiran Bedi	BJP	DL	Lt governor, puduchery
Supriyasule	NCP	MH	Member, loj sabha
Mehbooba Mufti	PDP	JK	Ex CM, J&k
Smriti Irani	BJP	UP	Minister for women & Child Development and minister for textiles
Harsimrat Kaur Badal	SAD	PB	Minister of food processing industries

Election commission of India – 2019

The Women belonging to various political parties' where occupied key positions namely President of India, prime Minister, Chief Minister, Cabinet Minister, Speaker and head of the political party are given in Table - 1. Ex Chief Ministers Vasunda Raj form Rajasthan and Mamata Benerge of West Bengal has been inspiration for all women leaders in India. Narendra Modi led NDA 2.0 Government has made six women as ministers. In those, three have been given cabinet rank

Male and Female Representation

The male and female representation in state legislature is furnished table -2. It is proposed considering the recent elections held in Andhra Pradesh state. This shows that the women representation is very low. In Tripura, Meghalaya and Nagaland the recent election reveals marginalization of women M.L.A's in

North Eastern States. Only six representatives had been elected three each Tripura and Meghalaya. No women M.L.A was found in Mizoram and Nag assembly out of 100 seats in legislative assemblies as per its last electi (2013) (2018) respectively. Highest percentage (14.44) of 13 women M were elected for Haryana assembly in 2014 election. There are 14 women el to 15th Assembly election held 2019 year which constituted 8 per cent strength. Only three women MLAs are elected from 60 member assemb Arunachal Pradesh in May 2019 elections. The mainstream political parties not fielded good number of women to contest assembly election.

Table – 2

Male and Female Representatives in State/UTs Assemblies in India

State/UTs	Latest year	Total	Male	Female	Percenta
Andhra Pradesh	'2019	175	161	14	8
Arunachal Pradesh	2014	60	57	3	5
Assam	2016	126	118	8	6.34
Bihar	2015	243	215	28	11.52
Chhatisgarh	2018	90	77	13	14.44
Delhi	2015	70	64	6	8.57
Goa	2017	40	38	2	0.5
Gujarat	2017	182	166	13	7.0
Haryana	2014	90	77	13	14.44
Himachal Pradesh	2017	68	65	4	5.0
Jarkhand	2014	77	69	8	10.38
J&K	2014	87	85	2	2.29
Karnataka	2018	225	216	9	4.0
Keral	2016	140	132	8	5.71
M.P	2018	230	209	21	9.13
Maharastra	2014	288	268	20	6.94
Manipur	2017	60	58	2	3.3
Meghalaya	2018	60	57	3	3.3
Mizoram	2018	40	40	0	0
Nagaland	2018	60	60	0	0
Odisha	2014	147	136	11	7.48
Pondicherry	2016	30	26	4	13.33
Punjab	2017	117	111	6	5.45
Rajasthan	2018	200	177	23	11.5
Sikkim	2014	32	29	3	9.37
Tamil Nadu	2016	225	204	21	9.33
Tripura	2018	60	57	3	3.3
U.P	2017	403	356	38	10.55
Uttarakhand	2017	70	65	5	7.14
West Bengal	2016	294	260	34	11.56
Telangana	2018	119	113	6	5.04

Source: Election commission o f India, 2019.

Status of Women Representation

The table .3 shows the women representation status in PRI's isIndia. There are nine states who have given more than (50 per representation in rural local governance.

Tale - 3

Status of representation of Women in PRIs – 2017

States/UTs	Representatives	Women representatives	Women representatives (%)
Andhra Pradesh	156049	78025	50
Arunachal Pradesh	9356	3094	33.07
Assam	26844	13422	50
Bihar	163325	70400	51.64
Chhatisgarh	158776	87549	55.14
Dadra & Nagar haveli	136	47	34.56
Daman & Diu	97	28	28.87
Goa	1559	514	32.97
Gujarat	132726	43670	32.9
Haryana	68152	24876	36.5
Himachal Pradesh	27832	13947	50.11
J&K	33282	11169	33.56
Jharkhand	51327	30373	59.18
Karnataka	95307	50892	53.4
Kerala	19089	9897	51.85
M.P	396819	198409	50
Maharastra	203203	101466	49.93
Manipur	1784	868	48.65
Odisha	100791	49697	49.31
Punjab	97180	33609	34.58
Rajasthan	121008	70531	58.29
Sikkim	1099	549	49.95
Tamil Nadu	119399	39975	33.48
Telangana	103468	46702	45.14
Tripura	10939	3930	35.93
Uttar Pradesh	718667	297235	41.36
Uttarakhand	61451	35537	57.83
West Bengal	59296	29579	49.88
Total	2911961	1345900	46.14

Source: Women's Link Vol.23, No. 40CT-Dec 2017

It is high in case of Jharkhand (59.18 per cent) followed by Rajasthan and Uttarakhand state with 58.29 and 57.83 percent respectively . 46.14 per cent of women have been elected to rural local bodies in India. Only few i.e. Gujarat, Goa, Arunachal Pradesh, Jammu and Kashmir and Tamil nadu had representation of 33 per cent.

election as women candidates have a better winning chances compared to the In each parliament election women's winning difference is better than men. The average difference stands at 10 per cent up to last election. Regardless of increasing the number of women candidature: there is o substantial rise in the number of women representatives to parliament. It is interesting to note that more often the political parties are reluctance to field women candidates and offered losing seats to women candidates even though they had better winning chances. Political parties were not encouraging the women to contest elections as these parties fell the mostly women are incapable of winning elections compared to men. This reveals that despite an odd political environment women have demonstrated the capability to win their seat. In Andhra Pradesh General elections of 2019, 12 women have been contested from four big parties. The Biju Janta Dal Party from Odissa allotted 33 per cent reservation, while All India Trinamool Congress (AITC) fielded 20 women out of 42 Lok Sabha seats.

Article 325 and 326: They guarantee political equality, equal right to participate in political activity and right to vote, respectively. Similarly Article 243 (D): It provides for the political reservation to women in every panchayat elections. It has extended this reservation to elected office as well as. In spite of the above provisions, participation of women in politics has not improved significantly. These are contradictory to the aims of the constitution.

Conclusion

The issue of political empowerment of women has become the agenda of various international conferences and symposiums. The issue came at the forefront at the time of the Fourth World Conference on Women held at Beijing in 1995. At the conference, it was declared that women's political participation is crucial for the general process of the advancement of women. This conference stressed the importance of women's political participation not only because of their points of view and talents, but also as a matter of their human rights. Political parties should come forward to increase women representatives. The absence of critical mass of women representatives has pushed women to the fringes in power sharing and has adverse impacts on their overall political status. Gender stereotypes which perceive women as weak representatives should be changes through awareness and education. Efforts need to be taken to enhance the participation of women in governance in large numbers. Women Reservation Bill which reserves 33% of seats for Indian women at the legislatures has to be passed soon in the Parliament. Women's leadership and communication skills need to be enhanced by increasing female literacy especially in rural areas. They should be empowered in order to break social cultural barriers and improve their status in the society. Women panchayat members have to be trained to analyse and understand their roles and responsibilities given in the 73rd amendment act.

References

1. Usha Narayanan, "Women's Political Empowerment: Imperatives and Challenges", Mainstream, April 10, 1999, p.7.
2. Lester W Milbrath, Goel, M.L., Political participation: How and Why Do People Get Involved in Politics, Rand McNally, Chicago, 1977, p.48.
3. Raj Bala, The Legal and Political status of Women in India, Mohit Publications, New Delhi, 1999 p.3.
4. Niroj Sinha (ed.), Women in Indian Politics, Empowerment of Women through Political Participation, Gyan Publishing House, New Delhi, 2000, p.17-18.
5. William Henry Chafe, The American: Her Changing Social, Economic and Political Roles, Oxford University Press, New York, 1972, p.46-47.
6. Imtiaz Ahmed, "Women in Politics", in Devaki Jain (ed.), Indian Women, Publications Division, New Delhi, 1975, p.312.
7. Singh, J.P., "Indian Democracy and Empowerment of Women," The Indian Journal of Public Administration, Oct-Dec, Vol.XLVI, No.4., 2000, p.619.
8. Rashmi Srivastava, "Empowerment of Women Through Political Participation with Special Reference to Madhya Pradesh" in Niroj Sinha (ed.) Women in Indian Politics, Anmol Publications, New Delhi, 2000, p.206.
9. Huntington, S,P,, Nelson, J.M., No Easy Choice: Political Participation in Developing Countries, Cambridge, 1976, p.4.
10. Bridges, F., Appel, L., and Grossklags, J. (2012). Young adults' online participation behaviors: An exploratory study of web 2.0 use for political engagement. Information Polity, 17, 163-176 doi: 10.3233/IP-2012-0271.
11. https://en.wikipedia.org/wiki/Women%27s_political_participation_in_India.

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CHALLENGES OF STUDENT- CENTRED LEARNING – A NOTE

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Introduction

Education enables individuals and societies to become all round thinkers by providing knowledge, ability, skill and attitude, thereby it strengthen their problem solving ability. In order to achieve this, learners should be taught in a way that enables them to really engage themselves in the process. In teacher-centred method, student put all their focus on the teacher. In the teacher-centred method during activities in classroom, the only participants are teachers. In teacher-centred method, the primary sources of responsibility for the learning path in the hands of students. Learning is not a spectator sport. Students do not learn much just by sitting in class listening to teachers, memorizing pre-packaged assignments, and spitting out answers. They must talk about what they are learning, write about it, relate it to past experiences, and apply it to their daily lives. They must make what they learn part of themselves knowledge are teachers.

Student-centred method on the other hand, enables students to put all their focus on their knowledge. Student-centred method also known as learner-centred is a method of teaching in which the focus of instruction is shifted from the teacher to the students. In original usage, student-centred learning aims to develop learner autonomy and independence by putting

The concept of student–centred learning has been credited as early as 1905 to Hayward and in 1956 to Dewey's work 1. Carl Rogers, the father of client–centred counselling, is associated with expanding this approach into a general theory of education2. The term student–centred learning was also associated with the work of Piaget and with Malcolm Knowles3, He describes the shift in power from the expert teacher to the student learner, driven by a need for a change in the traditional environment where in this 'so-called educational atmosphere, students become passive, apathetic and bored'. In the School system, the concept of child–centred education has been derived, in particular, from the work of Froebel and the idea that the teacher should not 'interfere with this process of maturation, but act as a guide4.

Student–Centred Learning

Student-centred involves providing opportunities for students to meaningfully talk and listen, write, read, and reflect on the content, ideas, issues, and concerns of an academic subject. The student-centred learning approach is diametrically opposed, in its ethos, to the philosophy underlying the conventional method of learning. By its very nature, student-centred learning allows students to shape their own learning paths and places upon them the responsibility to actively participate in making their educational process a meaningful one. The student-centred learning experience is not a passive one, as it is based on the premise that "student passivity does not support or enhance..... Learning' and that it is precisely "active learning" which helps students to learn in dependently. Within student-centred learning, students are given options in shaping their courses and in choosing particular units within their study program. Some proponents of student-centred learning assert that rather than devoting so much effort to teaching students what to think, student-centred learning is based on the idea of teach them how to think'5. According to Weimer (2002) there are five practices that need to change to achieve learning teaching: (a) the functions of content, (b)the role of the teacher, (c)the responsibility for learning, (d) the processes and purposes of evaluation, and (e) the balance of power

Kember (1997) described two broad orientations in teaching: the teacher centred/content oriented conception and the student centred/learning oriented conceptions. In a very useful breakdown of

these orientations he supports many other authors views in relation to student–centred view including: that knowledge is constructed by students and that the lecturer is a facilitator of learning rather than a presenter of information. Choice in the area of the learning is emphasised by Burnard, as he interprets Rogers' ideas of student–centeredness as 'students might not only choose what to study, but how and why that topic might be an interesting one to study6. He also emphasises Rogers' belief that students' perceptions of the world were important, that they were relevant and appropriate. This definition therefore emphasises the concept off students having 'choice' in their learning.

Harden and Crosby (2000)7 describe teacher–centred learning strategies as the focus on the teacher transmitting knowledge, from the expert to the novice. In contrast, they describe student–centred learning as focusing on the students' learning and 'what students do to achieve this, rather than what the teacher does'. This definition emphasises the concept of the student 'doing'.

Other authors articulate broader, more comprehensive definitions. Lea et al. (2003)8 summarises some of the literature on student–centred learning to include the followings tenets:

1. 'the reliance on active rather than passive learning,
2. an emphasis on deep learning and understanding,
3. increased responsibility and accountability on the part of the student,
4. an increased sense of autonomy in the learner
5. an interdependence between teacher and learner,
6. mutual respect within the learner teacher relationship,
7. and a reflexive approach to the teaching and learning process on the part of both teacher and learner.'

Gibbs (1995)9 draws on similar concepts when he describes student–centred courses as those that emphasise: learner activity rather than passivity; students' experience on the course outside the institution and prior to the course; process and competence, rather than content; where the key decisions about learning are made by the student through negotiation with the teacher. According to Brandes and Ginnis (1986) 10 the main principles of student–centred learning are:

a) The learner has full responsibility for her/his learning
b) Involvement and participation are necessary for learning
c) The relationship between learners is more equal, promoting growth, development
d) The teacher becomes a facilitator and resource person
e) The learner experiences confluence in his education
f) The learner sees himself differently as a result of the learning experience.

Characteristics of Student-Centred Teaching

According to Weimer and Maryellen there are five characteristics of student-centred teaching.

1. student-centred teaching engages students in the hard, messy work of learning.

2. student-centred teaching includes explicit skill instruction.

3. student-centred teaching encourages students to reflect on what they are learning and how they are learning it.

4. student-centred teaching motivates students by giving them some control over learning processes.

5. student-centred teaching encourages collaboration

One come across many challenges in the implementation of student-centred in in relation to the required standard they are expected to achieve. Based on the analysis of students and teachers response to questionnaire it was found that they face different problems while implementing student-centred method. The challenges were lack of interest and confidence, lack of teachers and

students interest, class size, students feel discomfort when they work with others. In addition to this, from the analysis of teachers' interview, it was found that the students lack the skill of expressing their idea in English and students show a disciplinary problem when they are engaged in various techniques of student centred method. With the recent changes in educational laws and new classroom management opportunities, students are becoming more actively engaged in instruction. Student-centred classroom management is rare because it requires, "a willingness to give up some control and let students take some ownership, which requires guts as well as talent". Teachers have a tremendous impact on a child's life and their success starts in a student centred classroom.

School administrators need to support student-centred management and provide training and skills to their teachers because teachers and their students will have higher outcomes and success. Because classrooms are diverse and have students of all learning styles and learning needs, teachers need to implement individualized student-centred classroom management techniques to ensure all students will be successful. Teacher-centred classroom management, such as using classroom wide behavior management systems, or following scripted lessons are not effective ways to meet the needs of individual students.

Implications for curriculum design

In relation to curriculum design, student–centeredness includes the idea that students have choice in what to study, how to study. However, to what extent can this be carried out in the structures of today's Universities? Modularisation, which will be expected in all European undergraduate courses by 2006, provides a structure that allows students an element of choice in what modules they study. Donnelly and Fitzmaurice (2005) in their chapter in this collection on 'Designing Modules for Learning' highlight the importance of attempting to focus on the needs of the students at the early stage of curriculum design. Choice in the curriculum is not without its difficulties and Edwards argues about the dangers of individuality in the concept of the social learner and how this can in a seemingly contradictory way lead to disempowerment (2001).

One student–centred approach to curriculum design, Problem–Based Learning (PBL), allows for some choice within a programme of areas that students may study. It allows students to set some of their own learning objectives/outcomes, dependent on prior knowledge. ProblemBased Learning, through the use of problems/issues/triggers, encourages the students to develop their own learning goals, thereby filling in the gaps in their knowledge or understanding (Boud and Feletti 1997). This element of choice or control is referred to in many of the definitions of student–centred learning. This aspect of responsibility aligns with the Lea et al. (2003) view that student–centred learning involves 'increased responsibility and accountability on the part of the student'. These approaches are more controlled by the teacher in their presentation and outcome (Davis and Harden 1999). However, they are useful in addressing the active learning aspect of student–centred learning. Other approaches to curriculum design also support the idea of student choice and activity in learning, for example, the systems–based approach, resource–based learning, and experiential/ personal relevance approach (Toohey 2000).

A growing practice in course design internationally is the writing of learning outcomes/objectives focusing on what the student will be able to do, rather than on the content being covered by the teacher (UCD Centre for Teaching and Learning 2005). This practice is an example of the move towards student–centred learning in the curriculum and helps to shift the emphasis on the learner as opposed to a coverage model by the teacher. Donnelly and Fitzmaurice (2005) re-iterate the importance of this shift in emphasis. This is also reflected in Gibbs' (1995) definition, i.e. an emphasis on the process and competence, rather than content..

Implications for teaching/learning methods

The University of Glasgow (2004) identified four main strategies in a study on student–centred learning practices in their University. The first strategy was to make the student more active in

acquiring knowledge and skills and might include exercises in class, fieldwork, use of CAL (computer assisted learning) packages etc. The second strategy was to make the student more aware of what they are doing and why they are doing it. A third strategy is a focus on interaction, such as the use of tutorials and other discussion groups. The final strategy is the focus on transferable skills. This last strategy is not mentioned in other definitions of the student–centred learning but does look beyond the immediate course requirements to other benefits to the student in later employment.

The effectiveness and critiques of student–centred learning

The use of student–centred learning appears to be reflective of today's society where choice and democracy are important concepts. Lea et al. (2003) reviewed several studies on student–centred learning and found that overall it was an effective approach. A six-year study in Helsinki, which compared traditional and activating instruction, found that the activating group developed better study skills and understanding, but were slower in their study initially11. Equally, Hall and Saunders found that students had increased participation, motivation and grades in a first year information technology course (1997). In addition, 94% of the students would recommend it to others over the more conventional approach 12. Students in a UK University elaborated on the impact of student–centred learning on them, i.e. they felt there was more respect for the student in this approach, that it was more interesting, exciting, and it boosted their confidence13.

Student–centred learning, despite its popularity, is not without its critics. The main critique of student–centred learning is its focus on the individual learner. In addition, there are some difficulties in its implementation, i.e. the resources needed to implement it, the belief system of the students and staff, and students' lack of familiarity with the term.

Conclusion:

The changing demographics of the student population and the more consumer/client–centred culture in today's society have provided a climate where the use of student–centred learning is thriving. The interpretation of the term 'student–centred learning' appears to vary between authors as some equate it with 'active learning', while others take a more comprehensive definition including: active learning, choice in learning, and the shift of power in the teacher–student relationship. It is used very commonly in the literature and in University policy statements, but this has not necessarily transferred into practice.

Student–centred learning is not without some criticism but in general it has been seen to be a positive experience, for example, Edwards (2001) emphasises the value of student–centred learning: 'Placing learners at the heart of the learning process and meeting their needs, is taken to a progressive step in which learner–centred approaches mean that persons are able to learn what is relevant for them in ways that are appropriate. Waste in human and educational resources is reduced as it suggested learners no longer have to learn what they already know or can do, nor what they are uninterested14.

Today, information and communications technology (ICTs) is increasingly being employed in classrooms across the globe. Owing to this, educators face growing challenges as they teach a very "different" and "wireless" generation of students who use technology which is evolving every day. With over 50 percent of India below 25 years of age, the need for a skill set that adds to their digital profile is of utmost importance.

Consequently, the need for active learners who have acquired the skills of problem-solving, independent thinking and autonomous learning, as well as the ability to work cooperatively has increased dramatically. Therefore, teaching methods, pedagogies and educational goals must be directed at producing individuals who have faith in their abilities and who will work at developing their capabilities throughout their lives.

It is important to note that in the 21st century, learning is not "one size fits all" approach. This era has become more about developing skills than just

absorbing information. The most important skills is self-development, which includes knowing one's strengths and weaknesses, and finding ways to mitigate gaps in personal development. This can be achieved by implementing student-centred learning practices, thereby letting students take charge of their learning. This method prepares individuals to take responsibilities and makes them independent. It helps develop the characteristics of lifelong learning – motivation, self-evaluation, time management and skills to access information. The 21st Century skills comprise the 4cs namely, (a)critical thinking, (b)creativity; (c) communication and (d)collaboration.

Student-centred learning is an approach in which students influence the content, activities, materials and pace of learning. This learning model places the students at the centre of the learning process. The instructor provides students with opportunities to learn independently, and from one another, and facilitates the development of skills that they need to do so effectively.

References:

1. O'Sullivan, M. (2003). The reconceptualisation of learner-centred approaches: A Nambian case study, *International Journal of Educational Dedvelopment*.
2. Burnard, P. (1999). Carl Rogers and postmodernism: Challenged in nursing and health sciences. Nursing and Health Sciences 1, 241–247. Geraldine O'Neill and Tim McMahon 37.
3. Rogoff, B. (1999). Cognitive development through social interaction: Vgotsky and Piaget. In P. Murphy (Ed.), Learners, Learning and Assessment. London: Open University Press.
4. Rogers, C. R. (1983a). As a teacher, can I be myself? In Freedom to Learn for the 80's. Ohio: Charles E. Merrill Publishing Company.
5. Simon, B. (1999). Why no pedagogy in England? In J. Leach and B. Moon (Eds.), Learners and Pedagogy. London: Sage Publications.
6. Taylor, P. G. (2000). Changing Expectations: Preparing students for Flexible Learning. The International Journal of Academic Development 5(2), 107–115.
7. Rogers, C. R. (1983b). The politics of education. In Freedom to Learn for the 80's. Ohio: Charles E. Merrill Publishing Company
8. Harden, R. M. and J. Crosby (2000). AMEE Guide No 20: The good teacher is more than a lecturerthe twelve roles of the teacher. Medical Teacher 22(4), 334–347.
9. Lea, S. J., D. Stephenson, and J. Troy (2003). Higher Education Students' Attitudes to Student Centred Learning: Beyond 'educational bulimia'. Studies in Higher Education 28(3), p.322.
10. Gibbs, G. (1995). Assessing Student Centred Courses. Oxford: Oxford Centre for Staff Learning and Development.
11. Brandes, D. and P. Ginnis (1986). A Guide to Student Centred Learning. Oxford: Blackwell. Bredo, E. (1999). Reconstructing educational psychology. In P. Murphy (Ed.), Learners, Learning and Assessment. London: Open University Press.
12. Broadfoot, P. (1999). Assessment and the emergence of modern society. In B. Moon and P. Murphy (Eds.), Curriculum in Context. London: Sage Publications
13. Lonka, K. and K. Ahola (1995). Activating instruction: How to foster study and thinking skills in Higher Education. European Journal of Psychology of Education 10, 351–368.
14. Hall, J. and P. Saunders (1997). Adopting a student-centred approach to management of learning. In C. Bell, M. Bowden, and A. Trott (Eds.), Implementing Flexible Learning. London: Kogan Page
15. Lee et al, 2003, *Op.Cit.*
16. Edwards, R. (2001). Meeting individual learner needs: power, subject, subjection. In C. Paechter, M. Preedy, D. Scott, and J. Soler (Eds.), Knowledge, Power and Learning. London: SAGEp.37.

ORIGINAL RESEARCH PAPER

Political Science

KEY WORDS: Women, Political Participation, Local Bodies, Andhra Pradesh

WOMEN POLITICAL PARTICIPATION IN LOCAL BODIES OF ANDHRA PRADESH

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ABSTRACT

The paper critically analyzes as well as evaluates the emergence of women leadership in Pachayati Raj Institutions particularly after 73rd Constitutional Amendment Act in Andhra Pradesh. It also covers the socio-economic determinants of the elected women and its consequent influence on the emerging women leadership in Andhra Pradesh and examines how far the socio-economic aspects have impaired or given opportunities to raise the issues and other social and economic problems in a formal forum, and thereby adequately ensure the improvement in all the spheres of life. This analysis establishes the fact that many myths associated with the entry of women into leadership positions that would be hindered by socio-economic profile in terms of their age, occupation, family income, land-holdings and education have been disproved. Moreover, it uses state-level variation in the implementation of the 73rd amendment in India to observe the relationship between political reservations for women in local government. Finally the constraints on female representatives and their level of effectiveness in executing pro-female policies given the political and social environment are examined.

INTRODUCTION

The term 'political participation' has a very wide meaning. It is not only related to 'Right to Vote' but also simultaneously relates to participation in decision-making process, political activism, political consciousness etc. Women participate in voting, run for public offices and political parties at lower levels more than men. To combat gender inequality in politics, the Government has instituted reservations for women in local governments. Women comprise more than 50 per cent of the world's population, yet they are consistently and copiously underrepresented as voters, political leaders and elected officials. But, there is almost universal agreement that gender equality is one of the ultimate developmental goals, as well as a major driver of human development. A certain ratio of women in political participation and decision-making positions is colossal crucial, not just for equity reasons, but also for immensely incorporating women's perspectives into policy and decision-making and amply achieving the envisaged positive changes (**Praveen Rai, 2011**).

Gender inequality leading to deprivation of power among women continues to be a political reality in India today. Women are perpetually excluded from decision-making at every rung of the ladder, starting from the household to the top layer of policy making. Although the Constitution of India attempts for removing gender inequalities by interdicting discrimination based on sex and class, and enshrining fundamental rights for all citizens, women still have only *de jure* rather than *de facto* access to these rights. There is no denying the fact that greater participation of women in the political process would be a pre-condition for their economic and social emancipation. However, even though a significantly large number of women vote in the country, on the contrary, only a few of them assume the reins of power. Paradoxically, though women have held the posts of President and Prime Minister as well as Chief Ministers of various states in India, the country ranks 20th from the bottom in terms of representation of women in Parliament (**World Economic Forum's Global Gender Gap Report, 2012**).

True democracy is a system which in Abraham Lincoln's words is a government of the people, by the people and for the people. The problem of low participation of women is of special concern for democracies and without changing that true democracy can never be achieved. Women's participation in decision-making is essential for women's interests to be incorporated into governance. It has been widely experienced that governance structures which do not provide for adequate participation of women, often suffer from state interventions which are neither inclusive nor democratic. Including women, especially in local governments are an essential step towards creating gender equal opportunities and gender sensitive policies. Since women have different needs and perspectives on social and political issues, it is important to involve women in governments to incorporate all of the societal viewpoints in policy and decision-making processes. Women are actively involved in household and community work and hence well aware of real issues faced by common people. This gives them insight and perspective which can be instrumental in sustainable overall development (**Besley, Pande and Rao, 2005**).

Reservation For Women

In addition to devolving powers to the Panchayat, the 73rd Amendment also required one-third of the GP Ward Member positions, as well as one-third of the Pradhan positions, to be reserved for women. Seats and Pradhan positions are also reserved for the two disadvantaged minorities in India, "scheduled castes" (SC) and "scheduled tribes" (ST), in the form of mandated representation proportional to each minority's population share in each district. Reservations for women have been implemented in all major states except Bihar and Uttar Pradesh (which have only reserved 25 per cent of the seats for women in the 1995/96 elections). Reservation has drastically increased the number of women in the village councils. **WOMEN POLITICAL PARTICIPATION IN INDIAN POLITICS: HISTORICAL CONTEXT**

The movement for women's suffrage began in the early 1900s in response to a national movement for suffrage, even though vast majority of neither men nor women had a right to vote the British colonial rule before 1947. After Indian independence from Britain, the Indian Constitution in 1950 officially granted women and men suffrage. Prior to universal suffrage, provincial legislatures had granted women the right to vote. Madras was the first to grant women's suffrage in 1921, but only to those men and women who owned land property. The rights granted in response to the movement towards suffrage were limited to qualifications of education and property ownership, including property ownership of husbands. This excluded vast majority of Indian women and men from voting, because they were poor. This changed in 1950 when universal

suffrage was granted to all adult Indian citizens (**Mithra, 2009**).

In 1950, universal suffrage granted voting rights to all women. This was enshrined in Article 326 in the constitution. India is a parliamentary system with two houses: Lok Sabha (Lower House) and Rajya Sabha (Upper House). Rates of participation among women in 1962 were 46.63 per cent for Lok Sabha elections and rose to a high in 1984 of 58.60 per cent. Male turnout during that same period was 63.31per cent in 1962 and 68.18 per cent in 1984.The gap between men and women voters has narrowed over time with a difference of 16.7per cent in 1962 to 4.4 per cent in 2009(**Chief Electoral Officer, Voting Percentage in Various Lok Sabha Elections**).

Voter turnout for national elections in the past 50 years has remained stagnant with turnout ranging between 50 and 60per cent. State elections have seen a growing trend in women's participation, and in some cases, women's turnout is exceeding male turnout (**Rukmini, 2014**) Increased turnout of women was reported for the 2012 Vidhan Sabha elections (legislative/state assemblies) with states such as Uttar Pradesh reporting 58.82 per cent to 60.29 per cent turnout. In the 2013 assembly elections, women's overall turnout was reported to be 47.4per cent and male turnout was 52.5per cent. Indian states of Arunachal Pradesh, Goa, Kerala, Manipur, Meghalaya, Mizoram, Daman and Diu, and Pondicherry all reported higher turnouts among women than men in 2013(**Election Commission of India, 2013**).The presence of women in local governments serves as an encouragement for other women to enter diverse professions and leads to breaking stereotypes of women's roles in society and public space. People had gained lot of confidence in women as good public administrators and local government representatives after seeing women making a positive difference in other people's life. The society acknowledges the sincerity and utmost commitment of women to their duties and their resistance to criminalization of politics. The measurement of women political participation is essential to identify the need of policy intervention to improve the same The process of capturing women participation at the local level is an evolving area and efforts are on to improve the present situation.

Women Reservation Bill

India experienced centralization of planning which resulted in higher inequality in political decision making at the various levels. While Government was deeply concerned of issues of gender equality, women were not always a part of such decision making. Although, India has seen women participating in politics as the longest serving Prime minister, as chief ministers of various states, members in national parliament and state legislative assemblies in large numbers, yet the occurrence of such incidents has not been commensurate to their population. For enabling better women participation in active politics, governments are trying to put in reservation for women but have not been successful in true terms due to non-support from some of the regional parties. Back in history, one of the prominent members of freedom struggles, Sarojini Naidu rejected reservation for women, citing that women are not week, timid and meek. She claimed that the demand for granting preferential treatment to women is an admission on her part of her inferiority and there has been no need for such a thing as the women have always been by the side of men. The issue of women's reservation again came to limelight in 1973 with voices recommending reservation for women in at least one third of the seats and eventually statutory women's panchayats at the village level were recommended to take care of the neglect of women in rural development programs through 73rd & 74th constitutional amendments in 1993.

Women's Reservation Bill or the Constitution (108th Amendment) Bill, is a pending bill in India which proposes to reserve 33 per cent of all seats in the Lower house of Parliament of India, and in all state legislative assemblies for women. The seats to be reserved in rotation will be determined by draw of lots in such a way that a seat shall be reserved only once in three consecutive general elections. Women's Reservation Bill was passed in Rajya Sabha on March 9, 2010. But Lok Sabha could not clear the bill due to resistance of some regional parties on certain provisions of the bill.

Panchayati Raj Reforms

Indian Constitution made provisions relating to the establishment, powers and responsibilities of the panchayats through the 73rd Amendment in 1993 with three tier system, viz. panchayats (village governance bodies) at the village, intermediate and district levels in every state, except provision of skipping intermediate level in states with less than twenty lakh population. The states have been empowered through law for the composition of panchayats. The reform provided for reservation of both seats and leadership positions for the Scheduled Castes, tribes and women. A normal duration of five years for panchayats has been provided with the authority of preparing the electoral rolls and conducting elections in the state Election Commission. The state government is also empowered for making laws providing criteria for disqualification of candidature from panchayat elections and also to legislate with respect to maintenance of accounts by the panchayats and their audit.

Apart from providing political empowerment, the Panchayati Raj reforms endow the panchayats with necessary powers and authority for enabling them to function as institutions of self-government. This has helped all the sections of the society particularly the weaker sections including women to take part and to share the responsibility of governance and development at least at the sub-district levels. As the legislation provides for reservation for women, the number of women elected representative at local level has sharply increased. In India30-50 per cent of local level elected representatives are women. For improving the level of participation by women in deliberative bodies the government of India, in 1992, amended the constitution (73rd and 74th amendments) that would reserve a third of the seats in the Panchayats (local governments) including the chairpersonship of these local bodies for women (**Molyneux, 1994**).

Women and Panchayati Raj Institutions in Andhra Pradesh (AP)

The 1/3 reservation paved way for women to exercise their political rights in local self-governance and created an enabling environment for women's participation in the grass-root politics. Though women issues have been politically neglected and women were being treated as second-class citizens, with less attention paid for removal of inequality and subordination, women entry into provincial politics to certain extent challenge the social stigmas attached to political power (**Kaushik, 1993**). This necessitated a constant search for ways and means to empower, promote awareness and increase the participation of women in the decision-making levels of PRIs. Notwithstanding, women panchayati members in Andhra Pradesh have made attempts to break these social barriers and raise their voices. Andhra Pradesh (AP) was one of the first two states in India to establish the PRIs in the year 1959. The State Legislature created a three-tier Panchayati Raj (PR) set-up under Andhra Pradesh Praja Samiti and Zilla Parishad Act, 1959, provided for co-option of two women. The Andhra Pradesh Gram Panchayat Act, 1964, governed the composition and functions of the Gram Panchayat in Andhra Pradesh. This Act provided for reservation of two seats for women if the total strength of the gram panchayati is 9 or less, 3 seats if the strength is between 10 and 15 and 4 seats if the strength is more than 15 (**Mishra, and Singh, 1993**). It is clear

that the Act ensured between 22 per cent and 25 per cent of representation of women in these bodies. In the middle level bodies, till 1986, women's representation was governed by the Andhra Pradesh Praja Samiti and Zilla Parishad Act of 1959, which provided for co-option of two women for each samiti in addition to the possibility of women members finding membership in PS through the electoral process. Based on the recommendations of Ashok Mehta Committee Report, the Mandal Praja Parishad, Zilla Praja Parishad Abhivrudhi Mandal Bill was introduced in legislative Assembly in July 1986 which completely altered the PRIs set-up in AP and in 1987, a new reservation set-up was introduced for Backward Caste and Women, i.e. 9 per cent. Thus, women got further encouragement to share the democratic decision-making process of PRIs.

Keeping in view the New Constitutional (73rd) Amendment Act of 1993, state government of Andhra Pradesh has enacted a new Law thereby making the previous PR system in-operative. In this new arrangement, the representation of women at Gram Panchayat (GP) level was 33.84 per cent, at Panchayat Samiti (PS) level it was 37.01per cent, and at Zilla Parishad (ZP) level it was 33.38 per cent after reservation (2001), which demonstrated visible enhancement of women's participation especially in the rural areas to get access to decision-making process thereby contribute for economic development, social reconstruction and distribution of scarce resources. As reservation opened new avenues for rural women to empower themselves, hopefully political power would enable numerous women to improve their socio-economic conditions, participatory abilities and thereby tap leadership potentialities.

73rd Constitutional Amendment

The 73rd Amendment and Women's Representation in India The passage of the 73rd amendment (Panchayati Raj) in 1993 in India has been acclaimed as an important step in the devolution of power to local governments. It established local governance at the rural level and local bodies at the village (panchayat), intermediate, and district levels with mandatory direct elections every five years (**Kalsi, 2017**). The amendment was intended to further the administration of development programs at the local level and improve the efficiency of resource distribution through the process of decentralization. Additionally, the amendment mandated panchayats to have at least 33% reservation for women in the council and leadership as well as reservations for scheduled castes and candidates for reserved seats would have to be nominated by their political parties and would also go through the process of campaigns and elections. The amendment required reservations to be randomly assigned to panchayats based on their serial registration number and female chairperson reservations to be rotated between panchayats every election cycle. The idea for this decentralization and women's representation at the grassroots level came to fruition because of the failure of the panchayats to be effective administrators of public service in decreasing poverty and inequality before the 1990s (**Chaudhuri, 2003**).

Prime Minister Rajiv Gandhi spearheaded the passage of the amendment and gave a speech in 1989 to promote a resurgence of focus on the Panchayati Raj. Gandhi and other elites in power made the case that centralization was leading to the delayed implementation of programs on the ground, impeding economic development, and decreasing the effectiveness of poverty reduction. The fact that the amendment was spearheaded by elites in parliament was beneficial because it was legitimized and had powerful national backing (**Duflo, 2004**).

However, there was no grassroots movement pushing for effective administration so states began implementation at different times and to different levels. Lack of a grassroots movement meant that there was no ground-level push for substantive representation. Even though elites like Gandhi wanted to help the "weaker sections of society" it was hard to hold them accountable for the implementation of concrete change. Therefore, it is helpful to test women's abilities to participate effectively in order to analyze the level of true decentralization and empowerment of underrepresented groups initially promised by these elites (**Gandhi, 1989**).

The national government used the 73rd amendment to revitalize local government and target development with reservations and decentralization. Proponents of the amendment realized biased perceptions deter socially disadvantaged groups from breaking into politics without any affirmative action policies, thus making reservations an important part of the new amendment. Reservations for women are important because men and women have different preferences, which result in a different allocation of public goods. Minority groups, like women, are likely to provide minority or low spill over goods such as transfers, rations, and water connections, which only benefit other women. Additionally, political parties are less likely to field candidates from disadvantaged groups because they do not believe that they will win the majority vote. It was important for reservations to be included in the amendment in order to target the development of women and other minorities. As a result, after the first round of elections between 1993 and 1994, "almost 800,000 women were brought into local politics" in a country with no prior history of that level of women's political participation (**Duflo, 2012**).

SUGGESTIONS

Political commitments and reservations aside, nearly all the world's women face major challenges in securing political participation at national and local levels. Women's lack of access to education and to economic and political engagements is often deeply rooted in and hampered by cultural, religious or traditional norms and values. Hence, women's empowerment not only requires socio-political reforms and technical capacity development, but also often requires men and women to change their mindset. Based on our experience, we are aware that changes in attitudes and behaviours require time and patience. Our programmes and activities are politically and culturally sensitive and must take the specific conditions in our partner countries into account without compromising on women's rights.

CONCLUSION

Women make up more than 50 per cent of the world's population, yet they are consistently underrepresented as voters, political leaders and elected officials. Gender inequality leading to deprivation of power among women continues to be a political reality in India today. Women are perpetually excluded from decision-making at every step of the ladder, starting from the household to the top layer of policy making. The 73rd amendment to the Constitution of India in 1993 established the framework of a three-tiered local government (Panchayat) system, with regular elections every five years, throughout India. The 73rd Constitutional Amendment Act has mandated not less than 1/3 reservation for women. Reservations have been provided for women not only at the level of members, but not less than 1/3 of the chairpersons are to be women. It is worth mentioning that the provision of reservation has truly helped women in increasing political participation throughout the state and country. It has been able to bring many positive changes in the status of rural women. It imbibed among women the qualities like the enjoyment of power, self-confidence, political awareness and affirmation of identity. However, there is still a long way to go to make women empowered and politically participative. The mere enactment of legislation and affirmative actions are not the solution. The need of the hour is to get people involved from every corner including civil society organizations, non-governmental organisations, community based

organisations, policy-makers, lawyers and most importantly family members to create a barrier free enabling environment for women.

REFERENCES

1. Praveen Rai (2011). "Electoral Participation of Women in India: Key Determinants and Barriers", Economic and Political Weekly, 14th January 2011, XVIJ (3): 47–55.
2. World Economic Forum's Global Gender Gap Report 2012.
3. Besley, T., R.Pande and V.Rao (2005) 'Political Selection and the Quality of Government: Evidence from South India.' Mimeo, Yale University.
4. Mithra, H.N. (2009). The Govt of India ACT 1919 Rules Thereunder and Govt Reports 1920, ISBN 978-1-113-74177-6
5. Chief Electoral Officer. "Voting Percentage in Various Lok Sabha Elections", Government of Uttarakhand, India. Retrieved 22 March 2014.
6. Rukmini, S. (2014) Who is behind the rise in voter turnouts?, The Hindu. Retrieved 22 March 2014.
7. Election Commission of India, 2013
8. Molyneux (1994) *New Panchayati Raj in Action*, Mittal Publications, New Delhi.
9. Kaushik (1993) Recent reforms in the Panchayat system in West Bengal: Toward greater participatory governance?' *Economic and Political Weekly*. pp.45–58.
10. Mishra and Singh (1993)Impact of 73rd Amendment Act on Women's leadership in the Punjab, International Journal of Rural Studies (*IJRS*), Vol.15 No.1 April, p.2
11. Kalsi (2017) Opportunities and Challenges of Women's Political Participation in India : A Synthesis of Research Findings from Select Districts in India, pp. 1-2.
12. Chaudhuri, Shubham (2003) 'What difference does a constitutional amendment make: the 1994 Panchayati Raj Act and the attempt to revitalize rural local government in India', Mimeo, Columbia University.
13. Duflo, E. and P.Topalova (2004) 'Unappreciated Service: Performance, Perceptions and Women Leaders in India.' Mimeo, MIT.
14. Gandhi (1989)"Impact of Women's Participation in Local Governance in Rural India", Jain L.C. (Ed), Decentralisation and Local Government, Orient Longman, New Delhi.
15. Duflo (2012) Unappreciated Service: Performance, Perceptions and Women Leaders in India', Mimeo2, MIT.

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श्रीभाष्यं विजयसारथेः मन्दाकिनी काव्यस्य महौत्कृष्टत्वम्

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प्रस्तावना

मन्दाकिनी विजयसारथेः कृतिषु प्रौढतरं काव्यम्। मन्दं अकति इति ञिच्छति इति। मन्दाकिनी शब्दस्य यौगिकाऽर्थे सत्यपि गङ्गायाः नामसु मन्दाकिनी शब्दः रूढः। गङ्गायाः उपनदीषु भागीरथी, मन्दाकिनी इत्यादि नामाभिः शाखा नदीनां प्रसिद्धिः तासां हिमालयेषु तत्र तत्र संगमः सङ्गमस्थलानां प्रयागशब्देन व्यवहारः अद्यापि प्रसिद्धः एव देवप्रयागः , रुद्रप्रयागः, कर्णप्रयागः इत्यत्र यथा -- यावत् अलहाबाद् नगरे यमुनायाः सङ्गमः तावत्पर्यन्तं बहवः प्रयागाः वर्तन्ते। बदरिका क्षेत्रे गङ्गायाः नाम मन्दाकिनी प्रसिद्धा। तथापि मन्दाकिनी शब्दः समग्रायाः गङ्गायाः पर्यायतया रूढः।

मन्दाकिनी शब्दोत्पत्तिः महौत्कृष्टत्वम् ----

संस्कृतवाङ्मये समग्र गङ्गापर्यायतया मन्दाकिनी शब्दः बहुषु काव्येषु प्रयुक्तः दृश्यते। यथा नैषधीयचरिते ---

अश्रान्त श्रुतिपाठपूतरसनाविर्भूतभूरिस्तवा
जिह्वाब्रह्ममुखोद्यविघ्नितनस्वर्ग क्रिया र्कलिना।
पूर्वं गाधिसुतेन सामिघटिता मुक्ता नु मन्दाकिनी
यत्प्रासाददुकूलवल्लिरनिलादोलैरखेलद्दिति।। [1]

रामायणेपि मन्दाकिनी शब्दः गङ्गा पर्यायतया प्रसिद्धः विशेषतया आकाशात् हरशिरसि अवतरन्ती गङ्गा मन्दाकिनी नाम्ना काव्येषु व्यवहृता। गङ्गायाः अवतरण कथा संस्कृतवाङ्मये प्रथमतया बालकाण्डे श्रीमद्रामायणे पञ्चसु सर्गेषु वर्णिता।
सूर्य वंशजः सगरः अश्वमेधेन इष्टवान् तस्य एकस्यां भार्यायां षष्टिसहस्र पुत्राः द्वितीयस्यां एकः पुत्रश्च बभूवुः।
तेषु द्वितीयस्याः पुत्रः नाम्ना कर्मणापि असमुञ्जसः पित्र सज्यात् बहिष्कृतः। तदानीं तस्य पत्नी अन्तर्वन्ती आसीत्। तस्याः अंशुमाक्षः नामकः। सगरस्य षष्टिः पुत्र सहस्राणि यागाश्वरक्षकाः यागाश्वमन्वयुः। इन्द्रोऽपि सगरस्य शततमं अश्वमेधं सान्तरायं कर्तुं यज्ञाश्वं जहार पाताले नारायणावतारस्य कपिलस्याश्रमे गोपायितवान् च। सगरकुमाराः अश्वं अदृष्टवा भूमिं खनित्वा पातालं ययुः कपिलाश्रमे अश्वं ददृष्टुः कपिलमेव अश्वस्य हर्तरं मत्वा तर्जयामास। ततः कुपितेन कपिलेन ते भस्मसात्कृतः।

अभ्यथावन्त संक्रुद्धास्तिष्ठ तिष्ठेति चाब्रुवन्।
अस्माकं त्वं हि तुरगं यज्ञियं हृतवानसि।। [1]
दुर्मेधस्त्वं हि संप्राप्तान्विद्धि नः सगरात्मजान्।

तेषां पारलौकिकाः क्रियाः कर्तुं कस्यापि अवकाशः नासीत् अनन्तरं सगरस्य पौत्रः अंशुमान् कपिलाश्रं गत्वा कपिलं प्रसाद्य अश्वमनीय यज्ञसमाप्तिं कृतवान्। तत्र सलिलस्य अलभ्यत्वात् सलिलक्रियाः कर्तुमपि अशुंमान् न पारितवान्। अशुंमान् दिलीपः भगीरथः इति क्रमेण सर्वे सगरवंशजाः गङ्गां पालमानेतुं तपस्यां चक्रुः। तेषु अंशुमतः पौत्रः भगीरथः एव सफलो बभूव। वतरिष्यमाणां गङ्गां धर्तुं शिव एव समर्थः इति ज्ञात्वा भगीरथः शिवं प्रसाद्य गङ्गां धारयितुं प्रार्थितवान्। प्रथमं शिवजटाजूटे अवतीर्णे गङ्गा शिवशिरसि गर्वेण विहरणं कृतवती शिवेन सा जटाजूटे बद्धा शिवोपि भगीरथ मनुसृत्य सप्तधा गङ्गां बिन्दुसरसि पातयामास। भागीरथ मनुसरन्ती गङ्गा क्रमेण पातालं प्रविश्य त्रिपथगा बभूव। इमां कथां आधारीकृत्य केवल गङ्गावतरण वर्णनात्मकं वर्णनमेव आदाय विजयसारथिः मन्दाकिनी काव्यं रचितवान्। कविना ईषत् परिवर्तिता श्रीमद्रामायण शता मूलकथा स्वकाव्ये अभिवर्णिता। मन्दाकिन्याः रचनोद्देशः कविना स्वयं एवमाविष्कृतः।

[1]अभिजानन्त्येव समेऽपि सुमनसो विद्वांसः शङ्करजगन्नाथनीलकण्ठ प्रमुखा महापुरुषाः भगवतीं भागीरथीं स्तुत्वा तस्या अनुग्रहविशेषेण परमांप्रतिष्ठाम् अवापुरिति। तेषाम् अनुकरणेन अनुसरणेन च वयं आत्मनः कृतार्थतां मन्यामहे। ऐहिकामुष्मिकफल- धात्रीति गङ्गेयं बहुपुराणेषु बहुधाऽजेगीयत। भगवत्या स्सुरसरितः पममावतरणकथा गीतेषु ग्रथिता मया। औचित्यदृशा किञ्चिदिव परिवर्तितापि कथा। काव्येऽस्मिन् विषयेयं एव सन्दब्धोस्ति प्रादान् मन्दाकिनी मात्मसुतां परिवहराजः सागराय। सङ्गस्तु मुत्कण्ठ तेसागरोऽपितया। कल्लोलैरभ्रङ्कषैः तां बलात् ग्रहीतुं काऽमोपि शासनात् भगवतो विभाय। वेलेयं न कदाऽपि भवताति- क्रम्यतामिति तं भगवानपि बलवद् आदिदेश। गङ्गाया बलवद- वतारणेन लोकस्यापकृतिर्भूयसीभवेदित्युभयधा सन्त्रस्त इव सागरः सौवादुद्यमात् व्यरंसीत्। विरहवेदनाम सहमानोऽपि सोऽयं किं कर्तव्यताविमूढेन मनसा क्षणमिवविचिन्त्य भगीरथं स्वपरमसखं राजप्रतिनधिम् आकार्य तं प्रार्थयामा सैवम् सखे। भवता कथमपि प्रेयसी मम सेयं सुरसरित् आनीयतामिति सोऽपि भगीरथः स्वमनोरथं सफलमेवमन्वानः कपिलेन शापाग्निना भस्मसात् कृतान् स्ववंशीयान् गङ्गाजलाप्लावनेन परलोकम् अवापयितुं सागराभीष्ट- ञ्चसफलं कर्तुं तामानेतुं समुद्यतः सागरानुमत्या स्वर्लोकमयासीत्। आतिर्थयविधिना सत्कृतेन भगीरथेन प्रार्थिता कथितमात्मप्रियस्य सागरस्य सन्देशमन्यमुदन्दञ्च समाकर्ण्य गाढं निःश्वस्य च तं प्रत्येयमवोचतं सा सुरनदी --- भ्रात अहमपि सागरं प्राणेश्वरमुपगन्तुमुत्कण्ठ्या समुदीक्षे --- प्रत्युतमत्प्रवाहसंपातं भूमाता न कदाऽपिसोढु शक्नुयात्। यदि नाम भगवान् गिरिशो मां वोढुं नियतमभ्युपागमिष्यत् तदा ममावतरणं निर्बाधम् अभिविष्यत् अतः कार्यमिदं साधयितुं तमेव शिवम् अभ्यर्थयस्व। तदा भगीरथेन तेन संप्रार्थितो भगवान् सदाशिवः सुपर्वापगां दरीधर्तुमुररीचकार। सैषा परिवहमण्डलादात्ममन्दिरादवततार शिवशिरसि सङ्गोचिर्तनैव स्त्रोतसा। फालानलेन विपानलेन चाशीविपानलेन पोप्लुष्यमाणवपुः सोपि सुरधुनी-शिशिरवारिस्पर्शमात्रेण अननुभूतपूर्वं सुखं किमप्यवाप्नुवालः तां सर्वदा निजकपर्द एव बन्दीकर्तुं यावदियेष तावदेवास्य भावमवगम्य वर्धित दुर्भरधाराबहुलेन स्त्रोतः स्वरूपेण विज्रंभमाणा तस्य जटाजूटमयीं कारां निर्भिद्य बहिर्निर्गमत्। अवतीर्य च भुविसप्तधाविभक्तैः स्त्रोत्रोभिः सकलमपि भूमीतलं पुनती भगीरथ पथमनुरुन्धाना तपोवनानि सादरमभिनन्दन्ती, वनानि सिञ्चन्ती, भूधरान् भिन्दन्ती, क्षेत्राणि तर्पयन्ती, ग्रामान् पुराणि पल्लीश्च रञ्जयन्ती प्रकृतिमखिलां पुलकयन्ती, सागरद्वितीभिरापगाभिर्मलन्ती पुण्यानिकिरन्ती दुरितान्य-मनुदन्ती ऊष्मणं चूषन्ती तीर्थानि सार्थयन्ती परासून् भगीरथवंशान् लेखलोकमवापयन्ती प्रजामखिलामाबालगोपालमामोदयन्तीचान्तेसागरेण प्रियेण मिलित्वा तदन्तरङ्गे निलीना भवति। इदमप्यपरमवधेयमस्ति सादरं सुमनसाम् यत् अधुना विविधा सुभाषासु गीतस्यैव संरंभो दृश्यते। प्रतिभाषं अपि नवीना साहिती प्रक्रिया नूत्नेन पथा नरीनृत्यते। सारस्वते सर्वतः कोऽपि सरसस्समुन्नमेषो देदिवीति। लोकगीतनाम्ना प्रचलितस्य गीतमयस्यैव साहितीविशेषस्य साम्राज्यं प्रचलति। संस्कृते वचन प्रक्रियाया इव गीतप्रक्रियाया अपि वराक्येव स्थितिः वरीवर्ति। इदानीन्तनाः कवयोपि वचनदारिद्र्यमिव गीतादारिद्र्यं व्यपोहितं समुत्सहन्ते समुद्यतन्ते च। अमुनैवलक्ष्येण गङ्गावतरण कथा पञ्चमात्रकेषु गीतेषु निबद्धा। प्रवाहस्यगतेः ध्वनेश्च वैविध्यं वैदग्ध्यं चाभिव्यञ्जयितुं प्रायोगमनार्थका ध्वन्यर्थकाश्च धातवः प्रयुक्ताः काव्येस्मिन्। नियतं सर्वाण्यपि गीतानि रचनासौन्दर्येण जीवन्ति। तथापि यथामति काव्येस्मिन् रसस्यभावस्य रीतेः नीतेः शब्दस्यार्थस्य च प्रामुख्यं सर्वधा परिकल्पितम्। प्रवाहगताविव अत्रापि क्वचिद् विलंबः क्वचिद् ऋजुता क्वचिद् विरल्यः क्वापि लघुता क्वापि गुरुता क्वाप्योजस्विता क्वापि प्रसन्नता चेत्यादयो गुणरीतयः समुदञ्चन्ति।

महाकवेरस्य छन्दसि रचनायां सामर्थे वर्तमानोपि संस्कृतभाषायां गीताप्रक्रिया न चिररूढा इति गेय काव्यरूपेण काव्यमिदं रचितमपि पाण्डित्यपूर्णं कृतम्। पाणिनीयधातुपाठे न्यूनाधिकाः द्विसहस्रं धातवो वर्तन्ते धातुपाठे दशविंशति धातून् गुच्छतया निबध्य सिद्धान्त कौमुद्यां गत्यर्थाः भाषार्थाः इति निरूपणं कृतम्। अपि च गेयस्य लयः जीवतुः एवं तालल्यान्वय अनुकूलान् धातून् मात्रा च्छन्दसि निबद्धान् एव गृहीत्वा एकत्र ग्रथनं सामान्यस्य कवेः असाध्यम्। विद्वानपि कवितानुकूलं तान् उपकर्तुं न शक्नोति।

अतः विजयसारथिः विद्वत्कविः। आन्ध्रदेशे आधुनिक महाकविषु जलप्रवाह वर्णनात्मकं किन्नेकसानि पाटलु इति खण्डकाव्यं विश्वनाथ सत्यनारायणेन कृतम्। किन्नेरसानी अद्यतन खम्मम् मण्डले गोदावर्यां मिलन्ती काचन

गिरिनदी। कविसाम्राट् विश्वनाथसत्यन्नारयणः तस्याः स्त्रीरूपं आपाद्य गृहिण्याः अवस्थां तस्यामारोप्य वर्णितवान्। श्वशुरगृहे भर्तुः मात्र कृतां पीडां असह्यमाना किन्त्रसानी भार्यायाः व्यथां दृष्ट्वापि मातरं किमपि वक्तुम् असमर्थः पतिं परित्यज्य श्वशुरगृहं त्यक्त्वा पितुः गृहं प्रस्थितवती। तस्याः गतिमपि विश्वनाथसत्यन्नारायणः लयानुकूलेन वर्णितवान्। काशीभट्ल सत्यन्नारायण महोदयेन कवेरस्य (विजयसारथेः) स्वर्णोत्सवसञ्चिकायां किन्नेरसानी गीतानि एव मन्दाकिनीं रचयितुं कवेः प्रेरणं अनुकुर्वन्तिति अभिप्रायं प्रकटीचक्रुः। तत्सादृश्यं द्योतयितुं काशीभट्ल महोदयः काव्यद्वयात् प्रत्येकं द्वित्राणि गीतानि उदाहृतवान्। आन्ध्रभाषा अनवगतापि तस्याः अर्थं अजानन्तोऽपि शब्दश्रवणमात्रेण तालल्या - नुकूलत्वे द्वयोः सादृश्यं ज्ञातुं प्रभवन्तीति एकमुदाहरणं दीयते।

కెరటాలలో నుర్యు
తెరచాలలో నీటి
పొరజాలులో కిన్నె
రటు కదలి యిటు కదలి
చిటి తరంగాలలో -- పోటి తరంగాలలో
నటనాలు మొదలెట్టేనే -- క్రొన్నీటి [1]
తుటుములాకదలాడేనే
పువు కన్నెలా జగా
రవ నవ్వులా వేల్పు
కవ చివ్వలా కన్నె
రటులూగి ఎటులూగి
చిరుగాలిలో నూగు [1]
కొఱవంపులో నీటు
తటి తీపులో కిన్నె
కటు లేచి ఇటు లేచి [1]

తదనుసారी मन्दाकिनी नृत्यं यथा
అతి మఞ్జుల పుష్ట
వన వఞ్జుల దీప్త
సుమకిఞ్జులాలో
కవదూర్మి పూరేణ------ కృతమఞ్జల పల్ల
విత జఞ్ఝల వీచి
మయ కంగుల పాస్త
గద జఞ్జుల

पराशक्तिस्वरूपिणी मन्दाकिनी गूढार्थरूपेण अत्र वर्णिता इति काशीभट्लमहोदयैः एव प्रतिपादितम्। अत्र तात्विकदृष्टिः प्रवाहेन सममुपवर्णिता ।

यथा --- **ओंकार गम्भीर मञ्जुलस्तानितेन**
विमलामृतासार पूरमा कल्पयति
कदम्बिनीव सा मन्दाकिनी
सा पराशक्तिरिवमन्दाकिनी।। 1

कदम्बनी मेघमाला तस्याः स्तनितमिव गङ्गाप्रवाह ध्वनिरपि भवति स च ध्वनिः प्रमवात्मक एव । तया स्वरूपा सा परत्वं परमात्मत्वं च भक्तस्य कल्पयति सा च पराशक्ति- रूपा कालत्रयामात्य क्रमस्वरूपा । एवं मन्दाकिनी महाशक्त्या अभिन्नां कविः ददर्श । राधाकृष्णयोः अद्वैतत्वे राधापि मन्दाकिन्येव

यथा --- **मुग्धमूलाधारमञ्जीरथा मञ्जु**
षड्वलादिष्ठानरशनयामणिपूर
वैजयन्त्याऽनहताब्ज
कन्धरामङ्गल सूत्रतायाब्ज
फलभूषणयेन्दु मिहिरताटंकया
कुण्डलीन्याख्यया श्रीपरशक्त्येव
विरहमाणः परन्धामा शिरोधाग्नि ।

मन्दाकिनी मूलधारं परिषिध्य पृथिवी तत्वं अमृतमयं कृत्वा अवगाढं अमृतं रकोति । एवं काशीभट्लमहोदयैः बहुधा मन्दाकिनी श्लाघिता । न. च रघुनाथाचार्यः अपि मन्दाकिनी परमात्मतत्वभूषिता इति प्रशशंसुः । कोराड सुब्रह्मण्य महोदयैः काव्यस्य अलङ्कारशैली बहुधा विवृता । अपि च पाणिनीय प्रयोगविशेषाः अपि बहवः अत्र दृश्यन्ते । यङन्ताः यङलुगन्ता क्यजन्ता क्यङन्ताः कृदन्ताश्च प्रयोगविशेषाः बहवः अस्मिन् काव्ये दृष्टाः । एतान् प्रयेगन् अर्थतः शब्दाश्च ज्ञातुं पाणिनीय प्रवीणाः नैरुक्ताः एव समर्थाः इति सुब्रह्मण्येन यदुदाहृतं तदक्षरशः सत्यम् ।

यथा ---- 1. विजहरीति ---- यङलुगन्तः
2. दंद्रम्ते , जंहम्यते ----- इति यङन्तौ
3. शब्दायते , वैरायते --- शब्दवैरकलहाभ्र कण्वर्महेभ्यः करणे (3-1-17) इति सूत्रं समर्थो प्रयोगौ । [1]
4. परिचाय्य - अग्नौ परिचाय्योपचाय्य समूह्याः (3-1-131) इति सूत्रम् । अनिविशेषे प्रयोगे निपातितः ।
5. सान्ताय्य - सान्नाय्यं हविर्विशेषः पाय्य सान्नाय्य निकाय्य

धाय्य - इति सूत्रेण (3-1-129) इत्यादयो ज्ञेयाः ।

एवं कवेः अभिनन्दनसञ्चिकायामेव काव्यमिदं

बहुभिः बहुधा प्रशस्तमिति अलं विस्तरेण ।

उपयुक्तग्रन्थसूची

1. बाणः, नैषधीयचरितम् , स –2, श्लो – 102
2. वाल्मीकिः, रामायणम् , बालकाण्डः, श्लो - 40
3. वाल्मीकिः, रामायणम, बालकाण्डः, श्लो- 29-30
4. श्री.भा.वि.सा, मन्दाकिनी, अवतारिका, पृ. सं - 15
5. श्री. भा.वि.सा, कृति, स्वर्णोत्सव संचिका, पृ .सं - 8
6. श्री.भा.वि.सा, कृति, स्वर्णोत्सव संचिका, पृ .सं - 8
7. श्री.भा.वि.सा , स्वर्णोत्सवसंचिका , पृ.सं - 11
8. श्री.भा.वि.सा, स्वर्णोत्सवसंचिका , पृ.सं - 11.

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श्रीविष्णु महापुराणे कलिधर्म निरूपणम्

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प्रस्तावना

वेदवाङ्मयमिव पौराणिक साहित्यमपि भारतीय संस्कृतेः प्राच्यधर्म परम्परायाश्च ज्ञानाय परमोपदेयत्वमुपकल्पते । पुराणिना पुरातन विषयप्रतिपादकानि प्राच्य विधानाञ्च कोशरूपाणि । वेदानामात्मस्वरूपत्वेन अङ्गीक्रियते । आत्मा पुराणानां वेदानाम् इति वचनप्रामाण्यात् । यत्र पुराणेषु भूगोलेतिहासविधिविज्ञानादाय आधुनिक विषयाः सूक्ष्मतया दुरीदृश्यन्ते । तत्र कथाख्याधिका देव-माहात्म्य व्रतनियम धर्मशास्त्र तीर्थ माहात्म्य सृष्टि प्रलयराजवंश मन्वन्तर वंशानुचरितानि सुष्टुतया प्रतिपादितानि सन्ति । अपि च व्याकरणछन्दः काव्यनाट्यालङ्कारायुर्वेदानीति ज्योतिष कर्मकाण्ड विविध दर्शन विषयाः पुराणेषु प्रचुरतया वर्तन्ते । एतस्मादेव उक्तं यत् "पुराणविद् सर्वविद् भवति" इति । पुराणादृते न कश्चित् अन्यो विषयः प्रवर्तते । वेदज्ञानमपि पुराणज्ञानेन विना न पोपुष्यते । वेदार्थकस्य सम्यक् ज्ञानाय पौराणिक साहित्यं नितरांमपेक्ष्यते । वेदस्य गूढार्थपरिज्ञानं पुराणाध्ययनात् विना न भवितुमर्हति । "इतिहास पुराणाभ्यां वेदं समुपबृंह्येत् " इति कथनात् वेदोपबृंहणं पौराणिक साहित्येन विहितम् ।

पुराणशब्दस्य व्युत्पत्तिः

"पुरा भवं पुराणम् "इत्यर्थे पुरा इत्यव्ययशब्दात् "सार्वचिरं प्राह्वेप्रगेइव्ययेभ्यष्ट्युट् युला तुट् च" इति (अष्टाध्यायी 4—6—23) सूत्रात् ट्युप्रत्ययान्तरं इत् संज्ञस्य टकारस्य लोपे कृते युवोरजाकौ इति सूत्रेण योरनादेशे च विहिते तुडागमाच्च पुरातन इति शब्दो निष्पन्नो भवति ।

पुराणस्य लक्षणम् --- लोके केश्चित् पुराणानां वर्ण्यविषये यद्यपि पञ्चभिर्दशभिर्वा लक्षणानि उक्तानि । तथापि पुराणानां पञ्च लक्षणानि विद्वत्सु प्रथितानि सन्ति । एतस्मादेव कारणात् प्रायः सर्वेषु पुराणेषु पञ्च लक्षणमेव समुल्लिखितमस्ति ।

सर्गश्च प्रचिसर्गश्च वंशो मन्वन्तराणि च ।
वंशानुचरितं चैव पुराणं पञ्चलक्षणम् ।। [1]

अत्र सर्गशब्दस्यार्थः सृष्टिरिति विज्ञेयः । प्रतिसर्गेण सम्पूर्णस्य जगतः संहारः परिकीर्तितः । देवर्षिराज्ञां च उत्पत्तिपरम्परा जगतः वंश शब्दतो निर्दिष्टास्ति । सृष्ट्यादीनां कालव्यवस्था मन्वन्तरमित्यभिधीयते । तत्तजन्वयजातानां देवर्षिराज्ञां वृत्तान्तानि वंशानुचरितानि पदेन कथितानि वर्तन्ते ।

पुराणानां विभाजनम्

पुराणसाहित्यस्य प्रायः महापुराणोपपुराणभेदात् सर्वत्र विविध विभाजनमुपलभ्यते ।

मद्वयं भद्वयं चैव ब्रत्रयं वचतुष्टयम् ।
अनापलिङ्गकूस्कानि पुराणानि पृथक् पृथक् ।। [1]

अत्रास्य श्लोकस्याहभोहयमसि यत् मकाराद्याक्षरेण द्वे भकाराद्यक्षरेण द्वे, ब्रकाराद्याक्षरेण त्रीणि, वकाराद्यक्षरेण चत्वारि महापुराणानि बोधव्यानि तथा च अ -- ना --- प --- लि ---ग --- कू -- स्काद्याक्षरैः पृथक्त्वेनैनैकैके महापुराणं विजानीयात् ।

पुराणेषु विष्णुपुराणम् -

अष्टादश पुराणेषु विष्णुपुराणस्य स्थानं तृतीयम् । एतच्च भगवतो विष्णोः दक्षिणबाहुत्वेन स्वीक्रियते । मत्स्य पुराणानुकारं पुराणेञ्च श्लोकसंख्यामिमां समर्थयति । किन्तु तन्मते पुराणमिदं भागद्वयात्मकं पूर्वभागः उत्तरभागश्चेति । तत्र पूर्वभागे षडंशाः वर्तन्ते । ये तावत् शक्ति पुत्रेण पराशरेण मैत्रेयाय कथिताः सन्ति । उत्तरभागश्च तदन्तरं वर्तते । यत्र सूतः शौनकादिभिः प्रैरितः सन् विष्युधर्णोत्तरनामकाः नानाधर्मकथाः प्रोक्तवान् इति ।

विष्णुपुराणे -- प्रथमेऽशे --- 22 अध्यायाः ।
द्वितीयेऽशे --- 26 अध्यायाः ।
तृतीयेऽशे --- 27 अध्यायः ।
चतुर्थेऽशे --- 24 अध्यायाः ।
पञ्चमेऽशे ---- 36 अध्यायाः ।
षष्ठेंऽशे --- 07 अध्यायाः सन्ति ।

वर्ण्यविषयाः

अधुना उपलभ्यमानञ्च विष्णुपुराणस्य प्रथमेंऽशे विस्तरेण सृष्टेः वर्णनं वर्तते । द्वितीयेंऽशे भूगोलस्य वर्णनं तृतीये चतुर्णामाश्रमाणां चतुर्थे राजवंशवर्णनं पञ्चमे श्रीकृष्णस्य चरितं षष्ठे कलियुगस्वरूपम्, प्रायश्चित्त विधानम्, योगमायनिरूपणम्, ज्ञानेन सह भक्तेः सामञ्जस्य प्रतिपादनं चेति विषयाः सम्यक् वर्णिताः सन्ति । अत्र विष्णुपुराणे भगवतो विष्णोः सामप्रदायिक संकीर्णतां विस्मृत्य श्रद्धया सह पुराणखायास्य मतं स्वीकुर्वन्ति । अतः पुराणस्यास्य मुख्यवर्ण्यविषयोऽस्ति विष्णोः उपासना भक्तिर्वा । भक्तिद्वारा स्वाभाविकतया साधकस्य मनः भगवद्वुन्मुखं जायते । नवथा भक्तेः विस्तृत प्रति पादनं पुराणस्यास्य वैशिष्ट्यमस्ति । भगवतः विष्णोः नाम -- गुण -- लीलादि स्मरणस्य माहात्म्यं पराशरेण प्रदर्शितं यत् विष्णुस्मरणेन समस्तपापराशिः भस्मसाद्भवति । मोक्षश्च अनायासेन प्राप्यते ।

किन्तु स्वर्गप्राप्तिः तत्र विघ्नरूपानुमीयते ।
विष्णु संस्मरणा तद्यक्षीण समस्त केश संशयः ।
मुक्तिं प्रयाति स्वर्गाप्तिस्तस्य विघ्नोऽनुमीयते ।। [1]

न केवलं विष्णोः स्मरणं अपि तु तद्यशोगुणादीनां श्रवणं, कीर्तनं,

कृतस्य कर्मणः तत् समर्पणं च अत्यावश्यकमिति मत्वा पुराणेऽस्मिन् त्याग -- संयम - साधनादीनां महत्वपूर्णं चित्रणं कृतं वर्तते । एतेषां त्यागादीनां दृष्टान्तरूपेण ध्रुव प्रह्लाद -- सगर -- भगीरथ - नहुष -- ययाति -- जमदग्नि -- विश्वामित्र – केशिध्वजादीनां चरितं समुपन्यस्तमस्ति । सर्वेषु चरितेषु प्रह्लादचरितम् अत्र भक्तिकससवितततया समुपस्थापितं वर्तते । एतच्चरितमाध्यमेन भक्तेः सुमहात्म्यं प्रकाशितम् । भगवतश्च भक्तिप्रियत्वं प्रतिपादितमस्ति ।

कलिधर्मनिरूपणम्

विष्मुपुराणे षष्ठेऽशे प्रथमोध्याये मैत्रेयाय पराशर महर्णिणा कृत कलिधर्मनिरूपणं विद्रीयते -- कलियुगे मनुष्याणां प्रवृत्तिः न भवति। दाम्पत्यजीवनं यज्ञयागादि कर्मण्यापि च समीचीनरूपेण प्रचलन्ति ।

विवाह न कलौ धर्म्या न शिष्यगुरु संस्थितिः ।
न दाम्पत्यक्रमो नैव वहिनिदेवात्मकः क्रमः ।। [1]

कलियुगे यः बलवान् सैव नीचकुलोद्भवो सन्नपि सर्वेषां स्वामीरूपेण विराजते । कलियुगे यस्य कस्यापि जनस्य मुखात् यत्किमपि वाक्यं निस्सरति तदेव प्रमाणमित्यङ्गीक्रियन्ते । कलियुगे सर्वे क्षुद्रदेवाः प्रमुखदेवतारूपेण पूज्यन्ते । उपवासाः तीर्थाटनं धनत्यागः इत्यादि कार्याण्येव तपोरूपेण स्वीक्रियन्ते । स्वचित्तानुसारिण क्रियमाण-कार्यमेव धर्मः इति कथ्यन्ते । मदगर्वो जायते । स्वकेशानुगुणेनैव स्त्रियः स्वलावण्यालङ्कारं क्रियन्ते । निर्धनः पतिः सर्वथा त्याज्य एवा धनवानेव पतित्वेनाङ्गीक्रियन्ते । अधिकधनदानमात्रेणैव स्वामीति कथ्यन्ते । न तत्र कुलस्य विषयः । गृहकार्येष्वेव धनं सर्वं व्ययीक्रियन्ते । अपि तु सर्वदा धनार्जने एव रताः भवन्ति । न तु आत्मज्ञानार्जने । अतः अतिथिं सत्कारः प्रायः न स्यात् ।

गुहान्ता द्रव्यसङ्घाता द्रव्यान्ता च तथामतिः ।
अर्थाश्चात्मोप भोग्यान्ता भविष्यन्ति कलौयुगे ।। [1]

स्त्रियस्तु सुन्दरपुरुषकामुकाः स्वेच्छाचारिणीव भ्रमन्ति । पुरषाः अक्रममार्गे धनार्जने इच्छुकाः भवन्ति । शूद्राः अपि ब्राह्मणैस्सह सञ्चरन्ति । क्षीरसमाप्यनन्तरं गाः हनन्ति । अन्नावृष्टिकारणात् सर्वे जनाः भयभीताः सन्तः बुभुक्षया पीडितः तपस्विजनाः इव कन्दान् फलानि भक्ष्यन्ते । अनावृष्टिकारणात् आत्महत्यार्थं प्रयत्नन्ते । कलियुगे असमर्थाः सर्वदा आनन्देन जीवनं यापयन्ति । कलिसङ्क्रमण काले विना स्नानेन भोजनं कुर्वन्ति । अग्नि -- देवता अतिथीनां पूजां न कुर्वन्त्येव । पितृ-देवतेभ्यः श्राद्धादि कार्याण्यपि सम्यक् न निर्वहन्ति । गुरूणां सम्मानं प्रायः न भवत्येव । कलियुगे स्त्रियः क्षुद्रस्वभावयुक्ताः शारीरिक संस्कार रहिताः अपवित्राः असत्यवादिनः भवन्ति ।

स्वपोषणपराः क्षुद्राः देहसंस्कार वर्जिताः ।
परुषानृत भाषिण्यो भविष्यन्ति कलौस्त्रियः ।। [1]

ब्रह्मचारिणः वेदाध्ययनं विना कालं यापयन्ति । गृहस्थाः अपि विना अग्निहोत्रादिना सत्कर्मकार्याचरणेन सदाचारेण व्यर्थरूपेण कालं यापयन्ति । सन्यासिनः अपि ग्रामेष्वेव निवसन् मित्रैस्सह सत्सम्बन्धान् व्यवहरन्ति । कलियुगे राजानः प्रजा-रक्षणं विना करग्रहणे एव उत्सुकाः भवन्ति । धनवान् राजा इति दरिद्रः भृत्याः इति च विज्ञायते । कृषिकाराः वैश्यादि जनाः स्वकर्मणा विना अन्यकार्येषु निमग्नाः भवन्ति । शूद्रगणाः सन्यासिवेषं धृत्वा भिक्षा-वृत्तिमारभन्ति । जनानाम् आयुरपि न्यूनं भवति । बाल्यावस्थायामेव मरणं प्राप्यन्ते केचन द्वादश वर्षे केशाः फलिताः भवन्ति । विंशंति वर्षाणि एव आयुर्दायं भवति ।

पलितोद्भवश्च भविता तथा द्वादशवार्षिकाः ।
नातिजीवति वै कश्चित् कलौ वर्षाणि विंशतिः ।। [1]

कलियुगे जनाः मन्दबुद्धियुक्ताः व्यर्थचिह्नानि धृत्वा मरणात्यन्तिक समीपे निवसन्ति । महापुरुषाः यदि सन्दिग्धा भवन्ति तर्हि चिन्तनीयं कलियुगस्य आरम्भो जाता इति । कलिप्रभावेन मेघाः अत्यन्त न्यूनाः तथैव व्यवसाय क्षेत्राण्यपि अल्प फलदायिनः भवन्ति ।

कलि प्रभावेण धान्यमपि स्वल्पं भवति । दुग्धमपि न्यूनं भवेत् । श्वश्रूखशुरश्च गुरुरूपेण विराजते । शालाद्याः एव मित्रिणि रूपाणि आभान्ति । मातापितरौ त्यक्त्वा चित्तभ्रान्त्या सञ्चरन्ति । सर्वदा पाप-कर्मणि एव रताः भवन्ति । निर्लज्ज मनुष्याः सर्वदा दुःखगामिनः भवन्ति ।

वाङ्मनः कायजेंदोषरभिभूताः पुनः पुनः ।
नराः पापान्यनुदिन करिष्यन्त्यल्पमेथसः ।। 1

स्वाध्याय रहिते स्वाहारहिते संसारे धर्मः नश्यत्येव । परन्तु अत्र महान् विषयोऽयं भवति यत् किञ्चित् प्रयासेन एव बृहत्पुण्यं प्राप्तुं शक्यते कलियुगे ।

उपसंहारः

महापुराणेषु विष्णुपुराणम् अत्यन्तसुप्रसिद्धमस्ति । विष्णुमहापुराणे अनेके धार्मिक विषयाः उक्ताः तत्र कलिधर्मविषयाः महोन्नत रूपेण समुपवर्णिताः ।

उपयुक्तग्रन्थसूती

1. अष्टाध्यायी --------- पाणिनीमहर्षिः ।
2. कूर्मपुराणम् -------- वेदव्यासः ।
3. देवीभागवतम् ------ वेदव्यासः ।
4. विष्णुपुराणम् ----- वेदव्यासः ।

संधर्व सूची

1. कूर्मपुराणम्, श्लो --- 1, पृ --12
2. देवीभागवतम्, अध्या --- 1, श्लो ---3, पृ --2
3. विष्णुपुराणम्, 6—1--11
4. विष्णुपुराणम् --- 6—1--20
5. विष्णुपुराणम् --- 6 –1--30
6. विष्णुपुराणम् -- 6 –1—42
7. विष्णुपुराणम् --- 6 ---1---57

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प्रस्तावना

1. **गजः** - गजस्य मांसः न भोजनोपयोगि किन्तु गजवाहनेन उल्लाघः(रोगशक्तिः) त्रिदोषशआन्तिश्च भवति । गजक्षीरं च प्रमेहः, कुष्ठः,गुल्मः इति रोगान् निरयति ।

हस्तीद्विपोगजोनाग इभश्चद्विरदः करी ।
वारणः कुञ्जरोदन्ती मातङ्गष्षष्ठिहायनः ।।
दन्तावलो गजश्चैव पद्मीस्तम्भेरमोमतः ।
न एकलभः प्रोक्तः कुम्भीचकथितो बुधैः ।।
कुञ्जरारोहणं कुर्यास्त्रिदोषशमनं परम्
दुग्धं प्रमेहशमनं कुष्ठुगुल्महरंपरम् ।। [1]
धन्वन्तरिनिघअमटुः, सुवर्णादिवर्गः, पृ -- 254

2. **अश्वः** - अश्वस्यापि आरोहेण मूत्रव्याधिः नश्यति । व्याधेः नाम मूत्रकुच्छ्रः मूत्रस्य विसर्गे यः क्लेशः सनिवारितो भवतीति भावः। "स्यात् कष्टं कृच्छमाभीलं " इत्यमरः अपि च त्रिदोषाः अश्वयानेन नष्टाः भवन्ति । अश्ववेगेन शरीरस्य चलनाधिक्यत् एवं भवेत् । अश्व बऽबा क्षीरं तु चित्तभ्रम निवारकम् ।

घोटाश्वतुरगोवाजी हरिरुसस्तुवङ्गमः ।
शालिहोत्रोजवीसप्तिस्तुरङ्गस्सैन्धवोहयः ।।

अश्वीरोहणमात्रन्तु मूत्रकृच्छ्रविनाशयेत् ।
त्रिदोषशमनंबलं तद्दुग्धंभ्रमनाशकम् ।। [1]

3. **उष्ट्रः**- उष्ट्रस्य क्षीरं शीतनाशकं अर्शसः उपशमकं क्रिमिनाशकं महोरशमकं भवति ग्रहपीडां च निवारयति । वीर्य क्षणतां निरूध्य धातुपुष्ठिकरं भवति ।

उष्ट्रः क्रमेलकोधूम्रः करभोदीर्घमार्गगः ।
द्विककृच्छं कुनासश्च दीर्घग्रीवोघ म्रकः ।।
क्रमेलदुग्धं शीतघ्नमर्शः क्रियामबुदरपहम् ।
ग्रहभूतप्रशमनं नष्टेद्रियकरं परम् ।। [1]

4. **गर्दभः** - गर्दभस्य क्षीरं शरीरबलवर्धकं भ्रमं मूत्रकृच्छं निवारयति ।

बालानामपि रोगहारकम् ।
गर्दभश्शदुकर्णश्च बालोयोशसभःखरः ।
भारवाहीभूरिगामो धूसरोरेणु भूषितः ।।
खरदुग्धं भ्रमहरं कायसिद्धिकरं परम् ।
कामिलामूत्रकृच्छघ्नं बलरोगहरंपरम् ।। [2]

5. **छागलः** – छागलस्य मांसः गुरुः स्निग्धश्च भवति । श्वास रोगान् कासरोगान् वातरोगान् प्रदररोगान् शोषकि कफदोषं च जेतुं प्रभवति।

छागलोबर्कश्चागः चलपादस्तुबस्तकः ।
वोलकश्चैडकोजाच लम्बकर्णस्तथैवच ।।
अजमांसं गुरुस्निग्धं श्वासकासानिलापहम् ।
प्रदरंहन्ति गुरुस्निग्धं च पित्तघ्नं कफदोषजित् ।। [1]

6. **मेषः**- मोषमांसं मधुरं शीतवीर्यं गुरुं च भवति । त्रिदोषहारकं भवति ।
प्रतिश्चायं ज्वरं च निवारयति शरीरकान्तिं वृद्धिं करोति ।

तगरुर्मेद्रं मोषा च ह्युरभ्र उदणस्तथा अतिलोमस्तभोर्णाम् युरेडकोवृष्टिरेव च।
तगरोर्मधुरं मांसं शीतवीर्यन्त्रिदोषजित् प्रतिश्चायाज्वरहरं गुरुकान्ति प्रदंनृणाम् ।। [2]

7. **महिषः** - महिष वृषभयोः मांसेन गुणः यद्यपि धन्वरिनिघण्टौ ऐदाहृतः तथापि बहुभिःजनैःतन्मांसं उपयुज्यते । त्वचा च दूनि प्रयोजनानि प्राकृ निरूपितानि । आरण्यकस्य धेनोः गावयस्य मांसगुणं तु कोशे उदाहृतः । गवस्य मांसं वीर्यवृद्धिकरं बलकरं स्वदुः च उदाहृतम् ।

महिषः कासरः श्रृंगी श्यामः कृष्णोलूलायकः ।
विषाणीकलुषो दंशी पीनस्कन्धोरजस्वलाः ।।
महिषः सलिलप्रीतिस्वशक्तिर्भारवाहकः ।
अनड्वांस्तुबलीवर्धः ककुद्वान्वृषभोवृषः ।
उक्षा चे सौरभेच्च गोपतिश्शंतर प्रियः ।।
प्रातर्दशीं च वृषभं कल्याणं च फलप्रदम् ।
ऐश्वर्यं लभते नित्यं शंकरस्यानुवाहनम् ।।
तस्यपुत्रीभानुमती गौरत्रसुरभिर्वसु माहेयी बहुलाधेनुस्सौरभेयी च नन्दिनी ।
गावस्य तु मांसन्तु वृष्यंबल्यंरुचिप्रदम् धातुवृद्धिकरंचैव मुनिभिः परिकीर्तितम् ।।

8. **व्याघ्रः** - व्याघ्रस्य चर्मणा प्रयोजनमेव धन्वन्तरि निघण्ढौ दृष्टम् । व्याघ्रचर्म यौगिभः आसनार्थं उपयुज्यते । व्यघ्रनखधारणं ग्रहदोष निवारकं आयुः बुद्धिकरं देहवृद्धिम् च ।

व्याघ्रः पञ्चनखोव्यालश्शार्दूलश्च गुहाशयः
पुण्डरीकस्तीक्ष्णद्रष्ट्रस्तरस्वीघोरदर्शनः।।
सुखासनं व्याघ्रचर्म भिक्षुप्रीतिकरं तथा
तन्नखोग्रहदोषगघ्नस्तथायुर्देहवर्धनः ।। [1]

9. **वराहः** - वराह मांसं बहुजनप्रियम् । वातघ्नं बलवर्धकं कान्तिकरं च । वराहमांस धूपः ज्वरनाशकः भूतबादा निवारकः दुष्टव्रणहरश्च ।

सूकरोवज्रदंष्ट्राश्च बराहोमोमशः किटिः दंष्ट्री दंष्ट्रायुधः क्रोडो वीरस्कन्धो बहुप्रजः ।।
स्तब्धरोमाबलीकोलो घृष्टिः प्रोत्रीकिरस्मृतः स एव कथितो घोणी पृथुवाराहनामतः ।।
किटिमांसं वातकरं बलवर्णकरंपरम् तद्धमोज्वरभूतघ्नो दुष्टव्रणहरः परम् ।। [2]

10. **मृगः** - मृगमांसस्य जनप्रीतिकरत्वं प्रसिद्धमेव । मधुरगुणं लघुबलकरं वीर्यवृद्धिकरं त्रदोषहरं च ।

मृगोघहरिणोन्यङ्कः सारङ्गः पृषतोरुरुः ।
एणश्शबलपृष्ठश्च पिशङ्गश्चारुलोचनः ।।
न्यङ्कस्वादुर्लघुर्बल्योवृष्योदोषत्रयापहः ।। [1]

11. **शुनकः** - श्व मांसः यद्यपि मासाहारैः अपि नादृतः । तथापि वातनाशकरः उदाहृतः विपाकदशायां उष्णकरश्च ।

कुक्कुरः सारमेयश्च भीषणः शुनकस्तथा ।
भूस्तरोभषकरम्वास्यादलरको रात्रिजागरः ।।
कौलेयोमृगदंशश्च वान्तादः सुमुखस्तथा
सूकरारिः कृतज्ञश्च मृगयालुशलस्तथा ।। [2]

12. **बिडालः** - माराजालचर्मणा धूमेण ग्रहपीडा भूतपीडा ज्वरदोषः कुष्ठः कण्डूः दद्रः त्यादि रोगाः शान्ताः भवन्ति ।

बिडालोविषदंष्ट्रश्च मूषिकारिर्बिडालकः वृषदंशश्चमार्जालो मायावीदीप्तलोचनः ।।
मार्जालचर्मजोधूपो ग्रहघ्नो भूतनाशनः ज्वरदोषहरः कुष्ठ कण्डूदद्रुविनाशनः ।। [1]

13. **श्रृगालः** - श्रृगालमांसस्यापि गुणाः धन्वतिरिणा उदाहृताः । किन्तु श्रृगालमांसस्य विनियोगः न श्रूयते । तथापि औषधरूपेण तस्य गुणाः दर्शिताः । गुणेन कटुः समस्तवातरोगनिवारिकः क्षयरोगनिवारकश्च । धातूनां पुष्टिं शरीरपुष्टिं त करोति।

श्रुगालोजम्बुकः क्रोष्टा गोमायुः क्राष्टकः शिवा ।
वञ्चकोमृगधूर्तश्च भूरिमायोधभौरवः ।।
श्रुगालमांसं कटुकं गन्धर्वव्याधि नाशनम् ।
वृष्यं पुष्टिकरं बल्यं सर्ववातक्षयापहम् ।। [2]

14. **मर्कटः** - मर्कटस्य क्षीरगुणः एव दर्शितः । क्षीरं कटुकं भवति । सन्याज रोगस्य श्वासरोगस्य च निवारकं भवति ।

मर्कटो वानरः कीशो करिरशाखा मृगः कपिः ।
प्लवङ्गमो वनैकाश्च प्लवङ्गः प्लवगः प्लवः ।।
प्लवङ्गगं क्षीरं कटुकं कांससन्यास श्वासजित् ।। [1]

एवं सर्वेषां पशूनां लोकोपकारकत्वं अस्मिन् प्रकरणे निरूपितम् । जीवन्ता एताः अपि पशवः मानवजातिः बहुधा उपकारकाः इति स्पष्टम्। विशेषतः कृषिकर्माणि पशूनां विनियोगः मानवप्राणरक्षायाः जीवातुभूतः। गतां महिषां च क्षीरं मेषादीनां मांसं आहाररूपेण प्राणदायको भवत्येव । अपि च कैश्चित् औषध उपयोगोपि दृष्टः ।

एतान् उपकारान् अतीत्य पशूनां उत्सर्गैः अपि प्रयोजनं बहुधा दृष्टम् । मृगमलादयः पदार्थाः सुगन्धद्रव्यरूपेण औषधरूपेण उपयुज्यन्ते । एतेषां द्रव्याणां कृतं तत् घारकाः मृगाः हन्यन्ते च यथा-

चर्मणि द्वीपिनं हन्ति दन्तयो र्दन्ति कुञ्जरम् ।
बालेषु चमरीं हन्ति सीम्नि पुष्कलको हतः ।। [1]
सीमाः अण्डकोशः पुष्कलकः गन्धमृगविशेषः ।

गन्धमृगाश्च यथा ---

चित्ररेखाकरीधूर्णी मार्जालीगन्धमार्जनी ।
सुगन्धगात्रावसना कस्तूरीरोमशाल्यपि ।।
मुगन्ध पूतवृषणी बिडालीकाण्डचालिनी ।
तत्कोशंवृष्यमायुष्य सौरभंगात्रगन्धजित् ।। [2]

15. **कस्तूरी मृगः** - पुष्कलकः एव कस्तूरी मृगः नाम्ना प्रसिद्धः । प्रायशः हिमवत्पर्वत प्रान्तेषु एव एते चरन्ते । एतेषां सीम्नः कस्तूरी लभ्यते । शीतज्वरार्तानां औषधार्थं उपयुक्तः । अद्यापि सद्यः प्रसूताभ्यः गृहेषु प्रसूति वातनिवारणाय कस्तूरी दीयते । भगवत्सेवासु चन्दनेन मिश्रितोऽयं उपयुक्तः । "कस्तूरी तिलकं" - इत्यादि स्तोत्राणि प्रसिद्धान्येव । चन्दनेन तं मिश्रणात् यक्षकर्दमनामकः अनुलेपः क्रियते । एवं बहूपकारकाणां पशूनां प्रस्तावः संस्कृत वाङ्मये कीदृशो वर्तते इति विचारः एवं अत्र शोधनिबन्धे विवक्षितः ।

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HINDU COLLEGE :: GUNTUR

3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
2021-22									
S. No.	**Title of paper**	**Name of the author/s**	**Department of the teacher**	**Name of journal**	**Year of publication**	**ISSN number**	**Link of Journal**	**Link of Publication**	**Link to the recognition in UGC enlistment of the Journal**
1	*Comparative Evaluation of Efficacy of Hydroxychloroquine versus Methotrexate in Patients with Rheumatoid Arthritis*	M.V.S.L. Priyanka	Department of Biomedical Sciences	International Journal of Modern Pharmaceutical Research (IJMPR)	2021	2319-5878	https://www.ijmpronlin	https://ijmpronline.com/download/article/469/1627635108.pdf	NA
2	*A Brief Review on Efavirenz and it's Adverse Effects*	M.V.S.L. Priyanka	Department of Biomedical Sciences	World Journal of Pharmaceutical Research	2022	2277-7105	https://www.sriramachandra.edu.in/university/departments.php?cid=5&did=39	https://wjpr.net/abstract_show/18659	NA
3	*Optimization of Cultural Conditions to Enhance Bioactive Metabolite Production by Streptomyces Olivoverticillatus VLK – 15*	Dr. N. Krishna	Department of Botany	**International Journal of Pharmaceutical Sciences and Research**	**2021**	**E-ISSN: 0975-8232; P-ISSN: 2320-5148**	https://wellcomeopenresearch.org/	https://ijpsr.com/bft-article/optimization-of-cultural-conditions-to-enhance-bioactive-metabolite-production-by-streptomyces-olivoverticillatus-vlk-15/	NA

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DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
4	*Deterioration and Aggregation of a Few Heavy Metals in Agricultural Soils and Dicoccum of Industrial Zone*	Dr. T. Umamaheswara Rao	Department of Chemistry	JOURNAL OF OPTOELECTRONICS LASER	2022	1005-0086	http://gdzjg.org/index.php/JOL	http://gdzjg.org/index.php/JOL/search/index?query=Deterioration+and+Aggregation+of+a+Few+Heavy+Metals+in+Agricultural+Soils+and+Dicoccum+of+Industrial+Zone+&dateFromYear=&dateFromMonth=&dateFromDay=&dateToYear=&dateToMonth=&dateToDay=&authors=	NA
5	*Development of Method and Validation for Assay in Cyclizine Hydrochloride Injection by Applying Stability Indicating HPLC Methodology by HPLC*	Dr. T. Umamaheswara Rao	Department of Chemistry	Forest Chemicals Review	2022	1520-0191	http://www.forestchemicalsreview.com/index.php/JFCR	http://forestchemicalsreview.com/index.php/JFCR/article/view/1199/1207	NA
6	*Role of Autonomous Colleges in Improving Quality of Higher Education in India in the Light of NEP - 2020*	Dr. P. Venugopal	Department of Economics	International Journal of Multidisciplinary Educational Research	2020	2277-7881	http://www.ijmer.in/	http://s3-ap-southeast-1.amazonaws.com/ijmer/pdf/volume9/volume9-issue12(8)/25.pdf	NA
7	*Marshall – Olkin Stereographic Circular Logistic Distribution*	Dr. S.V.S. Girija Sri Y. Sreekanth	Department of Mathematics	YMER	2022	0044-0477	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII	https://ymerdigital.com/uploads/YMER210692.pdf	https://ugccare.unipune.ac.in/Apps1/User/Web/CloneJournalsGroupII
8	*Estimation of Parameters of Pert Distribution by using Maximum Product of Spacings Method*	Dr. N. Viswam	Department of Statistics	International Journal of Creative Research Thoughts (IJCRT)	2021	2320-2882	https://www.ijraset.com/	https://ijcrt.org/papers/IJCRT2109281.pdf	NA

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3.3.1 Number of papers published per teacher in the Journals notified on UGC website during the last five years

DIGITAL PUBLICATIONS 2017 - 22

S. No.	Title of paper	Name of the author/s	Department of the teacher	Name of journal	Year of publication	ISSN number	Link of Journal	Link of Publication	Link to the recognition in UGC enlistment of the Journal
9	*Estimation of Parameters of Pert Distribution by using Method of Moments*	Dr. N. Viswam	Department of Statistics	International Journal for Research in Applied Science and Engineering Technology (IJRASET)	2021	2321-9653	https://www.ijraset.com/	https://ijcrt.org/papers/IJCRT2109281.pdf	NA
10	*Classification of Brain Stroke using Data Mining Techniques – A Comparative Analysis*	Dr. N. Viswam	Department of Statistics	International Research Journal of Modernization in Engineering Technology and Science	2023	e-ISSN: 2582-5208	https://www.irjmets.com	https://www.irjmets.com/uploadedfiles/paper//issue_1_january_2023/33286/final/fin_irjmets1675411490.pdf	NA
11	*Estimation of Parameters of Pert Distribution by using Maximum Likelihood Method of Estimation*	Dr. N. Viswam	Department of Statistics	IJSRD - International Journal for Scientific Research & Development|	2021	ISSN (online): 2321-0613	https://www.ijsrd.com/	https://www.ijsrd.com/articles/IJSRDV9I70113.pdf	NA
12	*Pert Distribution and its Properties*	Dr. N. Viswam	Department of Statistics	International Research Journal of Modernization in Engineering Technology and Science	2021	e-ISSN: 2582-5208	https://www.irjmets.com/	https://www.irjmets.com/uploadedfiles/paper/volume_3/issue_10_october_2021/16799/final/fin_irjmets1635145380.pdf	NA

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COMPARATIVE EVALUATION OF EFFICACY OF HYDROXYCHLOROQUINE VERSUS METHOTREXATE IN PATIENTS WITH RHEUMATOID ARTHRITIS

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ABSTRACT

Aim: The aim of the study is to compare the efficacy of Methotrexate(MTX) versus hydroxychloroquine(HCQ) in patients with Rheumatoid arthritis. **Materials and methods:** This prospective study involves 100 patients who were diagnosed with rheumatoid arthritis and those who met the study criteria were enrolled for the study.Relevant data such as demographic details, past medical history,drug name were collected from out- patient profile form and by patient interview. **Statistical Analysis:** All the raw data was collected, entered in Excel sheet 2010 in windows 10 version, the statistical analysis was done in SPSS 16.0 Software by an appropriate statistical method, paired sample T-test for knowing the significant p-value <0.05(confidence interval 95%). **Results:** Among 100 patients, 50% of patients have received the drug, Hydroxychloroquine(HCQ) (400 mg/day p.o). 50% patients have received the drug, Methotrexate(MTX)(15 mg/week in divided doses p.o).The study shows that Hydroxychloroquine group having remission rate of 4%(2 patients out of 50) with the remission value <2.6 and shown to be significant with p value < 0.05. Methotrexate group being assessed having remission rate of 8% (4 patients out of 50) with the remission value of <2.6 which is found to be statistically significant (p<0.05) and there is significance decrease in Erythrocyte sedimentation rate(ESR). **Conclusion:** In our study we have observed that methotrexate was more effective in managing rheumatoid arthritis when compared to hydroxychloroquine by comparing the DAS28 score of both groups.Pain and inflammation also reduced by avoiding further exacerbation of condition.

INTRODUCTION

Rheumatoid arthritis (RA) is a chronic, systemic autoimmune disease that characterized by inflammatory arthritis resulting in pain, swelling, and stiffness in the joints, and may cause severe joint damage, loss of function and disability. The disease may last from months to a lifetime, and symptoms may improve and worsen over time.[1] It is estimated to have a prevalence of about 1% of the population in India. It affects roughly three times as many women as men. People tend to develop rheumatoid arthritis between 40 and 60 years of age, although it can arise at any age.[2]



Fig. 1: Healthy joint and Rheumatoid arthritis affected joint.

Rheumatoid arthritis (RA) if left untreated, ultimately causes functional impairment.[3] Non-steroidal anti-inflammatory drugs (NSAIDs) help only in symptomatic relief, corticosteroids have problems with chronic use and disease-modifying anti-rheumatic drug (DMARD)

have promised to reduce the disease activity in a better way. Many new drugs were introduced into the market for the treatment of rheumatoid arthritis; however, their efficacy is yet to be established. Studies are necessary in Indian patients to compare the efficacy with methotrexate, an age old DMARD. Present study was aimed to evaluate the efficacy of methotrexate and hydroxychloroquine, an anti-malarial drug orally in the management of rheumatoid arthritis.

MATERIALS AND METHODS

This is a prospective observational study which doesn't involve any type of invasive techniques and blood samples purely for the study purpose. All the parameters were taken by seeing the data and reports available from the patient which have already been suggested by the physician.The study was carried out for a period of six months i.e. from October,2019 to March,2020.This study was performed in outpatient department of an ORTHOPAEDIC HOSPITAL, Guntur, Andhra Pradesh.

Inclusion criteria

I. Patients suffering from Rheumatoid arthritis.
II. Both males and females
III. Patients using Methotrexate or Hydroxychloroquine
IV. Patients of age group 16 – 70 years who are willing to participate in the study

Exclusive criteria

1. Pregnant women and women who trying to get pregnant
2. Lactating women
3. Patients of age group below 16
4. Patients of age group above 70
5. Patients with active infective diseases
6. Patients with renal dysfunction and hepatic diseases

Patients who are diagnosed with rheumatoid arthritis and who met the study criteria were enrolled for the study. Relevant data such as demographic details ,disease history, diagnosis, drug name, dose, laboratory data are collected from medical records of the patient and by patient interview where ever required were collected and documented. A suitable data collection form is designed and used in the study. Monitored the clinical status of the patient for 3 months.

All the necessary data was collected from - Interviewing the out patients and their care takers, Patient's prescriptions, Laboratory findings & Any other relevant sources.Before and after the treatment the patient clinical condition was checked.Advised the patients regarding their medication regimen and importance of medication adherence.

DAS28 score: The DAS28 is a measure of disease activity in rheumatoid arthritis (RA). DAS stands for 'disease activity score' and the number 28 refers to the 28 joints that are examined in this assessment. The DAS28 is a composite score derived from 4 of these measures. This '28' version is a simplification of the original DAS score, which requires 52 joints to be counted. ESR value and global health assessment value ((from 0=best to 100=worst).

DAS28 is a composite formula. Four parameters are used to calculate a disease severity score:

A) Number of swollen joints out of a total of 28 specified joints

B) Number of tender joints out of a total of 28 specified joints

C) Erythrocyte sedimentation rate

D) Patient's interpretation of well being, with 0 being at their best and 100 their worst

a. High disease activity: DAS28 of >5.1
b. Moderate disease activity: DAS28 of >3.2 to 5.1
c. Low disease activity: DAS28 of 2.6–3.2
d. Remission: DAS28 of <2.6

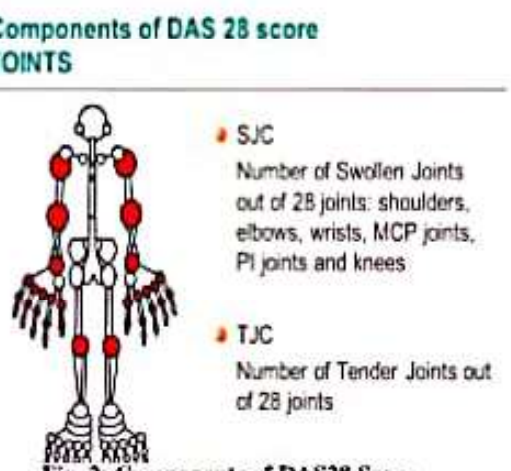

Fig. 2: Components of DAS28 Score.

- The 28 tender or swollen joint scores target the same joints (shoulders, elbows, wrists, metacarpophalangeal joints, proximal interphalangeal joints and the knees).
- These results are then fed into a complex mathematical formula to produce the overall disease activity score.
- The computation of the score is done through the following equation:[4]

$$DAS28 = 0.56 \times \sqrt{TJC28} + 0.28 \times \sqrt{SJC28} + 0.70 \times \ln(ESR) + 0.014 \times GH$$

Morisky medication adherence scale (MMAS) was used to calculate the medication compliance which contains 8 questionnaries and the information was gathered during patient counselling.This scale is used in our study to know the significance of medication adherence. Scores are summed and range from 0-8.

MMAS SCORING

- MMAS Score ≥8 : High Adherence

- MMAS Score 6 - < 8 : Medium Adherence
- MMAS Score < 6 : Low Adherence

If a patient scores higher on the scale, they are evaluated as more adherent. If they score lower on the scale, they are presumed to be struggling with low adherence.[8]

RESULTS

- Initially 110 patients were included in study, out of which 6 patients were excluded, because of no proper follow up due to lack of response and patient's disinterest towards the study; 4 patients were excluded because of infections that were developed for them during the study.
- Finally 100 patients were included in the study. Among 100 patients 50% patients i.e 50 patients were treated with Hydroxychloroquine and another 50% are treated with Methotrexate.
- Before and after patient clinical condition was checked, follow up was done for about 3 months.

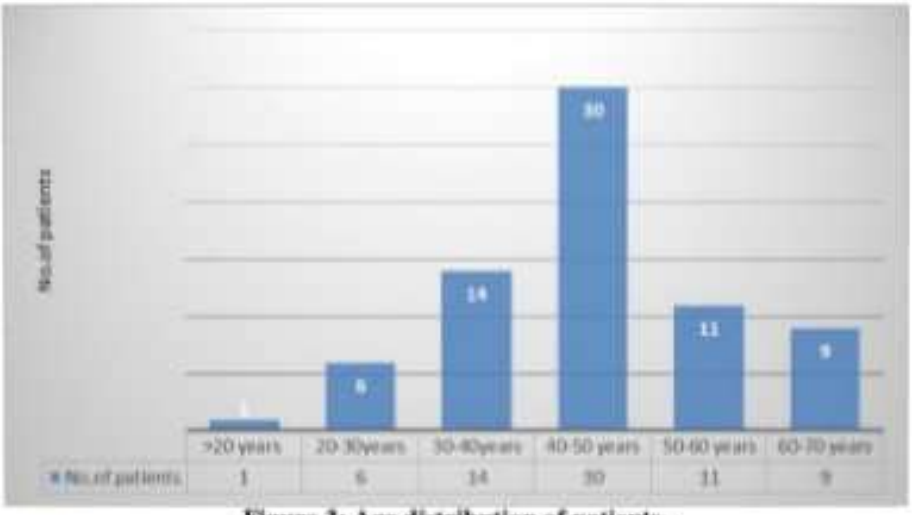

Figure 3: Age distribution of patients.

- Age distribution of patients - 39% of patients were between the age of 40-50 years, followed by 28% of patients were between the age of 50-60 years were 17% of patients were between the age of 30-40 years, 9% of patients were between age of 60-70 years were 6% of patients between the age of 20-30 years and 1% of patients below the age of 20years.
- According to our study, we have observed that most of the people who were affected with rheumatoid arthritis were from 40-50 age group.

Table 1: Gender differentiation.

GENDER	NUMBER OF PATIENTS	PERCENTAGE
Male	10	10%
Female	90	90%
Total	100	100

Table 1 shows the patient characteristics based on gender out of 100 patients 90 patients were females (90%) , 10 patients were males(10%).According to our study, we have observed that females are more prone to rheumatoid arthritis compared to males.

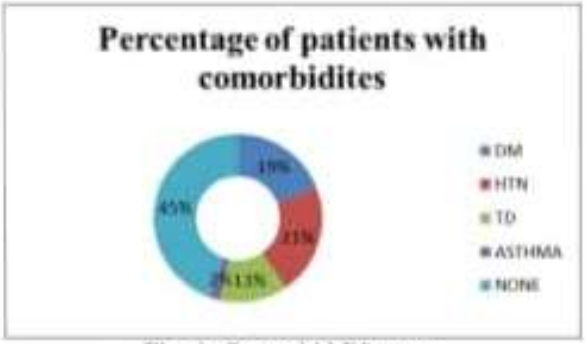

Fig. 4: Comorbid Diseases.

In our study most of the patients (45%) were with no comorbid diseases. And 19% of patients were reported with having DM, 21% of patients were reported with HTN, 13% patients reported with thyroid disease and 2% patients were reported with comorbidity of asthma.

Table 2: Rheumatoid Factor (RF) for patients using Hydroxychloroquine (HCQ) Drug.

Rheumatoid Factor (RF)	Before Treatment	After Treatment
0-30	39	31
30-60	6	12
60-90	1	5
>90	4	2

- Table 2 shows the pre and post treatment for rheumatoid factor of 50 patients using HCQ drug are included in the study. Out of 50 patients before treatment 39 patients were between RF value (0-30) and after treatment 31 patients were between the RF value (0-30). Before treatment 6 patients were between the RF value (30-60) and after treatment 12 patients were between RF value (30-60). Before treatment 1 patient had a RF value between (60-90) and after treatment 5 patients were between RF value (60-90). Before treatment 4 patients are with the RF >90 and after treatment 2 patients with RF>90.
- This shows that there was a significant increase in the RF value 30-60 and 60-90 patients when compared with pre treatment. And gradually there is a decrease in RF value 0-30 and >90 patients when compared with pre-treatment.

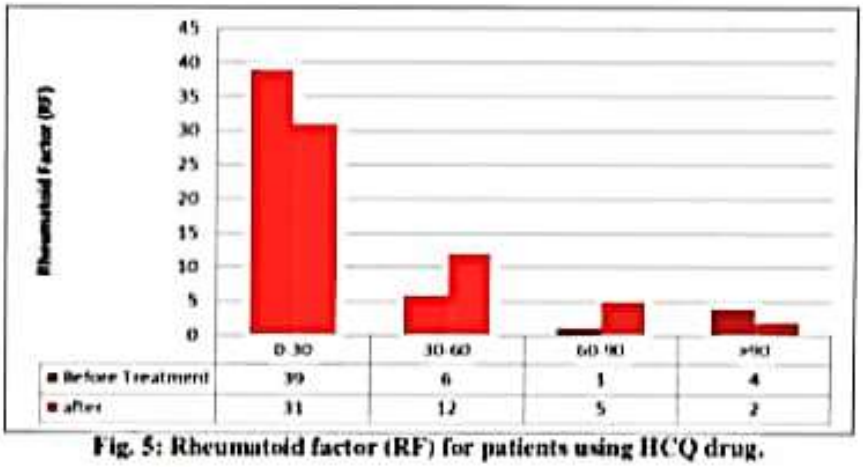

Fig. 5: Rheumatoid factor (RF) for patients using HCQ drug.

Table 3: ESR for patients using HCQ drug.

ESR (mm/hr)	Before treatment	After treatment
20-40	11	19
40-60	20	22
60-80	11	09
80-100	8	0

Table 5.5 shows the pre and post treatment ESR for 50 patients using HCQ drug. Out of 50 patients 11 patients were between 20-40 ESR and after treatment 19 patients were between 20-40 ESR. Before treatment 20 patients were between ESR 40-60 and after treatment 22 patients were between ESR 40-60. Before treatment 11 patients were between ESR 60-80 and after treatment 9 patients were between ESR 60- 80. Before treatment 8 patients were between ESR80-100 and after treatment there were no patient between ESR 80-100.

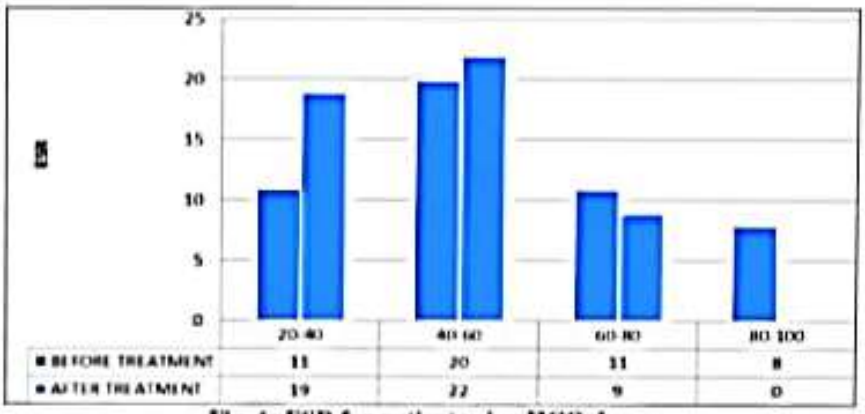

Fig. 6: ESR for patient using HCQ drug.

Table 4: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before treatment	58.0000	50	20.99563	2.96923
	After treatment	46.6000	50	16.45774	2.32748

Table 5: Paired Samples Correlations.

		N	Correlation	Sig.
Pair 1	Before treatment & After treatment	50	.706	.000

Table 6: Paired Samples Test.

		Paired Differences					t	Df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Before treatment – After treatment	11.40000	14.95026	2.11429	7.15118	15.64882	5.392	49	.000

Table 4 shows that pre and post treatment ESR for HCQ drug of 50 patients included in the study, the mean of the pretreatment ESR for HCQ drug was 58.000±20.995 and the mean of ESR for HCQ drug after treatment was 46.600± 16.457. This shows that there was a significant decrease in ESR for HCQ i.e., 11.4 ± 4.538 in treatment of HCQ drug. There was a significant difference in reduction of pain having p-value <0.05(95% confidence interval). By conventional criteria this difference is considered to be statistically significant.

Table 7: DAS28 for patients Using HCQ drug.

DAS28 score	Before treatment	After Treatment
2.6	0	2
2.6-3.2	11	19
3.2-5.1	17	14
>5.1	22	15

Table 7- Shows the pre and post treatment DAS28 for patient using HCQ drug. Out of 50 patients after treatment there were 2 Patients at low disease activity score which include 28 joints. Before treatment 11 patients were between the das28 2.6-3.2 and after treatment 19 were between disease activity score between2.6-3.2. Before treatment there were 17 patients between DAS3.2-5.1and after treatment were 14 patients between DAS3.2-5.1. Before treatment 22 patients were high disease activity score >5.1 and after treatment there 15 patients were at high disease activity score >5.1.

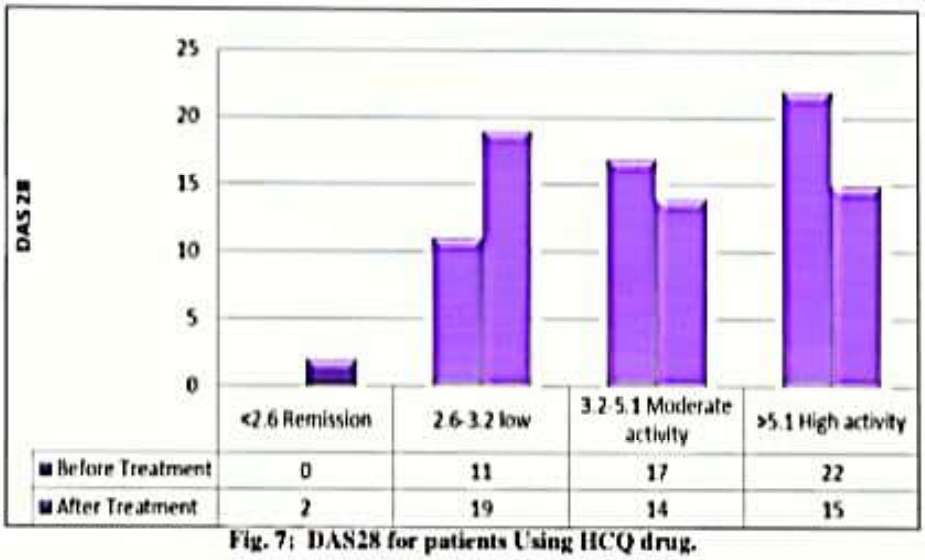

Fig. 7: DAS28 for patients Using HCQ drug.

Table 8: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	BEFORE TREATMENT	5.3960	50	1.37766	.19483
	AFTER TREATMENT	4.0860	50	1.26806	.17226

Table 12 shows the pre and post treatment ESR for patients using MTX drug. Out of 50 patients 12 patients were between ESR 20-40 and after treatment 22 patients were between ESR 20-40. Before treatment 13 patients were between ESR 40-60 and after treatment 22 patients were between ESR 40-60. Before treatment 17 patients were between ESR 60-80 and after treatment 5 patients were between ESR 60-80.Before treatment 8 patients were between ESR 80-100 and after treatment 1 were ESR 80-100.

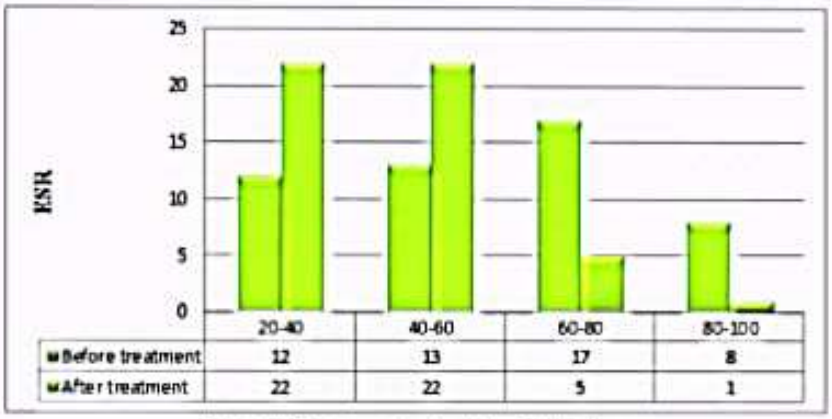

Fig. 9: ESR for patients using MTX drug.

Table 13: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before treatment	60.6000	50	19.93971	2.81990
	After treatment	41.3000	50	17.31491	2.44870

Table 14: Paired Samples Correlations.

		N	Correlation	Sig.
Pair 1	Before treatment & After treatment	50	.745	.000

Table 15: Paired Samples Test.

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Before treatment - After treatment	19.30000	13.51530	1.91135	15.45899	23.14101	10.098	49	.000

Table 15 shows that pre and post treatment ESR for MTX drug of 50 patients included in the study. The mean of pretreatment ESR for MTX was 60.600±19.93971 and the mean of ESR for MTX drug after treatment was 41.300±17.31491. This shows that there was a significant decrease in ESR for MTX drug i.e., 19.300±26248. There was a significant difference in reduction of ESR having p-value <0.05(95% confidence interval). By conventional criteria this difference is considered to be statistically significant.

Table 16: DAS28 for patients using MTX drug.

DAS28	Before treatment	After treatment
<2.6	0	4
2.6-3.2	12	17
3.2-5.1	17	20
>5.1	21	9

Table 16 shows the pre and post treatment DAS 28 for patients using MTX drug. Out of 50 patients after treatment 4 patients were between the DAS less than 2.6. Before treatment 12 patients were between the DAS 2.6-3.2 and after treatment 17 patients were between the DAS 2.6-3.2. Before treatment 17 patients were between DAS 3.2-5.1 and after treatment 20 patients were between DAS 3.2-5.1. Before treatment 21 patients were DAS >5.1 and after treatment 9 patients were DAS >5.1.

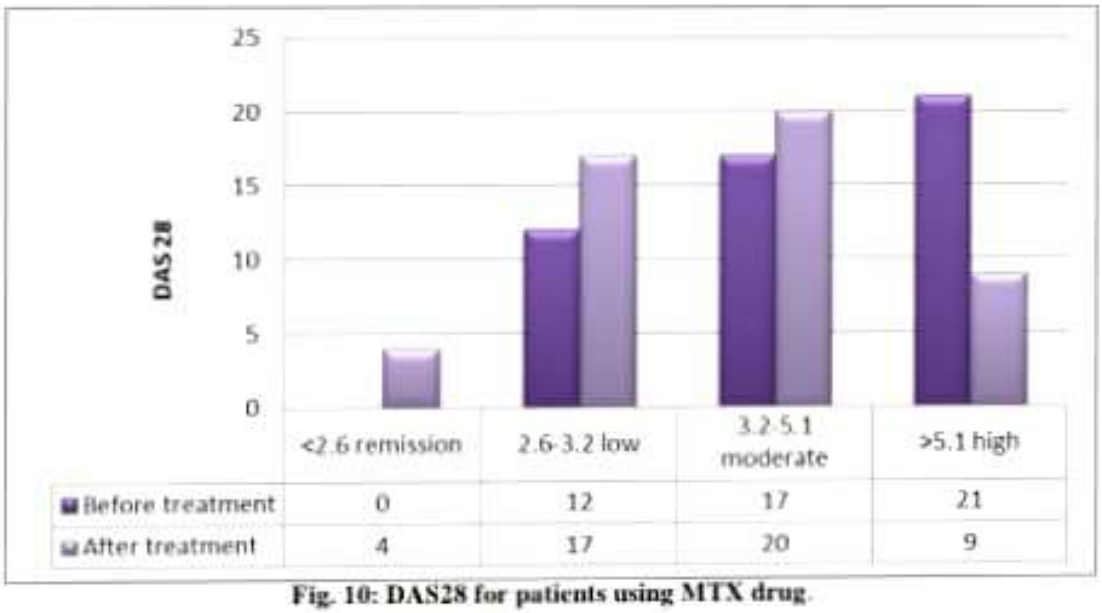

Fig. 10: DAS28 for patients using MTX drug.

Table 17: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before treatment	5.0260	50	1.27922	.18657
	After treatment	3.9320	50	1.10186	.16148

Table 18: Paired Samples Correlations.

		N	Correlation	Sig.
Pair 1	Before treatment & After treatment	50	.922	.000

Table 19: Paired sample test.

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Before treatment – After treatment	1.0940	.51564	.07292	.94746	1.24054	15.002	49	.000

Table 17 shows that pre and post treatment DAS28 for MTX drug of 50 patients included in the study. The mean of pre-treatment DAS 28 for MTX drug was 5.0260±1.27922 and the mean of DAS 28 for MTX after treatment was 3.9320±1.10186. This shows that there was a significant decrease in DAS 28 for MTX drug i.e., 1.0940±0.1350. There was a significant difference in reduction of DAS 28 for MTX drug having p-value <0.05(95% confidence interval). By conventional criteria this difference is considered to be statistically significant.

CO-Efficient of Variation (CV %)

It is a statistical measure of the dispersion of data points in a data series around the mean. Lower the CV value more consistent the results.

$$\text{CO-EFFICIENT OF VARIATION (CV)} = \frac{\text{SD X 100}}{\text{MEAN}}$$

Table 20: Co-efficient of variation for DAS28 of Hydroxychloroquine and Methotrexate.

Drugs	Standard deviation	Mean	Co-efficient of variation
HYDROXYCHLOROQUINE	1.26806	4.0860	31
MERHOTREXATE	1.10186	3.932	28

Medication adherence

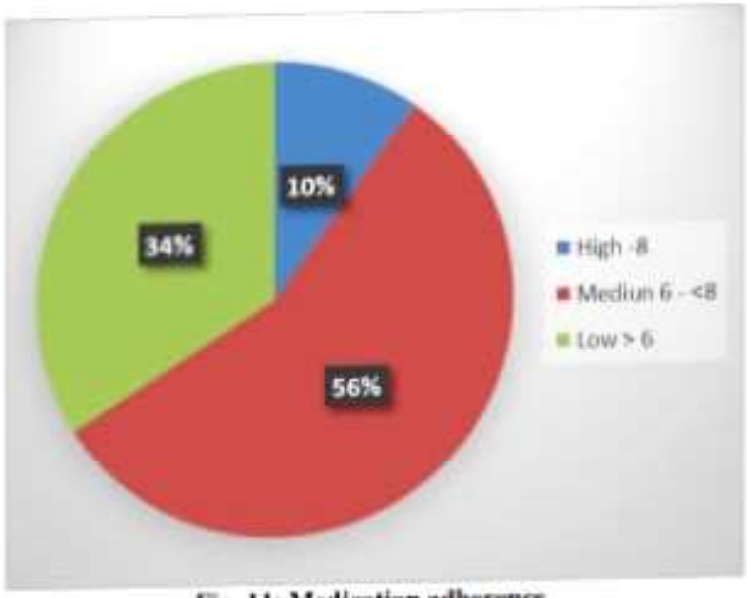

Fig. 11: Medication adherence.

- Out of 100 patients 50 patients are using MTX drug and 50 patients are using HCQ drug are included in study.
- In HCQ drug using patients out of 50 patients 17 patients are low adherent to medication and 28 patients are medium adherent to medication and 5 patients are high adherent to medication.
- In MTX drug using patients out of 50 patients 18 patients are low adherent to medication and 23 patients are medium adherent to medication and 13 patients are high adherent to medication.
- According to our study, we have observed that Patient medication adherence was moderately adherent.

DISCUSSION

With the disease modifying therapy currently available, complete remission of rheumatoid arthritis are almost rare. A variety of disease modifying anti rheumatic drugs(DMARDS) are available to control the clinical activity of rheumatoid arthritis, methotrexate an analogue of folic acid and hydroxychloroqine are the most commonly used drugs in monotherapy of rheumatoid arthritis.

Very little research was done before regarding the monotherapy of MTX and HCQ in RA patients making the comparison very difficult. We sought to design and conduct a study that would reflect clinical practice and yield results applicable to patients with RA and their physicians, therefore we allowed patients to continue taking NSAID's and other drugs prescribed to them.

Early Placebo controlled trails demonstrated that weekly low dosage methotrexate produced early symptomatic improvement in most RA patients.[5]

A double blind placebo controlled, crossover study of weekly pulse methotrexate dose(15mg), after 13 weeks of therapy patients receiving methotrexate showed greater improvement, judged by degree of joint swelling and tenderness, duration of morning stiffness and subjective assessment of clinical condition compared to those receiving placebo (p≤0.002).[6]

A 36 week randomised double-blind placebo controlled study done by HERA group regarding the usage of hydroxychloroqine of dose 400 mg in RA patients and it has been assessed by students "t" test. Over the course of study there was satisfactory significant improvement in the joint index (p=0.004,p=0.034), the pain index (p=0.007,p=0.001)and the physical index (p=0.020, p=0.011) with the group receiving hydroxychloroqine compared to placebo.[7]

We have done a prospective observational study with 100 patients in a time period of 6 months.Among 100 patients 50% patients i.e 50 patients were treated with Hydroxychloroquine and another 50% are treated with Methotrexate. All the patients are being diagnosed and assessed their condition by the various diagnosing tools like ESR, DAS 28, RF, and their adherence is being checked using Morisky's adherence scale.Both the group has been assessed by DAS28 score to check the remission rate after a 3 months period. Hydroxychloroquine group having remission rate of 4% (2 patients out of 50) with the value <2.6, and having standard mean deviation of 0.89881 and are shown to be significant with p value < 0.05. Another 50%, Methotrexate group being assessed having remission rate of 8% (4 patients out of 50) with the remission value of <2.6 and having standard mean deviation of 0.07292 and are shown to be significant with p value <0.05.

Morisky medication adherence scale was used to calculate the medication adherence.In our study the

patients having moderate compliance at an initial stage due to lack of knowledge about the comorbidities and usage of medications later, the compliance overcome by patients counselling about the medication use and their disease condition. And finally future studies are needed to provide information on the potential benefits of mono therapy of DMARD's like MTX and HCQ and others with respect to therapy.

CONCLUSION

Hydroxychloroqine and Methotrexate are the most commonly used primary drug of choice in the treatment of rheumatoid arthritis patients. In our 6 months study, it was Observed that Methotrexate is comparatively more efficacious than Hydroxychloroquine. Remission rate by methotrexate is slightly edge more over with hydroxychloroqine. However further work is required to determine the use of oral methotrexate and hydroxychloroqine in the therapeutic management of rheumatoid arthritis. Medication adherence among the patients was mostly moderate. Measures are to be taken to improve the medication adherence.Clinical pharmacist have major role in improving medication adherence through proper patient counseling.

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REFERENCES

1. Krati Chauhan; Jagmohan S. Jandu; Amandeep Goyal; Pankaj Bansal; Mohammed A. Al-Dhahir - Rheumatoid Arthritis.
2. Shichikawa K, Inoue K, Hirota S, et al. Changes in the incidence and prevalence of rheumatoid arthritis in Kamitonda, Wakayama, Japan, 1965–1996. Ann Rheum Dis., 1999; 58(12): 751–756.
3. Lipsky PE. Rheumatoid arthritis. In: Braunwald E,Fauci AS, Kasper DL, Hauser SL, Longo DL,Jameson JL, editors. Harrison's principles of internal medicine. 15th ed. McGraw Hill: New York, 2003; 1928-37.
4. DAS 28 -Disease activity score calculator for rheumatoid arthritis https://www.rheumakit.com /en/calculators/das28.
5. Early Placebo controlled trails demonstrated that weekly low dosage methotrexate produced early symptomatic improvement in most RA patients.
6. Bernard Bannwarth, Laurence Labat, Yola Moride, Thierry Schaeverbeke, 1994.
7. Clinical and Immunologic Effects in a Randomized, Double-Blind StudyWeekly Pulse Methotrexate in Rheumatoid Arthritis; PETER A. ANDERSEN, M.D., et al., 1985.
8. Treatment of rheumatoid arthritis with methotrexate alone, sulfasalazine and hydroxychloroquine, or a combination of all three medications.s.
9. James R O'Dell, Claire E Haire et al., New England Journal of Medicine, 1996; 33(20): 1287-1291.
10. Morisky DE, Ang A, Krousel-Wood M, Ward HJ. Predictive validity of a medication adherence measure in an outpatient setting. J Clin Hypertens (Greenwich), 2008; 10(5): 348–354.

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A BRIEF REVIEW ON EFAVIRENZ AND IT'S ADVERSE EFFECTS

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ABSTRACT

Efavirenz is a commonly used non-nucleoside reverse transcriptase inhibitor frequently prescribed in combination with other antiretroviral regimens (e.g., protease inhibitors) for the treatment of HIV. After its admission for pharmaceutical treatment in efavirenz has become a cornerstone for highly active antiretroviral therapies (HAART). Good CNS penetration has been seen as a major strength of antiretroviral regimens, particularly efavirenz. Accumulating evidence suggests neuroprotective effects against cognitive decline such as HIV-associated dementia and other aspects of neuropsychological functioning. Recent studies have focused on the relevance of plasma concentrations and genetic predispositions as moderators of neuropsychiatric complications. Nevertheless, its proven effectiveness as a first-line treatment in suppressing plasma viral load comes along with reports of adverse central-nervous and neuropsychiatric effects which have been frequently reported after exposure to efavirenz. The use of psychophysiological measures in the assessment process enables the quantification of treatment-related side effects with high temporal solution, comparative measurements, and sensitivity toward subclinical effects, and thus provides a new source of information.

KEYWORDS: Efavirenz, Neuropsychological effects, Antiretroviral regimens.

INTRODUCTION

Efavirenz (dideoxy inosine, ddI) is an oral non-nucleoside reverse transcriptase inhibitor (NNRTI). It is a synthetic purine derivative. Efavirenz was originally The use of highly active

antiretroviral therapy (HAART) has had an important impact on the course and treatment of disease and disease-related morbidity of HIV-infected patients, increasing their lifespan and quality of life.[1]

Efavirenz (EFV) is a non-nucleoside reverse transcriptase inhibitor (NNRTI) and is used as part of highly active antiretroviral therapy (HAART) for the treatment of a human immunodeficiency virus (HIV) type 1.[2] Currently, the CDC recommends that Efavirenz be given as part of a three-drug regimen that includes another nucleoside reverse transcriptase inhibitor (e.g., lamivudine, stavudine, zidovudine) and a protease inhibitor or efavirenz when treating HIV infection. It works by decreasing the amount of HIV in the blood. Although efavirenz does not cure HIV, it may decrease your chance of developing acquired immunodeficiency syndrome (AIDS) and HIV-related illnesses such as serious infections or cancer.

Mechanism of Action

Efavirenz inhibits the activity of viral RNA-directed DNA polymerase (i.e., reverse transcriptase). Antiviral activity of efavirenz is dependent on intracellular conversion to the active tri phosphorylated form. The rate of efavirenz phosphorylation varies, depending on cell type. It is believed that inhibition of reverse transcriptase interferes with the generation of DNA copies of viral RNA, which, in turn, are necessary for synthesis of new virions. Intracellular enzymes subsequently eliminate the HIV particle that previously had been uncoated, and left unprotected, during entry into the host cell. Thus, reverse transcriptase inhibitors are virustatic and do not eliminate HIV from the body. Even though human DNA polymerase is less susceptible to the pharmacologic effects of tri phosphorylated efavirenz, this action may nevertheless account for some of the drug's toxicity.

Low concentrations are associated with virologic failure, while higher concentrations are associated with increased adverse effects such as sleep disorders.[3]

Efavirenz is highly protein bound to human plasma proteins, predominantly albumin. Efavirenz converts to inactive hydroxylated metabolites by the action of the CYP3A4 enzyme.[5]

Dosing & Administration

Efavirenz is available by prescription only, and it is available as an oral capsule and an oral tablet. It is also available as a component of two combination drugs: efavirenz, emtricitabine, and tenofovir disoproxil fumarate, and efavirenz, lamivudine, and tenofovir disoproxil fumarate. The capsules are available as 200 mg and 50 mg, while the tablets are available as 600 mg. The recommended adult dose is 600 mg per day, but there has been data showing that 400 mg per day yields equivalent outcomes with fewer side effects.[6]

The administration of efavirenz with food has shown to increase the serum concentrations and the incidence of side effects. For this reason, patients should not take efavirenz with food.[4] Peak concentrations are reached by 5 hours following a single oral dose, with steady-state plasma concentrations achieved in 6 to 7 days. The half-life of efavirenz is roughly 45 hours, making once-daily dosing suitable.

Efavirenz should be taken on empty stomach, as increased efavirenz serum concentrations occur when taken with food. Increased serum concentrations can lead to increased adverse effects.

CONTRAINDICATIONS

The use of efavirenz is contraindicated in patients with previously documented hypersensitivity to efavirenz and patients concurrently receiving elbasvir/grazoprevir due to a CYP3A4 interaction. Efavirenz acts as a CYP3A4 inducer, in this case, by decreasing the serum concentrations of grazoprevir. The use of efavirenz is also contraindicated in pregnant women in the first trimester due to reports of neural tube defects.[7]

ADVERSE EFFECTS

Individuals with a human immunodeficiency virus (HIV) infection are at higher risk of developing adverse drug reactions. Multiple drugs are usually prescribed to patients with HIV infection for preventing the replication of HIV and for the treatment of the associated opportunistic infections.

Nervous system symptoms (including headache, dizziness, insomnia and fatigue) and dermatological effects (including maculopapular rash) appear to be the most common adverse events reported with efavirenz-containing antiretroviral regimens.[8]

❖ **Neuropsychiatric adverse effects associated with Efavirenz[9]**

Good CNS penetration has been seen as a major strength of antiretroviral regimens, particularly efavirenz. Accumulating evidence suggests neuroprotective effects against cognitive decline such as HIV-associated dementia and other aspects of neuropsychological functioning. Nevertheless, its proven effectiveness as a first-line treatment in suppressing plasma viral load comes along with reports of adverse central-nervous and neuropsychiatric effects which have been frequently reported after exposure to efavirenz.

Table 1: Neuropsychiatric effects associated with Efavirenz.

Neuropsychiatric effects associated with Efavirenz	Somnolence
	Suicidal ideation
	Impaired concentration
	Psychosis
	Depression
	Abnormal dreams
	Nervousness & Irritability
	Headache

❖ **Efavirenz-induced cutaneous reactions & skin eruptions[10]**

Efavirenz skin rashes are generally a mild-to-moderate diffuse maculopapular skin eruption or pruritic erythema.

❖ **Hemolytic anemia associated with Efavirenz[11]**

Efavirenz triggers suicidal cell death or apoptosis, an effect in part due to interference with mitochondrial potential. Causes of anemia include accelerated clearance of circulating erythrocytes. Even though lacking mitochondria, erythrocytes may enter suicidal erythrocyte death or eryptosis, which is characterized by cell shrinkage and cell membrane scrambling with phosphatidylserine translocation to the erythrocyte surface. Triggers of eryptosis include Ca2+ entry and increase of cytosolic Ca2+ activity ([Ca2+] i), oxidative stress, ceramide, as well as activation of p38 kinase, casein kinase 1α and/or cyclooxygenase.

❖ **Efavirenz induced liver injury[12]**

Efavirenz is associated with a low rate of serum enzyme elevations during therapy and is an uncommon, but well-established cause of clinically apparent acute liver injury.

❖ **Efavirenz induced gynecomastia[13]**

Gynecomastia has been recognized as an adverse event to efavirenz. It is defined as breast enlargement due to benign proliferation of glandular tissue and an association with efavirenz-based ART has been demonstrated by several studies. The exact mechanism underlying

gynecomastia remains unclear, but there are several hypotheses most of which are to do with estrogen hormone activation.

As per the review of literature, few researchers date back states as follows, According to Mukesh Dada etal, Efavirenz (EFV) is widely prescribed as part of antiretroviral therapy (ART) in South Africa, and it is most frequently prescribed in fixed-dose combination (FDC) at a dose of 600 mg.[14] Efavirenz has been linked to early (two to six weeks) transient as well as late neuropsychiatric effects, which include increased risk of suicidal ideation, encephalopathy, catatonia, psychosis and ataxia. All of these have been directly linked to EFV toxicity.[15] EFV has shown predominant effect on Neuropsychiatric effects.

Mahony etal,[16] Efavirenz induces depressive-like behavior, increased stress response and changes in the immune response in rats. Adverse central nervous system side effects such as headache, dizziness, insomnia, fatigue, severe depression and suicidal ideation are noted in patients receiving efavirenz.

Segamwenge stated that Acute Liver Failure among Patients on Efavirenz-Based Antiretroviral Therapy. Hepatotoxicity of Efavirenz is not as rare as previously described in the literature and does actually present with fatal outcomes. The frequent monitoring of liver enzymes should be done at initiation of antiretroviral therapy and should continue throughout the treatment period.

A case study has been conducted by Robert etal[17] Hemolytic anemia associated with efavirenz.

Gynecomastia in HIV-positive adult men receiving efa,virenz-based antiretroviral therapy has been reported at Newlands clinic, Harare, Zimbabwe[18] Sandra etal states, Gynecomastia is known to occur in some men taking an efavirenz-based antiretroviral therapy (ART) regimen. However, the incidence and outcomes of gynecomastia are not known in Zimbabwe. We described the characteristics and outcomes of gynecomastia among male patients on an efavirenz-based ART regimen.

Fulminant Liver Failure after Varying Periods of Exposure to Efavirenz

Throughout a 6-month amount, four patients while not different celebrated risk factors for acute liver disease bestowed with symptomatic drug-induced liver injury with variable symptoms and outcomes. The pattern of liver injury was hepatocellular for all the four cases.

Liver biopsies were in deep trouble all the four cases and also the results showed a significant mixed inflammatory cell infiltrate with eosinophils. for 3 patients withdrawal of Efavirenz from their antiretroviral program was enough to revive aminopherase levels to traditional and crystal rectifier to improvement of clinical symptoms. For one patient his clinical course was characterised by sudden liver failure and unsteady episodes of internal organ neurological disease that ultimately resulted in his death.[19]

Neuropsychiatric Adverse Events

Depression, suicidal ideation, aggressive/impulsive behavior, paranoid reactions, manic reactions and (largely anecdotal reports of) psychosis-like behavior, suicide, and severe delusions are among the most frequently reported psychiatric symptoms associated with efavirenz treatment. Dizziness, insomnia, impaired concentration, irritability, nervousness, somnolence, abnormal dreams, and hallucinations are usually classified as CNS symptoms. The differentiation between CNS-related and psychiatric symptoms is rather arbitrary and has not been consistent; in general, psychiatric symptoms include some sort of behavioral or conscious component. This differentiation aims to reflect differences at the symptom level but does not necessarily imply distinct causal mechanisms.Recent studies have focused on the relevance of plasma concentrations and genetic predispositions as moderators of neuropsychiatric complications.[20]

Efavirenz-Induced Hypersensitivity

Efavirenz hypersensitivity reactions generally embrace cutaneal reactions that ar ascertained within the 1st two weeks of treatment, ar usually delicate to moderate while not general manifestation, and improve with continuing medical care. Previously, triple-crown decrease protocols are delineate in patients receiving efavirenz United Nations agency developed rash while not general symptoms, however these protocols were dole out over seven or fourteen days.[21]

Efavirenz-Associated Gynecomastia

Gynecomastia has been recognized as associate adverse event to efavirenz. it's outlined as breast enlargement thanks to benign proliferation of organ tissue associated an association with efavirenz-based ART has been incontestible by many studies. the precise mechanism underlying abnormal condition remains unclear, however there ar many hypotheses most of that ar to try to to with internal secretion|steroid|sex hormone} hormone activation [22].Besides efavirenz, alternative medication have additionally been related to abnormal

condition. though benign, abnormal condition will cause vital psychological morbidity with potential negative implications on adherence to ART. Lipomastia (pseudogynecomastia), a breast enlargement thanks to central fat, could occur as a part of a fat distribution syndrome that has been related to highly active antiretroviral therapy regimens and several other unhealthful mechanisms are advocated in its development. Here we tend to report associate empiric longitudinal study of 5 patients diagnosed of abnormal condition related to efavirenz-based highly active antiretroviral therapy regimens[23].All cases reached triple-crown immunological and virologic responses to highly active antiretroviral therapy. The delay of look of abnormal condition from the start of highly active antiretroviral therapy ranged between four to fifteen months. altogether 5 cases, abnormal condition regressed when efavirenz withdrawal (mean amount of five months). In summary, we predict that highly active antiretroviral therapy induced abnormal condition ought to be suspected in HIV patients receiving efavirenz-containing regimens. Though pathological process is unclear, this study and a review of nation literature implicates 2 potential mechanisms: (a) immune restoration processes and (b) efavirenz mediate estradiol-like effects.[24]

Haemolytic Anaemia Related To Efavirenz

The polymerase matter efavirenz used for the treatment of human immunological disorder virus (HIV)-1 infection, triggers self-destructive necrobiosis or caspase-mediated cell death, a sway partly thanks to interference with mitochondrial potential. aspect effects of efavirenz embrace anemia.[25] Causes of anemia embrace accelerated clearance of current erythrocytes. albeit lacking mitochondria, blood cells could enter self-destructive erythrocyte death or eryptosis, that is characterised by cell shrinkage and semipermeable membrane scrambling with phosphatidylserine translocation to the blood cell surface. Triggers of eryptosis embrace Ca2+ entry and increase of cytosolic Ca2+ activity ([Ca2+]i), aerobic stress, ceramide, furthermore as activation of p38 enzyme, casein enzyme enzyme and/or enzyme.[26]

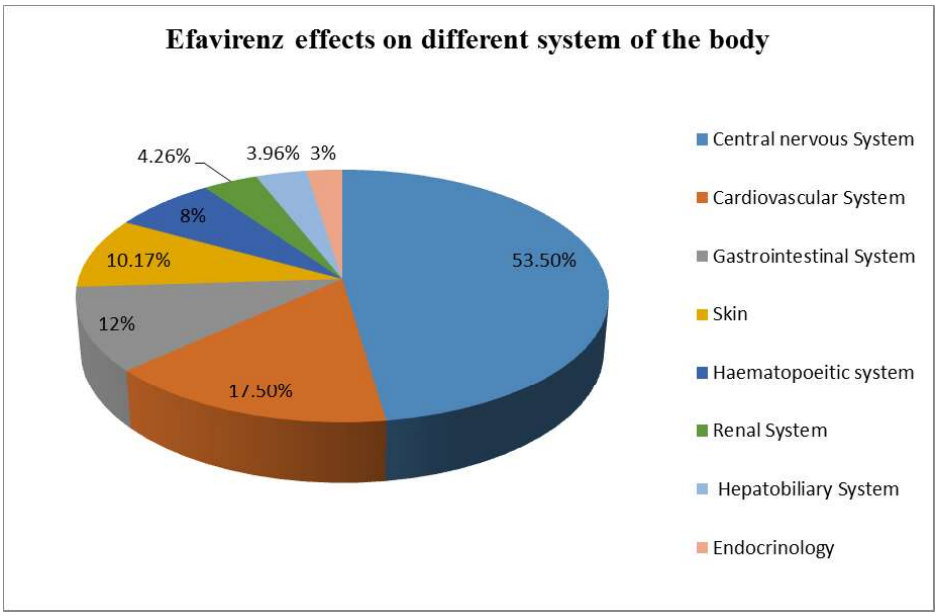

Figure 1: Efavirenz effects on various organ systems in our body.

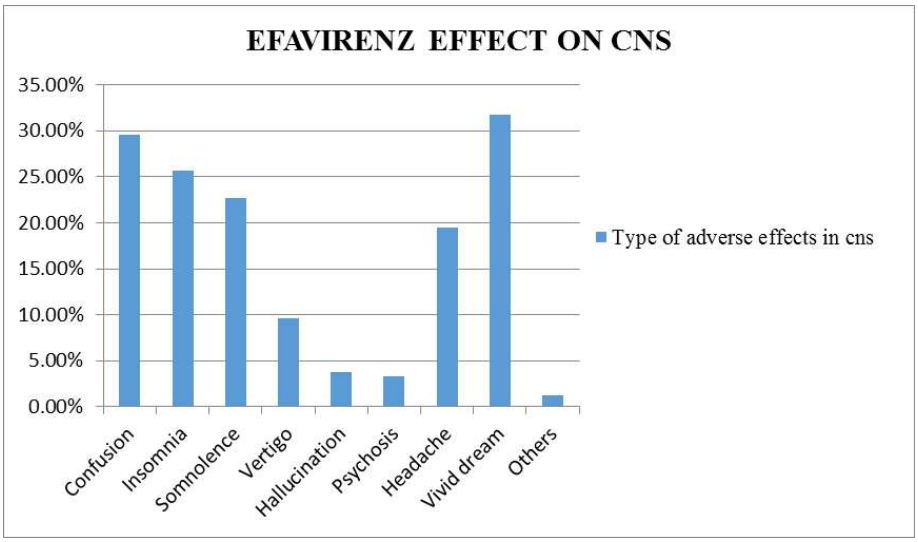

Figure 2: EFAVIRENZ EFFECTS ON CNS.

DISCUSSION

Hepatotoxicity of antiretroviral drugs is one of the more serious and life-threatening complications of antiretroviral drugs. Non-nucleoside reverse transcriptase inhibitors (NNRTIs) are the drugs most commonly implicated in hepatotoxicity yet these drugs are frequently used as part of the triple combination first-line ART regimen.[27] The greatest risk of NNRTI-associated severe hepatotoxicity are observed in patients taking Nevirapine, those with hepatitis B or C coinfection, and those co-administered protease inhibitors.[28] Severe hepatotoxicity is relatively uncommon among non-hepatitis C infected individuals and those not receiving protease inhibitor therapy.[29] Studies looking at grade 3-4[30] hepatotoxicity of

ART reported that most patients were asymptomatic and no deaths were due to liver-related events.[29,30,31] Fulminant hepatic failure due to Efavirenz leading to death is rare. Only one death due to fulminant liver failure after starting Efavirenz-based ART has been reported in the literature and two case reports of Efavirenz induced liver failure which required liver transplantation with good outcomes have been reported.[32,33,34] Hepatotoxicity due to Efavirenz has been described to occur between 100 days and 168 days (14 to 24 weeks).[28,35] It is possible that both immune mediated mechanisms and intrinsic toxic effects of the Efavirenz all had a role in the hepatotoxicity in patients.

CONCLUSION

Hepatotoxicity of Efavirenz is not as rare as previously described in the literature and does actually present with fatal outcomes. The greatest risk of Neuropsychiatric effects associated with Efavirenz. Fulminant hepatic failure due to Efavirenz leading to death is rare. The key message to note is that frequent monitoring of liver enzymes should be done at initiation of antiretroviral therapy and should continue throughout the treatment period. The use of psychophysiological measures in the assessment process enables the quantification of treatment-related side effects with high temporal solution, comparative measurements, and sensitivity toward subclinical effects, and thus provides a new source of information.

REFERENCES

1. Crum NF, Riffenburgh RH, Wegner S, et al. Comparisons of causes of death and mortality rates among HIV-infected persons: analysis of the pre-, early, and late HAART (highly active antiretroviral therapy) eras, J Acquir Immune Defic Syndr, 2006; 41: 194-200.
2. Efavirenz - Drug bank McDonagh EM, Lau JL, Alvarellos ML, Altman RB, Klein TE. PharmGKB summary: Efavirenz pathway, pharmacokinetics. Pharmacogenet Genomics, 2015 Jul; 25(7): 363-76. [PMC free article] [PubMed]
3. Robarge JD, Metzger IF, Lu J, Thong N, Skaar TC, Desta Z, Bies RR. Population Pharmacokinetic Modeling To Estimate the Contributions of Genetic and Nongenetic Factors to Efavirenz Disposition. Antimicrob Agents Chemother, 2017 Jan; 61(1). [PMC free article] [PubMed]
4. Maggiolo F. Efavirenz: a decade of clinical experience in the treatment of HIV. J Antimicrob Chemother, 2009 Nov; 64(5): 910-28.

5. ENCORE1 Study Group. Carey D, Puls R, Amin J, Losso M, Phanupak P, Foulkes S, Mohapi L, Crabtree-Ramirez B, Jessen H, Kumar S, Winston A, Lee MP, Belloso W, Cooper DA, Emery S. Efficacy and safety of efavirenz 400 mg daily versus 600 mg daily: 96-week data from the randomised, double-blind, placebo-controlled, non-inferiority ENCORE1 study. Lancet Infect Dis., 2015 Jul; 15(7): 793-802. [PubMed]
6. Rice DP, Faragon JJ, Banks S, Chirch LM. HIV/HCV Antiviral Drug Interactions in the Era of Direct-acting Antivirals. J Clin Transl Hepatol, 2016 Sep 28; 4(3): 234-240.
7. Adkins JC, Noble S. Efavirenz. Drugs., 1998 Dec; 56(6): 1055-64. discussion 1065-6. doi: 10.2165/00003495-199856060-0001
8. Neuropsychiatric Complications of Efavirenz Therapy: Suggestions for a New Research Paradigm Stefan Sütterlin Dipl.-Psych.Claus Vögele Ph.D.Siegfried Gauggel Ph.D., 2010.
9. Phillips EJ, Kuriakose B, Knowles SR. Efavirenzinduced skin eruption and successful desensitization. Ann Pharmacotherapy, 2002 Mar; 36(3): 430-2.
10. Freercks RJ, Mehta U, Stead DF, Meintjes GA. Haemolytic anaemia associated with efavirenz. *AIDS,* 2006; 20: 1212–1213.
11. Mechanistic insights into antiretroviral drug-induced liver injury Jamie N. Pillaye, Mohlopheni J. Marakalala, Nonhlanhla Khumalo, Wendy Spearman, Hlumani Ndlovu, 2008.
12. Braustein GD. Gynecomastia. N Engl J Med, 2007; 357: 1229–1237.
13. https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5843229/
14. Variava E, Sigauke F, Muchichwa P, Maartens G, Martinson N. *2016 Efavirenz toxicity manifesting as cerebellar ataxia: Case series from South Africa. Poster 963 CROI*. Dept. of Internal Medicine, Klerksdorp Tshepong Hospital Complex and University of the Witwatersrand, Klerksdorp, South Africa, 2016.
15. Efavirenz induces depressive-like behaviour, increased stress response and changes in the immune response in rats Siobhain M O'Mahony[1], Aye-Mu Myint, Harry Steinbusch, Brian E Leonard.
16. https://pubmed.ncbi.nlm.nih.gov/16691081/
17. https://bmcinfectdis.biomedcentral.com/articles/10.1186/s12879-019-4332-5
18. UNAIDS. AIDS epidemic update: December, 2007. [Accessed June 11, 2009]. Available at: http://data.unaids.org/pub/EPISlides/2007/2007_epiupdate_en.pdf.
19. Neuropsychiatric Complications of Efavirenz Therapy: Suggestions for a New Research Paradigm Stefan Sütterlin Dipl.-Psych., Siegfried Gauggel Ph.D.

20. Department of Health and Human Services. Panel on Antiretroviral Guidelines for Adult and Adolescents. *Guidelines for the use of antiretroviral agents in HIV-1-infected adults and adolescents,* 2008 Jan 29. [Accessed March 27, 2008].
21. Antiretroviral therapy for HIV infection in adults and adolescents. *Recommendations for a public health approach RW.* 2006. [Accessed March 27, 2008]. Available at: http://www.who.int/hiv/pub/guidelines/artadultguidelines.pdf.
22. UNAIDS. AIDS epidemic update, December 2007. [Accessed April 1, 2008.2007]. Available at http://data.unaids.org/pub/EPISlides/2007/2007_epiupdate_en.pdf.
23. Hauptfleisch MPK, Moore DP, Rodda JL. Efavirenz as a cause of ataxia in children. *SAMJ,* 2015; 105(11): 897–898. https://doi.org/10.7196/SAMJ.2015.v105i11.9451 [PubMed] [Google Scholar]
24. Desta Z, Saussele T, Ward B, et al. Impact of CYP2B6 polymorphism on hepatic efavirenz metabolism in vitro. *Pharmacogenomics*, 2007; 8(6): 547–558. https://doi.org/10.2217/14622416.8.6.547 [PubMed]
25. Nyakutira C, Röshammar D, Chigutsa E, et al. High prevalence of the CYP2B6 516G→T (*6) variant and effect on the population pharmacokinetics of efavirenz in HIV/AIDS outpatients in Zimbabwe. *Eur J Clin Pharmacol*, 2008; 64(4): 357–365. https://doi.org/10.1007/s00228-007-0412-3.
26. Mbuagbaw L., Mursleen S., Irlam J. H., Spaulding A. B., Rutherford G. W., Siegfried N. Efavirenz or nevirapine in three-drug combination therapy with two nucleoside or nucleotide-reverse transcriptase inhibitors for initial treatment of HIV infection in antiretroviral-naïve individuals, 2016; 2016(12). doi: 10.1002/14651858.CD004246.pub4. CD004246 [PMC free article] [PubMed] [CrossRef] [Google Scholar]
27. Sulkowski M. S., Thomas D. L., Mehta S. H., Chaisson R. E., Moore R. D. Hepatotoxicity associated with nevirapine or efavirenz-containing antiretroviral therapy: role of hepatitis C and B infections, 2002; 35(1): 182–189. doi: 10.1053/jhep.2002.30319. [PubMed] [CrossRef] [Google Scholar]
28. Sulkowski M. S., Thomas D. L., Chaisson R. E., Moore R. D. Hepatotoxicity associated with antiretroviral therapy in adults infected with human immunodeficiency virus and the role of hepatitis C or B virus infection, 2000; 283(1): 74–80. doi: 10.1001/jama.283.1.74.
29. U.S. Department of Health and Human Services. Division of AIDS (DAIDS) table for grading the severity of adult and pediatric adverse events.

30. Wit F. W. N. M., Weverling G. J., Weel J., Jurriaans S., Lange J. M. A. Incidence of and risk factors for severe hepatotoxicity associated with antiretroviral combination therapy, 2002; 186(1): 23–31. doi: 10.1086/341084. [PubMed]
31. Yimer G., Amogne W., Habtewold A., et al. High plasma efavirenz level and CYP2B66 are associated with efavirenz-based HAART-induced liver.

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COMPARATIVE EVALUATION OF EFFICACY OF HYDROXYCHLOROQUINE VERSUS METHOTREXATE IN PATIENTS WITH RHEUMATOID ARTHRITIS

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ABSTRACT

Aim: The aim of the study is to compare the efficacy of Methotrexate(MTX) versus hydroxychloroquine(HCQ) in patients with Rheumatoid arthritis. **Materials and methods:** This prospective study involves 100 patients who were diagnosed with rheumatoid arthritis and those who met the study criteria were enrolled for the study.Relevant data such as demographic details, past medical history,drug name were collected from out- patient profile form and by patient interview. **Statistical Analysis:** All the raw data was collected, entered in Excel sheet 2010 in windows 10 version, the statistical analysis was done in SPSS 16.0 Software by an appropriate statistical method, paired sample T-test for knowing the significant p-value <0.05(confidence interval 95%). **Results:** Among 100 patients, 50% of patients have received the drug, Hydroxychloroquine(HCQ) (400 mg/day p.o). 50% patients have received the drug, Methotrexate(MTX)(15 mg/week in divided doses p.o).The study shows that Hydroxychloroquine group having remission rate of 4%(2 patients out of 50) with the remission value <2.6 and shown to be significant with p value < 0.05. Methotrexate group being assessed having remission rate of 8% (4 patients out of 50) with the remission value of <2.6 which is found to be statistically significant (p<0.05) and there is significance decrease in Erythrocyte sedimentation rate(ESR). **Conclusion:** In our study we have observed that methotrexate was more effective in managing rheumatoid arthritis when compared to hydroxychloroquine by comparing the DAS28 score of both groups.Pain and inflammation also reduced by avoiding further exacerbation of condition.

INTRODUCTION

Rheumatoid arthritis (RA) is a chronic, systemic autoimmune disease that characterized by inflammatory arthritis resulting in pain, swelling, and stiffness in the joints, and may cause severe joint damage, loss of function and disability. The disease may last from months to a lifetime, and symptoms may improve and worsen over time.[1] It is estimated to have a prevalence of about 1% of the population in India. It affects roughly three times as many women as men. People tend to develop rheumatoid arthritis between 40 and 60 years of age, although it can arise at any age.[2]



Fig. 1: Healthy joint and Rheumatoid arthritis affected joint.

Rheumatoid arthritis (RA) if left untreated, ultimately causes functional impairment.[3] Non-steroidal anti-inflammatory drugs (NSAIDs) help only in symptomatic relief, corticosteroids have problems with chronic use and disease-modifying anti-rheumatic drug (DMARD)

have promised to reduce the disease activity in a better way. Many new drugs were introduced into the market for the treatment of rheumatoid arthritis; however, their efficacy is yet to be established. Studies are necessary in Indian patients to compare the efficacy with methotrexate, an age old DMARD. Present study was aimed to evaluate the efficacy of methotrexate and hydroxychloroquine, an anti-malarial drug orally in the management of rheumatoid arthritis.

MATERIALS AND METHODS

This is a prospective observational study which doesn't involve any type of invasive techniques and blood samples purely for the study purpose. All the parameters were taken by seeing the data and reports available from the patient which have already been suggested by the physician.The study was carried out for a period of six months i.e. from October,2019 to March,2020.This study was performed in outpatient department of an ORTHOPAEDIC HOSPITAL, Guntur, Andhra Pradesh.

Inclusion criteria

I. Patients suffering from Rheumatoid arthritis.
II. Both males and females
III. Patients using Methotrexate or Hydroxychloroquine
IV. Patients of age group 16 – 70 years who are willing to participate in the study

Exclusive criteria

1. Pregnant women and women who trying to get pregnant
2. Lactating women
3. Patients of age group below 16
4. Patients of age group above 70
5. Patients with active infective diseases
6. Patients with renal dysfunction and hepatic diseases

Patients who are diagnosed with rheumatoid arthritis and who met the study criteria were enrolled for the study. Relevant data such as demographic details ,disease history, diagnosis, drug name, dose, laboratory data are collected from medical records of the patient and by patient interview where ever required were collected and documented. A suitable data collection form is designed and used in the study. Monitored the clinical status of the patient for 3 months.

All the necessary data was collected from - Interviewing the out patients and their care takers, Patient's prescriptions, Laboratory findings & Any other relevant sources.Before and after the treatment the patient clinical condition was checked.Advised the patients regarding their medication regimen and importance of medication adherence.

DAS28 score: The DAS28 is a measure of disease activity in rheumatoid arthritis (RA). DAS stands for 'disease activity score' and the number 28 refers to the 28 joints that are examined in this assessment. The DAS28 is a composite score derived from 4 of these measures. This '28' version is a simplification of the original DAS score, which requires 52 joints to be counted. ESR value and global health assessment value ((from 0=best to 100=worst).

DAS28 is a composite formula. Four parameters are used to calculate a disease severity score:

A) Number of swollen joints out of a total of 28 specified joints

B) Number of tender joints out of a total of 28 specified joints

C) Erythrocyte sedimentation rate

D) Patient's interpretation of well being, with 0 being at their best and 100 their worst

a. High disease activity: DAS28 of >5.1
b. Moderate disease activity: DAS28 of >3.2 to 5.1
c. Low disease activity: DAS28 of 2.6–3.2
d. Remission: DAS28 of <2.6

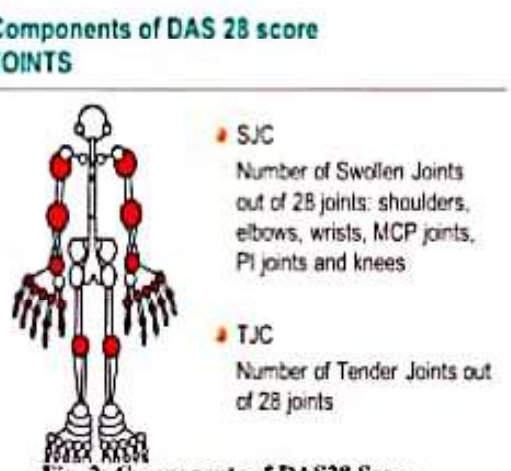

Fig. 2: Components of DAS28 Score.

- The 28 tender or swollen joint scores target the same joints (shoulders, elbows, wrists, metacarpophalangeal joints, proximal interphalangeal joints and the knees).
- These results are then fed into a complex mathematical formula to produce the overall disease activity score.
- The computation of the score is done through the following equation:[4]

$$DAS28 = 0.56 \times \sqrt{TJC28} + 0.28 \times \sqrt{SJC28} + 0.70 \times \ln(ESR) + 0.014 \times GH$$

Morisky medication adherence scale (MMAS) was used to calculate the medication compliance which contains 8 questionnaries and the information was gathered during patient counselling.This scale is used in our study to know the significance of medication adherence. Scores are summed and range from 0-8.

MMAS SCORING

- MMAS Score ≥8 : High Adherence

- MMAS Score 6 - < 8 : Medium Adherence
- MMAS Score < 6 : Low Adherence

If a patient scores higher on the scale, they are evaluated as more adherent. If they score lower on the scale, they are presumed to be struggling with low adherence.[8]

RESULTS

- Initially 110 patients were included in study, out of which 6 patients were excluded, because of no proper follow up due to lack of response and patient's disinterest towards the study; 4 patients were excluded because of infections that were developed for them during the study.
- Finally 100 patients were included in the study. Among 100 patients 50% patients i.e 50 patients were treated with Hydroxychloroquine and another 50% are treated with Methotrexate.
- Before and after patient clinical condition was checked, follow up was done for about 3 months.

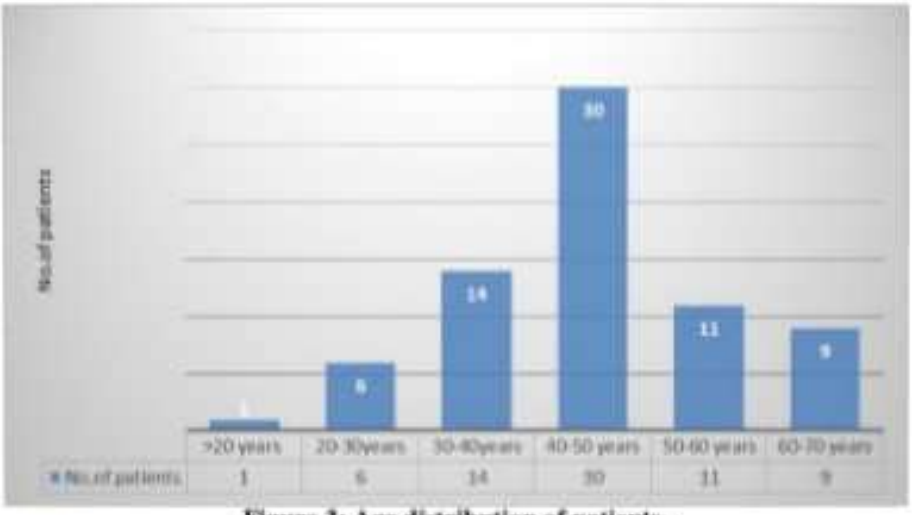

Figure 3: Age distribution of patients.

- Age distribution of patients - 39% of patients were between the age of 40-50 years, followed by 28% of patients were between the age of 50-60 years were 17% of patients were between the age of 30-40 years, 9% of patients were between age of 60-70 years were 6% of patients between the age of 20-30 years and 1% of patients below the age of 20years.
- According to our study, we have observed that most of the people who were affected with rheumatoid arthritis were from 40-50 age group.

Table 1: Gender differentiation.

GENDER	NUMBER OF PATIENTS	PERCENTAGE
Male	10	10%
Female	90	90%
Total	100	100

Table 1 shows the patient characteristics based on gender out of 100 patients 90 patients were females (90%) , 10 patients were males(10%).According to our study, we have observed that females are more prone to rheumatoid arthritis compared to males.

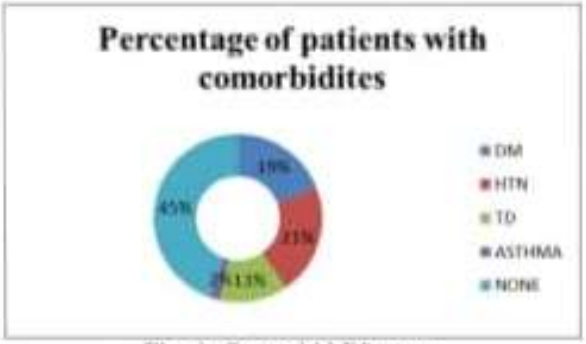

Fig. 4: Comorbid Diseases.

In our study most of the patients (45%) were with no comorbid diseases. And 19% of patients were reported with having DM, 21% of patients were reported with HTN, 13% patients reported with thyroid disease and 2% patients were reported with comorbidity of asthma.

Table 2: Rheumatoid Factor (RF) for patients using Hydroxychloroquine (HCQ) Drug.

Rheumatoid Factor (RF)	Before Treatment	After Treatment
0-30	39	31
30-60	6	12
60-90	1	5
>90	4	2

- Table 2 shows the pre and post treatment for rheumatoid factor of 50 patients using HCQ drug are included in the study. Out of 50 patients before treatment 39 patients were between RF value (0-30) and after treatment 31 patients were between the RF value (0-30). Before treatment 6 patients were between the RF value (30-60) and after treatment 12 patients were between RF value (30-60). Before treatment 1 patient had a RF value between (60-90) and after treatment 5 patients were between RF value (60-90). Before treatment 4 patients are with the RF >90 and after treatment 2 patients with RF>90.
- This shows that there was a significant increase in the RF value 30-60 and 60-90 patients when compared with pre treatment. And gradually there is a decrease in RF value 0-30 and >90 patients when compared with pre-treatment.

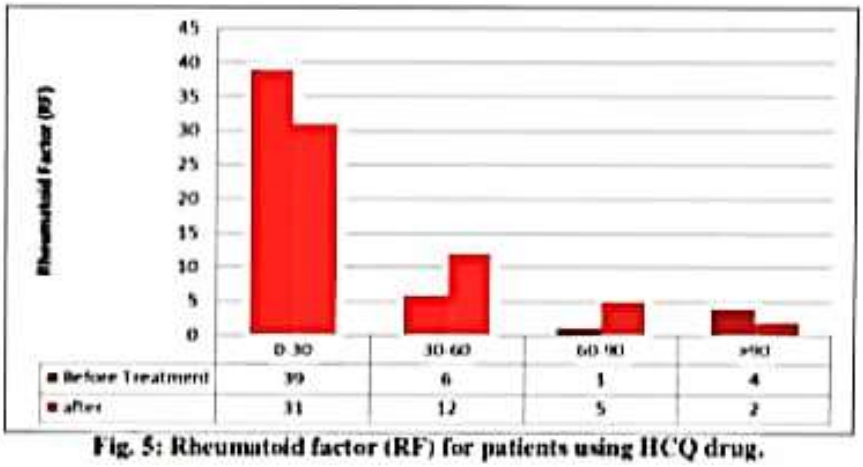

Fig. 5: Rheumatoid factor (RF) for patients using HCQ drug.

Table 3: ESR for patients using HCQ drug.

ESR (mm/hr)	Before treatment	After treatment
20-40	11	19
40-60	20	22
60-80	11	09
80-100	8	0

Table 5.5 shows the pre and post treatment ESR for 50 patients using HCQ drug. Out of 50 patients 11 patients were between 20-40 ESR and after treatment 19 patients were between 20-40 ESR. Before treatment 20 patients were between ESR 40-60 and after treatment 22 patients were between ESR 40-60. Before treatment 11 patients were between ESR 60-80 and after treatment 9 patients were between ESR 60- 80. Before treatment 8 patients were between ESR80-100 and after treatment there were no patient between ESR 80-100.

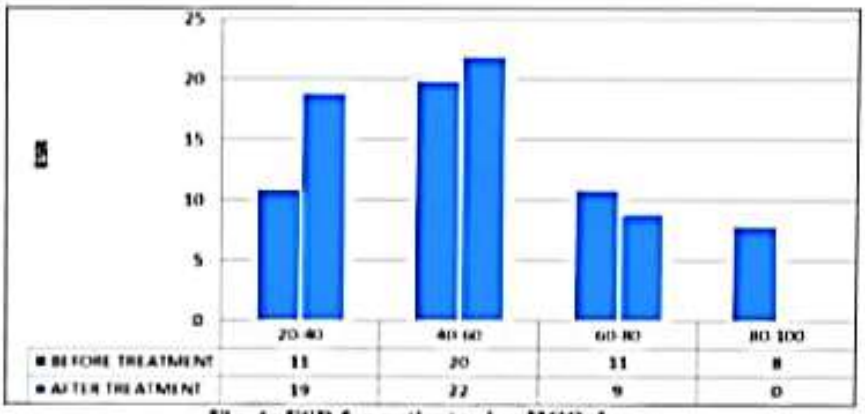

Fig. 6: ESR for patient using HCQ drug.

Table 4: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before treatment	58.0000	50	20.99563	2.96923
	After treatment	46.6000	50	16.45774	2.32748

Table 5: Paired Samples Correlations.

		N	Correlation	Sig.
Pair 1	Before treatment & After treatment	50	.706	.000

Table 6: Paired Samples Test.

		Paired Differences					t	Df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Before treatment – After treatment	11.40000	14.95026	2.11429	7.15118	15.64882	5.392	49	.000

Table 4 shows that pre and post treatment ESR for HCQ drug of 50 patients included in the study, the mean of the pretreatment ESR for HCQ drug was 58.000±20.995 and the mean of ESR for HCQ drug after treatment was 46.600± 16.457. This shows that there was a significant decrease in ESR for HCQ i.e., 11.4 ± 4.538 in treatment of HCQ drug. There was a significant difference in reduction of pain having p-value <0.05(95% confidence interval). By conventional criteria this difference is considered to be statistically significant.

Table 7: DAS28 for patients Using HCQ drug.

DAS28 score	Before treatment	After Treatment
2.6	0	2
2.6-3.2	11	19
3.2-5.1	17	14
>5.1	22	15

Table 7- Shows the pre and post treatment DAS28 for patient using HCQ drug. Out of 50 patients after treatment there were 2 Patients at low disease activity score which include 28 joints. Before treatment 11 patients were between the das28 2.6-3.2 and after treatment 19 were between disease activity score between2.6-3.2. Before treatment there were 17 patients between DAS3.2-5.1and after treatment were 14 patients between DAS3.2-5.1. Before treatment 22 patients were high disease activity score >5.1 and after treatment there 15 patients were at high disease activity score >5.1.

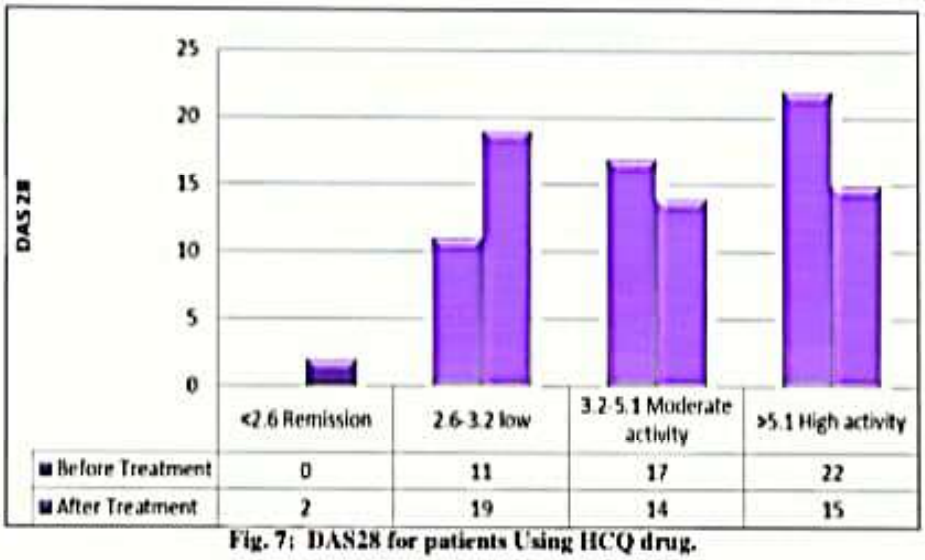

Fig. 7: DAS28 for patients Using HCQ drug.

Table 8: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	BEFORE TREATMENT	5.3960	50	1.37766	.19483
	AFTER TREATMENT	4.0860	50	1.26806	.17226

Table 12 shows the pre and post treatment ESR for patients using MTX drug. Out of 50 patients 12 patients were between ESR 20-40 and after treatment 22 patients were between ESR 20-40. Before treatment 13 patients were between ESR 40-60 and after treatment 22 patients were between ESR 40-60. Before treatment 17 patients were between ESR 60-80 and after treatment 5 patients were between ESR 60-80.Before treatment 8 patients were between ESR 80-100 and after treatment 1 were ESR 80-100.

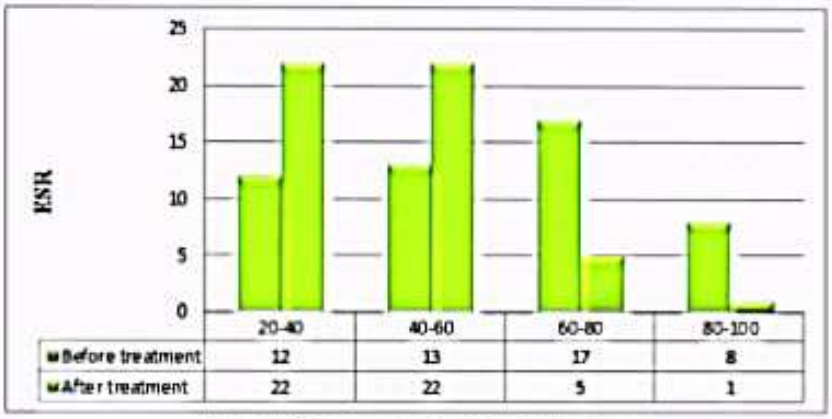

Fig. 9: ESR for patients using MTX drug.

Table 13: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before treatment	60.6000	50	19.93971	2.81990
	After treatment	41.3000	50	17.31491	2.44870

Table 14: Paired Samples Correlations.

		N	Correlation	Sig.
Pair 1	Before treatment & After treatment	50	.745	.000

Table 15: Paired Samples Test.

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Before treatment - After treatment	19.30000	13.51530	1.91135	15.45899	23.14101	10.098	49	.000

Table 15 shows that pre and post treatment ESR for MTX drug of 50 patients included in the study. The mean of pretreatment ESR for MTX was 60.600±19.93971 and the mean of ESR for MTX drug after treatment was 41.300±17.31491. This shows that there was a significant decrease in ESR for MTX drug i.e., 19.300±26248. There was a significant difference in reduction of ESR having p-value <0.05(95% confidence interval). By conventional criteria this difference is considered to be statistically significant.

Table 16: DAS28 for patients using MTX drug.

DAS28	Before treatment	After treatment
<2.6	0	4
2.6-3.2	12	17
3.2-5.1	17	20
>5.1	21	9

Table 16 shows the pre and post treatment DAS 28 for patients using MTX drug. Out of 50 patients after treatment 4 patients were between the DAS less than 2.6. Before treatment 12 patients were between the DAS 2.6-3.2 and after treatment 17 patients were between the DAS 2.6-3.2. Before treatment 17 patients were between DAS 3.2-5.1 and after treatment 20 patients were between DAS 3.2-5.1. Before treatment 21 patients were DAS >5.1 and after treatment 9 patients were DAS >5.1.

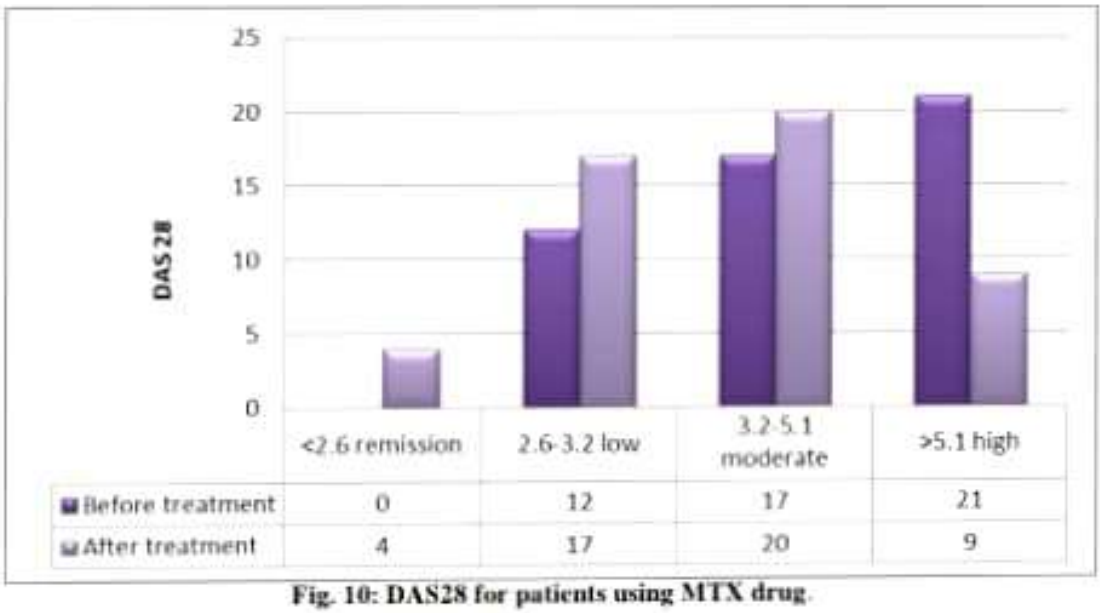

Fig. 10: DAS28 for patients using MTX drug.

Table 17: Paired Samples Statistics.

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before treatment	5.0260	50	1.27922	.18657
	After treatment	3.9320	50	1.10186	.16148

Table 18: Paired Samples Correlations.

		N	Correlation	Sig.
Pair 1	Before treatment & After treatment	50	.922	.000

Table 19: Paired sample test.

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Before treatment – After treatment	1.0940	.51564	.07292	.94746	1.24054	15.002	49	.000

Table 17 shows that pre and post treatment DAS28 for MTX drug of 50 patients included in the study. The mean of pre-treatment DAS 28 for MTX drug was 5.0260±1.27922 and the mean of DAS 28 for MTX after treatment was 3.9320±1.10186. This shows that there was a significant decrease in DAS 28 for MTX drug i.e., 1.0940±0.1350. There was a significant difference in reduction of DAS 28 for MTX drug having p-value <0.05(95% confidence interval). By conventional criteria this difference is considered to be statistically significant.

CO-Efficient of Variation (CV %)

It is a statistical measure of the dispersion of data points in a data series around the mean. Lower the CV value more consistent the results.

$$\text{CO-EFFICIENT OF VARIATION (CV)} = \frac{\text{SD X 100}}{\text{MEAN}}$$

Table 20: Co-efficient of variation for DAS28 of Hydroxychloroquine and Methotrexate.

Drugs	Standard deviation	Mean	Co-efficient of variation
HYDROXYCHLOROQUINE	1.26806	4.0860	31
MERHOTREXATE	1.10186	3.932	28

Medication adherence

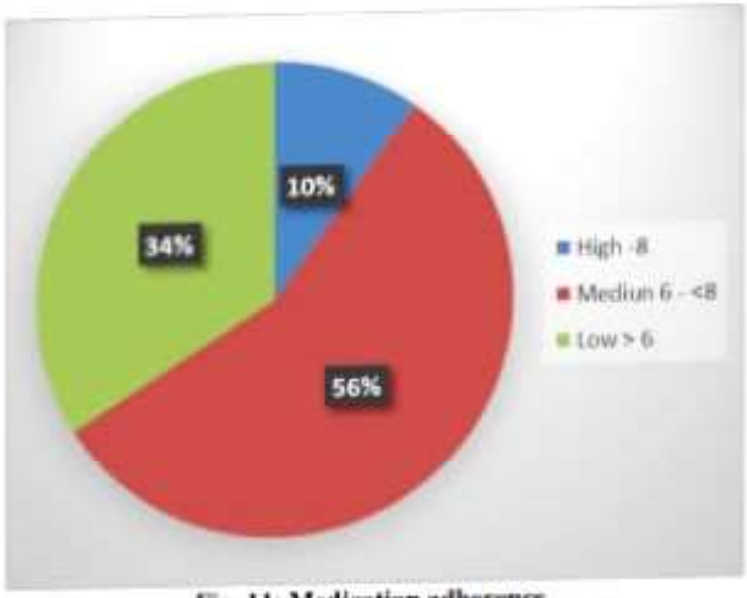

Fig. 11: Medication adherence.

- Out of 100 patients 50 patients are using MTX drug and 50 patients are using HCQ drug are included in study.
- In HCQ drug using patients out of 50 patients 17 patients are low adherent to medication and 28 patients are medium adherent to medication and 5 patients are high adherent to medication.
- In MTX drug using patients out of 50 patients 18 patients are low adherent to medication and 23 patients are medium adherent to medication and 13 patients are high adherent to medication.
- According to our study, we have observed that Patient medication adherence was moderately adherent.

DISCUSSION

With the disease modifying therapy currently available, complete remission of rheumatoid arthritis are almost rare. A variety of disease modifying anti rheumatic drugs(DMARDS) are available to control the clinical activity of rheumatoid arthritis, methotrexate an analogue of folic acid and hydroxychloroqine are the most commonly used drugs in monotherapy of rheumatoid arthritis.

Very little research was done before regarding the monotherapy of MTX and HCQ in RA patients making the comparison very difficult. We sought to design and conduct a study that would reflect clinical practice and yield results applicable to patients with RA and their physicians, therefore we allowed patients to continue taking NSAID's and other drugs prescribed to them.

Early Placebo controlled trails demonstrated that weekly low dosage methotrexate produced early symptomatic improvement in most RA patients.[5]

A double blind placebo controlled, crossover study of weekly pulse methotrexate dose(15mg), after 13 weeks of therapy patients receiving methotrexate showed greater improvement, judged by degree of joint swelling and tenderness, duration of morning stiffness and subjective assessment of clinical condition compared to those receiving placebo (p≤0.002).[6]

A 36 week randomised double-blind placebo controlled study done by HERA group regarding the usage of hydroxychloroqine of dose 400 mg in RA patients and it has been assessed by students "t" test. Over the course of study there was satisfactory significant improvement in the joint index (p=0.004,p=0.034), the pain index (p=0.007,p=0.001)and the physical index (p=0.020, p=0.011) with the group receiving hydroxychloroqine compared to placebo.[7]

We have done a prospective observational study with 100 patients in a time period of 6 months.Among 100 patients 50% patients i.e 50 patients were treated with Hydroxychloroquine and another 50% are treated with Methotrexate. All the patients are being diagnosed and assessed their condition by the various diagnosing tools like ESR, DAS 28, RF, and their adherence is being checked using Morisky's adherence scale.Both the group has been assessed by DAS28 score to check the remission rate after a 3 months period. Hydroxychloroquine group having remission rate of 4% (2 patients out of 50) with the value <2.6, and having standard mean deviation of 0.89881 and are shown to be significant with p value < 0.05. Another 50%, Methotrexate group being assessed having remission rate of 8% (4 patients out of 50) with the remission value of <2.6 and having standard mean deviation of 0.07292 and are shown to be significant with p value <0.05.

Morisky medication adherence scale was used to calculate the medication adherence.In our study the

patients having moderate compliance at an initial stage due to lack of knowledge about the comorbidities and usage of medications later, the compliance overcome by patients counselling about the medication use and their disease condition. And finally future studies are needed to provide information on the potential benefits of mono therapy of DMARD's like MTX and HCQ and others with respect to therapy.

CONCLUSION

Hydroxychloroqine and Methotrexate are the most commonly used primary drug of choice in the treatment of rheumatoid arthritis patients. In our 6 months study, it was Observed that Methotrexate is comparatively more efficacious than Hydroxychloroquine. Remission rate by methotrexate is slightly edge more over with hydroxychloroqine. However further work is required to determine the use of oral methotrexate and hydroxychloroqine in the therapeutic management of rheumatoid arthritis. Medication adherence among the patients was mostly moderate. Measures are to be taken to improve the medication adherence.Clinical pharmacist have major role in improving medication adherence through proper patient counseling.

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REFERENCES

1. Krati Chauhan; Jagmohan S. Jandu; Amandeep Goyal; Pankaj Bansal; Mohammed A. Al-Dhahir - Rheumatoid Arthritis.
2. Shichikawa K, Inoue K, Hirota S, et al. Changes in the incidence and prevalence of rheumatoid arthritis in Kamitonda, Wakayama, Japan, 1965–1996. Ann Rheum Dis., 1999; 58(12): 751–756.
3. Lipsky PE. Rheumatoid arthritis. In: Braunwald E,Fauci AS, Kasper DL, Hauser SL, Longo DL,Jameson JL, editors. Harrison's principles of internal medicine. 15th ed. McGraw Hill: New York, 2003; 1928-37.
4. DAS 28 -Disease activity score calculator for rheumatoid arthritis https://www.rheumakit.com /en/calculators/das28.
5. Early Placebo controlled trails demonstrated that weekly low dosage methotrexate produced early symptomatic improvement in most RA patients.
6. Bernard Bannwarth, Laurence Labat, Yola Moride, Thierry Schaeverbeke, 1994.
7. Clinical and Immunologic Effects in a Randomized, Double-Blind StudyWeekly Pulse Methotrexate in Rheumatoid Arthritis; PETER A. ANDERSEN, M.D., et al., 1985.
8. Treatment of rheumatoid arthritis with methotrexate alone, sulfasalazine and hydroxychloroquine, or a combination of all three medications.s.
9. James R O'Dell, Claire E Haire et al., New England Journal of Medicine, 1996; 33(20): 1287-1291.
10. Morisky DE, Ang A, Krousel-Wood M, Ward HJ. Predictive validity of a medication adherence measure in an outpatient setting. J Clin Hypertens (Greenwich), 2008; 10(5): 348–354.

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OPTIMIZATION OF CULTURAL CONDITIONS TO ENHANCE BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15

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Keywords:

Streptomyces olivoverticillatus, Mangrove ecosystem, Bioactive compounds, Optimization

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ABSTRACT: The study was designed to optimize cultural conditions for enhanced production of bioactive compounds by *Streptomyces olivoverticillatus* VLK-15 isolated from mangrove ecosystem of South Coast of Andhra Pradesh, India. The soil dilution plate technique was employed for the isolation of actinobacteria using starch-casein agar (ISP-6) medium. The potent strain was identified as *Streptomyces olivoverticillatus* VLK-15 based on polyphasic taxonomy. The productivity of the strain was enhanced by amending the medium (ISP-2) with mannitol (0.5%), peptone (0.5%), and K_2HPO_4 (0.05%) adjusted to pH 7.0 and incubated with the culture at 30ºC for ninety-six hours. The secondary metabolites produced under optimized conditions exhibited high antimicrobial activity against Gram-positive and Gram-negative bacteria as well as fungi. Hence, it could be a possible source for novel and potential antimicrobial compounds.

INTRODUCTION: The mangrove ecosystem provides a unique environment to support the growth of different microorganisms which are involved in the production of potential secondary metabolites possessing pharmaceutical properties. Mangrove ecosystems are found in tropical and subtropical intertidal regions around the world [1]. Only 5% of microbes, including bacteria, fungi, and actinobacteria isolated from these ecosystems have been studied [2]. Among the microorganisms, marine actinobacteria produce unique and novel secondary metabolites, which play an extensive role in the pharmaceutical and medical industry due to their capacity to produce secondary metabolites.

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More than 10,000 bioactive compounds have been reported from actinobacteria [3], of which the major share (70%) is contributed by the genus *Streptomyces* alone [4]. There is an urgent need to access new actinobacterial species for the exploration of novel bioactive compounds to hit multi-drug-resistant pathogens. Hence, we focused on the isolation and identification of novel species as well as optimization of cultural conditions to enhance the bioactive compounds from rare actinobacterial strains from the mangrove ecosystem of the south coast of Andhra Pradesh.

MATERIALS AND METHODS:

Microorganism: The strain VLK-15 was isolated from the mangrove ecosystem of the south coastal region of Andhra Pradesh, India, and identified as *Streptomyces olivoverticillatus* VLK-15 by using polyphasic taxonomy including molecular analysis (16S r RNA). The gene sequence of the strain VLK-15 is deposited in the GenBank database of NCBI with the accession number MG309759 [5].

Antimicrobial Activity: The antimicrobial activity of strain VLK-15 was tested against bacteria such as *Bacillus megaterium* (NCIM2187), *Staphylococcus aureus* (MTCC 3160), *Pseudomonas aeruginosa* (ATCC 9027), *Escherichia coli* (ATCC 35218) and *Candida albicans* (ATCC 10231) by agar well diffusion assay [6]. Nutrient agar and Czapek-Dox agar media were used for culturing the test bacteria and fungi, respectively. The culture filtrate was extracted with ethyl acetate, and the solvent extract (50 µL) was added to the well made on agar media seeded with test microbe. Wells with only ethyl acetate served as control. The diameter of the inhibition zone was measured after 24 h of incubation at 30°C for bacteria and 24-72 h for yeast.

Optimization of Cultural Conditions for Improved Production of Bioactive Compounds: An attempt was made to optimize the culture conditions such as pH, temperature, carbon sources, nitrogen sources and minerals to enhance the bioactive metabolite production by *Streptomyces olivoverticillatus* VLK-15. The secondary metabolite production by the strain was studied at regular intervals up to eight days. Fermentation was carried out in 250-mL Erlenmeyer flasks with constant shaking at 120 rpm.

The strain was cultured on ten different media such as ISP-1 to ISP-7 and Czapek-Dox, nutrient broth and yeast-starch broth to determine the ideal conditions for the maximum production of bioactive compounds. The bioactive metabolite production in each medium is evaluated [6].

The effect of initial pH on the bioactive compounds was determined by altering pH of the production medium from 4 to 10. Similarly, the optimal temperature for the production of bioactive compounds was evaluated by inoculating the strain at temperatures ranging from 25 to 45°C, while maintaining all other conditions at optimum levels [7].

The influence of carbon sources on bioactive compound production was determined by adding different carbon sources to the fermentation medium, such as dextrose, galactose, maltose, sucrose, mannitol, lactose, starch, cellulose, fructose, and xylose, each at a concentration of 0.4% (w/v) [8]. Impact of different concentrations of the best carbon source (0.1-1% w/v) on bioactive compound production was also studied.

The effect of various nitrogen sources such as ammonium nitrate, peptone, tryptophan, L-proline, tyrosine, urea, yeast extract, histidine, cysteine, and alanine were studied by supplementing nitrogen source (0.4%) to the medium. Further, the best nitrogen source (0.1–1% w/v) for improved production of bioactive metabolites was also recorded [9]. The optimized medium was supplemented with mineral salts such as KH_2PO_4, K_2HPO_4, $FeSO_4$, $ZnSO_4$ and $MgSO_4$ to measure the impact of minerals on the production of bioactive metabolites [10]. The metabolites produced by the strain grown under optimal conditions were tested for antimicrobial activity.

Statistical Analysis: Data obtained on antimicrobial activity under different culture conditions are statistically analyzed with one-way analysis of variance (ANOVA).

RESULTS AND DISCUSSION:

Influence of Incubation Time on Biomass and Antimicrobial Activity: The growth pattern and bioactive compound production by the strain VLK-15 were studied at regular intervals up to eight days. The stationary phase of *Streptomyces olivoverticillatus* VLK-15 continued from 72 h to 120 h of incubation **Fig. 1**. The bioactive compounds obtained from the four-day-old culture exhibited good antimicrobial activity against the test microorganisms. This is in conformity with the earlier reports that the extracts of 4-day old cultures of *Streptomyces lavendulocolor* VHB-9 [11], *S. cheonanensis* VUK-A [12], and *S. tendae* TK-VL_333 [13] exhibited high antimicrobial activity against the test microorganisms.

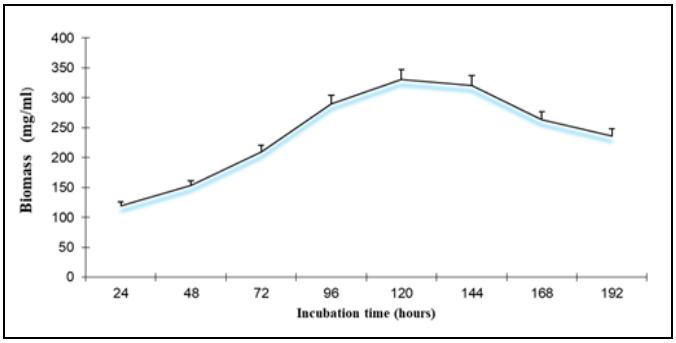

FIG. 1: GROWTH CURVE OF *STREPTOMYCES OLIVO-VERTICILLATUS* VLK-15. DATA ARE STATISTICALLY ANALYZED AND FOUND TO BE SIGNIFICANT AT 5%

Effect of Culture Media on Antimicrobial Compounds: Bioactive compound production by the strain VLK-15 was studied in different culture media **Fig. 2**. Among the ten media tested, YMD (ISP-2) supported high levels of bioactive metabolites followed by starch-casein (ISP-6) and nutrient broth.YMD broth for *Streptomyces violaceoruber* VLK-4 [6] and Czapek- Dox broth for *Streptomyces* sp. MNK-7 [14] supported high rates of antibiotic production.

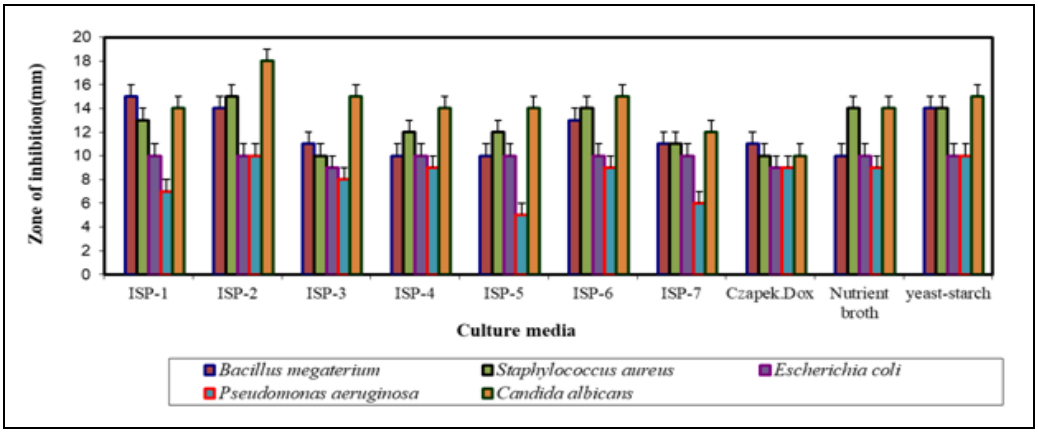

FIG. 2: EFFECT OF DIFFERENT CULTURE MEDIA ON BIOACTIVE COMPOUNDS PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

Impact of pH and Temperature on Bioactive Compounds: Maximum antimicrobial production by the strain VLK-15 was found at pH 7 **Fig. 3**. Bioactive metabolites obtained from *Streptomyces lavendulocolor* VHB-9 [11], *S. cheonanensis* VUK-A [12], *S. violaceoruber* VLK-4 [6,] and *Streptomyces* sp. VITSVK9 [7] exhibited good antimicrobial activity when grown at pH 7.

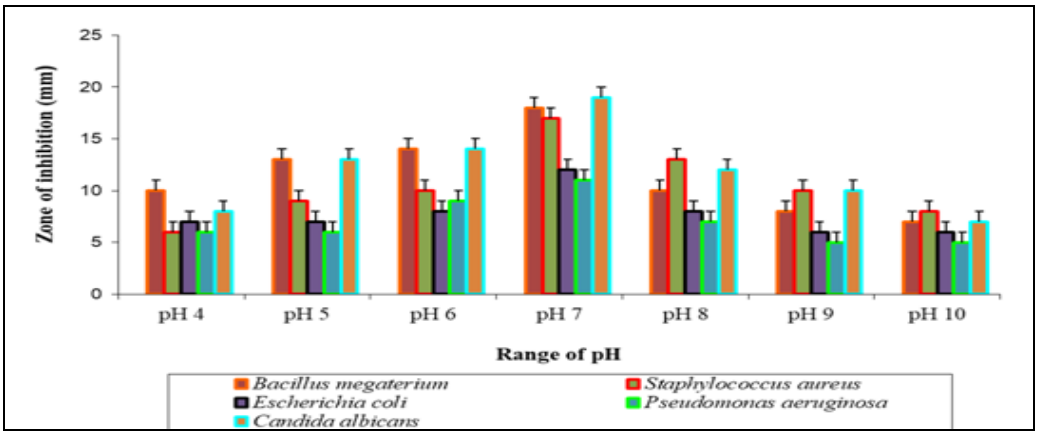

FIG. 3: INFLUENCE OF pH ON SECONDARY METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

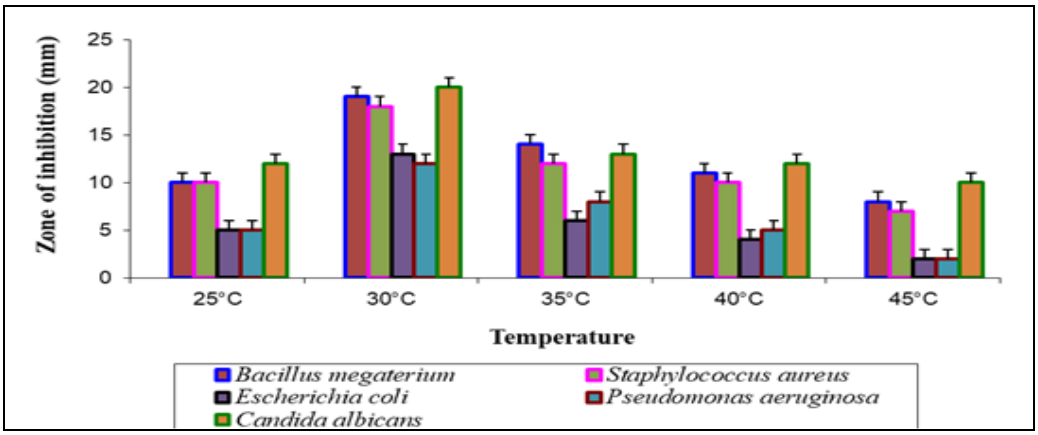

FIG. 4: EFFECT OF TEMPERATURE ON BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

The production of antimicrobial compounds by the strain VLK-15 increased with rise in the incubation temperature from 25 -30°C **Fig. 4**. Further increase in temperature (above 30°C) resulted in the decline in the production of bioactive metabolites. Earlier reports on *Streptomyces* species also indicated that culturing the strains at 30°C could lead to high levels of bioactive metabolites [11, 12, 15, 16].

Impact of Carbon and Nitrogen Sources on Antimicrobial Activity: The effect of carbon sources on production of secondary metabolites by the strain VLK-15 is shown in **Fig. 5**. Significant production of bioactive compounds was found in mannitol amended medium (ISP-2) followed by dextrose, lactose and starch. As mannitol appeared as the most favored carbon source for secondary metabolite production, different concentrations of mannitol (0.1-1%) were added to find the optimal concentration. Mannitol @ 0.5% exhibited high yields of bioactive compounds **Fig. 6**. Mannitol was reported as a best carbon source to enhance the bioactive metabolite production by *Streptomyces violaceoruber* VLK-4 [6], whereas *S. lavendulocolor* VHB-9 [11] and *Streptomyces cheonanensis* VUK-A [12] utilized lactose as carbon source for secondary metabolite production.

Different nitrogen sources were tested for their impact on bioactive metabolite production by the strain VLK-15. Among the nitrogen sources used, peptone followed by yeast extract and tryptophan was found to be the best nitrogen source for the production of bioactive compounds. As Peptone improved the bioactive compound production by the strain VLK-15, the influence of various concentrations of peptone was assessed, and 0.5% was found to be good for the production of bioactive metabolites **Fig. 8**. These results are comparable with *Streptomyces lavendulocolor* VHB-9 [11] and *Streptomyces cheonanensis* VUK-A [12].

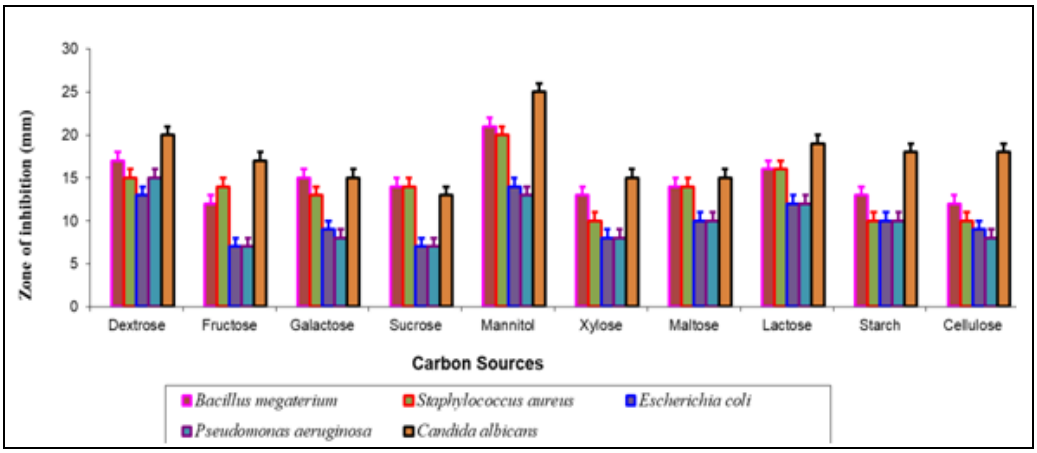

FIG. 5: EFFECT OF DIFFERENT CARBON SOURCES ON BIOACTIVE METABOLITES PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

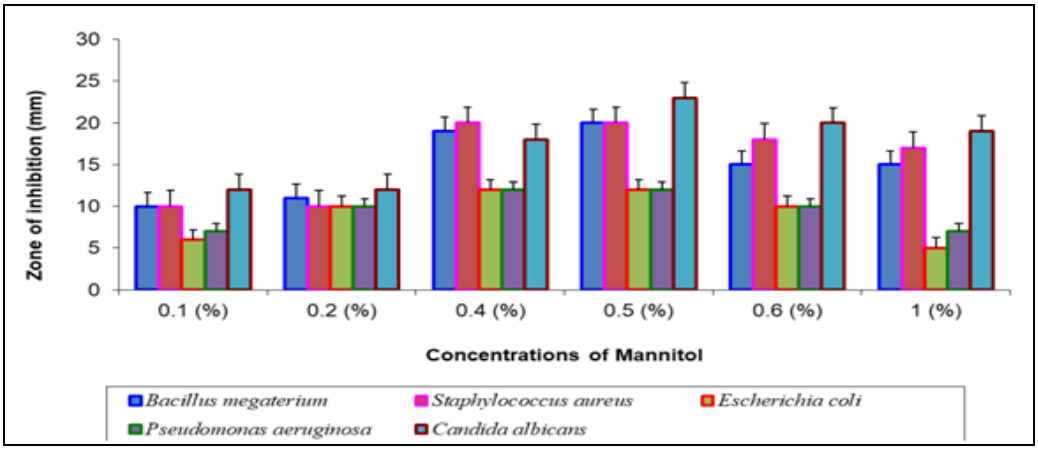

FIG. 6: IMPACT OF DIFFERENT CONCENTRATIONS OF MANNITOL ON BIOACTIVE METABOLITES PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

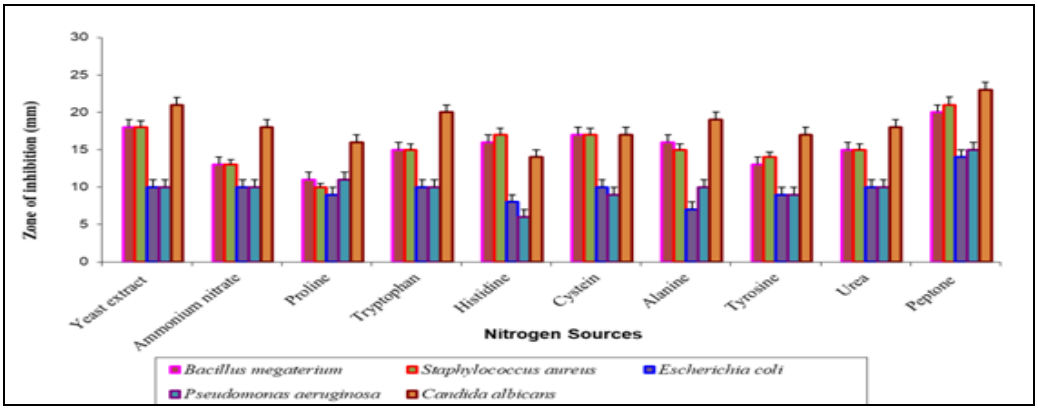

FIG. 7: EFFECT OF NITROGEN SOURCES ON SECONDARY METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

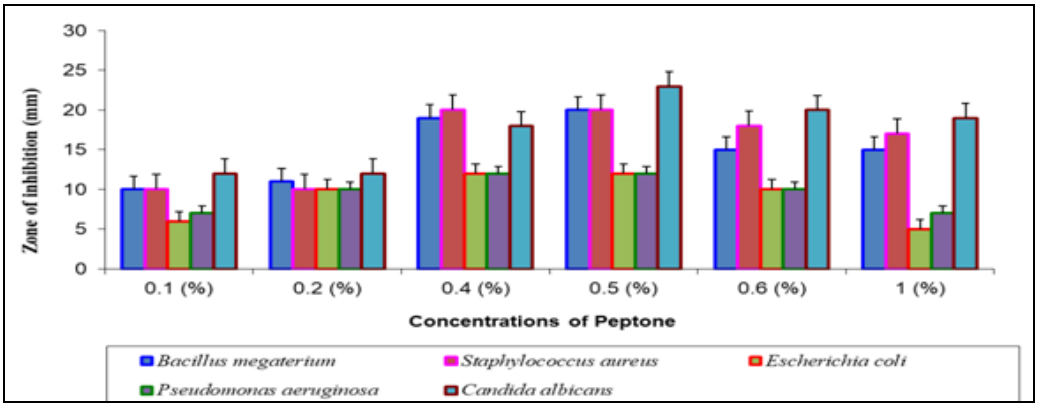

FIG. 8: INFLUENCE OF DIFFERENT CONCENTRATIONS OF PEPTONE ON BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

Influence of Minerals on Production of Antimicrobial Compounds: The effect of minerals on secondary metabolite production by the strain VLK-15 is shown in **Fig. 9**. Among the minerals tested, K_2HPO_4 followed by $FeSO_4$, KH_2PO_4 and $ZnSO_4$ supported a high yield of bioactive compound production. Similar results were found for *Streptomyces lavendulocolor* VHB-9 [11], *S. violaceoruber* VLK-4 [6,] and *Streptomyces cheonanensis* VUK-A [12].

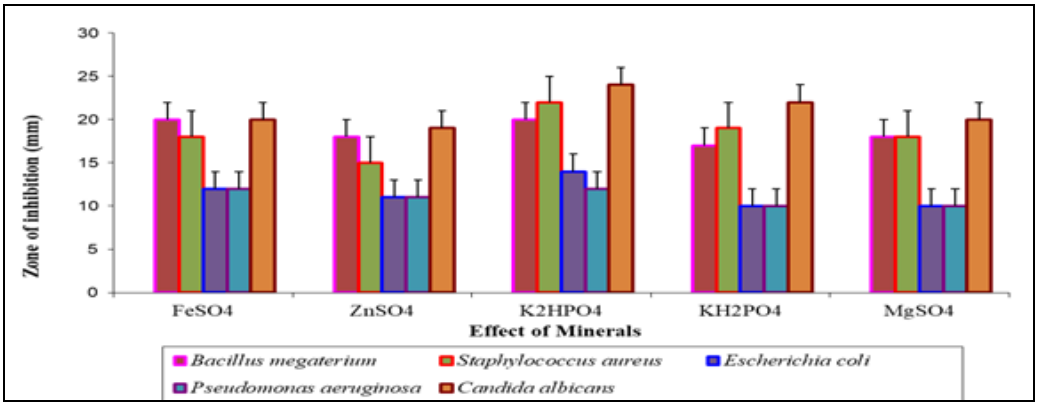

FIG. 9: IMPACT OF DIFFERENT MINERALS ON BIOACTIVE METABOLITE PRODUCTION BY *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15. Data are statistically analyzed and found to be significant at 5%.

The bioactive compounds obtained from optimized fermentation broth exhibited high antimicrobial activity against test bacteria and fungi **Table 1**. Among the bacteria tested, *Bacillus megaterium* was highly sensitive to the compounds produced by the strain VLK-15 followed by *Staphylococcus aureus*, *Pseudomonas aeruginosa* and *E. coli*. *Candida albicans* showed high sensitivity to the metabolites.

TABLE 1: ANTIMICROBIAL ACTIVITY OF ETHYL ACETATE EXTRACT OF *STREPTOMYCES OLIVOVERTICILLATUS* VLK-15 UNDER OPTIMIZED CONDITIONS

Test microorganisms	Zone of inhibition (mm)
Bacillus megaterium	32
Staphylococcus aureus	30
Pseudomonas aeruginosa	27
Escherichia coli	26
Candida albicans	30

CONCLUSION: In the present study a potent strain *Streptomyces olivoverticillatus* VLK-15 isolated from the mangrove ecosystem of the south coast of Andhra Pradesh, India, possessing antagonistic activity, revealed its potential to inhibit several pathogenic bacteria and fungi. Bioactive metabolite production was often influenced by various environmental factors, including nutrients (nitrogen and carbon source), temperature and pH. Maximum bioactive compound production by the strain was found when cultured on ISP-2 medium amended with mannitol @ 0.5% and peptone @ 0.5% adjusted to pH 7 for 96 hours at 30ºC. Further studies on purification, characterization, and identification of bioactive metabolites of *Streptomyces olivoverticillatus* VLK-15 are in progress. This is the first report on the optimization of culture conditions to enhance the bioactive metabolites by the strain *Streptomyces olivoverticillatus* VLK-15.

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CONFLICTS OF INTEREST: All authors declare that there are no conflicts of interest.

REFERENCES:

1. Ottoni CA, Amini MS, Alam I, Alzubaidy H, Mokhtar N, Archer JAC and Bajic VB: Soil and rhizosphere associated fungi in gray Mangroves (*Avicennia marina*) from the Red Sea-a metagenomic approach. Genom Prot Bioinform 2015; 13: 310-20.
2. Xu J: Bioactive natural products derived from mangrove-associated microbes. RSC Adv 2015; 5: 841-92.
3. Azman AS, Othman I, Fang CM, Chan KG, Goh BH and Lee LH: Antibacterial, anticancer and neuroprotective activities of rare Actinobacteria from mangrove forest soils. Indian J Microbiol 2017; 57: 177-87.
4. Zengler K, Parakar A and Keller M: New methods to access microbial diversity for small molecule discover," in Natural Products therapeutic Medicine, L. Zhang and A. L. Demain, Eds., 2005; 275–294. Humana Press, Totowa, NJ, USA.
5. Naragani K, Indupalli MD and Muvva V: Phylogenetic characterization of potential bioactive metabolites producing actinomycetes from mangrove sediments. Int J Pharm Sci & Res 2020; 11(12): 1000-05.
6. Krishna N, Munaganti RK and Muvva V: Optimization studies for enhanced bioactive metabolite production by *Streptomyces Violaceoruber* VLK-4 isolated from the south coast of Andhra Pradesh, India. Int J Pharm Sci Res 2014; 5: 4760-68.
7. Saurav K and Kannabiran K: Diversity and optimization of process parameters for the growth of Streptomyces VITSVK 9 sp. Isolation from Bay of Bengal. India J Nat Environ Sci 2010; 1: 56-65.
8. Elliah P, Srinivasulu B and Adinarayana K: Optimization studies on neomycin production by a mutant strain of *Streptomyces marinensis* in solid state fermentation process. Biochemistry 2000; 39: 529-34.
9. Kathiresan K, Balagurunathan R and Selvam MM: Fungicidal activity of marine actinomycetes against phytopathogenic fungi. Ind J Biotechnol 2005; 4: 271-6.
10. Kumar MR, Vijayalakshmi M, Saidulu K, Krishna N, kiranmayi MU, Reddy DK and Akkewar DM: Antimicrobial profile of *Arthrobacter kerguelensis* VL-RK_09 isolated from Mango Orchards. Brazilian Journal of Microbiology 2016; 47: 1030-38.
11. Bindu-Hima BSSN, Kumar MR, Vijayalakshmi M, Krishna N and Deepa IM: Optimization, isolation and characterization of bioactive compounds from *S. lavendulocolor* VHB-9. AJPClin Res 2018; 11: 361-68.
12. Kiranmayi MU, Sudhakar P, Krishna N and Vijayalakshmi M: Influence of cultural conditions for improved production of bioactive metabolites by *Streptomyces cheonanensis* VUK-A Isolated from Coringa Mangrove Ecosystem. Curr Tren Biotechnol Phar 2012; 6: 99-111.
13. Kavitha A and Vijayalakshmi M: Optimization and purification of L-asparaginase produced by *Streptomyces tendae* TK-VL_ 333. Z Naturforsch 2010; 65: 528-31.
14. Saha MR, Rifa FA, Islam MZ and Khondkar P: Optimization of conditions and in vitro antibacterial activity of secondary metabolite isolated from *Streptomyces* sp. MNK 7. J Appl Sci Res 2010; 6: 453-59.
15. Atta HM, Bahobail AS and El-Sehrawi MH: Studies on isolation, classification and phylogenetic characterization of antifungal substance produced by *Streptomyces albidoflavus*- 143. New York Science J 2011; 4: 40-53.
16. Ghosh U and Prasad B: Optimization of carbon, nitrogen sources and temperature for hyper growth of antibiotic producing strain *Streptomyces kanamyceticus* MTCC 324. The Bioscan 2010; 5: 157-58.

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Deterioration and Aggregation of A Few Heavy Metals in Agricultural Soils And Dicoccum of Industrial Zone

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ABSTRACT To assessing degree of adulteration and aggregation of soils as well as different parts of Dicoccum of the industrial area by heavy metals includes: cobalt (Co), Nickel (Ni), copper(Cu), the Iron (Fe), and total Manganese (Mn). Basic aim to this work is checking of whether pollution is occurred for metal of soils those are obtained by water from nearest river different activities occurring in industries includes the cultivating of dicoccum wheat. For this case sampling is distributed on total four stations of agricultural, surface as 0cm to 15cm along with depth as 15cm to 30cm are performed. Physicochemical analyses are performed. Different dosages, with the help of spectrometry by flame, are carried out over the samples of soil, various parts of wheat containing grains, leaves, roots as well as stems. Finally, values achieved revealed that decreasing metal pollution in soil, strength of heavy metals of soils along with plants of wheat. In some plots, average strength of Cu in soils surpass limit value and results are correlated from literature. The liquid effluents, industrial operations along with agricultural practices disproportionate may be in the beginning of aforementioned pollution by metal.

Keywords: Dicoccum wheat, Agriculture, Physico-chemical parameters, Soils, Pollution from Metals.

I. INTRODUCTION

In general heavy metals were present in the earth crust, by the activities performed by humans and naturally in agriculatural lands. In actual practice these wont undergoes biodegradation. These may present in soil as well as living organisms [1-2]. Mechanisms for absorption to trace elements which are present in roots of plants were not yet studied, maximum elements by plants takes free ionic species only[3]. Both metals like Cu, Zn get inside to leaf as Pb[4]. In solution, low strength for Zn contains pH as 7-8[5]. Degradation of organic matter are more crucial to soil-plant migration of Ni[6]. More than 10.00mg/kg MS, Ni become toxic for plants those are more sensitive greater to 50.00mg/kg MS to tolerant plants[7]. When compared with remaining elements, Cr consists of phyto avialability as very low [8-9]. Roots consists Cr^{+3}, Cr^{+6} is transferred for parts of aerial [8]. If it is a either basic or neutral Cr is acting as a more phyto available to that of acid in soil [9]. Average strength to Cr in soil is 50.00mg/kg MS [10]. In the range 0.01mg/kg – 5.00 mg/kg MS, Ni is noted as a required element. It preserves necrosis by association of urease, which is more responsible to hydrolysis of urea in level of leaves[11]. By considering all these author has chosen total four agricultural areas of Adayar subjected to campaign. In this present author has tried to assess various levels of accumulation of metals includes Co, Ni, Cu, Fe, Mn those are present in soils along different parts of industrial areas. Authors are studied physic-chemical parameters includes pH water, cationic exchange capacity (CEC), porosity, moisture, texture, moisture, pH of KCl, electrical conductivity (CE), content of organic matter. The objective of this work is to assess the degree of accumulation and pollution in soils of agricultural along with various regions dicoccum, by referring to values those are reported in literature.

II. MATERIALS AND METHODS

Context in Geology and hydrology

Adayar River is located in to the south on south–eastern coast of India in north–eastern part of Tamil Nadu. [12-13] this area is subjected to characterization with sedimentary as well as field of metamorphic. Which may consist of clays along with sands those contains agricultural lands. Quaternary recent related to sand as well as alluvial silts of Seybouse.

Agricultural activities are generally consuming nearly 60% of available water, meanwhile domestic as well as industrial sectors using 40%. 2/3rd groundwater is utilized to irrigation 3rd for both drinking, industrial purposes. Around 49.276 ha of agricultural land is situated nearer to this bay. This region consists of various industries includes steel, metal as well as business related to agriculture and different industries. Adyar zone consists of green cover as 30.1 percent, which is against 14.9 percent average to cities.

This industrial area of adayar is characterized by various crops those are citrus, tomatoes, water melons along with remaining vegetables those are generally sprinkled by system of irrigation. Some area of non irrigated land is occupied with different crops referred as grain along with pasture to cattle.

Analysis and Sampling

Present area is located in downstream of river named as adayar, among the sea; In the west area Tharamani, In the south area Thiruvanmiyur , In East Besant Nagar, North west as Kotturpuram along with North past as Raja Annamalai puram the Adyar River. The study area is represented in Fig:1.



Fig: 1 Schematic view of Adayar

Adyar estuary consists of hundreds of birds species [14] now days which is effected with urbanization activity.[15] For studying origin of contaminations of metal soils, different samples are collected on a plot in control soil, located nearer to 6 km by industrial zone[16].

Authors are carried total two sample carrying campaigns were done. Samples of wheat are collected in same spot of land like agricultural. Samples were subjected to deposition in various plastic bags; those were placed in lab. Temperature is maintained 4°C. Latter samples are subjected to washing and applied for drying process to about 3mm. Basic idea behind this research is for studying aggregation of various elements noted as CO, Ni, Cu, Fe and Mn those are present in soils also various areas of Wheat. For perform, require to identify amount of item which is taken from plant of dicoccum wheat in final research process.Four samples of soil are taken those are within radius of 15m is taken by a Auger stainless steel as 0cm to 20cm and 20cm to 40cm. Nearly, 400grams, depth as same along with samples of same horizon were thoroughly mixed, placed in plastic bags those were numbered.

Soil samples those are collected are processed as per the standard ISO 11 464. These were subjected to drying in air, after that disassembled finally sieved as 2mm in room temperature.

III. SOIL SAMPLES

In order to prepare solution of soil as per the standard procedure authors are followed the same protocol NF ISO 11466 (X 31-415). Described operation was deliberated enough for analyzing metals those are in the state as heavy in both soils as well as sediments [17]. Samples those are collected were subjected to mineralize with the help of HCl+HNO_3 as 4v/v. Mineralization's were skilled over total 4 samples to 0.25gm ± 0.015gm as absorption. After that added 1.90ml - 2.9ml triple distilled water, latter 7.40mL acid as Hydrochloric acid at 36% m/m, density of 1.21gm/mL after that 2.60ml HNO_3 to 66%m/m, density 1.38gm/mL are procured from industry. Final mixture is closed by watch glass, kept in room temperature for 10hrs. Solution is kept over hot plate for about 180minutes with the help of reflux heating. At that instance cooled furthermore filtered over filter paper which is freed from dust. At the final stage attained volume is accustom to 50.00ml by triple condensed water. With the help of atomic absorption spectrometry the solution is renewed to examination and determination of metals.

Samples collected by plant are consists of Co, Ni, Cu, Fe and Mn present in samples collected from various plants. For this analysis authors are adopted the protocol [18].

Distinctive samples those are taken from various stems, roots after that subjected to drying with help of 60°C to 80hr to get mass as less than to that of 5% in total loss. Obtained samples are dry in condition those are applied for crushing in room temperature to pick up particles those consists diameter as less than to 450µm. 0.30gm ± 0.003gm of smashed material from the plant is brought out into standard flask accompanying with 4.50mL of nitric acid to 55% m/m, density as 1.28gm/mL and 6.00ml H_2O_2 - 35%m/m, density 1.21gm/ml. After that the obtained sample is kept

for 14h to degassing below hoot, latter heated over griddle at a temperature of 95°C to 1h 20min. after that at 160°C for 1h 45min. latter to this time period the sample is subjected to cooling purpose, obtained sample is subjected to filtration with the help of whatman filter paper after that relocated to a vial of 45.00ml, outright to mark at 25°C by triply distilled water. After that solution is discharged in 50.00 mL tube after that at a temperature of 4°C the sample is stored. To this reaction a blank solution is considered to each series of mineralization.

Overall strengths to Co, Ni, Cu, Fe along with Mn occurred in soils as well as various regions of dicoccum are evaluated with the help of AAS with help of flame Air-C_2H_2 nor N_2O-C_2H_2, as system which is a double beam, this is furnished with the help of hollow lamps. The range to spectra broadens at least as 180nm - 900nm. Both the blank as well as standards utilized to calibration are repeatedly tested. At last, study of interference all long dosing to each and every analyzed element is carried out. Various values obtained in this study are represented in Table:1 and Table:2.

Table 1. Textural classification.
Here= Actual density (*Ad); Apparent density (*AD); porosity (*P)

Horizons	Depth (cm)	Coarse sand (%)	Sand - Fine (%)	Slit - Coarse(%)	Silt -Fine (%)	(%)Clay	Ad*(g/cm^3)	AD* (g/cm^3)	P* (%)
P1	5 - 25 cm	39.90	15.00	12.09	0.56	26.85	2.35	1.54	37.37
	25 to 45 cm	41.19	16.65	9.51	0.46	29.81	-	-	-
P2	5 - 25 cm	41.66	13.06	12.12	0.71	28.11	2.93	1.53	36.42
	25 to 45 cm	42.61	14.34	09.15	0.41	31.74	-	-	-
P3	5 - 25 cm	39.90	16.86	16.89	0.35	24.42	2.46	1.59	38.30
	25 to 45 cm	43.01	15.02	11.12	0.26	27.62	-	-	-
Control soil	5 - 25 cm	36.80	11.02	7.43	1.43	39.68	2.74	1.71	29.63
	25 to 45 cm	37.91	10.51	7.61	0.21	41.82	-	-	-

Table 2. In different months studied Physico-chemical parameters of soils

Horizons	P1	P2	P3	Control soil
H (%)	11.49 ± 0.23	10.99 ± 0.53	11.03 ± 0.27	9.31 ± 0.19
H (%)	16.01 ± 0.03	14.59 ± 0.32	14.00 ± 0.26	10.45 ± 0.29
MO (%)	1.29 ± 0.40	1.31 ± 0.12	1.29 ± 0.15	1.69 ± 0.15
MO (%)	1.51 ± 0.30	1.29 ± 0.11	1.42 ± 0.40	1.79 ± 0.16
pH Water	7.41 ± 0.42	6.41 ± 0.20	7.40 ± 0.12	6.79 ± 0.26
pH Water	7.27 ± 0.23	6.39 ± 0.29	7.31 ± 0.52	6.67 ± 0.26
pH KCL	7.52 ± 0.19	6.30 ± 0.15	7.37 ± 0.18	6.55 ± 0.21
pH KCL	7.13 ± 0.18	7.29 ± 0.02	7.29 ± 0.11	6.76 ± 0.21
EC (mS per cm)	0.22 ± 0.12	0.29 ± 0.31	0.31 ± 0.49	0.32 ± 0.22
EC (mS per cm)	0.46 ± 0.15	0.39 ± 0.41	0.50 ± 0.49	0.29 ± 0.29
CEC (meq per100 gm.)	17.66 ± 1.41	18.11 ± 1.39	15.09 ± 1.42	22.49 ± 1.45
CEC (meq per100 gm.)	16.15 ± 1.21	17.79 ± 1.43	16.79 ± 1.26	23.11 ± 1.29

Horizons Parameters	P1	P2	P3	Control soil
H %	9.21 ± 0.21	7.67 ± 0.08	7.16 ± 0.20	6.88 ± 0.22
H%	11.31 ± 0.03	9.20 ± 0.16	9.50 ± 0.32	8.78 ± 0.18
MO%	1.45 ± 0.19	2.54 ± 0.14	2.60 ± 0.19	2.10 ± 0.15
MO %	1.49± 0.20	2.25 ± 0.09	2.80 ± 0.16	1.98± 0.20
pH Water	7.81 ± 0.11	6.26 ± 0.19	6.31 ± 0.21	6.78 ± 0.19
pH Water	7.30 ± 0.19	6.53 ± 0.31	6.30 ± 0.20	7.86 ± 0.19
pH KCL	7.59 ± 0.10	6.20 ± 0.32	6.31 ± 0.09	6.50 ± 0.11
pH KCL	7.21± 0.21	6.19 ± 0.32	6.20 ± 0.41	6.47± 0.10
EC (mS per cm)	0.58 ± 0.12	0.29 ± 0.19	0.30 ± 0.19	0.31 ± 0.50
EC (mS per cm)	0.49 ± 0.19	0.32 ± 0.30	0.29 ± 0.21	0.54 ± 0.49

CEC (meq per 100 g)	16.29 ± 1.31	17.80 ± 1.19	13.30 ± 1.20	23.67 ± 1.50
CEC (meq per 100 g)	17.83 ± 1.30	18.40 ± 1.29	15.60 ± 1.20	23.87 ± 1.30

IV. RESULTS AND DISCUSSION

Texture and porosity

The fractionation covenant enforcing total twointegral approaches includes split by dry appropriate80 µm latter separation by gravimetric up to 2µm with the help of sedimentation particles by considering the stocks law which allows for classifying particles confer totheir diameter.

In the table 1 values for sedimentation to the total two layers 5-25cm and 25 - 45cmare recorded which demonstrates that horizons for soil like P1, P2 and P3. Due to the presence of physicochemical properties to clay they plays a crucial role for the availability of metals like heavy. As per the study authors are finally concluded that heavy metals are absorbed just like to complexes with organic matter of soil forminglatter with complex of organo-metallic.

Overall porosity to the soil represented as "% P" which is determined by apparent density represented as "AD" along with actual density which is noted as "ad'. To measure actual density that is purely dependent over volume which may be associated with solid. Cylinder process is adopted for measuring the apparent density. In this case, authors are performed a single measure over samples of soil for deep as 5.0cm – 25.0cm which is interrelated to area of labour.

Total four horizons are consists of reasonably very fine consistency which is distinguished by strong enough enforcement 37.37 porosity; 36.42; 38.30 and 29.63 to P2, P1, P3 as well as ground control.

In general practice solution of the soil accommodates ions of mineral as well as very tiny in size molecules of organic fluctuating in composition along with its mobility also even its obsession over particles of solid.

V. PHYSICOCHEMICAL PARAMETERS

Physicochemical parameters for studied soil with surface area as 5.00cm to 25.00cm along with depth as 25.00cm to 45.00cm are tabulated in the table 2. By considering standard NF ISO 11465 to level of humidity as H% to soils. The obtained samples are applied for drying with the help of oven at a temperature of 105°C ± 5°C in oven as far as mass is stable. Moisture varies from 6.88% to 11.49% in surface from 5.00cm to 25.00cm and 8.78%to 16.01% as 25.00cm to 45.00cm in depth.

Values so obtained reveal that moisture is highly accessible over 25cm to 45cm whenever compared with 5.00cm to 25.00cm. In general practice wheat evokes higher water over the edge of 5.00cm to 25.00cm.

Authors are determined the rate of organic matter (MO%) with the help of Walkey and Black method includes organic fraction of potassium dichromate 1.50N in presence of acidic medium along with a titration in return by Mohr salt at 0.9N.

Soils named as P1, P2 and P3 which are composed over the surface and in depth, consist of very less organic matter. Rate of organic matter varies as 1.31% - 2.10% in both surface as well as 1.29% - 1.98% in depth. In addition to this, identified readings didn't correspond by rate of clay this may changes as 24.42% - 28.11% at deeper of 5.00cm to 25.00cm as well as 27.62; 41.82% at a deeper of 25.00cm to 45.00cm. Soils studied are very poor in the form of organic matter, this is mainly due to non-favorable factors includes climatic, circumstances of topography etc.,

The calculated pH alters as 6.78 to 7.40 for the 5.00cm to 25.00cm layer and 7.86 to 7.31 to the 25cm to 45cm layer. To KCl pH, it is 6.50 to 7.37 to 5cm to 25cm layer as well as 6.47 to 7.29 for layer 25cm to 45cm. Finally, the obtained values reveal that environment is reasonably alkaline in nature, by very low reserve acidity, which is distinguished by deviation not more than to that of 0.5. Metals like heavy are able to absorb, undergoes immobilization by minerals of clay those are also able to form complexes with organic matter to soil which forms organo-metallic complex.

VI. RESULTS OF THE ANALYSES OF SOILS

Enlargement to median strengths in terms of units like mg/kg MS for the heavy metals is depicted from Fig. 2-5.

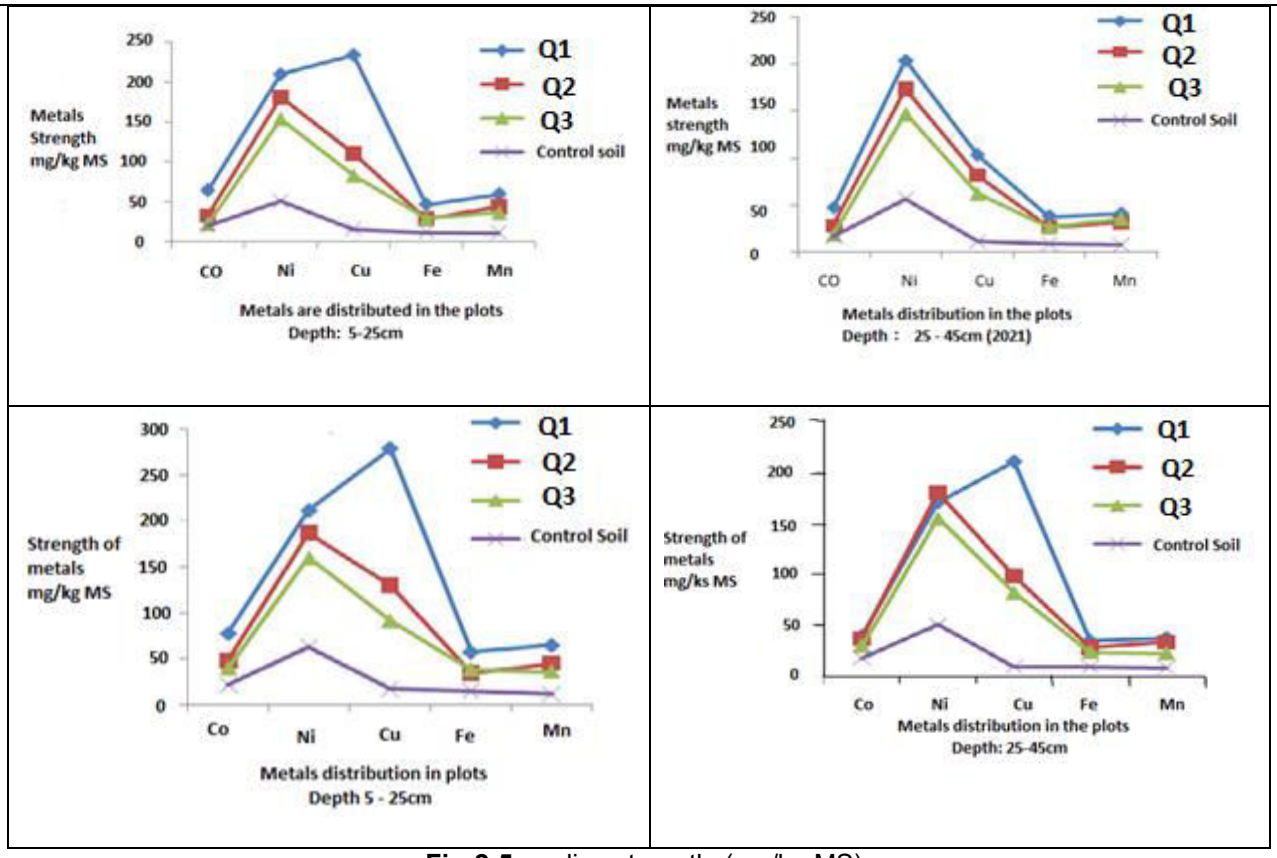

Fig:2-5 median strengths(mg/kg MS)

The results by this work were reported in table 3 and 4. Obtained values reveals that the strength of Co, Ni, Cu, Fe and Mn in soil during dry period should increases marginally by soil composed in wet period, this is mainly due to an increase in temperature as 28°C - 45°C especially in summer generation a pre-strength of soil metals both in as well as solutions from the waters present in nearby industries. The reproducibility of values which are noted in the table 3 are appraised which is depend on coefficients of variation (CV) among the strengths those are achieved by total four samples of soils.

The percentage of CV to total values of assays to metals provides a distribution of homogeneous as <10%. CV as 9.63 in soil named as P2 in measurement of Cu at deep as 25.00cm to 45.0 cm. CV as 1.7 is in control soil in measurement of Mn as deep: 25.00cm - 45.00cm. Strengths obtained by surface as 5.00cm to 25.00cm in soils as P1 and P2 were 243.70 ± 16.01(7.30) and 109.37 ± 8.19 (7.5) and 102.90 ± 6.76 (6.6) and 81.40 ± 4.50 (5.53). 29.09 ± 1.88 (6.89) At a deep 20.00cm to 40.00cm, Pb strength is identified in soil (P1) is 83.49 ± 5.19(6.29).

Heavy metals strength for these soils shows significant increase in the form as Co>Ni>Cu>Fe>Mn; to four areas. Samples are collected at a depth of 5.00cm to 25.00cm, give an identical classification: Co>Ni>Cu>Fe>Mn; for P1, P2 and P2 and P3. In same depth control soil has provided Ni>Co>Fe>Cu>Mn and Co>Ni>Fe>Cu>Mn.

Soil named as P3 were very lowly contaminated. In other hand P2 and P3 provides the values to depth of 5.00cm to 25.00cm along with P1 and P2 to deep as 25.00cm - 45.00 cm. Obtained contamination is generally due to Cu. The values obtained are represented in the Table: 3, 4 and 5.

Table 3. Heavy metal contents in mg/kg.MS by average of 4 measures

	Q1	Q2	Q3	Control soil
	Mean ± SD	Mean ± SD	Mean ± SD	Mean ± SD
Co	62.79 ± 3.10 (6.64)	33.52 ± 2.30 (7.10)	22.43 ± 1.59 (7.59)	18.58 ± 1.37 (7.21)
Ni	209.19 ± 12.09 (6.19)	179.72 ± 10.49 (5.9)	149.29 ±10.36 (6.59)	49.27 ± 2.74 (5.29)

Cu	243.70 ± 16.01 (7.30)	109.37 ± 8.19 (7.5)	83.49 ± 5.19 (6.29)	16.20 ± 0.89 (6.50)
Fe	47.09 ± 5.29 (9.29)	26.16 ± 2.48 (9.20)	29.09 ± 1.88 (6.89)	12.29 ± 0.86 (6.5)
Mn	59.89 ± 4.09 (5.2)	44.90 ± 1.87 (4.2)	36.59 ± 1.75 (4.6)	11.21 ± 0.61 (5.7)
IP	1.09	0.63	0.49	0.21

Table 4. Heavy metal contents in mg/kg.MS as average of 4 measures

	Q1	Q2	Q3	Control soil
	Mean ± SD	Mean ± SD	Mean ± SD	Mean ± SD
Co	46.71 ± 3.21 (6.9)	27.62 ± 2.46 (8.9)	18.10 ± 1.30 (7.20)	15.50 ± 1.12 (7.23)
Ni	204.20 ± 12.87 (6.3)	173.84 ± 11.95 (6.9)	147.34 ± 10.63 (7.21)	56.11 ± 3.80 (6.8)
Cu	102.90 ± 6.76 (6.6)	81.40 ± 4.50 (5.53)	62.17 ± 2.74 (4.41)	10.16 ± 0.85 (4.92)
Fe	37.43 ± 1.55 (4.14)	26.08 ± 1.27 (4.9)	26.31 ± 1.80 (6.84)	7.68 ± 0.12 (8.17)
Mn	40.18 ± 1.76 (4.4)	31.55 ± 2.65 (8.4)	35.37 ± 1.72 (4.9)	6.62 ± 0.11 (1.7)
IP	0.59	0.48	0.41	0.13

Table 5. Evolution of metal concentrations as mg/kg in various areas of dicoccum wheat

		P1	P2	P3	Témoin	Reference values
	Organe	**Mean ± SD**	**Mean ± SD**	**Mean ± SD**	**Mean ± SD**	
Co	roots	6.43 ± 0.26	6.37 ± 0.23	6.15 ± 0.20	6.1 ± 0.21	6.5
		-4.04	-3.61	-3.25	-3.44	
	stems	2.5 ± 0.11	2.8 ± 0.13	2.77 ± 0.16	2.5 ± 0.12	2.8
		-4.4	-4.64	-5.8	-4.8	
	leaves	2.98 ± 0.13	3.12 ± 0.14	3.08 ± 0.11	2.97 ± 0.10	3.1
		-4.4	-4.5	-3.6	-3.4	
	grains	4.25 ± 0.21	2.78 ± 0.12	2.23 ± 0.11	1.15 ± 0.09	4.5
		-5	-4.32	-4.93	-7.83	
Ni	roots	4.96 ± 2.49	45.19 ± 2.09	33.69 ± 2.59	37.87 ± 2.59	396
		-4.81	-4.71	-7.74	-7.03	
	stems	14.7 ± 0.85	13.1 ± 0.60	11.83 ± 0.97	12.20 ± 0.80	17
		-5.8	-4.6	-8.2	-6.6	
	leaves	10.20 ± 0.56	8.76 ± 0.41	10.20 ± 0.54	10.13 ± 0.43	12
		-5.5	-4.7	-5.3	-4.24	
	grains	23.65 ± 1.12	21.38 ± 0.87	22.41 ± 1.06	20.50 ± 0.91	25
		-4.74	-4	-4.73	-4.44	
Cu	roots	0.55 ± 0.08	0.38 ± 0.06	0.27 ± 0.04	0.19 ± 0.008	6.40
		-14.54	-15.79	-14.81	-4.21	
	stems	0.33 ± 0.04	0.14 ± 0.008	0.11 ± 0.009	0.12 ± 0.009	<6,0
		-12.12	-5.71	-8.18	(7.5)	

	leaves	0.36 ± 0.07	0.19 ± 0.05	0.17 ± 0.009	0.15 ± 0.008	<6.0
		(15.55)	(13.88)	-6.92	-5.71	
	grains	< DL	< DL	< DL	< DL	0.50
Fe	roots	4.44 ± 0.23	4.11 ± 0.21	3.2 ± 0.12	1.56 ± 0.08	8.60
		-5.2	-5.11	-3.75	-5.13	
	stems	0.18 ± 0.009	0.96 ± 0.03	0.22 ± 0.01	0.7 ± 0.02	< 1.0
		-5	-3.12	-4.54	-2.86	
	leaves	0.86 ± 0.06	0.73 ± 0.02	0.6 ± 0.03	0.3 ± 0.01	
		-6.98	-2.74	-5	-3.33	1.40
	grains	0.14 ± 0.008	0.12 ± 0.009	< DL	< DL	0.30
		-5.71	-7.5			
Mn	roots	2.45 ± 0.14	1.71 ± 0.10	1.44 ± 0.09	0.23 ± 0.01	6.0
		-5.71	-5.85	-6.25	-4.35	
	stems	1.17 ± 0.08	1.09 ± 0.06	1.12 ± 0,05	0.10 ± 0.008	<6.0
		-6.84	-5.5	(4.46)	-8	
	leaves	1.80 ± 0.10	1.40 ± 0.09	1.28 ± 0.07	0.15 ± 0.009	<6.0*
		-5.55	-6.43	-4.47	-6	
	grains	0.20 ± 0.09	< DL	< DL	< DL	0.10
		-6.25				

VII. CONCLUSION OF ANALYSES OF PLANTS AND SOILS

According to the results the strength identified in various parts of dicoccum were almost same. This information reveals that roots of wheat contains strength for Ni in the order of 396 $mg.kg^{-1}$, values provided varying levels of 4.96 ± 2.49; 45.19 ± 2.09; 33.69 ± 2.59 and 37.87 ± 2.59. The grains contain a Ni concentration slightly less which is of 25 mg/kg. To Cu, strengths identified various parts of dicoccum were weak which comparison is done by various studies. In this work soils contains Cu amount higher than to that of 100 $mg.kg^{-1}$, Cu strength in dicoccum grains were less to that of limits of detection by AAS flame. Fe, readings are found in normal range. But, in dicoccum, value found 8.60 mg/kg which is more to our readings.

Soil named as P3 along with control, strength in Ni were not identified by using AAS flame. Strength in Mn in roots and leaves are minimum to that of actual value. Mn 0.20 ± 0.09, which is maximum to that $0.09mg/kg^{-1}$. Readings of coefficients of variation (CV%) of dicoccum wheat such as roots, stems, leaves along with grains show excellent reproducibility. Obtained values were lower to that 10. Authors are identified that there is minimum strength of metals in various parts of dicoccum wheat in control soil compared to soils named as P1, P2 and P3. Present work revealed an assessment of level to contamination as well as aggregation of ETM in different soils present in agricultural land along with various parts of dicoccum wheat (*Triticum dicoccum* Desf). Authors concluding that arrangement of metals those are heavy present in soils purely dependent over texture as well as physico-chemical parameters. Values also revealed that there is a less pollution by metal in soil, those provides an anthropogenic area of described elements. Different organs of wheat accumulates less strengths in heavy metals those are compared with different studies. Majority of plots, concentration for Cu, Fe and Mn in grains of wheat were lower to limit of detection with atomic absorption spectrometry.

VIII. REFERENCES

[1] Tiwary, R.K. and Dhar, B.B. “Effect of coal mining and coal based industrial activities on water quality of the river Domodar with specific reference to heavy metals”. Int. J. Surf. Mining reclamation Environ. 1994; 8 : 111-115.

[2] Bennasser, L., Fekhaoui, M., Benoit-Guyoud, J.L. and Merlin, G. “Influence of tide on water quality of lower Sebou polluted by Gharb plain wastes (Morocco)”. Wat. Res. 1997; 31 : 867-867.

[3] Bargagli, R.. Trace elements in terrestrial plants. An ecophysiological approach to biomonitoring and biorecovery. Springer. 1998; 324.

[4] Kabata-Pendias, A. and Pendias, H. “Trace elements in soils and plants”; 1992; 2nd edition, CRC Press, Boca Raton. Etats-Unis.
[5] Barber, S.A. Soil nutrient bioavailability, a mechanistic approach. John Wiley and Sons, New York, Etats-Unis. 1995
[6] Siberlin, N.. Bioavailability of trace elements in cultivated soil: Study of transfer in a field corn crop and consequences on yield. Final dissertation, IUT Louis Pasteur, Schiltigheim. 1996.
[7] Krämer, U., Cotter-Howells, J.D., Charnock, J.M., Baker, A.J.M. and Smith, J.A.C. “Free histidine as a metal chelator in plants that accumulate nickel”; Nature. 1996; 379 : 635-638.
[8] Merian, E.. Metals and their compounds in the environment, occurrence, analysis, and biological relevance. VCH, Weinheim, Allemagne. 1991.
[9] Bonnard, R. Definition of impact assessment values for Cr. Working document of the MATE Group "Contaminated Sites/Soils/Public Health".1997
[10] Pichard, A., Bisson, M., Diderich, R., Houeix, N., Hulot, C., Lacroix, G., Lefevre, J.P., Leveque, S., Magaud, H., Morin Rose, A. and Pepin, G. Chromium and its derivatives, INERIS toxicological and Environmental data sheet for chemical substances. 2005.
[11] Seregin, I.V. and Kozhevnikova, A.D. “Physiological role of nickel and its toxic effects on higher plants”. Russ. J. Plant. Physiol. 2006; 53. 257-277.
[12] P. Periakali, T. Vengopal; Giridharan, L; Jayaprakash, M; Periakali, P. "Environmental impact assessment and seasonal variation study of the groundwater in the vicinity of River Adyar, Chennai, India". Environmental Monitoring and Assessment. 2009; 149 (1–4): 81–97.
[13] P. M. Velmurugan, T. Vengopal; Giridharan, L; Jayaprakash, M; Velmurugan, PM (). "A comprehensive geochemical evaluation of the water quality of River Adyar, India". Bull Environ Contam Toxicol.; 2009; 82 (2): 211–217.
[14] "How urbanisation watered down the natural wealth". The Hindu. 29 May 2007. Archived from the original on 13 December 2007. Retrieved 23 September 2009.
[15] "Death of an estuary". The Hindu. 12 January 2003. Archived from the original on 14 February 2011. Retrieved 23 September 2009.
[16] "Eco-park may cover all of Adyar Creek". The Hindu. 31 March 2007. Archived from the original on 9 December 2007. Retrieved 23 September 2009.
[17] Louhi, A., Hammadi, A. and Achouri, M. 2012. Determination of some heavy metal pollutants in sediments of the Seybouse River in Annaba, Algeria. Air, Soil Water Res. 2 : 91-101.
[18] Barbaste, M.. Analysis of metal contents in plants-Laboratory analysis practices. USRAVE, Bordeaux, 2004; 2 p..

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Development of method and validation for Assay in Cyclizine Hydrochloride Injection by Applying Stability Indicating HPLC Methodology by HPLC

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ABSTRACT

A validated HPLC method is developed to determine Cyclizine Hydrochloride (CYHC) in pharmaceutical formulation. Isocratic elution at a flow rate of 1.0 ml/min was employed on C18 5 µm (150 x 4.6 mm), 4µm or equivalent column, or similar is used for this chromatography analysis. Ammonimum acetate, acetonitrile, Glacial acetic acid, and triethylamine are used as a buffer solution. Mobile phase is 10mM ammonium acetate as 64v/v, acetonitrile as 35v/v, and triethylamine as 1v/v. The UV detection wavelength was 254nm, and 20.0µl sample was injected. The run time is 55 minutes for diluted standard and 30 minutes for sample, blank, placebo, system suitability, sensitivity solution. The retention time is ± 2.1min. The% R.S.D CYHC is identified. The mean Percentage recovery for Irinotecan is found within the specification limit. Thus, the proposed HPLC method can be successfully applied for routine quality control analysis formulations. The method developed is simple and is better than the methods reported in the literature.

Key words: RP-HPLC Refractive index detector, CYHC, flow rate, column, ICH Guidelines, USP reference.

Introduction

The molecular Formula for Cyclizine Hydrochloride (CYHC) is $C_{10}H_{16}ClNO$. This CYHC is a medication utilized for treating as well as preventing problems like nausea, vomiting as week as dizziness can be obtained as a result of neither motion sickness nor vertigo[2]. This CYHC should also be utilized for nausea latter common anesthesia because whatever is developed by opioid utilization.[1][2] This medicine is taken as oral i.e., by mouth, in rectum, nor passed as an injection into the body as a vein. [3] This CYHC consists of minor side problems which includes effecting sleepiness, mouth converting into dry, constipation, as well as trouble with vision.[4]

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Another name containing medicine, Diconal is in associated with cyclizine along with the opioid dipipanone.[5] This dipipanone was treated as a schedule I controlled substance/compound in the country US.[6] This drug is generally appearing as magazine hydrochloride, also initiated for selling in US under brand name Marezine. Selling began in France as Marzine in 1965. [7][8] Maha Mohamed Abdelrahman et.al.,[9] have described CYZ based on routes like oral or intravenous, to euphoric or hallucinatory variations. Neven M Habib et al. [10] proposed TLC and performed chromatographic separation by silica gel 60 F254 utilizing a developing system containing methylene chloride 7.0v/v acetone 1.0v/v methanol 0.50v/v scanning, which divided at 220nm. 1.00mL/min is the rate of flow—wavelength as 220nm. The linear relationship obtained among peak area of integrated as well as strength as 10µg/mL -50µg/mL to PYH, 10-50µg/mL to CYH, and 7-50µg/mL to MEH. GislaineKuminek Hellen K et al., [11] developed with the help of C18 column with methanol 80v/v water 20v/v at a pH range of 2.6 as mobile phase. The authors identified rate of flow s of 1.00 mL/min. Wavelength is identified at 230 nm. Amina M. Abass et al.,[12] characterized sensors revealed Nernstian gradients of 59.61 ± 0.50, 57.71 ± 0.23, 53.01 ± 0.14, also 53.20 ± 0.35 mV per decade by pH range as 2.5–5.5 in DBPH along with 3.5–5.0 in *O*-NPOE electrodes, and 4.0–5.5 to DOPH as well as DBP plasticized film-based sensors.

EXPERIMENTAL

LC 20AT pump and UV-Visible detector with flexible wavelength program and injector named Rheodyne utilized in a present developmental method by the author. C18 5 µm (150mm x 4.6mm) or equivalent was used for this chromatography analysis. With the help of Loba ultrasonic bath sonicator mobile phase containing the gas was separated. The common balance to measure the drugs is employed. From the local market sample, CYHC is procured. In this present method Ammonium acetate, Acetonitrile, triethylamine, acetic acid, Trifluoroacetic acid, and O-phosphoric acid were used as AR grade; Acetonitrile and water collected as HPLC grade are procured and utilized in this process by the author. Exactly 10.00g ammonium acetate is transferred in a 100.00mL standard flask. To this, 50.00mL Acetonitrile is added after that is subjected to dissolution. Again, for this Solution, 10.00mL glacial acetic acid and 2.00mL of triethylamine is added. After that subjected to dilution to a suitable volume by the required diluent. The mobile phase is prepared by dissolving 10mM ammonium acetate as 64v/v, Acetonitrile as 35v/v and triethylamine as 1v/v. Whose pH is adjusted to 3.7 by using approx. 90.00ml acetic acid per liter. 10 mM Ammonium Acetate was prepared by taking 0.77gm Ammonium Acetate is dissolved in suitable solvent latter, it is diluted up to mark in 1000ml of water. Standard Solution is prepared by taking 80mg CYHC standard into a 100ml classic flask. Dissolve in and makeup to suitable volume by suitable solvent. It is finally filtered by 0.45µm filter, discarding the first 5.00ml filtrate. A sample solution is prepared by taking 20 tablets after determining the mass average. Grinded these 20 tablets until the fine powder was obtained. Latter weighed 200mg of powdered sample in 100.00ml standard flask. Added 80.00ml solvent after that sonicated to a time interval of 5 minutes. In this case, the precaution is taken as the product doesn't allow get warm. It diluted the volume with a suitable solvent and mixed thoroughly. It was filtered the same by 0.45µm filter paper and discarded the first 5ml of filtrate.

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METHOD DEVELOPMENT

To 10ppm solution of CYHC with the help of UV spectrophotometer spectrum in Acetonitrile is recorded individually. 254nm is the maximum absorbance wavelength. Maximum absorbance showed spectra for CYHC. Essential dissolution and peak performance were identified with the help of C18 5 µm (150mm x 4.6mm), 4µm, or equivalent columns. 10mM ammonium acetate as 64v/v, Acetonitrile as 35v/v with triethylamine as 1v/v was consumed as eluent. To identify optimum separation, measuring reaction course between 0.50mL/min – 1.50mL/min is enforced. Using a reaction 1.00mL/min rate of flow is adequate for the analyte separation as successful.

Proposed Method Validation and requirements

System Suitability

% RSD of peak responses as per CYHC to six replicate injections is 2.0 %. The tailing factor for the peak is mainly CYHC should not be more than that 2.0. The theoretical plate count is less than 2000. A total of six replicate injections to working standard solution were passed as per the analysis method. % RSD to peak responses is measured. Analytical system complies with requirements that are specified by system suitability. System suitability results are tabulated in Table 1.

Sample	CYHC Area	CYHC Tailing	CYHC Tangent
1	1228700	0.9	5552
2	1225786	0.9	5583
3	1224860	0.9	5574
4	1224390	0.9	5581
5	1221977	0.9	5609
6	1223832	0.9	5589
Mean	1223754	0.9	5581
% RSD	0.2		

Table 1:system suitability results

Specificity

Carried out this experiment by passing Blank injecting Blank as the diluent, placebo preparation, diluted standard solution, Monohydroxy, Dihydroxy, isopropyl ester, the dimer of CYHC, prepared sample, spiked sample by known foreign substances which documented retention times are as different. Peaks of blank as diluents, impurity, and Placebo should not interfere with the peak of CYHC. Specificity criteria is performed by introducing Blank as the diluent, preparation of Placebo, regular solution, Monohydroxy CYHC, Dihydroxy CYHC, CYHC isopropyl ester, CYHC Dimer, Sample, sample which is spiked by known foreign substances in the chromatographic system and recorded retention times. From the below results, it can be concluded that there is no retardation

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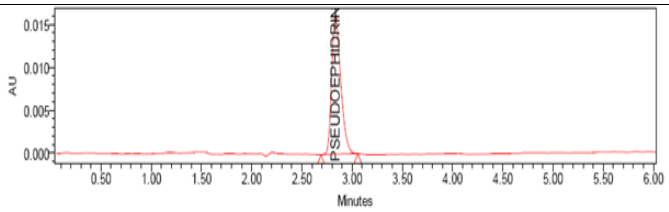
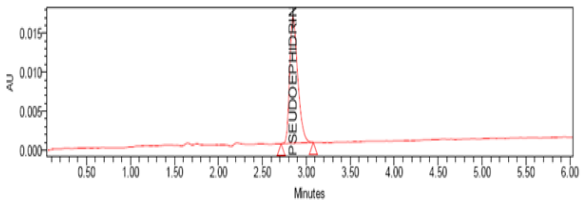
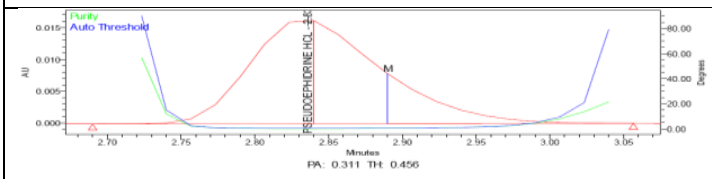
of other peaks of placebo & impurity peaks, including the CYHC peak. Chromatograms to solvent, drug active, product, Drug Active andproduct UV-stressed, and placebo peaks are denoted in Figure 1.



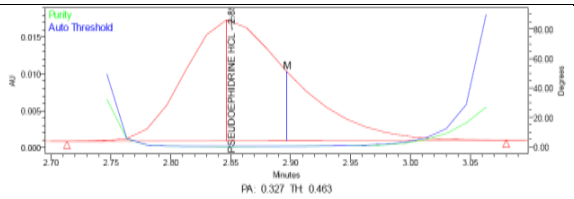
Drug active – Peak due to Cyclizine HCl



Product – UV stress: Peak due to Cyclizine HCl



Drug active Purity angle < Threshold = 0.311< 0.456



Purity angle < Threshold = 0.285 < 0.42

Figure:1

STRESSED CONDITION STUDIES:

A Stressed condition of CYHC injection should be performed to finalize either information of stability through its shelf life or any non-persistent compound that may be assessed that should not combine with the help of peak obtained by injecting CYHC. Along with the above, a stressed condition study should be helpful to know high about degradation type pathways other than different parameters. These include oxidants, alkali solution, acidic, neutral, heat, or in the presence of light to every degradant. For these studies authors are used Preparedness of Sample, Preparedness of Placebo, Acid Stressed sample (1.0N HCl), Alkali Stressed sample (1.0N NaOH), 3.0% w/v Peroxides of Hydrogen Stressed (3.0%w/v H2O2), Neutral Stressed, UV light exposed, Photo stability exposed, Sunlight exposed, Thermal Stressed (Dry heat) sample, Alkali Stressed sample (1.0N NaOH) at 25°C for 5 minutes, Neutral Stressed sample for 2 hours. Sample is found to be degrading in both circumstances of Alkali as well as neutral conditions. They were slightly degrading in Hydrogen peroxide, state of sunlight. No other degradation is identified in various conditions like Photo stability, acidic, thermal and UV light stressed conditions. Anyhow, unknown foreign substances, related substances and degradation foreign substances are scattered by CYHC peak. CYHC peak is purer, which is finalized by Chrome Leon software. Hence these related substance circumstances are completed as product release ad shelf-life.

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Intermediate Precision

% RSD due toCYHCconcentration to six samples are 2.0 %. Mean results obtained in the repeatability; also, intermediate precision must not vary by more than 3.0%. %RSD of impurities 0.05% latter above total 6 measurements were NMT 15.0%RSD of total impurities to total 6 measurements is NMT 10.%RSD of e related compounds 0.05% as well as above to12 preparation NMT is 15.0. %RSD to total impurities is NMT 10.0. Therefore, finally obtained results by this procedure is rugged.% RSD for intermediate precision is 1.5%. Intermediate precision as well as repeatability comply as they differ by 2.0 %. Finally obtained values are tabulated in table: 4.3.% RSD due toCYHCstrengthto total six samples must be less than or equal to 2.0%. Six separate sample preparations of batch 240598 are analyzed according to the method of analysis. The % RSD due toCYHCstrengthto assay meets requirements to reproducibility as 0.7 % respectively. Finally, obtained values are tabulated in the table: 2.

Equipment number	LC 5
HPLC column	C18 5 µm (4.6 x 150 mm)
Sample	Results (mg/tab)
	Cyclizine Hydrochloride
1	47.6
2	48.5
3	48.5
4	48.7
5	49.5
6	49.6
Mean	48.7
% RSD	1.5
Sample	Mean Results (mg/tab)
	CYHC
Repeatability	50.1
Intermediate Precision	48.7
Mean	49.4
% RSD	2.0

Table 2: Intermediate precision Results

LINEARITY:

The correlation coefficient of the regression line to CYHC is within range, i.e., 0.99. The Y-intercept of the line should not be significantly different from zero, i.e., z falls within specified limits only when +2 > z > -2. Total five solutions containing 50, 75, 100, 125, and 150 % of CYHC, relative to working strengths of 0.8000 mg/ml, are prepared later passed as per rules and regulations in the analysis method. A linear regression curve was constructed, and finally, R2and assessment values were measured. R^2 for CYHC is 1.000. The obtained plot is a straight line shown in figure 2.

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Noted z is 0 for CYHC.

CALIBRATION CURVE: y = Bx + A, R^2 = coeff. of determination

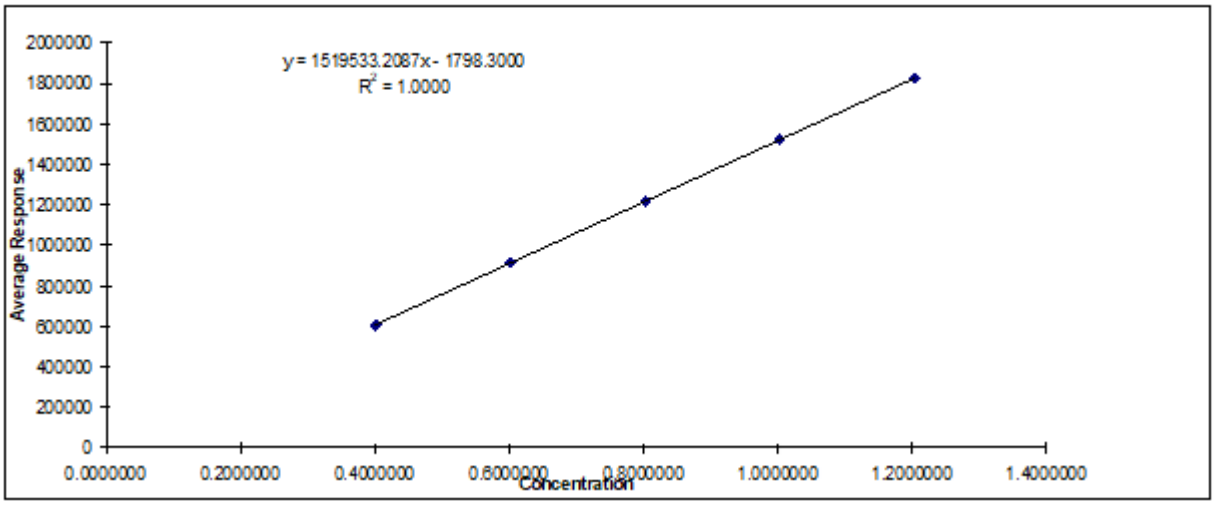

Figure2: Linearity Graph for CYHC

ACCURACY

A spiked known quantity of Monohydroxy CYHC, standard in a sample. Spike known quantity of CYHC into placebo at specification level of unknown foreign substances. The percentage recovery of active compounds/substances for each solution prepared is 98.0 – 102.0 % of the actual amount. Sample solutions are spiked by known strengths of CYHC to result in strength of 0.4000mg/ml, 0.6000mg/ml, 0.8000mg/ml, 1.0000mg/ml, and 1.2000mg/ml representing respectively 50, 75, 100, 125, as well as 150 % of CYHC relative to working strength as 0.8000mg/ml. These samples were passed in duplicate according to the method of analysis. By accuracy results below, percentage recovery values to CYHC satisfy acceptance criteria to accuracy across 50 % - 150 %. % of individual and mean recovery at every stage is 95 to 105 to the known foreign substance to Monohydroxy CYHC. % individual and mean recovery at every stage varied between 80 to 120 to Known impurity. % individual and mean recovery at every stage varied between 80 to 120 for Unknown foreign substances. Results were represented in table 3.

Sample %	Theoretical	Actual	% Recovery	Average % Recovery
50	39.39	38.74	98.3	98.2
50		38.62	98.0	
75	59.09	58.50	99.0	99.1
75		58.60	99.2	

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100	78.78	78.51	99.7	99.7
100		78.49	99.6	
125	98.48	98.91	100.4	100.4
125		98.81	100.3	
150	118.17	118.03	99.9	100.0
150		118.31	100.1	

Table 3: Accuracy results for CYHC

RANGE:

Derived specified range from linearity and accuracy studies. %RSD obtained for all accuracy range measurement is NMT 10.0. Correlation and regression coefficient is NLT 0.995 to Linearity as well as accuracy range parameter. With the help of obtained values finally,concluded this method proposed is varying LOQ Level as 300% of target strength to monohydroxy CYHC, dihydroxy CYHC, Isopropyl ester as well asCYHC. Range of method is varying LOQ to 400% of target strength to CYHC dimer.Based on accuracy values, range to assay of used Emitex tablets is 25 – 75mg/tab of CYHC, this denoted 50 % - 150 % of working strength. From these obtained values are represented in Table: 4&5 and graphs obtained are represented in Figure 3.

Level (in %)	**Peak area counts from Linearity**
50	964332.559
100	2004482.159
200	4273985.563
300	6444178.663
Coefficient of Correlation	**1.000**
Coefficient of Regression	**1.000**

Table 4: Coefficient of correlation and Regression

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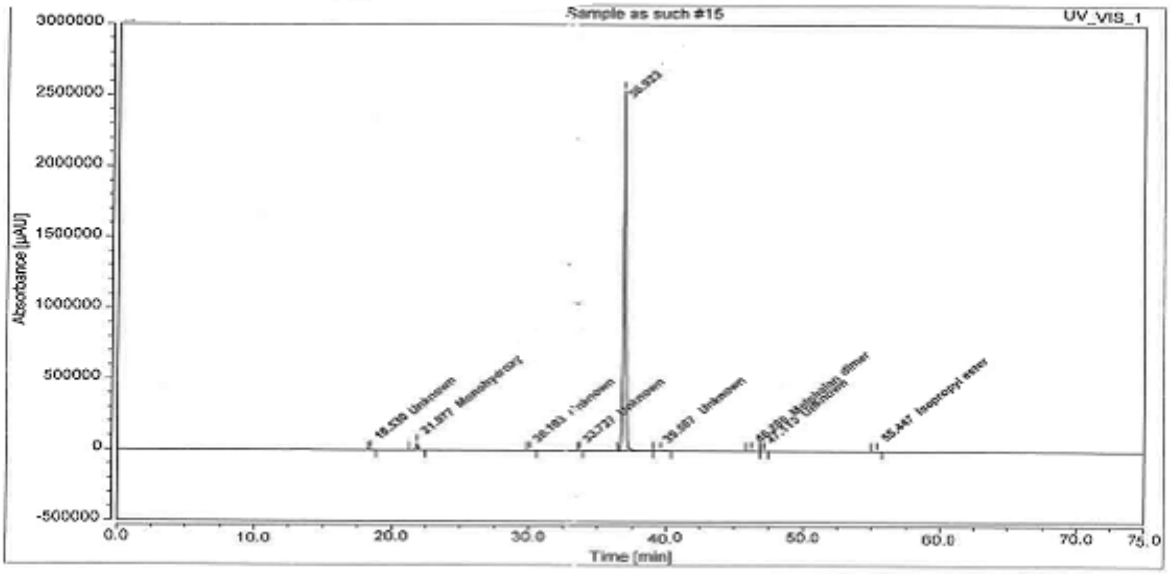

Figure 3: Chromatogram for CYHC

S.No	Name of Peak	Rt. In min.	Area µAU*Sec	Height µAU	Area %	Match Factor	Purity Limit	Results
1	Unknown	17.690	33806.555	3587	0.11	1000	990	Pure
2	Monohydrate CYHC	19.788	40439.496	47631	1.31	1000	990	Pure
3	Unknown	29.201	8169.863	749	0.03	990	990	Impure
4	Unknown	37.633	3927.433	480	0.01	998	990	Pure
5	CYHC	41.896	30270960.812	2537533	99.11	1000	990	Pure
6	Unknown	40.600	33059.100	919	0.22	995	990	Pure
7	CYHC dimer	49.300	48421.804	3499	0.09	999	990	Pure
8	Unknown	48.009	6678.950	511	0.05	982	990	Impure
9	Isopropyl ester	55.399	15947.630	1262	0.07	997	990	Pure
Total					100.00			

Table 5: Results of purity

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Robustness:

This parameter Robustness is an analytical procedure by a tiny portion of capability for remain those are not affected by very minute also calculates differentiation in this method parameters also provides a note of its reliability course of regular utilization. We changed the oven temperature to ±5°C. System suitability criteria compulsory obey total experimental processes. Total known impurities were separated by each other, and impurities spiked a peak of CYHC in the sample. This process is finalized as robust with minimal variation in process parameters. The Robustness obtained values were represented in table 6.

			Theoretical Plates NLT 2000	**Tailing factor is 2.0**	**%RSD is 5.0**
	Actual parameters		278389	1.13	0.6
	Abnormality in flow rate	-0.2 ml/min	298988	1.14	0.6
		+0.2 ml/min	264186	1.09	0.7
	Temperature	-5°C	254612	1.20	0.6
		+5°C	285999	1.11	0.6
	Wavelength	-5nm	258939	1.19	2.7
		+5nm	258988	1.29	1.2
	Difference in organic solvent as Methanol	+2%	296782	1.09	3.7
		-2%	296028	1.09	1.1
	Different in organic solvent as Acetonitrile	+2%	267958	1.22	0.4
		-2%	267982	1.22	0.4

Table 6: Robustness Results for CYHC

RESULTS AND DISCUSSION

For this procedure author has used mobile phase as 10 mM ammonium acetate as 64v/v acetonitrile as 35v/v along with triethylamine as 1v/v for this pH is adjusted as 3.7 with approx.90 ml acetic acid per liter gradient elution with 1.00ml/min flow rate is with peak shape. The column used in this measurement is C18 5 µm (150 x 4.6 mm). Run time is 55 minutes for diluted standard and 30 minutes for sample, blank, placebo, system suitability, and sensitivity solution. % RSD of the peak responses as per CYHC to six replicate injections should be less than nor equal to 2.0 %. The tailing factor for the peak is mainly CYHC should not be more than 2.0. The theoretical plate count should not be less than 2000. No components were identified to co-elute with the CYHC peak, and the peak purity results indicate that the CYHC peak can therefore be considered spectrally pure. %RSD of impurities 0.05% latter above six measurements are NMT 15.0. %RSD of total impurities to the total of 6 measurements is NMT 10. Impurity is more minor than LOQ, which may not be considered an

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entirely foreign substance. Therefore, the final results obtained by this procedure are rugged. % RSD for intermediate precision is 1.5%. % RSD due to CYHC strength to assay meets requirements to reproducibility as 0.7 % respectively. A linear regression curve is constructed. By accuracy results below, percentage recovery values to CYHC satisfied acceptance criteria to accuracy across 50 % - 150 %. % of individual and mean recovery at every stage lie as 95 to 105 to known impurity to Monohydroxy CYHC. % individual and mean recovery at each stage varied between 80 to 120 to Known contaminant. The correlation and regression coefficient are NLT 0.995 for Linearity and accuracy range parameters. This proposed method is linear in the specified range for CYHC and its related foreign substances. Accuracy, LOD & LOQ of this method is established to CYHC for 50mg/ml injection. Hence, by using this process, it is concluded that this method stands validated and may be utilized in routine with stability analysis.

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Conflicts of interest:
There are no conflicts of interest among the authors who were done this present work.

References

1. "Cyclizine 50mg Tablets - Summary of Product Characteristics (SPC) - (eMC)". www.medicines.org.uk. 27 March 2015. Archived from the original on 20 December 2016. Retrieved 13 December 2016.
2. M. Feldman; L. S. Friedman; L. J. Brandt; "Sleisenger and Fordtran's Gastrointestinal and Liver Disease: Pathophysiology, Diagnosis, Management", 2015; Elsevier Health Sciences; p. 218. ISBN 9781455749898. Archived from the original on 20 December 2016.
3. World Health Organization (2021). "World Health Organization model list of essential medicines: 22nd list 2021". Geneva: World Health Organization. hdl:10665/345533. WHO/MHP/HPS/EML/2021.02.
4. "Cyclizine Side Effects in Detail - Drugs.com". www.drugs.com. Archived from the original on 21 December 2016. Retrieved 13 December 2016.
5. "Diconal Tablets by Amdipharm". Electronic Medicines Compendium. Datapharm. Archived from the original on 1 April 2008. Retrieved 5 August 2008.
6. "Conversion Factors for Controlled Substances". DEA Diversion Control Division. Retrieved 26 March 2020.
7. W. Sneader; "Drug Discovery: A History", 2005; John Wiley & Sons. Ltd; p. 404. ISBN 0-471-89979-8. Archived from the original on 10 September 2017.
8. Marshall Sittig; "Pharmaceutical Manufacturing Encyclopedia, Volume 1", 1988; Noyes Publications; p. 406. ISBN 0-8155-1144-2. Archived from the original on 10 September 2017.

Forest Chemicals Review
www.forestchemicalsreview.com
ISSN: 1520-0191
September-October 2022 Page No. 1016 – 1026
Article History: Received: 06 April 2022, Revised: 28 April 2022, Accepted: 04 May 2022, Publication: 15 May 2022

9. M. M. Abdelrahman; M. Y. Fares; N. S. Abdelwahab; M. A. Hegazy; G. M. El-Sayed; "Ecofriendly Validated Chromatographic Methods for Quantitation of Cyclizine and Its Toxic Impurities in Its Parenteral Formulation", 2021; 84; 155-165. https://doi.org/10.1007/s10337-020-03985-x

10. N. M. Habib; M. M. Abdelrahman; N. S. Abdelwhab; N. W. Ali; "Validated Chromatographic Methods for the Analysis of Two Binary Mixtures Containing Pyridoxine Hydrochloride", Journal of AOAC International; 2017; 100(2); 414–421. https://doi:10.5740/jaoacint.16-0213

11. K. Gislaineet.al. "Development and Validation of a Stability-Indicating HPLC Method for the Determination of Buclizine Hydrochloride in Tablets and OralSuspension and its Application to Dissolution Studies", Quím. Nova; 2012; 35(1); 207-212. https://doi.org/10.1590/S0100-40422012000100036.

12. M. A. Amina; S. M. Sahar; S. H. Omar; A. Ahmed; "Novel potentiometric sensors for determination of ondansetron hydrochloride in pure and dosage form", RSC Adv.; 2021; 11; 34820-34827. https://doi.org/10.1039/D1RA03268B

Recovery of Indian Economy during Post-Covid :Impact of Stimulus Packages

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"What happens in India has a big impact, both in the region and in the world," Luis Breuer, IMF's senior resident representative to India, said in a recent IMF podcast. "You're talking about a large slice of humanity and the global economy."

Introduction:

Corona viruses are a large family of viruses which may cause illness in animals or humans. In humans, several corona viruses are known to cause respiratory infections ranging from the common cold to more severe diseases such as Middle East Respiratory Syndrome (MERS) and Severe Acute Respiratory Syndrome (SARS). This new virus and disease were unknown before the outbreak began in Wuhan, China, in December 2019. COVID-19 is now a pandemic affecting many countries globally (WHO-2020). WHO is working closely with global experts, governments, and other health organizations to provide advice to the countries about precautionary and preventive measures.

Inspite of best measures taken by the concerned Government all over the world, it spread quickly across the globe in the first months of 2020, reaching 10 million confirmed cases towards the end of June. By the end of first week of July, 213 countries and territories around the world have reported a total of 11,231,289 confirmed cases of the virus and 529,909 deaths. It is shocking to notice that the WHO says the pandemic has not yet reached its peak in Central and South America, where Brazil has been the worst-hit so far. It was observed that Brazil is only the second country in the world, after the US, to have confirmed more than one million cases and its death toll stands at more than 60,000. In Qatar, percentage of persons infected to total population is highest, however, percentage of deaths to total infected and to total population is comparatively low when compared with other many countries. In UK, Mexico, Italy and France the percentage of deaths to total infected persons is more than One percent and in Spain, UK, Italy, Sweden, the percentage of deaths to total population is more than 0.05 percent. Europe and North America saw the first major outbreaks in April but as they began to ease, Latin America and Asia started seeking an increase in the cases. The surge in cases in Asia is largely down to widespread outbreaks in India and Pakistan.

Though, In India, Percentage of cases to total population and percentage of deaths to total population is very low when compared most of the other countries of the world, the virus is spreading continuously in major urban centers of the country. There are about 648 thousand confirmed cases where 394 thousand were recovered. The State of Maharashta continues to remain the worst affected with highest number of active cases with about 32 percent of total cases of the country followed by Delhi, Tamil Nadu, Gujarat, Uttar Pradesh. The recovery rate has a positive sign. It is found that about half of the national cases accounted from Mumbai, Delhi, Ahmadabad, Chennai and Thane.

IMPACT ON ECONOMY

Covid-19 is spreading rapidly in India, so to reduce the effect of Covid-19, the period of lock down has been extended till 31st may 2020 and policymakers are worried about how to fight the virus and minimize its impact on the economy therefore, several relief funds have been announced by the Indian Government. Undoubtedly, this Corona virus has put the Indian economy at a major risk. Corona virus ravages the economic foundations of India's trade. According to many experts, economists, policy makers have estimated that its most economic impact can be seen which is as follows.

a) Loss of income due to lack of jobs

b) Possibility of deprivation due to lack of work

c) Production disruption in many areas

d) Due to Global Lockdown Exports and Imports are declined

e) Decline in price of petrol, diesel due to lack of demand

f) According to an SBI research report, the GDP growth rate for the 2020 financial year is currently down 1.1 percent

Initiatives by Government of India:

Government of India has been taking all the necessary steps to ensure that we are prepared well to face the challenge and threat posed by the growing pandemic of COVID-19 the Corona Virus. With active support of the people of India, GOI has been able to contain the spread of the Virus in our country. The most important factor in preventing the spread of the Virus locally is to empower the citizens with the right information and taking precautions as per the advisories being issued by Ministry of Health & Family Welfare.

The outbreak of the Covid-19 pandemic is an unprecedented shock to the Indian economy. To combat with COVID-19, Indian Government initiated lockdown from 23rd March, 2020. The economy was already in a parlous state before Covid-19 struck. With the prolonged country-wide lockdown restrictions for about 100 days, global economic downturn and associated disruption of demand and supply chains, the economy is likely to face a protracted period of slowdown. The magnitude of the economic impact will depend upon the duration and severity of the health crisis, the duration of the lockdown and the manner in which the situation unfolds once the lockdown is lifted. (Mahendra Dev & Rajeswari Sengupta, 2020. "Covid-19 Impact on the Indian economy," Indira Gandhi Institute of Development Research, Mumbai Working Papers 2020-013, Indira Gandhi Institute of Development Research, Mumbai).

According to KPMG, the lockdown in India will have a sizeable impact on the economy mainly on consumption which is the biggest component of GDP. Reduction in the urban transaction can lead to a steep fall in the consumption of non-essential goods. It can be severe if disruption causes by the long lockdown and affect the availability of essential commodities. Due to weak domestic consumption and consumer sentiment, there can be a delay in investment which further add pressure on the growth. We can't ignore that post-COVID-19, some economies are expected to adopt de-risking strategies and shift their manufacturing bases from China. This

can create opportunities for India. According to KPMG, opportunities will largely depend on how quickly the economy recovers and the pace at which the supply chain issues are addressed. According to Du & Bradstreet, COVID-19 no doubt disrupted human lives and global supply chain but the pandemic is a severe demand shock which has offset the green shoots of recovery of the Indian economy that was visible towards the end of 2019 and early 2020. The revised Gross Domestic Product (GDP) estimates for India downwards by 0.2 percentage points for the fiscal year 2020 to 4.8 per cent and by 0.5 per cent for the fiscal year 2021 to 6 per cent. Further, it is stated that the extent of the actual impact will depend upon the severity and duration of the outbreak. Further, according to the World Bank's assessment, India is expected to grow 1.5 per cent to 2.8 per cent. And IMF projected a GDP growth of 1.9 per cent for India in 2020 because the global economy is affected by the COVID pandemic, the worst recession since the Great Depression in the 1930s. Also, we can't ignore that the lockdown and pandemic hit several sectors including MSME, hospitality, civil aviation, agriculture and allied sector.

Prime Minister of India, Narendra Modi announced the first 21 days of India's lockdown on 24 March. During this address to the nation he said, "*Jaan hai toh jahaan hai*" (Only if there is life there will be livelihood). On 11 April, in a meeting with the Chief Minister's of India, the Prime Minister said "Our mantra earlier was *jaan hai toh jahaan hai* but now it is *jaan bhi jahaan bhi* (Both, lives and livelihood matter equally)." On 14 April, another address to the nation was made by Modi in which he extended the lockdown, with adjustments, to 3 May. In the Prime Minister's fifth meeting with the Chief Ministers on 11 May, the Prime Minister said that Indians must prepare for the post corona virus pandemic world, just as the world changed after the world wars. During the meeting Modi said "*Jan se lekar jag tak*" (From an individual to the whole of humanity) would be the new principle and way of life. On 12 May, the Prime Minister addressed the nation saying that the corona virus pandemic was an opportunity for India to increase self-reliance. He proposed the *Atmanirbhar Bharat Abhiyan* (Self-reliant India Mission) economic package. The following day the Finance Minister started laying out the details of the Prime Minister's vision which would continue into the next few days. The Finance Minister stated that the aim was to "spur growth" and "self-reliance", adding that, "self-reliant India does not mean cutting off from rest of the world" (source : Wikipedia).

Stimulus Packages :

India's economy is facing an unprecedented crisis in the wake of coronavirus pandemic, which has prompted the government to come up with a series of economic stimulus packages since the nation-wide lockdown was imposed on March 25. Economic stimulus package is announced by a government at a time when there's a deep slowdown or a nation is facing recession. India has been under lockdown, which forced industries and business houses to shut all kinds activities, bringing the economic growth engine to a grinding halt. The lockdown has not only caused immense problems for the common man but has led to shutting down of small businesses, job losses across the industry, stopping of air and rail travel and has restricted movement of people and goods.

Several economists have already said that India may have entered the recession as 40-day lockdown has caused a major downfall in consumption activity, a major driver of economic

growth. The lockdown has also led to fears of banks' non-performing assets rising manifold in the months to come. As per a Reuters poll, for the first time since the 1990s, India's gross domestic product is expected to shrink. The government is now considering another second dose of relief measures for the common man and a stimulus package for India Inc, which could be significant considering losses suffered in the lockdown period. Rating agency Icra has also estimated the country's GDP might contract by as much as 20 per cent in the June quarter and by 2 per cent in FY21.

Government kept on responding to the unprecedented challenge thrown by the pandemic ever since it announced nationwide lockdown in March 2020. On March 27, 2020 A package of Rs.1.74 lakh crore was announced. This related to freed food supply and direct benefit transfer. The measures included the following:

- 5 kg wheat or rice per person from April to June for 3 months benefiting 800 million people.
- 1 kg pulses for each household for 3 months
- Rs.500 pm for 3 months to be transferred to 200 million women Jan Dhan accounts
- Free Gas cylinders to 80 million poor families for 3 months
- MGNREA wages increased from Rs.182 to Rs.202 per day
- Payment of Rs.2000 top 80 million farmers under PM KISAN
- Ex-gratia of Rs.1000 to 30 million poor senior citizens, widows and differently-abled

RBI announced the following measures

- CRR reduced from 4% to 3% (Liquidity enhancement : Rs1.37 lakh crores
- Repo rate reduced from 5.55 % to 4.40 %
- MSF raised from 2% of SLR to 3% up to June, 2020
- Targeted Long Term Repo Operation Auctions – Rs.2.0 lakh crores
- Liquidity Coverage Ratio reduced from 100% to 80% (till April 2021)(Liquidity: Rs.4.74 lakh crores)
- Banks to avail Rs.1.0 lakh cr at fixed rate of 4.4% for investments
- Special Liquidity facility of Rs.50,000 cr to NBFCs

Atma Nirbhar Bharat Yojana

On May 13, 2020 the PM announced a relief package called Atma Nirbhar Yojana of Rs.20 lakh crores.

With the world's worst outbreak of COVID pandemic stalling a nascent economic recovery, the government has begun assessing the impact of the second wave of infections on different sectors and may look at providing support at an appropriate time to segments requiring

fiscal help. Recently, the RBI also announced a loan restructuring scheme for small borrowers amid the pandemic.Fiscal stimulus package for supporting growth may focus on the following three priorities: (a) income support measures for the vulnerable rural and urban populations, (b) additional vaccination expenditure in view of the Center's recent commitment for financing 75% of India's total vaccine procurement and (c) additional capital expenditure directed mainly towards expanding health sector infrastructure. The additional support for these three priorities may respectively be kept at INR1.0 lakh crore for (a), INR0.35 lakh crore for (b) and at INR1.0 lakh crore for (c) in view of the limited fiscal space. Together, these add to INR2.35 lakh crore of which around INR0.65 lakh crore can be accessed by restructuring of budgeted expenditure on other heads leaving a balance of INR1.7 lakh crore which would constitute an additionality of 0.8% points of GDP on the budgeted fiscal deficit of 6.7% of estimated GDP. Thus, the fiscal deficit would need to be increased to 7.9% of GDP in FY22 so as to cover the revenue shortfall of 0.4% points and the expenditure additionality of 0.8% points of GDP. Some of the economic indicators, including the Goods and Services Tax (GST) collections, still provide confidence and incoming data will throw some more light on the state of the economy, sources said. However, the sources said that fiscal stimulus would be effective only once local lockdowns ease and restrictions on business due to curfews are lifted.

Services sectors like hospitality, tourism and aviation which had just started recovering were hit hard by the second wave of COVID, the sources said, adding these segments might need some support on an urgent basis from the government. Besides, MSME sector, the second highest employer after agriculture, has been impacted very much and would require financial assistance. Most of the states have imposed curfews in their states to contain spiralling COVID infections and deaths. Also, the second COVID wave has dealt a blow to both consumer and investor sentiments, which also need to be lifted.

Both monetary and fiscal policies are directed towards boosting the Indian economy which is expected to grow in double digits during the current financial year despite the devastating second wave as per the revised outlook by many organisations, including the Reserve Bank of India (RBI), the sources pointed out. The RBI retained its growth forecast at 10.5 per cent for the current financial year while ADB projected a growth rate of 11 per cent for this fiscal. However, rating agencies have lowered growth forecasts for India saying that the second wave of infections will hamper economic recovery. They, however, projected that the negative impact on economic output will be limited to the April-June quarter.

As GDP shrunk by 23.9 percent in this quarter, the share of the less affected sectors in total GVA have also increased (Table 1). Some sectors like agriculture and tourism are seasonal, and therefore less impact on some of these may be owing to seasonal factors. However, manufacturing, trade, hotels, transport, communication and broadcasting services, and construction are the key sectors which have to drive the revival. Under the current structure of Indian economy, nobody would expect the revival to be driven by agriculture and/or public utility services and/or public administration services.

TABLE 1: Share of sectoral GVAs in total GVA in Q1 (April-June) of 2019-20 and 2020-21 (at 2011-12 prices)					
	2019-20		2020-21		
GVA at basic prices from	Q1	Share in total GVA (%)	Q1	Share in total GVA (%)	20-21Q1 growth direction
Agriculture, forestry & fishing	439842.6	13.3	454657.9	17.8	⇑
Mining & quarrying	92806.5	2.8	71208.9	2.8	⇓
Manufacturing	578936.5	17.5	351395.6	13.8	⇓
Electricity, gas, water supply& other utility services	81627.9	2.5	75876.7	3.0	⇓
Construction	262827.8	7.9	130749.6	5.1	⇓
Trade, hotels, transport, communication and services related to broadcasting	630860.5	19.1	334284.4	13.1	⇓
Financial, real estate & professional services	803322.2	24.3	760490.9	29.8	⇓
Public administration, defence & other services	417483.0	12.6	374656.2	14.7	⇓
GVA at Basic Price	3307707	100.0	2553320	100.0	⇓
* Absolute GVA figures are in Rs. crore					
Data Source: Ministry of Statistics and Programme Implementation (MoSPI)					

To sum up the evaluation of the key sectors of the economy:

- Agriculture, forestry and fishing is the only sector with positive growth. This is a good development for the economy, but revival cannot be agriculture-based under current circumstances. Manufacturing has to come out of its slowdown.
- Financial, real estate and professional services are relatively less badly affected and should recover in reasonable time. But the recovery will be dependent on industrial and consumer demand recovery.
- The same is true for construction and internal trade, tourism, transport, communication sectors. Though construction shows signs of rebound, tourism is likely to suffer the most. That may eclipse some possible gains in communication during the pandemic. However, till the detailed sectoral data are available, overall revival of this bouquet of some important large sectors of the economy will be dependent on real sector activities and how these sectors cope up with periodic disruptions due to the pandemic.

- Electricity, gas, water and public utility services and public administration, defence other services are likely to get back to normalcy sooner than other sectors. But these can propel economy into the desired growth path alone, and once again long-term stability these sectors will depend upon overall revival.

A Proof of Economic Revival

Prevailing global trade tensions of the last two years and the huge external shock of t COVID-19 pandemic have initiated discussions on self-reliance. Propagating a self-reli Indian economy assumes an inward-looking future global economy. However, there is contradiction in that proposition. If every country strives to import less and export more, th international trade would be shrinking. That further implies reduction of exports for all maj exporting countries. A deeper introspection is required. India posted the world's third highe growth rate in overall and manufacturing exports, since the early 1990s. Contrary to popul perceptions, India did reasonably well in high-skill manufacturing exports while underperformed in low-skill manufacturing exports. The country is still in a position to gai future export opportunities, particularly in low-skill manufacturing like clothing and footwea Grabbing those future opportunities entail more global integration and trade engagement Overemphasis on self-reliance may be inimical to that. Losing export as an important futur driver of growth would be imprudent, to say the least.

- Nomura's India Business Resumption Index (NIBRI), a measurement for tracking the exten of normalisation in the economy, hit 98.1 points in February 2021. It had hit a record low a 44.8 in April-June during the national lockdown.
- The economy has reflected growth prospects due to fiscal activism.
- FDI equity inflow in India stood at US$ 49.97 billion in 2019-20.
- Foreign Portfolio Investment has been Rs. 12.9 trillion (US$ 174.31 billion) in India between 2020 and 2021 (as of September 2020).
- The National Statistical Office estimated real GDP growth to be (-) 7.7% as opposed to (-) 10.3% projected by IMF in October 2020.
- RBI's monetary policy committee had projected a GDP of (-) 7.5%.
- Merchandise imports experienced a growth of 7.6 per in December 2020.
- The rise in the imports of pearls and precious stones, machinery and electric goods shows the revival of Domestic Economic Activities.
- The BSE index jumped 91 per cent from a record low of 25,881 in over 10 months.
- The GST collection was Rs 1.15 lakh crore in December 2020.
- The collection was the highest since the GST implementation. Read more on Goods and Services Tax (GST).

It is observed that, in India, a little bit of complacency due to fewer Covid positive cases during January to March and not highlighting to the public the spread of the United Kingdom,

Brazil, and South Africa mutant virus variants has resulted in the present situation. There is evidence in history that a hundred years ago during the 1918 Spanish Flu pandemic a similar more dangerous second wave was witnessed due to similar complacency at that point of time. It is time to implement the learnings from history. Therefore, the needs of the hour are – all steps to be taken to control infection, strict containment in localities having a large number of positive cases, following Covid appropriate behaviors, no crowding, increasing the vaccination drive, and strengthening the hospital infrastructure such as providing more number of hospital beds and ensuring the supply of oxygen. These are the panacea needed urgently to bring the economy back on track for higher growth.

Conclusion:

However, the most worrying factor at present is rising poverty globally as well as in India for the first time in the last 20 years due to disruption by the pandemic. The number of people below the poverty line (BPL) increased to 50 million in India and 95 million globally. A two-decade-long trend of poverty reduction has reversed. As per the World Bank's estimates, global poverty is expected to rise to 150 million by the end of 2021 depending on overall economic contraction. Extreme poverty, which is defined as living on less than $1.90 a day, is likely to affect between 9.1 percent and 9.4 percent of the world population. There is an urgent need for targeted and localised lockdowns only. Revenue expenditure needs to be increased in India to target income support measures, which is extremely important now to maintain livelihood and contain poverty. Growth is expected in certain sectors whereas poverty is a reality. As per the IMF's forecast, there will be a 9 percent hit to per capita income from 2020 to 2022.

However, there is hope for economic recovery and stabilization. Because, unlike the first wave, we have vaccines this time. It is reasonable to expect that the pace of new infections will slow down as vaccinations pick up. Whether or not India's favourable GDP revisions undergo a downgrade will depend on how fast vaccinations pick up, which will determine the time it will take to flatten the second wave. Fiscal support through an institutional mechanism – for instance through the creation of special-purpose vehicles, is required to support stages of vaccine production; its distribution through a decentralised supply-chain process for all demographic groups, and a fund to provide money to those in the private sector who can produce vaccines on a large scale.

References :

1. Dev & Rajeswari Sengupta, 2020. "*Covid-19: Impact on the Indian economy*," Indira Gandhi Institute of Development Research, Mumbai Working Papers 2020-013, Indira Gandhi Institute of Development Research, Mumbai).
2. Souvik Dubey, Payel Biswas and Carl J. Lavie. "*Psychological impact of COVID-19*", Elsevier, nibi.nlm.nih.gov, 2020
3. https://home.kpmg/in/en/home/insights/2020/03/the-business-implications-of-coronavirus.html Du & https://www.dnb.co.in/perspective/covid-19
4. https://covid19.who.int/?gclid=Cj0KCQjw9IX4BRCcARIsAOD2OB1KDxBeRsF21tnfJbAKmqd_vOXihBUEd-DHtGOIyFl57lBbYUWoNpsaAkqOEALw_wcB

https://wikipedia+on+coronavirus&oq=wikipedia+on+orona&aqs=chrome.1.69i57j0l5.1166 8j0j4&sourceid=chrome&ie=UTF-8

5. www.worldometers.info/https://www.who.int/emergencies/diseases/novel-coronavirus-2019/question-and-answers-hub/q-a-detail/q-a-coronaviruses
6. https://en.wikipedia.org/wiki/Coronavirus_disease_2019
7. https://en.wikipedia.org/wiki/COVID-19_pandemic_in_India
8. https://prsindia.org/Covid-19/cases
9. https://timesofindia.indiatimes.com/city/bengaluru/worst-case-scenario-india-to-have-351-cases-by-sept-1/articleshow/76987434.cm
10. https://www.deccanherald.com/national/coronavirus-india-march-25
11. www.covid19.who.int
12. Aatmanirbharbharat.mygov.in (2020)

Marshall-Olkin Stereographic Circular Logistic Distribution

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Abstract

Marshall and Olkin (1997) proposed an interesting method of adding a new parameter to the existing distributions. The resulting distributions are called the Marshall-Olkin distributions, these distributions include the original distributions as a special case and are more flexible and represent a wide range of behavior than the original distributions. In this paper, a new class of asymmetric stereographic circular logistic distribution is introduced by using Marshall-Olkin transformation on stereographic circular logistic distribution (Dattatreyarao et al (2016)), named as Marshall-Olkin Stereographic Circular Logistic Distribution. The proposed model admits closed form density and distribution functions, generalizes the stereographic circular logistic model and is more flexible to model various types of data (symmetric and skew-symmetric circular data).

Keywords:*Characteristics, Stereographic circular logistic distribution, circular data, Marshall-Olkin transformation, l-axial data.*

1. Introduction

Directions in two-dimensions can be represented as points on the circumference of a unit circle and models for representing such data are called circular distributions. There are many areas that deals with directional/circular data such as animal, birds navigation, sound waves in physics, wind direction in meteorology, image analysis in computer science and vector cardiology in medical science, to name some of them. For modeling circular data, Dattatreyarao et al (2007), Jammalamadaka and SenGupta (2001)etc derived many models for such data. Most of these models are typically symmetric around some center and very few asymmetricaldistributions are available for describing circular data. In this paper, we describe a broad class of model for the asymmetric case and discuss anapplication.

Phani (2013) constructed new circular models, Stereographic Circular Logistic, Stereographic Extreme-Value, Stereographic Double Exponential and some Stereographic Semicircular models by applying Inverse Stereographic Projection on linear models.Adding parameters to a well-established distribution is a time honored device for obtaining more flexible new families of distributions. Marshall and Olkin(1997) proposed an interesting method of adding a new parameter to the existing distributions. The resulting distributions, called the Marshall-Olkin (MO) distributions, which includes the parent distributions as a special case and gives more flexible to model various types of data. In this paper, we use the Marshall-Olkin transformation to derive a new flexible circular model, so called The Marshall-Olkin Stereographic Circular Logistic Distribution, which generalizes the baseline distribution (i.e., Stereographic Circular Logistic Distribution) for modeling circular data.

2. Methodology of Marshall-Olkin Transformation

An ingenious general method of adding a parameter to a family of distribution is introduced by Marshall-Olkin (1997). Here we adapt the idea of Marshall-Olkin to circular case for deriving more flexible circular models. For a circular random variable θ with a distribution function $F(\theta)$, we can obtain a new family $G(\theta)$ which contains one more parameter is given by $$G(\theta)=\frac{F(\theta)}{\alpha+(1-\alpha)F(\theta)},\ \theta\in[-\pi,\pi)\ , \qquad (1)$$

Where $F(\theta)$ is a distribution function and $\alpha>0$ .If $\alpha=1$ then we have $G(\theta)=F(\theta)$ and the probability density function corresponding to (2.1), say $g(\theta)$ takes the form $$g(\theta)=\frac{\alpha f(\theta)}{\left(1-(1-\alpha)(1-F(\theta))\right)^2},\ -\pi\le\theta<\pi. \qquad (2)$$

Where $F'(\theta)=g(\theta)$ is the baseline density function.

3. Marshall–Olkin Stereographic Circular Logistic Distribution

Here probability density and distribution functions of Stereographic Circular Logistic Distribution are revisited Phani (2013).

Stereographic Circular Logistic Distribution

A random variable θ on unit circle is said to have stereographic circular logistic distribution with location parameter μ and scale parameter $\sigma>0$ denoted by SCLG(μ,σ) , if the probability density and cumulative distribution functions are respectively given by

$$f(\theta)=\frac{1}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right),$$

Where $\sigma>0$ and $-\pi\le\theta,\mu<\pi$

$$F(\theta)=\left[1+\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1}\ ,\text{where } \sigma>0,\ -\pi\le\theta<\pi.$$

Then by applying Marshall-Olkin transformation on stereographic circular logistic distribution, we obtain more flexible asymmetric circular distribution, we call it as Marshall-Olkin Stereographic Circular Logistic Distribution.

Definition:

A random variable θ on unit circle is said to have Marshall-Olkin stereographic circular logistic distribution with location parameter μ, scale parameter $\sigma > 0$ and tilt parameter $\alpha > 0$ denoted by MOESCLG(μ, σ, α), if the probability density and cumulative distribution functions are respectively given by

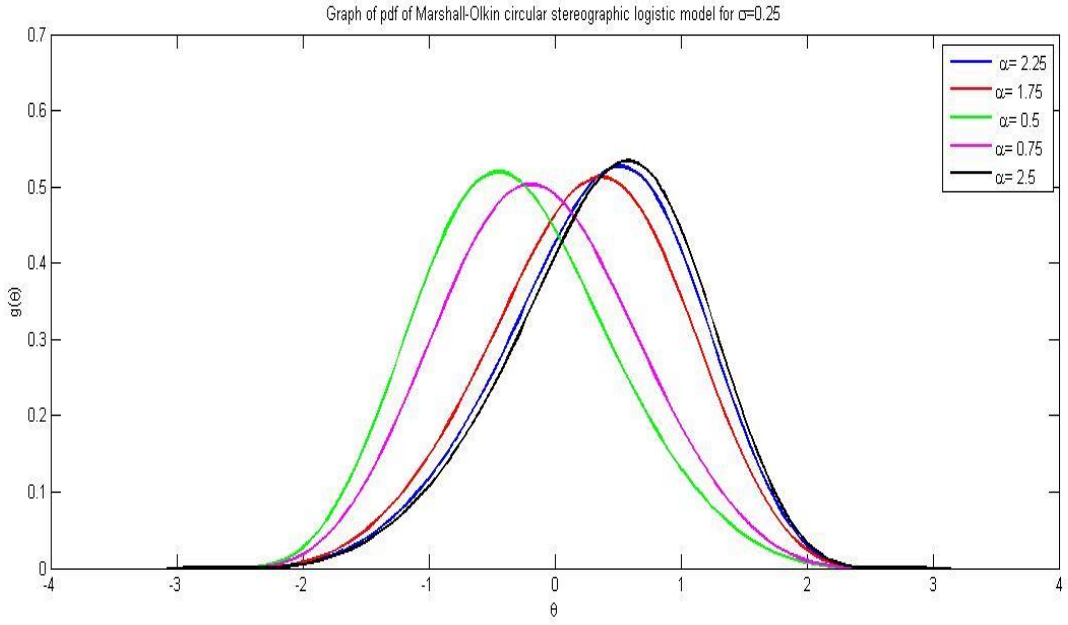
$$g(\theta) = \frac{\alpha}{2\sigma}\sec^2\left(\frac{\theta}{2}\right)\left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-2}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)-\mu}{\sigma}\right)\right), \qquad (3)$$

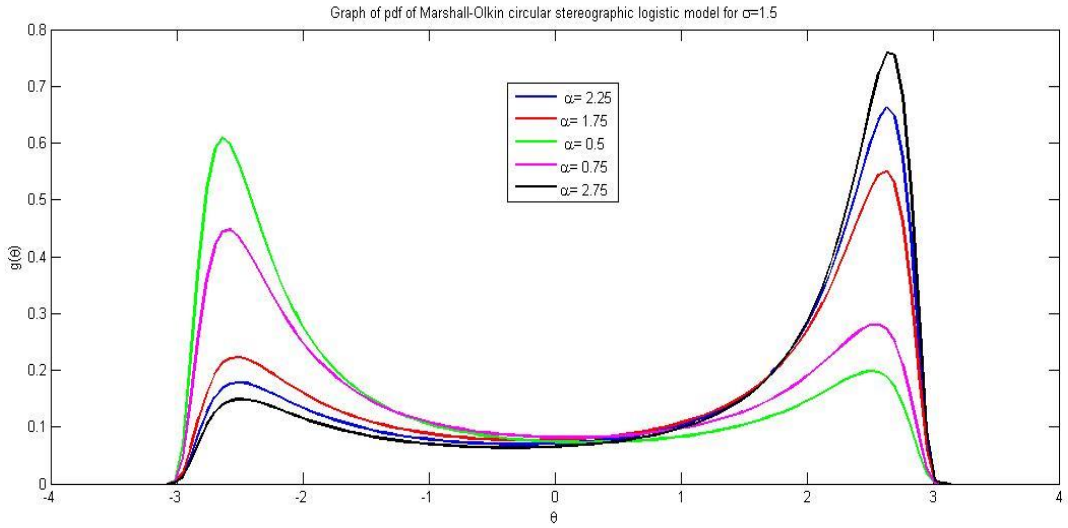
Where $\sigma, \alpha > 0$ and $-\pi \le \theta, \mu < \pi$

$$G(\theta) = \left[1+\alpha\exp\left(\frac{-\left(\tan\left(\frac{\theta}{2}\right)-\mu\right)}{\sigma}\right)\right]^{-1}, \text{where } \sigma, \alpha > 0, -\pi \le \theta < \pi. \qquad (4)$$

This distribution is asymmetric for $\alpha \neq 1$ and symmetric for $\alpha = 1$. Marshall-Olkin Stereographic logistic distribution is **unimodal** if $\sigma < 0.5$ and **bimodal** if $\sigma > 0.5$

Graphs of probability density function of Marshall-Olkin Stereographic Circular Logistic Distribution for various values of σ, α and $\mu = 0$ are presented here.





4. Extension to l-axial Marshall-Olkin Stereographic Circular Logistic Distribution

We extend the proposed model to the l -axial distribution, which is applicable to any arc of arbitrary length say π/l for $l=1,2,...,$ so it is desirable to extend the Marshall-Olkin stereographic circular logistic distribution. To construct the l -axial Marshall-Olkin stereographic circular logistic distribution, we consider the density function of Marshall-Olkin stereographic circular logistic distribution and use the transformation $\phi=\theta/l$, $l=1,2,...,$. The probability density function of ϕ is given by

$$g(\phi)=\frac{\alpha l}{2\sigma}\sec^2\left(\frac{l\phi}{2}\right)\left[1+\alpha e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}\right]^{-2}e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}},\text{ where }\quad -\frac{\pi}{l}<\phi<\frac{\pi}{l} \tag{5}$$

Case (1) when $l=1,$ the probability density function (6.1) is the same as that of Marshall-Olkin Stereographic Circular Logistic Distribution.

$$g(\phi)=\frac{\alpha}{2\sigma}\sec^2\left(\frac{\phi}{2}\right)\left[1+\alpha e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}\right]^{-2}e^{-\frac{\tan\left(\frac{\phi}{2}\right)}{\sigma}}\quad\text{where}\quad -\pi<\phi<\pi \tag{6}$$

,

Case (2) When $l=2$, the probability density function (6.1), the probability density function is

$$g(\phi)=\frac{\alpha}{\sigma}\sec^2(\phi)\left[1+\alpha e^{-\frac{\tan(\phi)}{\sigma}}\right]^{-2}e^{-\frac{\tan(\phi)}{\sigma}}\quad\text{where}\quad -\frac{\pi}{2}<\phi<\frac{\pi}{2} \tag{7}$$

,

It is called as Marshall-Olkin Stereographic Semicircular Logistic Distribution.

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References

1. DattatreyaRao, A.V., RamabhadraSarma, I. and Girija, S.V.S. (2007), On Wrapped Version of Some Life Testing Models, *Comm Statist,-Theor.Meth.* , 36, issue # 11, pp.2027-2035.
2. Dattatreyarao A.V, Phani.Y and Girija, S.V.S., (2016), Stereographic Logistic Model – Application to Noisy Scrub Birds Data, Chilean Journal of Statistics, Vol. 7, No. 2, 69-79.
3. Gradshteyn and Ryzhik (2007). Table of Integrals, series and products, 7th edition, Academic Press.
4. Jammalamadaka S. Rao and Sen Gupta, A. (2001), Topics in Circular Statistics, *World Scientific Press, Singapore.*
5. Mardia, K.V. andJupp, P.E. (2000), Directional Statistics, *John Wiley, Chichester.*
6. Marshall A.W. and I. Olkin, A new method for adding a parameter toa family of distributions with application to the exponential and Weibullfamilies, Biometrika 84 (1997), no. 3, 641-652.
7. Phani Y. (2013), On Stereographic Circular and Semicircular Models, Thesis submitted to AcharyaNagarjuna University for the award of Ph.D.
8. Toshihiro Abe, Kunio Shimizu and Arthur Pewsey, (2010), Symmetric Unimodal Models for Directional Data Motivated by Inverse Stereographic Projection, *J. Japan Statist. Soc.*, Vol. 40 (no. 1), pp. 45-61.
9. Wenhao Gui (2013). Marshall-Olkin Extended Log-Logistic Distribution and its Application in Magnification Processes, Applied Mathematical Sciences, Vol. 7, , no. 80, 3947-3961.

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ESTIMATION OF PARAMETERS OF PERT DISTRIBUTION BY USING MAXIMUM PRODUCT OF SPACINGS METHOD

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ABSTRACT

The PERT Distribution is one of the most important distributions for practical use in business, because it is widely used to generate random values within a range in the financial models and simulations in the area of processes and analysis in general. Present Maximum Product Spacings of the unknown parameters of PERT distribution using Newton-Raphson iterative procedure. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Efficiency (RE) for both the parameters under sample based on 10,000 simulations to assess the performance of the estimators. A simulation study is conducted to evaluate the performance of the Maximum Product of Spacings estimates.

Keywords: PERT distribution, Maximum Product Spacings, Averages, Parameters.

INTRODUCTION

The PERT distribution (also known as the Beta-PERT distribution) gets its name because it uses the same assumption about the mean as PERT networks. It is a version of the Beta distribution and requires the same three parameters as the Triangle distribution, namely minimum (a), mode (b) and maximum (c). The PERT method implies overweighting the 'most likely' estimate. It transforms the three-point estimate into a bell-shaped curve and allows determining probabilities of ranges of expected values. The PERT Distribution was originally developed within the 1950s for the Polaris weapon to calculate a probable time-frame for completion of the project supported optimistic, pessimistic, and mode likely time frames. Nowadays, it is used for project completion time analysis in Program Evaluation and Review Technique (PERT). PERT may be a modeling technique to estimate completion time or other desired event, bases on best estimates for the minimum, maximum, and presumably values for the event. The PERT distribution also uses the foremost likely value, but it's designed to get a distribution that more closely resembles realistic probability distribution. Depending on the values provided, the PERT distribution can provide an in depth fit the traditional or lognormal distributions.

The PERT distribution came out of the necessity to explain the uncertainty in tasks during the event of the Polaris missile (Clark, 1962). The PERT distribution, a bit like Triangulum , will produce only one shape from its three parameters. Thus, we are restricted to accepting this interpretation, or creating our own. The modified-PERT distribution may be a quick alternative approach first proposed in Vose (2000) . Malcolm et al studied the generalized biparabolic distribution (GBP) as a good candidate

to be utilized as the distribution underlying to PERT. The Pert Distribution is one among the foremost important distributions for practical use in business, because it's widely wont to generate random values within a variety within the financial models and simulations within the area of processes and analysis generally . Distribution Pert, also as a variant called Modified Pert Distribution, may be a particular case of Generalized Beta Distribution, encompassing a good range of distributions with values within the defined range. The mean formula in PERT, where may be a weighting where mode influences twice than the ends. Note that mean is different from mode. If the mode is closer to the minimum, the tail is longer to the utmost direction, bringing mean for the utmost side and the other way around . This distribution is widely wont to model project duration in PERT analysis, where its name originates. In this model, the user specifies mode (most common value), minimum and maximum. From these data, the distribution is totally defined. Pert are often wont to estimate project duration, costs, margins, markups, turnover and, finally, many variables within the business world.

Torabi (2008) proposed a general method of Minimum Spacings Distance Estimators and a related method of hypothesis testing based on Spacings. Consider different parameter estimation methods in seventy three extensive Generalized Half Log Logistic distribution based on Complete and Censored Data by Torabi and Bagheri (2010). Ramamohan *et al* (2011) studied using Minimum Spaceing Square Distance Estimation Method from an optimally constructed grouped sample for Estimation of Scale parameter (σ) when Shape parameter (β) is known in Log Logistic Distribution. We know that the Maximum Likelihood method of estimation and the Moments method of estimation are most general methods of estimation. Although Maximum Likelihood Estimation method is advantageous in the good judgment of its efficiency and has good theoretical properties, there is confirmation that it does not execute well, in particular in the case of small samples.

The method of moments is simply applicable and often gives precise forms for estimators of unknown parameters. There are, however, cases where the method of moments does not give explicit estimators (e.g., for the parameters of the Gompertz and Weibull distributions).This spacings-based estimation process provides an alternative to the fixed parametric estimation methods like the method of moments, minimum two, Maximum Likelihood (ML), and so on. Cheng and Amin (1979, 1983) studied the estimation method that generalizes the initiative contained in the Maximum Spacings Estimator (MSPE) and separately discussed by Ranneby (1984) and enjoys similar compensations. Cheng and Amin (1983) communicate that in such situations as a three-parameter Lognormal Gamma, Weibull distribution where the ML method breaks down due to unboundedness of the likelihood, the Maximum Spacings Estimation (MSPE) method produces reliable and asymptotically resourceful estimators. In some situations like mixtures of normals where the MLE is known to turn out inconsistent estimators, the MPS estimators are consistent (see Ranneby, 1984). Kaushik Ghosh (2001) considered a general estimation method using spacings it is shown that the Maximum Spacing Estimator is asymptotically most efficient within the subclass of spacings based estimators. Ehab Mohamed Almetwally *et al* (2019) calculated the Maximum Product Spacings and Bayesian Method for Parameter Estimation for Generalized Power Weibull Distribution under Censoring Scheme. Yongzhao Shao (2001) deliberated the Consistency of the Maximum Product of Spacings Method and Estimation of a unimodal distribution.

Recently many of authors studied some of the different estimation procedures like Vijaya lakshmi, Raja Sekharam and Anjaneyulu (2018) studied Estimation of Scale (λ) and Location (μ) of two-parameter Rayleigh distribution by using Median Ranks estimated method. Vijaya lakshmi, Raja Sekharam and Anjaneyulu (2019) studied Estimation of Scale (θ) and Shape (α) parameters of Power Function Distribution By Least Squares Method using Optimally Constructed Grouped data. Vijaya lakshmi and Anjaneyulu (2019) studied estimation of Location (μ) and Scale (λ) for two-parameter Half Logistic Pareto Distribution (HLPD) by Least Square Regression Method. Vijaya lakshmi and Anjaneyulu (2019) studied Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method.

Rajwant Kumar Singh Kumar Singh, Sanjay (2016) discussed the Method maximum of product of spacings is used to estimate the parameters of the model along with reliability and hazard functions. The proposed estimators are compared with the corresponding maximum likelihood estimators on the idea of Monte Carlo Simulation study.

E. M. Almetwally & H. M. Almongy & M. K. Rastogi & M. Ibrahim, (2020) The adaptive type-II progressive censoring schemes of maximum product spacing will be discussed. This article discusses the estimation of the Weibull parameters using the maximum product spacing and the maximum likelihood estimation methods. We also discuss the construction of reliability estimation of adaptive type-II progressively censored reliability sampling schemes for the Weibull distribution to determine the optimal adaptive type-II progressive censoring schemes.

In this chapter, we discuss about the estimation procedure for the unknown parameters for PERT distribution. The idea behind the maximum product spacings parameter estimation is to determine the parameters for given the sample data. We present MPS of the unknown parameters of PERT distribution using Newton-Raphson iterative procedure. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Efficiency (RE) for both the parameters under sample based on 10,000 simulations to assess the performance of the estimators. A simulation study is conducted to evaluate the performance of the Maximum Product of Spacings estimates. Finally, the proposed estimation method is applied on real and generalized data sets the results are given. Which illustrate the maximum product of spacings is a powerful alternative to maximum likelihood estimation of unknown parameters for PERT distribution.

A random variable X ~ PERT (a, b, c) has probability density function and is in the form

$$f_{PERT}(x;\ a,b,c) = \frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}};\ \text{a < x < c} \qquad \dots (1)$$

$$\alpha = \frac{4b+c-5a}{c-a}\ ;\quad \beta = \frac{5c-a-4b}{c-a}$$

(a, b, c) are parameters of PERT distribution.

A random variable X ~ PERT (a, b, c) has cumulative distribution function and is in the form

$$F_{PERT}(x;\ a,b,c) = \frac{(-1)^{\alpha}\beta(\frac{z}{z-1};\ a,\ 1-a-b)}{\beta(\alpha,\ \beta)};\ \text{a < x < c} \qquad \dots (2)$$

Here, $z = \frac{x-a}{c-a}$

A random variable X ~ PERT (a, b, c) has Quantile function and is in the form
The pth quantile x_p of PERT distribution is of the equation.

$$x_p = a+(c-a)\frac{\alpha+(p-\frac{5}{6})}{\alpha+\beta+\left(p-\frac{7}{6}\right)} \qquad \dots (3)$$

Let U ~ U(0,1), then equation (5.3) can be used to simulate a random sample of size 'n' from the PERT distribution as follows.

$$x_i = a+(c-a)\frac{\alpha+(u_i-\frac{5}{6})}{\alpha+\beta+\left(u_i-\frac{7}{6}\right)},\ i = 1, 2, \dots, n. \qquad \dots (4)$$

AXIMUM PRODUCT SPACESTIMATION OF PARAMETERS OF PERT DISTRIBUTION MINGS (MPS) METHOD

Let $D_i = F_{PERT}(X_i) - F_{PERT}(X_{i-1})$ for i = 1 to n, be the uniform spacings of a random sample from the PERT distribution,

Where

$F_{PERT}(X_0) = 0$

$F_{PERT}(X_{n+1}) = 1$ and

$$D_i = \begin{cases} D_1 = F_{PERT}(X_1) \\ D_i = F_{PERT}(X_i) - F_{PERT}(X_{i-1}) = F_{PERT}\left(X_{(2:m)}\right);\ i = 2, \dots, m. \\ D_m = 1 - F_{PERT}(X_m) \end{cases}$$

Clearly$\sum_{i=1}^{n+1} D_i = 1$. The MPS estimates, $\hat{a}_{MPS}^2$ and $\hat{c}_{MPS}$ are obtained by maximizing the Geometric mean of spacings,

$$G = [\prod_{i=1}^{n+1} D_i]^{1/n+1} \quad \ldots (5)$$

With respect to $\alpha,\ \beta$ or, equivalently by maximizing the logarithm of the Geometric mean of sample spacings:

$$H(\alpha,\ \beta) = \frac{1}{n+1}\sum_{i=1}^{n+1} log\ D_i(\alpha,\ \beta) \quad \ldots (6)$$

The estimates $\hat{\alpha}_{MPS}^2$, $\hat{\beta}_{MPS}^2$ and of the parameters $\alpha,\ \beta$ can be obtained by solving the following non-linear equations

$$H(\alpha,\ \beta) = \frac{1}{n+1}\sum_{i=1}^{n+1} \log\left[\frac{\beta(z_i;\ \alpha,\ \beta)-\beta(z_{i-1};\ \alpha,\ \beta)}{\beta(\alpha,\ \beta)}\right] \quad \ldots (7)$$

$$\frac{dH(\alpha,\beta)}{d\alpha} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\{[\Psi_0(\alpha)-\Psi_0(\alpha+\beta)]-[\Psi_0(z_i;\ \alpha)-\Psi_0(z_i;\alpha+\beta)]-[\Psi_0(z_{i-1};\ \alpha)-\Psi_0(z_{i-1};\alpha+\beta)]\} = 0 \quad \ldots (8)$$

$$\frac{dH(\alpha,\beta)}{d\beta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\{[\Psi_0(\beta)-\Psi_0(\alpha+\beta)]-[\Psi_0(z_i;\ \beta)-\Psi_0(z_i;\alpha+\beta)]-[\Psi_0(z_{i-1};\ \beta)-\Psi_0(z_{i-1};\alpha+\beta)]\} = 0 \quad \ldots (9)$$

Let we take,

$\Delta_1 = \Psi_0(\alpha)-\Psi_0(\alpha+\beta)$

$\Delta_2 = \Psi_0(z_i;\ \alpha)-\Psi_0(z_i;\alpha+\beta)$

$\Delta_3 = \Psi_0(z_{i-1};\ \alpha)-\Psi_0(z_{i-1};\alpha+\beta)$

$\Delta_4 = \Psi_0(\beta)-\Psi_0(\alpha+\beta)$;

$\Delta_5 = \Psi_0(z_i;\ \beta)-\Psi_0(z_i;\alpha+\beta)$

$\Delta_6 = \Psi_0(z_{i-1};\beta)-\Psi_0(z_{i-1};\alpha+\beta)$

$$\Psi_0(\alpha) = (-1)^{n+1}n!\sum_{k=o}^{\infty}\frac{1}{(\alpha+k)^{n+1}} \quad \ldots (10)$$

$$\Psi_0(\alpha+\beta) = (-1)^{n+1}n!\sum_{k=o}^{\infty}\frac{1}{(\alpha+\beta+k)^{n+1}} \quad \ldots (11)$$

$$\Psi_0(z_i;\ \alpha) = (-1)^{n+1}n!\sum_{k=o}^{\infty}\sum_{i=1}^{n}\frac{z_i}{(\alpha+k)^{n+1}} \quad \ldots (12)$$

$$\Psi_0(z_i;\alpha+\beta) = = (-1)^{n+1}n!\sum_{k=o}^{\infty}\sum_{i=1}^{n}\frac{z_i}{(\alpha+\beta+k)^{n+1}} \quad \ldots (13)$$

$$\Psi_0(z_{i-1};\ \alpha) = (-1)^{n+1}n!\sum_{k=o}^{\infty}\sum_{i=1}^{n}\frac{z_{i-1}}{(\alpha+k)^{n+1}} \quad \ldots (14)$$

$$\Psi_0(z_{i-1};\alpha+\beta) = (-1)^{n+1}n!\sum_{k=o}^{\infty}\sum_{i=1}^{n}\frac{z_{i-1}}{(\alpha+\beta+k)^{n+1}} \quad \ldots (15)$$

The reduced form of equations (8) and (9) are becomes

$$\frac{dH(\alpha,\beta)}{d\alpha} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\{\Delta_1 - \Delta_2 - \Delta_3\} = 0 \quad \ldots (16)$$

$$\frac{dH(\alpha,\beta)}{d\beta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\{\Delta_4 - \Delta_5 - \Delta_6\} = 0 \quad \ldots (17)$$

But the equations have to be performed numerically using nonlinear optimization techniques.

Note that if $x_{i+k} = x_{i+k-1} = \ldots = x_i$. We get $D_{i+k-1}(\alpha, \beta) = \cdots = D_i(\alpha, \beta) = 0$. Therefore, the MPS estimators are sensitive to closely spaced observations, especially ties. When the ties are due to multiple observations, $D_i(\alpha, \beta)$ should be replaced by the corresponding likelihood $f_{PERT}(x_i, \alpha, \beta)$.

Since $x_i = x_{i-1}$. For the Exponentiated Exponential Gompertz (EEG) distribution, the MPS estimators are asymptotically normally distributed (see Cheng *et a*l (1983)) with joint bi-variate normal distribution given by

$$(\hat{\alpha}^2_{MPS}, \hat{\beta}_{MPS}) \sim N[(\alpha, \beta), I^{-1}(\alpha, \beta)] \text{ for } n \to \infty \quad \ldots (18)$$

Where $I(\alpha, \beta)$ is Fisher information matrix

$$I(\alpha, \beta) = -\begin{bmatrix} I_{11}(\alpha, \beta) & I_{12}(\alpha, \beta) \\ I_{21}(\alpha, \beta) & I_{22}(\alpha, \beta) \end{bmatrix} \quad \ldots (19)$$

$$I_{11}(\alpha, \beta) = \frac{dH(\alpha,\beta)}{d\alpha}$$

$$I_{22}(\alpha, \beta) = \frac{dH(\alpha,\beta)}{d\beta}$$

$$I_{12}(\alpha, \beta) = I_{21}(\alpha, \beta) = \frac{d^2H}{d\alpha d\beta}$$

$$\frac{d^2H}{d\alpha d\beta} = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{1}{D_i(\alpha,\beta)}\{[\Psi_0(\beta) - \Psi_0(\alpha+\beta)][\Psi_0(\beta) - \Psi_0(\alpha+\beta)] - \Psi_0(\alpha+\beta) -$$

$$[\Psi_0(z_i;\alpha) - \Psi_0(z_i;\alpha+\beta)][\Psi_0(z_i;\beta) - \Psi_0(z_i;\alpha+\beta)] - -\Psi_0(z_i;\alpha+\beta) - [\Psi_0(z_{i-1};\alpha) -$$

$$\Psi_0(z_{i-1};\alpha+\beta)][\Psi_0(z_{i-1};\beta) - \Psi_0(z_{i-1};\alpha+\beta)] - \Psi_0(z_{i-1};\alpha+\beta)\} = 0$$

…(20)

Here,

$$z_i = \frac{x_i - a}{c-a}$$

$$z_{i-1} = \frac{x_{i-1} - a}{c-a}$$

$$\alpha = \frac{4b+c-5a}{c-a} \; ; \quad \beta = \frac{5c-a-4b}{c-a}$$

SIMULATION STUDY

In this section, we develop a simulation study. The major goal of these simulations is to calculate the efficiency of the Maximum Product Spacings estimation method for the parameters of the PERT distribution. The subsequent procedure was adopted as follows:

Step 1: Set the sample size 'n' and the vector of parameter values $\Psi = (\alpha, \beta)$.

Step 2: Using the values obtained in step (2), compute $\hat{\alpha}^2_{MPS}$ and $\hat{\beta}_{MPS}$ through Maximum Product Spacings..

Step3: Repeat steps (2) and (3) N times

Step4: Using $\widehat{\Psi}$ of Ψ, compute the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\widehat{\Psi}_{lm}$ is Maximum Product Spacings estimate method of $\widehat{\Psi}_m$, m=1, 2 where $\Psi_m$is a general notation that can be replaced by $\Psi_1 = \alpha, \Psi_2 = \beta$ based on sample l, (l =1,2,…,r), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

Average Estimate $(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} \hat{\psi}_{lm}}{r}$

Variance$(\hat{\psi}_m)$ = $\frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}$

SD $(\hat{\psi}_m = \sqrt{\frac{\sum_{i=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}}$

Mean Absolute Deviation$(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} Med(\left|\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}\right|)}{r}$

Mean Square Error $(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} (\widehat{\psi}_{lm} - \psi_m)^2}{r}$

Relative Absolute Bias$(\hat{\psi}_m) = \frac{\sum_{i=1}^{r} |(\widehat{\psi}_{lm} - \psi_m)|}{r\psi_m}$

Relative Error$(\hat{\psi}_m) = \frac{1}{r}\left(\frac{\sum_{i=1}^{r} MSE\sqrt{(\widehat{\psi}_{lm})}}{\psi_m}\right)^2$

The results were computed using the software R (R Core Development Team). The seed used to generate the random values. The chosen values to perform this procedure were N = 10,000, and n = (20, 40, 60,…, 200). For different population parameter values.

APPLICATIONS

In this section, we considered two real data sets. Frist data set consists of 62 observations of the stren gths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. analyzed b y Smith and Naylor (1987). The second data set is presented by Boag [29] and is related to the ages (in months) of 18 patients who died from other causes than cancer.

In this Section, our simulation study indicated that the MPS estimators should be used for estimating th e parameters of the PERT distribution. Initially, we compared the estimates obtained from the different proced ures with the MPS estimator. Then, we compared the results obtained from the PERT distribution fitted by the MPS estimators with some common lifetime models, such as Rayleigh, Logistic, Gamma, Log normal, Weibul l, and Generalized Exponential distributions.

The Kolmogorov-Smirnov (KS) test is considered to check the goodness of fit. This procedure is based on the KS statistic $D_n = sup_x|F_n(x) - F_0(x)|$

Where sup_x is the supremum of the set of distances?

$F_n(x)$ is the empirical distribution function and $F_0(x)$ is cumulative distribution function. In this case, we test the null hypothesis that the data comes from $F_0(x)$ and with significance level of 5%, we will reject the

null hypothesis if p value is smaller than 0.05. As discrimination criterion method, we considered the AIC (Akaike Information Criteria) computed, respectively, by

AIC = $-2l(\widehat{\Psi}, x) + 2k$

Where k is the number of parameters fitted and $\widehat{\Psi}$ is estimate of Ψ.

The data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory with (α)= 4 and
(β) = 2. The data are:
4.99, 3.97, 2.18, 3.14, 2.19, 4.96, 2.66, 4.98, 3.37, 2.85, 4.88, 3.27, 4.29,3.29, 4.10, 4.76, 4.49, 4.24, 2.85, 3.16, 2.16, 2.34, 3.84, 4.52, 2.89, 4.87, 2.87, 2.40, 4.30, 3.73, 3.45, 4.98, 4.43, 2.09, 2.30, 2.89, 2.53, 2.01, 4.94, 2.23, 4.15, 2.73, 3.59, 3.27, 4.70, 2.14, 4.84, 4.46, 4.42, 2.57, 3.64, 3.54, 3.70, 3.95, 2.98, 4.23, 3.78, 4.84, 3.54, 3.03, 2.98, 3.89. These data have also been analyzed by Smith and Naylor (1987). We obtained

$\hat{\alpha}_{MPS}$ = 2.546 and $\hat{\beta}_{MPS}$ = 1.6254

Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	PERT	Uniform	Triangular
KS	0.5148	0.01254	0.1689
AIC	2015.23	2654.8	2421.13

Boag Data Set 2
The data set related to the ages (in months) of 18 patients who died from other causes than cancer extracted from Boag (1949), which considered the Alpha Logarithm Transformed Rayleigh distribution to describe such data.
0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162.
.0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162.

We obtained
$\hat{\alpha}_{MPS}$ = 1.265 and $\hat{\beta}_{MPS}$ = 2.3698
Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	PERT	Uniform	Triangular
KS	0.5487	0.0001	0.0028
AIC	1024.29	2354.01	1952.34

Comparing the empirical function with the adjusted distributions, a better fit for the PERT distribution among the chosen models can be observed. This result is confirmed from AIC, since PERT distribution has the minimum values among the chosen models. Moreover, considering a significance level of 5%, the PERT distribution was the only model in which p values returned from the KS test were greater than 0.05.
Maximum Product Spacings method for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MPSs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=5, b=6 and c=7 in Table 5.1.

TABLE-5.1

Maximum Product Spacings method for estimating the PERT (a=5, b=6, c=7)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	3.9857	0.8519	0.9234	0.7958	0.9945	0.8197	0.6985
	b	4.5045	0.7587	0.8945	0.6424	0.9501	0.7386	0.6214
	c	5.6365	0.8765	0.8865	0.8596	0.9958	0.7447	0.6987
40	a	3.4568	0.8467	0.8976	0.7524	0.9678	0.7694	0.6559
	b	4.6897	0.6987	0.7982	0.6524	0.8976	0.7358	0.5902
	c	5.8649	0.6021	0.6524	0.7156	0.9941	0.7428	0.6287
60	a	3.6571	0.7984	0.8345	0.6025	0.9532	0.7076	0.5987
	b	4.7852	0.6531	0.7298	0.5638	0.8654	0.7147	0.5548
	c	5.9324	0.5964	0.6124	0.6958	0.8965	0.6549	0.6587
80	a	3.9958	0.7087	0.7134	0.6124	0.7985	0.6839	0.5682
	b	4.9368	0.5576	0.6983	0.5124	0.6987	0.6637	0.4925
	c	6.0874	0.4986	0.5474	0.6587	0.8576	0.5791	0.4487
100	a	4.4268	0.6124	0.6987	0.4958	0.7754	0.5755	0.3257
	b	5.6986	0.5209	0.5974	0.4936	0.6532	0.6537	0.3254
	c	6.3245	0.4756	0.5638	0.6487	0.8169	0.6031	0.3065
120	a	4.4685	0.5986	0.6711	0.3685	0.6109	0.5331	0.2143
	b	5.7215	0.3981	0.4587	0.3985	0.5987	0.5863	0..2958
	c	6.4781	0.4684	0.5964	0.5968	0.7598	0.5265	0.2965
140	a	4.6789	0.4186	0.6034	0.3524	0.5955	0.4139	0.2014
	b	5.8965	0.3268	0.4983	0.2987	0.4954	0.4924	0.2098
	c	6.5685	0.3958	0.4012	0.5587	0.7147	0.4757	0.2547
160	a	4.8569	0.4587	0.5283	0.2931	0.5587	0.4018	0.1985
	b	5.8468	0.3198	0.3672	0.2025	0.3987	0.3659	0.1932
	c	6.9875	0.3224	0.3821	0.5187	0.6955	0.4494	0.2469
180	a	4.9012	0.3969	0.3056	0.2548	0.5076	0.3405	0.1025
	b	5.9974	0.2901	0.3367	0.1925	0.3542	0.3163	0.0954
	c	6.9987	0.3168	0.3897	0.4968	0.6158	0.37936	0.2147
200	a	5.0012	0.2987	0.3248	0.2252	0.3651	0.2949	0.0948
	b	6.0154	0.2451	0.3154	0.1245	0.2956	0.2088	0.0754
	c	7.0234	0.2987	0.4127	0.3174	0.5573	0.30179	0.2098

OBSERVATIONS:

1. Average Estimate (AE) of PERT parameters of estimated a, b, c by MLE are increased when sample size is increased.

2. Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Maximum Product Spacings method for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MPSs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=2, b=2.5 and c=3 in Table 5.2.

TAB LE- 5.2

Maximum Product Spacings method for estimating the PERT (a=2, b=2.5, c=3)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	0.9547	0.6854	0.7542	0.5542	1.8957	0.7854	0.7652
	b	1.025	0.6798	0.7542	0.5324	1.8765	0.7632	0.7542
	c	1.2654	0.5565	0.7435	0.498	1.7985	0.6421	0.6325
40	a	1.3547	0.5478	0.6187	0.5847	1.7023	0.6135	0.5962
	b	1.3873	0.2568	0.3187	0.5247	1.6897	0.632	0.6154
	c	1.3965	0.3658	0.5247	0.4958	1.6584	0.3451	0.3125
60	a	1.3968	0.4821	0.4987	0.4721	1.5987	0.5962	0.5547
	b	1.4156	0.4235	0.4878	0.5047	0.808	0.5147	0.5047
	c	0.4196	0.2547	0.2847	0.3956	0.8387	0.3004	0.2965
80	a	1.4635	0.4187	0.4487	0.4458	1.5732	0.4689	0.4587
	b	1.7893	0.4635	0.4754	0.4968	0.7187	0.4965	0.4532
	c	1.7854	0.2436	0.4335	0.3314	0.8058	0.2154	0.2047
100	a	1.5278	0.2054	0.2187	0.3987	1.5247	0.3475	0.3254
	b	2.1457	0.2958	0.3954	0.4235	0.6544	0.4865	0.4752
	c	2.6865	0.2354	0.2487	0.1247	0.7985	0.1254	0.1159
120	a	1.8656	0.1936	0.2056	0.3478	0.5247	0.3987	0.3587
	b	2.3546	0.2154	0.3745	0.3965	0.6289	0.4875	0.4765
	c	2.9658	0.2074	0.2257	0.1054	0.7254	0.1154	0.1098
140	a	1.9658	0.1657	0.1987	0.3147	0.3165	0.2635	0.2547
	b	2.4578	0.1847	0.2987	0.2474	0.2569	0.3564	0.3487
	c	2.9754	0.1254	0.2147	0.1008	0.3547	0.1065	0.1059
160	a	1.9854	0.1604	0.1836	0.3045	0.3587	0.2451	0.2254
	b	2.4879	0.1685	0.2769	0.2257	0.4986	0.2658	0.2532
	c	2.9867	0.1158	0.1987	0.9654	0.6954	0.1009	0.1001
180	a	2.0216	0.1158	0.1765	0.2998	0.2984	0.2187	0.2054
	b	2.5001	0.1398	0.2554	0.2135	1.0267	0.1968	0.1754
	c	3.0041	0.1075	0.1587	0.3054	0.5873	0.0958	0.0854
200	a	2.0164	0.1009	0.1587	0.1987	0.2785	0.1987	0.1754
	b	2.4998	0.1256	0.1693	0.1472	0.3568	0.1542	0.1427
	c	2.9987	0.1025	0.1387	0.1047	0.3325	0.0568	0.0457

OBSERVATIONS:

1. Average Estimate (AE) of PERT parameters of estimated a, b, c by MLE are increased when sample size is increased.
2. Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Maximum Product Spacings method for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MPSs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=3, b=4 and c=5 in Table 5.3.

TAB LE- 5.3

Maximum Product Spacings method for estimating the PERT (a=3, b=4, c=5)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	1.4587	0.5565	0.7521	0.9963	1.0023	0.9721	0.9765
	b	1.5765	0.3547	0.6058	0.9285	1.0254	0.9225	0.9335
	c	3.0547	0.5341	0.7498	0.9187	1.0965	0.9487	0.9285
40	a	1.5498	0.4183	0.6538	0.9465	1.003	0.9226	0.9154
	b	1.7683	0.2163	0.4732	0.9187	1.0165	0.9118	0.9032
	c	3.1368	0.3987	0.6354	0.8732	1.0087	0.8997	0.8965
60	a	2.1823	0.4381	0.6652	0.9183	0.9765	0.9008	0.8936
	b	2.2698	0.2536	0.5187	0.7765	0.9876	0.9002	0.8937
	c	3.4965	0.1889	0.4468	0.6989	0.9683	0.8965	0.8832
80	a	2.3865	0.2154	0.4732	0.8005	0.9828	0.8931	0.8897
	b	2.6847	0.1398	0.3822	0.7264	0.9732	0.8567	0.8447
	c	2.7376	0.1287	0.3657	0.5682	0.9531	0.8362	0.8229
100	a	2.5432	0.1098	0.3322	0.6554	0.9154	0.8669	0.8542
	b	2.6243	0.153	0.3941	0.4863	0.9585	0.7214	0.6487
	c	3.8038	0.0932	0.3025	0.3282	0.9221	0.6287	0.5968
120	a	2.6243	0.1021	0.3174	0.2831	0.6998	0.8247	0.7255
	b	2.7035	0.0498	0.2196	0.3006	0.8214	0.6258	0.5584
	c	3.9419	0.0658	0.2983	0.1965	0.7132	0.6116	0.4952
140	a	2.7736	0.9873	0.1893	0.1987	0.8365	0.7015	0.6532
	b	3.0985	0.0398	0.1851	0.2824	0.6248	0.5963	0.5221
	c	4.3587	0.0496	0.2341	0.1287	0.6854	0.5532	0.4938
160	a	2.8997	0.0129	0.1487	0.1996	0.7936	0.6127	0.5124
	b	3.254	0.0398	0.1801	0.2154	0.5542	0.5936	0.4432
	c	4.6689	0.0892	0.2162	0.1154	0.5539	0.4332	0.3965
180	a	3	0.0116	0.1021	0.1732	0.7163	0.6007	0.5214
	b	3.987	0.0287	0.1687	0.2114	0.4539	0.5165	0.4968
	c	4.8685	0.0856	0.2874	0.1136	0.4199	0.4117	0.2954
200	a	2.999	0.0103	0.1012	0.1421	0.6198	0.5968	0.495
	b	4.0004	0.0168	0.1365	0.0763	0.3765	0.3965	0.2963
	c	4.9867	0.0796	0.2734	0.0324	0.2365	0.4117	0.2965

OBSERVATIONS:

1. Average Estimate (AE) of PERT parameters of estimated a, b, c by MLE are increased when sample size is increased.
2. Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

We calculate the Maximum Product Spacings method for estimating the PERT (α, β). The MPSs and their Average Estimate (AE), Variance (VAR), Standard Deviation (Newton-Raphson iterative procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are considered. SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α =3 and β = 2.5 in Table 5.4

TABLE-5.4

Maximum Product Spacings method for estimating the PERT ($\alpha = 3$, $\beta = 2.5$)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	1.6215	0.8954	0.9625	0.5472	0.9914	0.9927	0.5289
	β	1.512	0.6923	0.7165	0.4258	0.7369	0.7452	0.3245
40	α	2.2013	0.9128	0.9546	0.5327	0.9768	0.9845	0.4572
	β	1.6369	0.5387	0.6034	0.4058	0.6985	0.7265	0.3165
60	α	2.2478	0.9234	0.9429	0.4925	0.9637	0.9732	0.4068
	β	1.8354	0.4598	0.5469	0.3985	0.6049	0.6954	0.2958
80	α	2.3547	0.8935	0.9168	0.3982	0.9537	0.9695	0.3978
	β	2.0245	0.3964	0.5735	0.3547	0.5956	0.6538	0.2589
100	α	2.5014	0.8893	0.9267	0.3524	0.9439	0.9536	0.3564
	β	2.1753	0.2876	0.4637	0.3219	0.4914	0.5368	0.2469
120	α	2.6412	0.7879	0.8182	0.2987	0.8845	0.9416	0.3187
	β	2.2288	0.2198	0.2984	0.2548	0.3549	0.4698	0.2147
140	α	2.7124	0.7523	0.8464	0.2546	0.8987	0.9302	0.2187
	β	2.2354	0.1996	0.2846	0.2154	0.2937	0.3648	0.1987
160	α	2.9429	0.6958	0.8067	0.1956	0.7956	0.8453	0.1784
	β	2.3368	0.1568	0.2165	0.1625	0.2987	0.3024	0.1524
180	α	3.004	0.4239	0.7035	0.1836	0.7265	0.8078	0.1658
	β	2.4693	0.1328	0.1956	0.1618	0.1964	0.2986	0.1423
200	α	3.017	0.2982	0.5473	0.1212	0.6958	0.7656	0.1147
	β	2.4952	0.1185	0.1356	0.1032	0.1528	0.2584	0.0987

OBSERVATIONS:

1. Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
2. Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

We calculate the the Maximum Product Spacings (MPS) method for estimating the PERT(α , β) Newton-Raphson simulation procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are taken. The MPSs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters $\alpha = 4.5$ and $\beta = 4$ in Table 5.5

TABLE-5.5

Maximum Product Spacings method for estimating the PERT ($\alpha = 4.5$, β =4)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	2.4987	0.9135	0.9574	0.5437	0.9965	0.9678	0.4879
	β	2.4154	0.6957	0.7563	0.5061	0.8547	0.7985	0.3954
40	α	2.9875	0.8955	0.9521	0.5167	0.9784	0.9545	0.3987
	β	2.5473	0.5987	0.6952	0.4982	0.7845	0.7485	0.2968
60	α	3.0586	0.8351	0.9379	0.4765	0.9712	0.9286	0.3526
	β	2.9873	0.5568	0.6241	0.4536	0.6986	0.6756	0.2435
80	α	3.1479	0.7989	0.8935	0.3985	0.9689	0.9168	0.2987
	β	3.0124	0.4989	0.4987	0.3873	0.5987	0.5457	0.2098
100	α	3.5568	0.7551	0.8351	0.2976	0.9547	0.8968	0.1984
	β	3.4875	0.2987	0.4542	0.2539	0.5864	0.4987	0.1976
120	α	3.6547	0.7438	0.8147	0.2254	0.9468	0.8542	0.1765
	β	3.5874	0.2761	0.4178	0.1998	0.5127	0.4982	0..1645
140	α	3.7451	0.7127	0.7727	0.1976	0.8976	0.7954	0.1209
	β	3.6581	0.1984	0.3473	0.1765	0.4986	0.3957	0.1134
160	α	3.9445	0.6982	0.7027	0.1098	0.8756	0.7183	0.1198
	β	3.8457	0.1547	0.2954	0.0987	0.4128	0.3854	0.1076
180	α	4.1958	0.5874	0.5987	0.0986	0.7956	0.6953	0.1034
	β	3.9784	0.1954	0.2423	0.0762	0.3964	0.2957	0.0987
200	α	4.4985	0.4921	0.4982	0.0698	0.7548	0.5682	0.0998
	β	4.0024	0.1654	0.1983	0.0543	0.3785	0.2741	0.0876

OBSERVATIONS:

1. Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
2. Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased

CONCLUSIONS

1. The Maximum Product Specings is the better one for estimating the parameters of the PERT distribution; Since Sample size increases Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters are decreases.
2. The Maximum Product Specings has the smallest Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters, proving to be the efficient method.

OBSERVATIONS FOR THE SIMULATION RESULT

1) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error (RE) of the estimators are dependent on the sample sizes.
2) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of the estimators are independent on the population parameter values.
3) The Average Estimate (AE) of Maximum Product Spacings a, b, c estimators is increased when sample size increased.
4) The Average Estimate (AE) of Maximum Product Spacings ($\hat{\alpha}$) and ($\hat{\beta}$) estimators is increased when sample size increased.
5) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of Maximum Product Spacings the estimators a, b, c are decreased when sample size are increased.
6) The Variance (VAR), Standard deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), Relative Error (RE) of Maximum Product Spacings ($\hat{\alpha}$) and ($\hat{\beta}$) estimators are decreased when sample size increased.

REFERENCES

1. Clark, CE (1962) The PERT model for the distribution of an activity. Operations Research 10, pp. 405406
2. Vose D (2000). Risk analysis, 2nd ed. J Wiley, Chichester, UK.
3. Malcolm, D., Roseboom, J., Clark, C., & Fazar, W. (1959). Application of a technique for research and development program evaluation. *Operations Research*, *7*, 646–669.
4. Torabi, H. (2008). A general method for Estimating and Hypotheses Testing using Spacings. Journal of Statistical Theory and Applications Vol. 8, No. 2, PP.163-168.
5. Torabi, H. and Bagheri, F. L. (2010). Estimation of Parameters for an Extended Generalized Half Logistic Distribution Based on Completed and Censord Data, Journal of Iranian Statistical Society, Vol. 9, No. 2, PP. 171-195.
6. Ramamohan, C. H. and Anjaneyulu, G.V.S.R. (2011). How the Least Square Method be good for Estimating the parameters of Two-Parameter Weibull Distribution from an optimally constructed grouped sample. International Journal of Statistics and Systems, Vol. 6, No. 4, PP. 525-535.
7. Cheng R.C.H.; Amin, N.A.K. (1979). Maximum product-of-spacings estimation with applications to the lognormal distribution, University of Wales Institute of Science and Technology, Mathematical Report, PP: 79-1
8. Cheng, R.C.H., Amin, N.A.K.,(1983). Estimating parameters in continuous univariate distributions with a shifted origin. Journal of Royal Statistical Society. Series: B 45, pp: 394–403.
9. Ranneby, B., (1984). The maximum spacing method: an estimation method related to the maximum likelihood method. Scandinavian Journal of Statististics 11, pp: 93–112.
10. Ranneby, B., Ekstrom, M., (1997). Maximum spacing estimates based on different metrics. Preprint.
11. Kaushik Ghosha, Rao Jammalamadaka, S., (2001). A general estimation method using spacings Journal of Statistical Planning and Inference, 93, pp:71-82.
12. Ehab Mohamed Almetwally and Hisham Mohamed Almongy, (2019). Maximum Product Spacing and Bayesian Method for Parameter Estimation for Generalized Power Weibull Distribution Under Censoring Scheme, Journal of Data Science,17(2). PP. 407 – 444.
13. Yongzhao Shao, (2001). Consistency of the Maximum Product of Spacings Method and Estimation of a Unimodal Distribution, Statistica Sinica, 11, pp:1125-1140

14. Vijaya Lakshmi, M. Rajasekhram, O. V. and Anjaneyulu, G. V. S. R. (2018). Estimation of Scale (λ) and Location (μ) of Two-Parameter Rayleigh Distribution Using Median Ranks method, International Journal of Science and Research (IJSR),1(1), PP. 1948-1952.
15. Vijaya Lakshmi, M. Rajasekhram, O. V. and Anjaneyulu, G. V. S. R. (2019).Estimation of Scale(θ) and Shape (α) Parameters of Power Function Distribution By Least Squares Method Using Optimally Constructed Grouped Data, Journal of Emerging Technologies and Innovative Research (JETIR), 6(6), PP. 320-328.
16. Vijaya Lakshmi, M. and Anjaneyulu, G. V. S. R. (2019), Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Least Square Regression Method, International Journal For Research and Applied Science and Engineering Technology (IJRASET), 7(8), PP. 2321-9653.
17. Vijaya Lakshmi, M. and Anjaneyulu, G. V. S. R. (2019), Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method, International Journal of Research and Analytical Reviews, 6, 2, PP. 558-567.

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Estimation of Parameters of Pert Distribution by Using Method of Moments

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Abstract: ***The method of moments has been widely used for estimating the parameters of a distribution. Usually lower order moments are wont to find the parameter estimates as they're known to possess less sampling variability. The method of moments may be a technique for estimating the parameters of a statistical model. It works by finding values of the parameters that end in a match between the sample moments and therefore the population moments (as implied by the model). the Method of moment Estimator is used to find out Estimates the parameters of PERT Distribution. We also compare equispaced and unequispaced Optimally Constructed Grouped data by the method of an Asymptotically Relative Efficiency. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample supported 1000 simulations to assess the performance of the estimators.***
Keywords: Method of Moments, PERT Distribution, equispaced and unequipped Optimal Grouped sample.

I. INTODUCTION

The method of moments is an alternative to the maximum likelihood method of estimating values of PERT density function parameters that describe the size distribution.

The method of moments of estimation was introduced by Karl Pearson (1894, 1895). The procedure consists of equating as many population moments to sample moments as there are parameters to estimate. Mathematical support for this procedure comes from the principle of moments as discussed intimately in Kendall and Stuart (1969). In essence, this principle says that two distributions that have a finite number of lower moments in common are going to be approximations of 1 another. Thus, the distribution of the data is approximated by equating the moments of a distributional form to the data moments. To see how this could be done with the three-parameter PERT distribution.

The method of moments considered that best estimates of the parameters of a probability distribution are those that moments of the PDF about the source are adequate to the corresponding moments of the sample data. Pearson originally considered only moments about the origin, but later it became customary to use the variance as the second central moment and the coefficient of sleekness as the standardized third central moment, to work out second and third parameters of the distribution if necessary.

In the method of moments approach, the parameters of a probability distribution model are estimated by matching the moments of the dataset thereupon of the candidate model. The number of moments required corresponds to the amount of unknown model parameters. Application of this method is simple, as closed-form expressions for the moments are often readily derived for many common distributions. However, the raw moments could also be biased thanks to the presence of outliers and/or the shortage of perfect agreement between the info and therefore the model.

The estimation of parameters of the three-parameter generalized exponential distribution introduced by Hossain and Ahsanullah [5] by using the utmost likelihood estimation and therefore the method of moments. Rameshwar D. Gupta (2000), And Debasis Kundu Studied. The Generalized Exponential Distribution: Different Method of Estimations. Recently a full derivation of the conditional moment equations was derived and numerical results show that when the utmost order of the considered moments is high, the amount of equations that have to be integrated is usually much smaller for the conditional moments approach and the resulting equations are less stiff(2013). Syed Afzal Hossain (2018) studied Estimating the Parameters of a Generalized Exponential Distribution, here discussed the maximum likelihood (ML) method and the method of moments to estimate the parameters.

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In this paper, we discuss about the estimation procedure for the unknown parameters for PERT distribution. The idea behind the Method of Moments (MoM) parameter estimation is to determine the parameters for given the sample data. We present MoM of the unknown parameters of PERT distribution using Newton-Raphson iterative procedure. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Efficiency (RE) for three parameters under sample based on 10,000 simulations to assess the performance of the estimators. A simulation study is conducted to evaluate the performance of the Method of Moment estimates. Finally, the proposed estimation method is applied on real and generalized data sets the results are given. Which illustrate Method of Moments estimation of unknown parameters for PERT distribution?

A random variable X ~ PERT (a, b, c) has probability density function and is in the form

$$f_{PERT}(x;\ a, b, c) = \frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}};\ a < x < c \qquad \ldots(1)$$

$$\alpha = \frac{4b+c-5a}{c-a}\ ;\quad \beta = \frac{5c-a-4b}{c-a}$$

(a, b, c) are parameters of PERT distribution

A random variable X ~ PERT (a, b, c) has cumulative distribution function and is in the form

$$F_{PERT}(x;\ a, b, c) = \frac{(-1)^{\alpha}\beta(\frac{z}{z-1};\ a,\ 1-a-b)}{\beta(\alpha,\ \beta)};\ a < x < c \qquad \ldots (2)$$

Here, $z = \frac{x-a}{c-a}$

A random variable X ~ PERT (a, b, c) has Quantile function and is in the form

The pth quantile x_p of PERT distribution is of the equation.

$$x_p = a+(c-a)\frac{\alpha+(p-\frac{5}{6})}{\alpha+\beta+\left(p-\frac{7}{6}\right)} \qquad \ldots (3)$$

Let U ~ U(0,1), then equation (4.3) can be used to simulate a random sample of size 'n' from the PERT distribution as follows

$$x_i = a+(c-a)\frac{\alpha+(u_i-\frac{5}{6})}{\alpha+\beta+\left(u_i-\frac{7}{6}\right)},\ i = 1, 2, \ldots, n. \qquad \ldots (4)$$

II. ESTIMATION OF PARAMETERS OF PERT DISTRIBUTION USING METHOD OF MOMENTS

If X follows the PERT distribution then K^{th} moment of PERT distribution is given by

$$E(X^k) = \int_a^c x^k f(a, b, c)dx \qquad \ldots(5)$$

The moments of PERT distribution as follows

$$E(X) = \frac{a+4b+c}{6} = \mu \qquad \ldots (6)$$

$$E(X^2) = \frac{a^2\beta(\beta+1)+2ac\alpha\beta+c^2\alpha(\alpha+1)}{42} \qquad \ldots (7)$$

$$E(X^3) = \frac{a^3\beta(\beta+1)(\beta+2)+3a^2c\alpha\beta(\beta+1)+3c^2a\alpha\beta(\alpha+1)+c^2\alpha(\alpha+1)(\alpha+2)}{336} \qquad \ldots (8)$$

$$E(X^4) = \frac{a^4\beta(\beta+1)(\beta+2)(\beta+3)+4a^3c\alpha\beta(\beta+1)(\beta+2)+6a^2c^2\alpha\beta(\alpha+1)(\beta+1)+4c^3a\alpha(\alpha+1)(\alpha+2)+c^4\alpha(\alpha+1)(\alpha+2)(\alpha+3)}{3024} \qquad \ldots (9)$$

By using the (5) and (6), we get

$$\text{Mean} = \hat{\mu} = \bar{x} = \frac{a+4b+c}{6} \qquad \ldots (10)$$

$$\text{Variance} = \hat{\sigma}^2 = \frac{(c-a)^2}{36} \qquad \ldots (11)$$

Solving (11), we have

$$\hat{a} = \frac{(c-\hat{\sigma})}{6} \qquad \ldots (12)$$

$$\hat{c} = \frac{6\bar{x}-7c+\hat{\sigma}}{24} \qquad \ldots (13)$$

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III. SIMULATION STUDY

In this section, we develop a simulation study. The major goal of these simulations is to calculate the efficiency of the Method of Moments estimation method for the parameters of the PERT distribution. The subsequent procedure was adopted as follows:

Step 1: Set the sample size 'n' and the vector of parameter values $\Psi = (\alpha, \beta)$.

Step 2: Using the values obtained in step (2), compute $\hat{\alpha}_{MoM}$ and $\hat{\beta}_{MoM}$ through Method of Moments.

Step3: Repeat steps (2) and (3) N times

Step4: Using $\widehat{\Psi}$ of Ψ, compute the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\widehat{\Psi}_{lm}$ is Method of Moments estimate method of $\widehat{\Psi}_m$, m=1, 2 where $\Psi_m$is a general notation that can be replaced by $\Psi_1 = \alpha, \Psi_2 = \beta$ based on sample l, (l =1,2,…,r), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

$$\text{Average Estimate } (\hat{\psi}_m) = \frac{\sum_{l=1}^{r} \hat{\psi}_{lm}}{r}$$

$$\text{Variance}(\hat{\psi}_m) = \frac{\sum_{l=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}$$

$$\text{SD } (\hat{\psi}_m = \sqrt{\frac{\sum_{l=1}^{r} (\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}})^2}{r}}$$

$$\text{Mean Absolute Deviation}(\hat{\psi}_m) = \frac{\sum_{l=1}^{r} Med(|\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}|)}{r}$$

$$\text{Mean Square Error } (\hat{\psi}_m) = \frac{\sum_{l=1}^{r} (\widehat{\psi}_{lm} - \psi_m)^2}{r}$$

$$\text{Relative Absolute Bias}(\hat{\psi}_m) = \frac{\sum_{l=1}^{r} |(\widehat{\psi}_{lm} - \psi_m)|}{r\psi_m}$$

$$\text{Relative Error}(\hat{\psi}_m) = \frac{1}{r}\left(\frac{\sum_{l=1}^{r} MSE\sqrt{(\widehat{\psi}_{lm})}}{\psi_m}\right)^2$$

The results were computed using the software R (R Core Development Team). The seed used to generate the random values. The chosen values to perform this procedure were N = 10,000, and n = (20, 40, 60,…, 200). For different population parameter values.

IV. APPLICATIONS

In this section, we considered two real data sets. First data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. analyzed by Smith and Naylor (1987). The second data set is presented by Boag (1949) and is related to the ages (in months) of 18 patients who died from other causes than cancer.

In this Section, our simulation study indicated that the MoM estimators should be used for estimating the parameters of the PERT distribution. Initially, we compared the estimates obtained from the different procedures with the MoM estimator. Then, we compared the results obtained from the PERT distribution fitted by the MoM estimators with some common lifetime models, such as Uniform and Triangular distributions.

The Kolmogorov-Smirnov (KS) test is considered to check the goodness of fit. This procedure is based on the KS statistic $D_n = sup_x|F_n(x) - F_0(x)|$

Where sup_x is the supremum of the set of distances,

$F_n(x)$ is the empirical distribution function and $F_0(x)$ is cumulative distribution function. In this case, we test the null hypothesis that the data comes from $F_0(x)$ and with significance level of 5%, we will reject the null hypothesis if p value is smaller than 0.05. As discrimination criterion method, we considered the AIC (Akaike Information Criteria) computed, respectively, by

$$\text{AIC} = -2l(\widehat{\Psi}, x) + 2k$$

Where k is the number of parameters fitted and $\widehat{\Psi}$ is estimate of Ψ.

The data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory with (α) = 4 and

(β) = 2. The data are:

4.99, 3.97, 2.18, 3.14, 2.19, 4.96, 2.66, 4.98, 3.37, 2.85, 4.88, 3.27, 4.29,3.29, 4.10, 4.76, 4.49, 4.24, 2.85, 3.16, 2.16, 2.34, 3.84, 4.52, 2.89, 4.87, 2.87, 2.40, 4.30, 3.73, 3.45, 4.98, 4.43, 2.09, 2.30, 2.89, 2.53, 2.01, 4.94, 2.23, 4.15, 2.73, 3.59, 3.27, 4.70, 2.14, 4.84, 4.46, 4.42, 2.57, 3.64, 3.54, 3.70, 3.95, 2.98, 4.23, 3.78, 4.84, 3.54, 3.03, 2.98, 3.89. These data have also been analyzed by Smith and Naylor (1987). We obtained

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$\hat{\alpha}_{MoM}$ = 2.247 and $\hat{\beta}_{MoM}$ = 1.5874.

Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	PERT	Uniform	Triangular
KS	0.5148	0.01254	0.1689
AIC	2015.23	2654.8	2421.13

Boag Data Set 2

The data set related to the ages (in months) of 18 patients who died from other causes than cancer extracted from Boag (1949), which considered the PERT distribution to describe such data.

0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162.

.0.3, 4, 7.4, 15.5, 23.4, 46, 46, 51, 65, 68, 83, 88, 96, 110, 111, 112, 132, 162.

We obtained

$\hat{\alpha}_{MoM}$ = 1.1236 and $\hat{\beta}_{MoM}$ = 2.2487

Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	PERT	Uniform	Triangular
KS	0.4965	0.0001	0.0018
AIC	1154.65	2587.25	2014.56

Comparing the empirical function with the adjusted distributions, a better fit for the PERT distribution among the chosen models can be observed. This result is confirmed from AIC, since PERT distribution has the minimum values among the chosen models. Moreover, considering a significance level of 5%, the PERT distribution was the only model in which p values returned from the KS test were greater than 0.05.

Method of Moments (MoM) for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=5, b=6 and c=7 in Table 4.1.

TABLE-4.1

Method of Moment for Estimating the PERT Distribution (a=5, b=6 and c=7)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	3.1354	0.8885	0.9154	0.7245	1.9194	0.9989	0.7985
	b	4.0245	0.8368	0.8574	0.6123	1.9213	0.9987	0.6958
	c	5.0198	0.8924	0.9121	0.4526	1.9417	0.9754	0.6932
40	a	3.2359	0.8847	0.9032	0.6958	1.90549	0.9687	0.6985
	b	4.1287	0.7651	0.7798	0.5964	1.9172	0.9621	0.6358
	c	5.1387	0.8832	0.902	0.4625	1.8316	0.9124	0.6158
60	a	3.3165	0.8775	0.8854	0.6178	1.8244	0.8932	0.6658
	b	4.2698	0.7184	0.7265	0.4968	1.8146	0.8721	0.5987
	c	5.2364	0.8638	0.8765	0.4785	1.8099	0.8997	0.5986
80	a	3.5689	0.8065	0.8245	0.5968	1.8081	0.8365	0.5721
	b	4.5364	0.6865	0.7085	0.4352	1.7646	0.8254	0.5687
	c	5.5024	0.8435	0.8521	0.3965	1.8101	0.8387	0.5123
100	a	3.9658	0.7543	0.7956	0.4752	1.8038	0.8254	0.5236
	b	4.7698	0.6547	0.6874	0.4132	1.7313	0.8054	0.5478
	c	5.9864	0.7786	0.7865	0.2968	1.7585	0.8198	0.4587
120	a	4.1028	0.6982	0.7145	0.4325	1.7039	0.8247	0.4265
	b	5.198	0.6448	0.6654	0.3965	1.7011	0.7154	0.5368

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	c	6.1586	0.7368	0.7587	0.2754	1.7033	0.7965	0.3254
140	a	4.3896	0.6462	0.7241	0.3658	1.7008	0.7154	0.5478
	b	5.2687	0.5867	0.6024	0.2965	1.6034	0.6587	0.4598
	c	6.3698	0.6458	0.6687	0.2636	1.6034	0.6687	0.3621
160	a	4.5473	0.6364	0.6879	0.2269	1.6128	0.6498	0.5147
	b	5.6935	0.4963	0.5784	0.2684	1.5035	0.5565	0.3254
	c	6.5368	0.5968	0.6178	0.2564	1.5037	0.5368	0.3124
180	a	4.7965	0.5736	0.6024	0.2154	1.6107	0.5471	0.3021
	b	5.7635	0.3987	0.4658	0.2487	1.5003	0.4921	0.2965
	c	6.7635	0.5598	0.5687	0.2015	1.5004	0.6789	0.2547
200	a	4.8657	0.5496	0.5748	0.2141	1.5151	0.5163	0.2531
	b	5.7632	0.3865	0.4187	0.1968	1.4053	0.41	0.2245
	c	6.6981	0.5269	0.5541	0.2001	1.4137	0.6352	0.2278

- *Observations*
- Average Estimate (AE) of PERT parameters by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Method of Moments (MoM) for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=2, b=2.5 and c=3 in Table 4.2.

TABLE-4.2

Method of Moment for Estimating the PERT Distribution (a=2, b=2.5 and c=3)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	0.8954	0.7102	0.8457	0.6587	1.9987	0.8145	0.7985
	b	0.8997	0.6754	0.7854	0.5587	1.9954	0.7798	0.7625
	c	1.1354	0.5867	0.7725	0.5841	1.9854	0.6653	0.9465
40	a	0.9584	0.5768	0.6658	0.6021	1.9354	0.6458	0.6357
	b	1.1254	0.4932	0.6987	0.5487	1.9287	0.6958	0.8706
	c	1.1524	0.3987	0.6125	0.5252	1.9232	0.4236	0.7049
60	a	1.1587	0.5829	0.5921	0.5028	1.8925	0.6254	0.617
	b	1.8457	0.5487	0.5353	0.5387	0.8547	0.5869	0.5647
	c	1.8546	0.3147	0.5147	0.4587	0.8547	0.4103	0.3996
80	a	1.2987	0.4657	0.4721	0.4998	1.8657	0.4987	0.4514
	b	1.9658	0.5252	0.5224	0.5147	0.7965	0.5364	0.5291
	c	1.8965	0.2874	0.4545	0.3658	0.8547	0.2935	0.2694
100	a	1.3254	0.2699	0.5014	0.4243	1.6854	0.3698	0.3557
	b	1.9857	0.4432	0.5024	0.4848	0.6958	0.5278	0.5123
	c	2.4781	0.2568	0.404	0.2487	0.8542	0.2547	0.2483
120	a	1.4216	0.2675	0.4487	0.3998	0.5587	0.3325	0.3017
	b	1.9936	0.4325	0.4757	0.4365	0.6684	0.5187	0.4915
	c	2.4179	0.1745	0.3998	0.1587	0.8083	0.2147	0.203
140	a	1.6254	0.2287	0.4121	0.3357	0.5587	0.2854	0.2455
	b	2.0147	0.3178	0.4128	0.3024	0.6287	0.3784	0.3563
	c	2.4587	0.1254	0.3856	0.1451	0.7245	0.1957	0.1841
	a	1.7235	0.2287	0.4018	0.3354	0.5124	0.2821	0.2273

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160	b	2.1021	0.1124	0.3958	0.2958	0.5247	0.2958	0.2278
	c	2.6578	0.1769	0.3364	0.1352	0.7154	0.1951	0.1922
180	a	1.7965	0.1387	0.3935	0.3232	0.4054	0.2487	0.2338
	b	2.1471	0.1287	0.3836	0.2641	0.5065	0.2547	0.2399
	c	2.7484	0.1636	0.3254	0.1547	0.6254	0.1887	0.1416
200	a	1.8045	0.1203	0.2741	0.2268	0.3698	0.2354	0.2094
	b	2.2968	0.1198	0.3134	0.1874	0.4787	0.2487	0.1802
	c	2.7689	0.1588	0.3009	0.1287	0.5247	0.1783	0.1182

- *Observations*
- Average Estimate (AE) of PERT parameters by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Method of Moments (MoM) for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=3, b=4 and c=5 in Table 4.3.

TABLE-4.3

Method of Moment for Estimating the PERT Distribution (a=3, b=4 and c=5)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	1.3568	0.6024	0.7635	0.9998	1.4457	0.9984	0.9978
	b	1.4758	0.3754	0.6288	0.9452	1.3982	0.9451	0.9532
	c	2.5684	0.5964	0.7621	0.9378	1.3378	0.9627	0.9598
40	a	1.3879	0.4754	0.6921	0.9753	1.3956	0.9458	0.9365
	b	1.50247	0.2351	0.4822	0.9398	1.2521	0.9358	0.9289
	c	2.6587	0.4154	0.6348	0.9287	1.2182	0.9583	0.9408
60	a	1.9015	0.4578	0.6692	0.9687	1.1578	0.9152	0.9056
	b	1.9682	0.2747	0.5127	0.9281	1.1286	0.9287	0.9158
	c	2.7548	0.2012	0.4537	0.8787	1.1175	0.9466	0.9358
80	a	2.3876	0.2966	0.5521	0.8987	1.2432	0.9118	0.9006
	b	2.4965	0.1582	0.3952	0.9077	1.1589	0.9158	0.9088
	c	2.8875	0.2147	0.4533	0.8547	1.1006	0.9337	0.9283
100	a	2.4754	0.1257	0.3662	0.8721	1.1985	0.9006	0.8998
	b	2.5164	0.1154	0.3391	0.8964	1.1268	0.9058	0.8997
	c	3.5472	0.1132	0.3295	0.8174	1.0065	0.9221	0.9118
120	a	2.5877	0.1178	0.3463	0.8521	1.1584	0.8933	0.8457
	b	2.6874	0.1025	0.3194	0.8732	1.1157	0.8965	0.8547
	c	3.8733	0.1097	0.3241	0.7985	0.9987	0.9157	0.9064
140	a	2.7015	0.1054	0.3138	0.8154	1.1068	0.8725	0.8257
	b	2.7845	0.1009	0.3126	0.8421	0.9658	0.8799	0.8331
	c	4.0157	0.0954	0.2993	0.7732	0.92543	0.9087	0.8959
160	a	2.79856	0.1124	0.3228	0.7938	0.9932	0.8598	0.8165
	b	2.8741	0.0958	0.3102	0.8365	0.9465	0.8566	0.8289
	c	4.2245	0.0942	0.3044	0.7521	0.8098	0.8354	0.8154
	a	2.8541	0.1098	0.3128	0.7784	0.9587	0.8068	0.7932

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180	b	3.5413	0.0945	0.3012	0.8164	0.9087	0.8165	0.8007
	c	4.3115	0.0931	0.3011	0.7468	0.7985	0.8057	0.8009
200	a	2.8965	0.0928	0.3087	0.7158	0.8547	0.7998	0.7542
	b	3.6874	0.0929	0.2994	0.7584	0.8356	0.6875	0.6458
	c	4.5687	0.0914	0.2984	0.6952	0.7721	0.7653	0.7054

- *Observations*
- Average Estimate (AE) of PERT parameters of estimated a, b, c by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

We calculate the Method of Moments (MoM) for estimating the PERT (α, β). Newton-Raphson iterative procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are considered. The MoMs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α =3.5 and β = 2.5 in Table 4.4.

TABLE-4.4
Method of Moments for Estimating the PERT Distributions (α = 3 , β =2.5)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	1.5965	0.9969	0.9972	0.9458	1.9989	0.9999	0.8998
	β	1.4469	0.9965	0.9956	0.9268	1.9968	0.9986	0.8759
40	α	1.6582	0.9921	0.9952	0.9169	1.9756	0.9965	0.8685
	β	1.5248	0.9154	0.9823	0.8965	1.9698	0.9954	0.8532
60	α	1.7156	0.9025	0.9753	0.8865	1.9154	0.9923	0.8456
	β	1.6052	0.8956	0.9523	0.8547	1.8964	0.9897	0.8532
80	α	1.8356	0.8754	0.9326	0.8469	1.8998	0.9752	0.8365
	β	1.6125	0.8721	0.9165	0.8421	1.8863	0.9568	0.8169
100	α	1.8465	0.8598	0.9026	0.7965	1.8546	0.9468	0.7986
	β	1.6358	0.8322	0.8973	0.7524	1.8769	0.9132	0.7854
120	α	1.9865	0.8169	0.8688	0.7436	1.8326	0.9056	0.7798
	β	1.6698	0.7963	0.8546	0.7165	1.8635	0.8965	0.7684
140	α	1.9966	0.7465	0.8324	0.7098	1.7965	0.8936	0.6998
	β	1.7098	0.7398	0.8147	0.6987	1.7532	0.8732	0.6854
160	α	2.1032	0.7169	0.7995	0.6856	1.7436	0.8569	0.6598
	β	1.8562	0.6958	0.7854	0.6784	1.7164	0.8469	0.6459
180	α	2.3698	0.6898	0.7098	0.6654	1.6953	0.8299	0.6398
	β	1.9965	0.6721	0.6954	0.6438	1.6623	0.8132	0.6275
200	α	2.6831	0.6588	0.6654	0.6198	1.6125	0.7986	0.6184
	β	2.1965	0.6487	0.6469	0.5968	1.5986	0.7211	0.6098

- *Observations*
- Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

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We calculate the Method of Moments (MoM) method for estimating the PERT(α , β) Newton-Raphson simulation procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are taken. The MoM s and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α = 4.5 and β = 4 in Table 4.5.

TABLE 4.5
Method of Moments for Estimating the PERT Distribution (α = 4.5 , β =4)

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	2.3345	0.9981	0.9977	0.8965	1.9987	0.9988	0.7845
	β	2.3243	0.9841	0.9854	0.84	1.8864	0.9854	0.8765
40	α	2.4254	0.9654	0.9735	0.8824	1.9752	0.9754	0.7564
	β	2.4032	0.9498	0.9441	0.7798	1.8765	0.9501	0.855
60	α	2.5489	0.9254	0.9365	0.8564	1.9546	0.9473	0.7211
	β	2.4583	0.9154	0.9154	0.7584	1.8547	0.9365	0.8547
80	α	2.6985	0.8904	0.8856	0.8369	1.9365	0.8954	0.6954
	β	2.5621	0.8762	0.8756	0.7432	1.8487	0.8801	0.8568
100	α	2.7698	0.8868	0.8632	0.8166	1.9147	0.8779	0.6687
	β	2.647	0.8854	0.8321	0.7285	1.8198	0.8564	0.8327
120	α	2.8647	0.8697	0.8564	0.7956	1.8989	0.8701	0.7587
	β	2.7654	0.8354	0.8187	0.7054	1.7954	0.8485	0.8147
140	α	3.1658	0.8451	0.7954	0.7564	1.8836	0.8532	0.7321
	β	2.8791	0.7264	0.7754	0.6987	1.7865	0.7765	0.7125
160	α	3.5517	0.8252	0.7548	0.7435	1.8324	0.8432	0.7121
	β	3.1478	0.7124	0.7321	0.6887	1.7654	0.7554	0.6884
180	α	3.9648	0.7927	0.7465	0.7288	1.7965	0.8177	0.7154
	β	3.4873	0.7064	0.7258	0.6754	1.7214	0.7658	0.6719
200	α	4.2956	0.7542	0.7154	0.7054	1.7436	0.7965	0.6921
	β	3.7658	0.6857	0.6987	0.6487	1.6987	0.7543	0.6154

- *Observations*
- Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
- Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

V. CONCLUSIONS

1) Method of Moments is the better one for estimating the parameters of the PERT distribution; Since Sample size increases Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters are decreases.

2) The Method of Moments has the smallest Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters, proving to be the efficient method.

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A. Observations For The Simulation Result

1) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error (RE) of the estimators are dependent on the sample sizes.

2) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of the estimators are independent on the population parameter values.

3) The Average Estimate (AE) of Method of Moments a, b, c estimators is increased when sample size increased.

4) The Average Estimate (AE) of Method of Moments $(\hat{\alpha})$ and $(\hat{\beta})$ estimators is increased when sample size increased.

5) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of Method of Moments the estimators a, b, c are decreased when sample size are increased.

6) The Variance (VAR), Standard deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB), Relative Error (RE) of Method of moments $(\hat{\alpha})$ and $(\hat{\beta})$ estimators are decreased when sample size increased.

REFERENCES

[1] Alexander Lück& Verena Wolf(2016),Generalized method of moments for estimating parameters of stochastic reaction networks,BMC System Biology 10(98),pp:1-2.
[2] Boag J. W. (1949). Maximum likelihood estimates of the proportion of patients cured by cancer therapy. Journal of the Royal Statistical Society, Series B, 11, PP: 15–53.
[3] Cheng, R.C.H., Amin, N.A.K.,(1983). Estimating parameters in continuous univariate distributions with a shifted origin. Journal of Royal Statistical Society. Series: B 45, pp: 394–403.
[4] Coher?, A. C. (1951). 'Estimating Parameters of Logarithmic-Normal Distributions by Maximum Likelihood,' J. her. Statist.
[5] Hossain, S. A. and Ahsanullah, M., 2010. "On Generalized Exponential Distributions." Advances and Applications in Statistical Sciences. Vol. 4, Issue 1, 1-22.
[6] Joseph J.Moder, E.G.Rodgers. Judgment Estimates of the Moments of PERT Type Distributions. Management Science 15, No. 2, B76-B83 (1968).
[7] Pearson, K. 1894. Contributions to the mathematical theory of evolution. II. On the dissection of asymmetrical frequency curves. Philosophical Transactions of the Royal Society of London, Series A, 185, pages unknown. (republished in Karl Pearson's Early Statistical Papers. Cambridge Univ. Press. 1948. pp. 1-41).
[8] Pearson, K. 1895. Contributions to the mathematical theory of evolution. II. Skew variations in homogeneous material. Philosophical Transactions of the Royal Society of London, Series A, 186, 343-414. (republished in Karl Pearson's Early Statistical Papers. Cambridge Univ. Press. 1948. pp. 41-112).
[9] Kendall, M.G.; Stuart, A. 1969. The advanced theory of statistics, Vol. 1. 3rd ed. New York: Hafner Publishing Company. 87 p.
[10] Keeefr.D.L. Verdini.W.A. "Better Estimation of PEPT Activity Time Parameters" Management science, vol. 39, pp. 1086-1091, September 1993.
[11] Kamburowski, J. (1997). New validations of PERT times. Omega, The International Journal of Management Science, 25(3), 323–328.
[12] Rameshwar D. Gupta, Debasis Kundu (2000), Generalized Exponential Distribution: Different Method of Estimations, J. Statist. Comput. Simul., 2000, Vol. 00, Pp. 1- 22.
[13] Hansen LP. Large sample properties of generalized method of moments estimators. Econometrica. 1982:1029–54.
[14] Syed Afzal Hossain (2018), Estimating the Parameters of a Generalized Exponential Distribution, Journal of Statistical Theory and Applications, Vol. 17, No. 3 (September 2018) 537-553.
[15] Smith, R.L., Naylor, J.C. (1987): A comparison of maximum likelihood and Bayesian estimators for the three-parameter Weibull distributuion. Journal of Applied Statistics, 36, pp: 358–369 .

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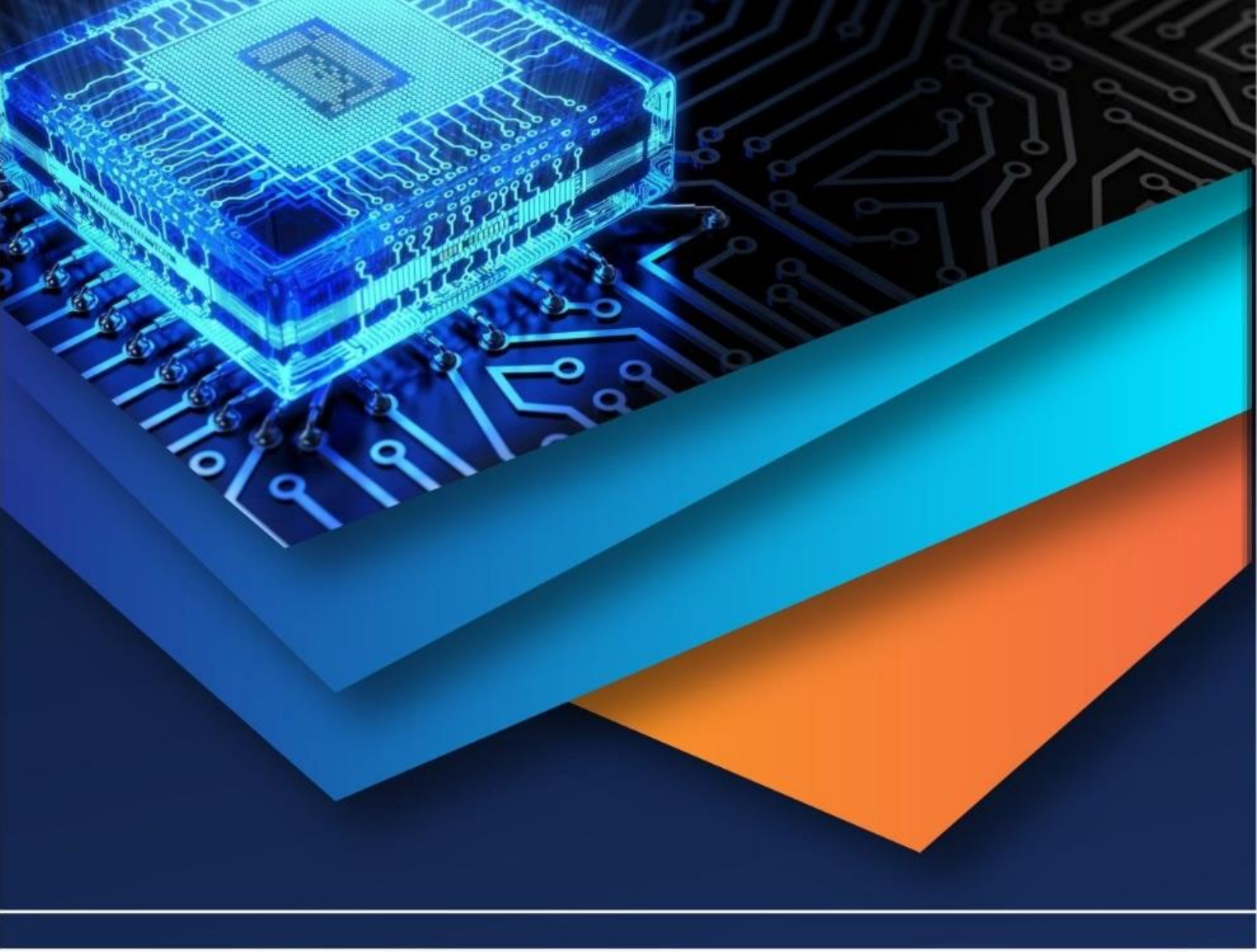

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CLASSIFICATION OF BRAIN STROKE USING DATA MINING TECHNIQUES-A COMPARATIVE ANALYSIS

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ABSTRACT

Brain Stroke is the most common malignancy disease that affects male and female population and the number of affected people is the second most common leading cause of cancer deaths among all cancer types in the developing countries. If a stroke is suspected, to show whether you have had an ischaemic stroke or a haemorrhagic stroke, there are two main types of scans used to assess the brain in people who have had a suspected stroke are a CT scan, MRI scan and MRA scan. This paper presented the classification method for. Four measures were used to evaluate performance in terms of accuracy, sensitivity, and privacy. The aim of the study is to determine the best decision tree classifier for medical datasets classification. Experimental results show that Random Forest has a better performance than ID3, J48, and LMT.

Keywords: Brain Stroke, Random Forest, ID3, J48, Classification.

I. INTRODUCTION

Brain Stroke is a life killing disease through the existence of debilitating growths influencing for male and female mostly after the age of 40 all over the world. Early diagnosis of the Brain Stroke by the CT scan and MRI scan reduces the death rate globally. Many techniques are available for the detection of Brain Stroke among which digital mammography is the familiar and successful technique currently used by CT scan, MRI scan and MRA scan.

Stroke is becoming an important cause of premature death and disability in low-income and middle-income countries like India, largely driven by demographic changes and enhanced by the increasing prevalence of the key modifiable risk factors. As a result developing countries are exposed to a double burden of both communicable and non-communicable diseases. The poor are increasingly affected by stroke, because of both the changing population exposures to risk factors and most tragically not being able to afford the high cost for stroke care. Majority of stroke survivors continue to live with disabilities, and the costs of on-going rehabilitation and long term-care are largely undertaken by family members, which impoverish their families.

Data Mining and Machine Learning are Siamese twins from which several insights can be derived through proper learning algorithms. There has been tremendous progress in data mining and machine learning as a result of evolution of smart and Nano technology which brought about curiosity in finding hidden patterns in data to derive value. The fusion of statistics, machine learning, information theory, and computing has created a solid science, with a firm mathematical base, and with very powerful tools.

Signs and Symptoms of Brain stroke:

Trouble speaking and understanding what others are saying. You may experience confusion, slur words or have difficulty understanding speech.

Paralysis or numbness of the face, arm or leg. You may develop sudden numbness, weakness or paralysis in the face, arm or leg. This often affects just one side of the body. Try to raise both your arms over your head at the same time. If one arm begins to fall, you may be having a stroke. Also, one side of your mouth may droop when you try to smile.

Problems seeing in one or both eyes. You may suddenly have blurred or blackened vision in one or both eyes, or you may see double.

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Headache. A sudden, severe headache, which may be accompanied by vomiting, dizziness or altered consciousness, may indicate that you're having a stroke.

Trouble walking. You may stumble or lose your balance. You may also have sudden dizziness or a loss of coordination.

II. LITERATURE REVIEW

Classification Algorithms

According to Classification algorithms which deals more with classification includes the following: ID3(Iterative Dichotomiser 3), J48, LMT, Linear Classifiers, Logistic Regression, Naïve Bayes Classifier, Perceptron, Support Vector Machine; Quadratic Classifiers, K-Means Clustering, Boosting, Decision Tree, Random Forest (RF); Neural networks, Bayesian Networks and so on. Of all the Classification algorithms mainly used in four algorithms for comparison purpose i.e ID3, J48, LMT and Random Forest.

Random Forest

Random Forest (RF) is an approach which has been proposed by Breiman for classification tasks. It mainly comes from the combination of tree-structured classifiers with the randomness and robustness provided by bagging and random feature selection. The classification is performed by sending a sample down in each tree and assigning it the label of the terminal node it ends up with. At the end, the average vote of all trees is reported as the result of the classification. Random forest is very efficient with large datasets and high dimensional data.

ID3

ID3 stands for Iterative Dichotomiser 3. This approach known as supervised and non-parametric decision tree type. Mostly, it is used for classification and regression. The ID3 algorithm is considered as a very simple decision tree algorithm developed by Quinlan in 1986. ID3 uses information gained as splitting criteria. The growing stops when all instances belong to a single value of target feature or when best information gain is not greater than zero. ID3 does not apply any pruning procedures nor does it handle numeric attributes or missing values. It only accepts categorical attributes in tree building. Also does not support noise data. To remove the noise preprocessing technique has used. ID3 algorithm cannot handle the continuous attributes for that discretization is used to convert continuous attributes to categorical attributes.

J48

J48 is based on a top-down strategy, a recursive divide and conquer strategy. You select which attribute to split on at the root node, and then you create a branch for each possible attribute value, and that splits the instances into subsets, one for each branch that extends from the root node. A decision tree is a predictive dependent on available data's varied attribute values. The internal nodes of a decision tree denote various attributes; inter-nodal branches reveal attribute's possible values in observed samples, while terminal nodes provide information of the dependent variable's final value.

LMT

logistic model tree (LMT) is a classification model with an associated supervised training algorithm that combines logistic regression (LR) and decision tree learning. The basic LMT induction algorithm uses cross-validation to find a number of LogitBoost iterations that does not overfit the training data. A faster version has been proposed that uses the Akaike information criterion to control LogitBoost stopping. Logistic model trees are based on the earlier idea of a model tree: a decision tree that has linear regression models at its leaves to provide a piecewise linear regression model (where ordinary decision trees with constants at their leaves would produce a piecewise constant model).

The research data was obtained from different source of online. This data was chosen because of its accuracy and has also been anonymized (de-identifed), therefore confidentiality is ensured. The number of Attributes is 8 with one class making it 9, Instances are 400 and test mode 10-fold cross-validation. All attributes are nominal-valued as follows: Gender, hypertension, heart disease, ever married, work type, Residence type smoking status, stroke.

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III. TESTING AND EVALUATION

To test and evaluate the performance of the proposed method, different quantitative measures have been used, such as accuracy, sensitivity and specificity. These can be machine-learning model to decide a new sample's target value (dependent variable) calculated by using mathematical equations shown in equations (1), (2) and (3)

Accuracy

It has been used and can be calculated by using mathematical equation:

$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN} \quad \text{... (1)}$$

Where TP is True positive, FP is false positive FN is false negative and TN is true negative.

Specificity

Ability of a classifier to identify the negative results is estimated as specificity, given as: testing phase. The results are presented in the upcoming section. To test the performance of the proposed method, We measure accuracy, sensitivity, and specificity, to show the performance of the proposed method.

$$Specificity = \frac{TN}{TN+FP} \quad \text{..... (2)}$$

Sensitivity

Sensitivity is the metric that evaluates a model's ability to predict true positives of each available category. Specificity is the metric that evaluates a model's ability to predict true negatives of each available category.

$$Sensitivity = \frac{TP}{TP+FN} \quad \text{..... (3)}$$

Experiment

To conduct experiments in the proposed method database was used. The dataset contains 400 records are used to extract all features methods. WEKA tools were used for classification with 60- 40% percentage split. 60% of the samples are used in the training phase and the remaining samples are used in the increased as shown in Tables.

Random Forest

Table-1: Random Forest Detailed Accuracy by Class

	TP Rate	FP Rate	Precision	Recall	F Measure	MCC	ROC Area	PRC Area	Class
	0.703	0.503	0.675	0.703	0.689	0.202	0.604	0.625	Yes
	0.497	0.297	0.530	0.497	0.513	0.202	0.604	0.531	No
Weight Average	0.620	0.420	0.616	0.620	0.618	0.202	0.604	0.603	

J48

Table-2: Table-2: J48 Detailed Accuracy by Class

	TP Rate	FP Rate	Precision	Recall	F Measure	MCC	ROC Area	PRC Area	Class
	0.929	0.708	0.661	0.000	0.929	0.295	0.582	0.659	Yes
	0.292	0.071	0.734	0.292	0.418	0.295	0.498	0.582	No
Weight Average	0.673	0.452	0.690	0.690	0.630	0.295	0.582	0.598	

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ID3

Table-3: ID3 Detailed Accuracy by Class

	TP Rate	FP Rate	Precision	Recall	F Measure	MCC	ROC Area	PRC Area	Class
	0.757	0.544	0.675	0.757	0.714	0.223	0.596	0.088	Yes
	0.456	0.243	0.557	0.456	0.502	0.223	0.582	0.461	No
Weight Average	0.637	0.423	0.628	0.637	0.629	0.223	0.585	0.575	

LMT (Logistic Model Tree)

Table-4: LMT Detailed Accuracy by Class

	TP Rate	FP Rate	Precision	Recall	F Measure	MCC	ROC Area	PRC Area	Class
	0.891	0.708	0.651	0.891	0.753	0.233	0.586	0.651	Yes
	0.456	0.243	0.557	0.456	0.502	0.223	0.582	0.581	No
Weight Average	0.650	0.467	0.648	0.650	0.611	0.233	0.663	0.661	

Table-5: Detailed Accuracy by Class Stratified Cross-validation-Summary

	Mean Absolute Error	Root Mean Squared Error	Relative Absolute Error	Root Relative Squared Error
Random Forest	0.4292	0.5154	89.2079 %	105.089 %
J48	0.4354	0.4722	90.0993 %	114.3597 %
ID3	0.4321	0.5598	95.11 %	103.24 %
LMT	0.4289	0.4682	89.1558 %	95.4601 %

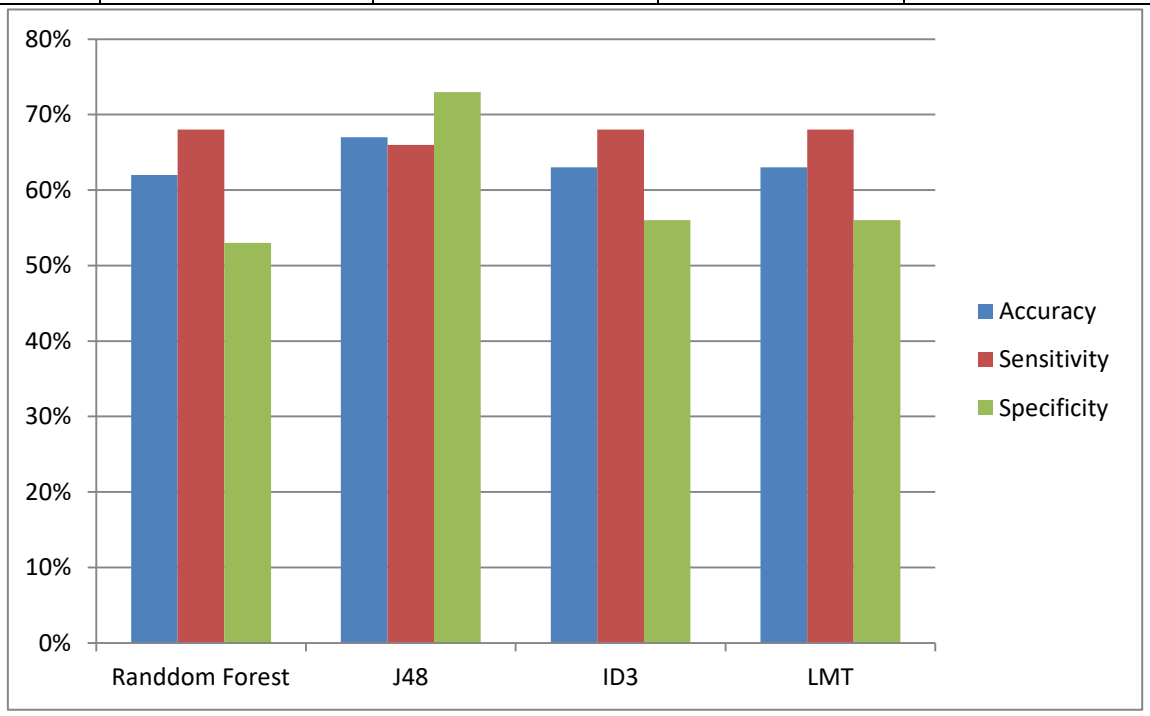

Figure: 1 Result of Classification

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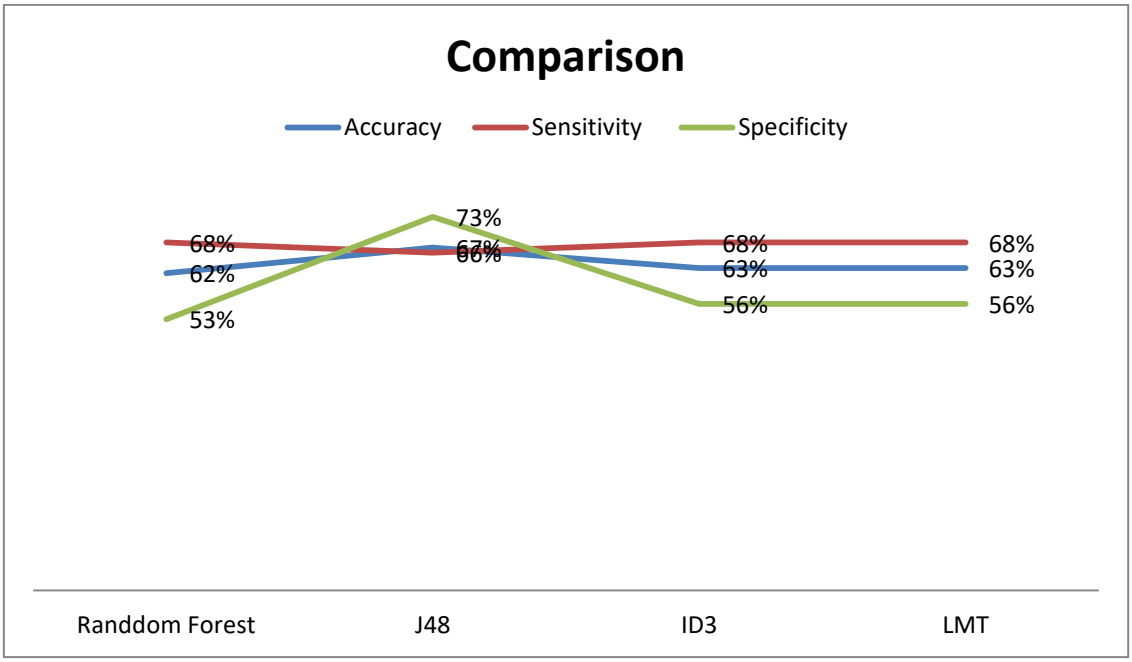

Figure 2: Results of Comparison

Table 6

Classifier	Accuracy	Sensitivity	Specificity
Random Forest	**62%**	**68%**	**53%**
J48	**67%**	**66%**	**73%**
ID3	**63%**	**68%**	**56%**
LMT	**63%**	**68%**	**56%**

IV. RESULTS AND DISCUSSION

In this study, the data set was used for three decision tree classifiers based on continues data set. The highest precision was given with a good for Random Forest accuracy 67.5%, Accuracy 62%, sensitivity 68.12%, Specificity 53.25%, while in Decision ID3 accuracy 63.19% sensitivity 68.52%, Specificity 56.06%, J48 67.00% accuracy, sensitivity 66.00%, Specificity 73.30% and LMT accuracy 65.00%, Sensitivity 65.02%, Specificity 64.25%. The highest Accuracy was 67% for J48 classification algorithm, a good sensitivity result is given by ID3 algorithm, the highest Specificity 73.3% is ID3 algorithm.

V. CONCLUSION

This study aimed to determine the best decision tree classifier using ID3 classifiers, Random Forest and J48 and LMT that all these decision tree algorithms are applied on the dataset. The study contains two main processes; the first one is build the classifier using the 60 percentage from the dataset. The second; building the classifier using the 40 percentage to test the classifier. The highest precision was given with a good for Random Forest accuracy 67.5%, Accuracy 62%, sensitivity 68.12%, Specificity 53.25%, while in Decision ID3 accuracy 63.19% sensitivity 68.52%, Specificity 56.06%, J48 67.00% accuracy, sensitivity 66.00%, Specificity 73.30% and LMT accuracy 65.00%, Sensitivity 65.02%, Specificity 64.25%. The highest Accuracy was 67% for J48 classification algorithm, a good sensitivity result is given by ID3 algorithm, the highest Specificity is 73.3%. So, in future we shall focus on performing the experiments with ensemble technique on the specified decision tree classifiers for further analysis. It can explore optimizing the classifiers for improving the accuracy.

VI. REFERENCE

[1] Mohamed Alhag Alobed, Ali Ahmed, Ashraf Osman Ibrahim, Classification of Breast Cancer Image Using Data Mining Techniques, American Journal of Data Mining and Knowledge Discovery, ISSN: 2578-7810 (Print); ISSN: 2578-7837.

[2] Pareek, A. and S. M. Arora, Breast cancer detection techniques using medical image processing. Breast cancer, 2017. 2 (3).

[3] S. Punitha, S. Ravi, et al., Breast Cancer Detection using Classification Techniques in Digital Mammography: International Science Press, I J C T A, 9 (7), 2016, pp. 3123- 3134, ISSN: 0974- 5572.

[4] Curtis, C., et al., The genomic and transcriptomic architecture of 2,000 breast tumours reveals novel subgroups. Nature, 2012. 486 (7403): p. 346-352.

[5] Perou, C. M., et al., Molecular portraits of human breast tumours. Nature, 2000. 406 (6797): p. 747-752.

[6] Tang, J., Rangayyan, R. M., Xu, J., El Naqa, I., & Yang, Y. (2009). Computer-aided detection and diagnosis of breast cancer with mammography: recent advances. Information Technology in Biomedicine, IEEE Transactions on, 13 (2), 23.

[7] Bonita R, Beaglehole R. Stroke prevention in poor countries. Time for action. Stroke. 2007;38:2871–2872. [PubMed] [Google Scholar]

[8] Pandian JD, Srikanth V, Read SJ, Thrift AG. Poverty and stroke in India. A time to act. Stroke. 2007;38:3063–3069. [PubMed] [Google Scholar]

[9] Abraham J, Rao PS, Inbaraj SG, Shetty G, Jose CJ. An epidemiological study of hemiplegia due to stroke in South India. Stroke. 1970;1:477–481. [PubMed] [Google Scholar]

[10] Prasad K, Vibha D, Meenakshi Cerebrovascular disease in South Asia - Part I: A burning problem. JRSM Cardiovasc Dis. 2012;1:20. [PMC free article] [PubMed] [Google Scholar]

[11] Smith, R. A., V. Cokkinides, and H. J. Eyre, American Cancer Society guidelines for the early detection of cancer, 2006. CA: a cancer journal for clinicians, 2006. 56 (1): p. 11-25. [7].

[12] Affar, M. A., Hybrid Texture based Classification of Brea Mammograms using Ad boost Classifier. International Journal of Advanced Computer Science and Applications, 2017. 8 (5). [8]

[13] Choi, J. P., T. H. Han, and R. W. Park, A hybrid bayesian network model for predicting breast cancer prognosis. Journal of Korean Society of Medical Informatics, 2009. 15 (1): p. 49-57. [9]

[14] Aswinikumarmohanty, Sukantakumar swain, Pratapkumarchampati, Sarojkumarlenka, "Image Mining for Mammogram Classification by Association Rule Using Statistical and GLCM features", IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 5, No 3, 2011.

[15] S. Aruna, Dr S. P. Rajagopalan and L. V. Nandakishore, 2011 Knowledge Based Analysis Of Various Statistical Tools In Detecting Breast Cancer. 2011.

[16] Y. J. Lee, O. L. M. W. H. W. Survival -Time Classification of Breast Cancer Patients. 2008 [cited 2017; Available from: http://www.cs.wisc.edu/dmi/annrev/rev0601/uj.ppt. 16. Clark, A. F. The mini-MIAS database of mammograms.

Estimation of Parameters of Pert Distribution by Using Maximum Likelihood Method of Estimation

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Abstract— The objective of this paper is to estimate the parameters and also construct an Optimal Grouped sample within the absence of prior knowledge or guess values of parameters. In this heuristic algorithm, the Maximum Likelihood Estimator Method is used to find out Estimates the parameters of PERT Distribution. We also compare equispaced and unequispaced Optimally Constructed Grouped data by the method of an Asymptotically Relative Efficiency. We also computed Average Estimate (AE), Variance (VAR), variance (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators.

Keywords: Maximum Likelihood Estimator, PERT Distribution, Equispaced and Unequispaced Optimal Grouped Sample

I. INTRODUCTION

PERT as a network analysis tool, gives the likelihood to research large projects task by task and to estimate the number of some time which can be spent on each task. It was first discovered in the 1950s by the US Navy, during the works on the Polaris missiles project. It was an outsized and sophisticated project involving multiple contractors and an efficient planning tool was a necessity to end the project successfully. The project has been finished long before the deadline and most of the people attributed this success to PERT.

The Program Evaluation and Review Technique (PERT) may be a method used in project management for planning, scheduling and coordinating the tasks of a project. It is supported a network diagram analysis of the various tasks that are involved. PERT is specifically fitted to large, complex and non repititive projects. The aim of this text is to offer an summary of this method with samples of applications and a comparison to other existing network based project management tools. It will also explore the usage of this method in today's projects, its advantages and limitations.

In probability and statistics, the PERT distribution is a family of continuous probability distributions defined by the minimum (a), most likely (b) and maximum (c) values that a variable can take. The Probability density function of PERT Distribution is

$$f(x)=\begin{cases}\dfrac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}} & if\ a\le x\le c\\ 0, \quad Otherwise\end{cases} \tag{1}$$

Where

$$\alpha=\frac{4b+c-5a}{c-a}=1+\frac{4(b-a)}{c-a}$$

$$\beta=\frac{5c-a-4b}{c-a}=1+\frac{4(c-b)}{c-a}$$

A. Estimation of Parameters of PERT Distribution

From theoretical point of view, the most general method of estimation known is the method of Maximum Likelihood Estimators (MLE) which was initially formulated by C.F. Gauss but as a general method of estimation was first introduced by Prof. R.A. Fisher an excellent English statistician, in 1912.

Maximum chances are a comparatively simple method of constructing an estimator for unknown parameter θ. Maximum likelihood estimation (MLE) are often applied in most problems; it's a robust intuitive appeal, and sometimes yields an inexpensive estimator of θ. Furthermore, if the sample is large, the tactic will yield a superb estimator of θ. For these reasons, the tactic of maximum chances is probably the foremost widely used method of estimation in statistics.

We have learned many various distributions for random variables, and every one of these distributions had parameters: the numbers that you simply provide as input once you define a variate. So far when we were working with random variables, we either were explicitly told the values of the parameters, or we could divine the values by understanding the process that was generating the random variables.

II. LIKELIHOOD FUNCTION

Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample of size n from a population with density function f (x, θ). Then the likelihood function of the sample values $x_1, x_2, \ldots\ldots, x_n$, usually denoted by L = L(θ) is their joint density function, given by

$$L = f(x_1,\theta)\,f(x_2,\theta).....\ f(x_n,\theta)$$

$$L=\prod_{i=1}^{n} f(x_i,\theta) \tag{2}$$

L gives the relative likelihood that the random variables assume a particular set of values $x_1, x_2, \ldots\ldots, x_n$. For a given sample $x_1, x_2, \ldots\ldots, x_n$, L becomes a function of the variable θ, the parameter. The principle of maximum

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likelihood consists in finding an estimator for the unknown parameter $\theta = (\theta_1, \theta_2, \ldots\ldots\theta_k)$, say, which maximizes the likelihood function $L(\theta)$ for variations in parameter i.e., we wish to find $\hat{\theta} = \left(\hat{\theta}_1, \hat{\theta}_2, \ldots\ldots, \hat{\theta}\right)$ so that

$$L\left(\hat{\theta}\right) > L(\theta) \qquad \forall\, \theta \in \Theta$$

$$L\left(\hat{\theta}\right) = \sup\ L(\theta) \quad \forall\, \theta \in \Theta$$

Thus if there exists a function $\hat{\theta} = \hat{\theta}\left(x_1, x_2, \ldots\ldots, x_n\right)$ of the sample values which maximizes L for variations in θ, then $\hat{\theta}$ is to be taken as an estimator of θ, $\hat{\theta}$ is usually called Maximum Likelihood Estimator (MLE). Thus $\hat{\theta}$ is the solution, if any, of

$$\frac{\partial L}{\partial \theta} = 0 \quad and \quad \frac{\partial^2 L}{\partial \theta^2} < 0 \tag{3}$$

Since L > 0, and logL is a non-decreasing function of L; L and log L attain their extreme values (maxima or minima) at the same value of $\hat{\theta}$. The first of the two equations in (2) can be rewritten as

$$\frac{1}{L}\frac{\partial L}{\partial \theta} = 0 \Rightarrow \frac{\partial \log L}{\partial \theta} = 0 \tag{4}$$

a form which is much more convenient from particular point of view. If θ is vector valued parameter, then $\hat{\theta} = \left(\hat{\theta}_1, \hat{\theta}_2, \ldots\ldots, \hat{\theta}_k\right)$, is given by the solution of simultaneous equations:

$$\frac{\partial}{\partial \theta_i} \log L = \frac{\partial}{\partial \theta_i} \log L\left(\hat{\theta}_1, \hat{\theta}_2, \ldots\ldots, \hat{\theta}_k\right) = 0;\ i = 1,2,\ldots\ldots, k \tag{5}$$

Equations (4) and (5) are usually referred to as the Likelihood Equations for estimating parameters. Maximum Likelihood Our first algorithm for estimating parameters is called maximum likelihood estimation (MLE). The central idea behind MLE is to select those parameters that make the observed data the most likely. For different values of parameters, the likelihood of our data will be different. If we have correct parameters, our data will be much more probable than if we have incorrect parameters. For that reason we write likelihood as a function of our parameters (θ).

III. MAXIMUM LIKELIHOOD ESTIMATOR

Let $x_1, x_2, x_3, \ldots\ldots x_n$ be a random sample drawn from a PERT distribution with parameters (a, c), then the likelihood function L of this sample is defined as

$$f(x_i, a, c) = f(x_1, a, c)\, f(x_2, a, c)\, f(x_3, a, c) \ldots\ldots\ldots f(x_n, a, c)$$

$$= \prod_{i=1}^{n} f(x_i, a, c)$$

$$= \prod_{i=1}^{n} \left[\frac{(x_i - a)^{\alpha-1} (c - x_i)^{\beta-1}}{\beta(\alpha, \beta)(c - a)^{\alpha+\beta-1}} \right] \tag{6}$$

Taking logarithms on both sides, we get

$$\log\ f(x_i, a, c) = \log \left[\prod_{i=1}^{n} \left\{ \frac{(x_i - a)^{\alpha-1} (c - x_i)^{\beta-1}}{\beta(\alpha, \beta)(c - a)^{\alpha+\beta-1}} \right\} \right]$$

$$= \sum_{i=1}^{n} \log \left[\frac{(x_i - a)^{\alpha-1} (c - x_i)^{\beta-1}}{\beta(\alpha, \beta)(c - a)^{\alpha+\beta-1}} \right]$$

$$= \sum_{i=1}^{n} \log (x_i - a)^{\alpha-1} (c - x_i)^{\beta-1} - n\left[\log \left\{\beta(\alpha, \beta)(c - a)^{\alpha+\beta-1}\right\}\right]$$

$$= \sum_{i=1}^{n} (\alpha - 1) \log (x_i - a) + (\beta - 1) \sum_{i=1}^{n} \log (c - x_i) - n \log [\beta(\alpha, \beta)] - n(\alpha + \beta - 1) \log (c - a) \tag{7}$$

Differentiating (7) Partially w.r.t 'a' and equating to '0' we get

$$\hat{c} = \frac{(\alpha - 1) \sum_{i=1}^{n} x_i (x_i - a)^{-1} + n\beta}{(\alpha - 1) \sum_{i=1}^{n} (x_i - a)^{-1}} \tag{8}$$

Again differentiating (7) partially w.r.t. 'c' and equating to '0' we get

$$\hat{a} = \frac{(\beta - 1) \sum_{i=1}^{n} x_i (c - x_i)^{-1} - n\alpha}{(\beta - 1) \sum_{i=1}^{n} (c - x_i)^{-1}} \tag{9}$$

In the above Estimates equations (8), (9) we can take n=1, we get both a and c values as follows

$$\hat{a} = \frac{x_1 (\beta - 1) + n\alpha (x_1 - c)}{(\beta - 1)}$$

$$\hat{c} = \frac{x_1 (\alpha - 1) + n\beta (x_1 - a)}{(\alpha - 1)}$$

A. APPLICATIONS

Real data set 1

1) QQ-Plot:

The data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. Unfortunately, the units of measurement are not given in the paper. The data are:
4.99, 3.97, 2.18, 3.14, 2.19, 4.96, 2.66, 4.98, 3.37, 2.85, 4.88, 3.27, 4.29,3.29, 4.10, 4.76, 4.49, 4.24, 2.85, 3.16, 2.16, 2.34, 3.84, 4.52, 2.89, 4.87, 2.87, 2.40, 4.30, 3.73, 3.45, 4.98, 4.43, 2.09, 2.30, 2.89, 2.53, 2.01, 4.94, 2.23, 4.15, 2.73, 3.59, 3.27, 4.70, 2.14, 4.84, 4.46, 4.42, 2.57, 3.64, 3.54, 3.70, 3.95, 2.98, 4.23, 3.78, 4.84, 3.54, 3.03, 2.98, 3.89. These data have also been analyzed by Smith and Naylor (1987). QQ-Plot given in graph-3.1.

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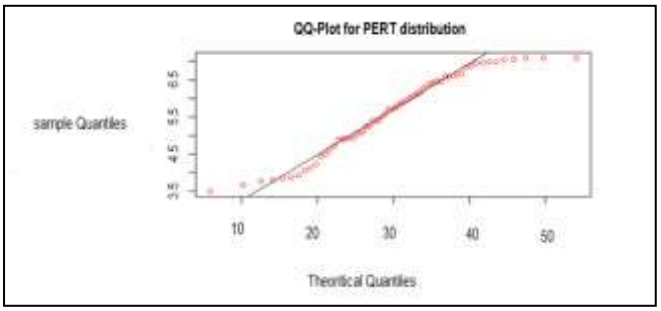


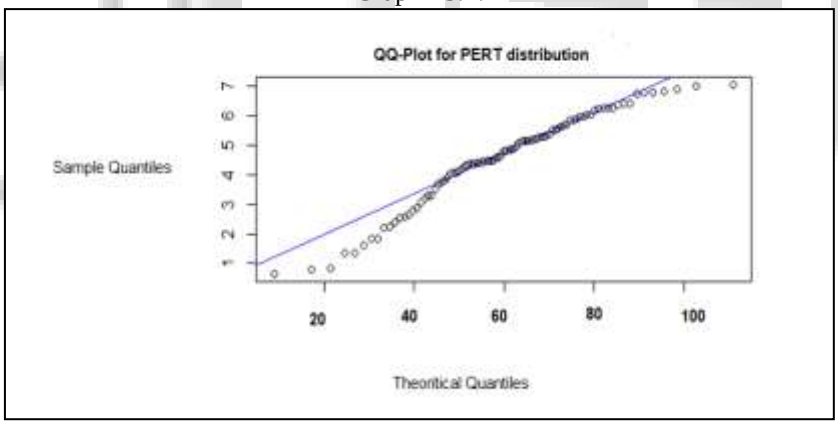
Graph - 3.1:

B. MLE Estimation for Real Data Set1:

The data set consists of 62 observations of the strengths of 3.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. Unfortunately, the units of measurement are not given in the paper. The data are:
4.99, 3.97, 2.18, 3.14, 2.19, 4.96, 2.66, 4.98, 3.37, 2.85, 4.88, 3.27, 4.29,3.29, 4.10, 4.76, 4.49, 4.24, 2.85, 3.16, 2.16, 2.34, 3.84, 4.52, 2.89, 4.87, 2.87, 2.40, 4.30, 3.73, 3.45, 4.98, 4.43, 2.09, 2.30, 2.89, 2.53, 2.01, 4.94, 2.23, 4.15, 2.73, 3.59, 3.27, 4.70, 2.14, 4.84, 4.46, 4.42, 2.57, 3.64, 3.54, 3.70, 3.95, 2.98, 4.23, 3.78, 4.84, 3.54, 3.03, 2.98, 3.89.
We obtained
$\hat{a}_{MLE}$ = 2.216 $\hat{b}_{MLE}$ = 4.5432 and $\hat{c}_{MLE}$ = 6.5486 by using MLE method
Generalized data set 2
The data set consists of 100 observations from the generalized data. The data are:

1.590207, 3.453602, 2.639683, 4.314286, 1.736906, 3.801353, 2.444123, 4.988625,
3.135345, 3.513842, 4.835063, 1.850554, 3.876850, 4.275311, 1.355567, 4.489725,
1.891095, 4.344346, 3.620824, 4.490531, 3.018184, 4.810403, 1.856308, 2.137707,
2.608060, 1.219972, 4.788248, 3.858185, 2.206794, 3.180953, 2.424288, 4.797195,
2.092996, 3.911932, 4.744874, 2.679947, 1.677493, 2.824798, 1.021153, 2.368581,
4.685041, 4.919465, 4.929624, 2.390345, 1.553020, 2.560500, 2.784886, 2.826569,
3.853104, 3.217942, 1.095691, 3.995635, 3.141613, 2.895459, 4.745624, 4.495316,
4.141889, 1.256580, 1.195974, 3.147275, 1.856513, 4.201841, 1.425041, 3.862838,
4.558851, 1.615452, 4.302462, 4.906739, 1.664893, 1.041652, 3.507625, 1.970019,
1.557807, 4.582408, 3.489803, 1.481760, 2.191980, 3.335935, 4.461020, 3.995509,
1.507663, 1.869488, 3.988402, 4.310966, 1.198871, 2.512960 4.943203, 4.084483,
4.331192, 2.376588, 4.35175, 4.12862,4 1.287679, 2.847278, 2.011727, 2.402388,
1.319219, 4.257921, 1.511085, 1.113146.

Graph - 3.2:



C. MLE Estimation for Generalized Data Set2:

The data set consists of 100 observations from the generalized data. The data are:

1.590207, 3.453602, 2.639683, 4.314286, 1.736906, 3.801353, 2.444123, 4.988625,
3.135345, 3.513842, 4.835063, 1.850554, 3.876850, 4.275311, 1.355567, 4.489725,
1.891095, 4.344346, 3.620824, 4.490531, 3.018184, 4.810403, 1.856308, 2.137707,
2.608060, 1.219972, 4.788248, 3.858185, 2.206794, 3.180953, 2.424288, 4.797195,
2.092996, 3.911932, 4.744874, 2.679947, 1.677493, 2.824798, 1.021153, 2.368581,
4.685041, 4.919465, 4.929624, 2.390345, 1.553020, 2.560500, 2.784886, 2.826569,
3.853104, 3.217942, 1.095691, 3.995635, 3.141613, 2.895459, 4.745624, 4.495316,
4.141889, 1.256580, 1.195974, 3.147275, 1.856513, 4.201841, 1.425041, 3.862838,
4.558851, 1.615452, 4.302462, 4.906739, 1.664893, 1.041652, 3.507625, 1.970019,
1.557807, 4.582408, 3.489803, 1.481760, 2.191980, 3.335935, 4.461020, 3.995509,
1.507663, 1.869488, 3.988402, 4.310966, 1.198871, 2.512960 4.943203, 4.084483,
4.331192, 2.376588, 4.35175, 4.12862,4 1.287679, 2.847278, 2.011727, 2.402388,
1.319219, 4.257921, 1.511085, 1.113146.

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We obtained
$\hat{a}_{MLE}$ = 1.0184, $\hat{b}_{MLE}$ = 5.154 and $\hat{c}_{MLE}$ = 7.0325 by using MLE method

IV. SIMULATION STUDY

In order to obtain the Maximum Likelihood estimation of Location Parameter (a) and Scale Parameter (c) and study the properties of their estimate through the Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If ξ_{lm} is the Maximum Likelihood Method estimation of ξ_m m=1,2 where ξ_m is the general notation hat can be replaced by ξ_1 = a, and ξ_2 = 2, based on the sample 1, (l=1,2,3,.....r), Then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square (MSR), Relative Absolute Basis (RAB) and Relative Error (RE) are given respectively by

$$\text{Average Estimate } (\hat{\psi}_m) = \frac{\sum_{l=1}^{r} \hat{\psi}_m}{r}$$

$$\text{Variance } (\hat{\psi}_m) = \frac{\sum_{l=1}^{r} \left(\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}\right)^2}{r}$$

$$\text{Standard Deviation } (\hat{\psi}_m) = \sqrt{\frac{\sum_{l=1}^{r} \left(\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}\right)^2}{r}}$$

$$\text{Mean Absolute Deviation} = \frac{\sum_{l=1}^{r} Med\left(\left|\hat{\psi}_{lm} - \overline{\hat{\psi}_{lm}}\right|\right)}{r}$$

$$\text{Mean Square Error } (\hat{\psi}_m) = \frac{\sum_{l=1}^{r} (\hat{\psi}_{lm} - \psi_m)^2}{r}$$

$$\text{Relative Absolute Bias } (\hat{\psi}_m) = \frac{\sum_{l=1}^{r} |\hat{\psi}_{lm} - \psi_m|}{r.\psi_m}$$

$$\text{Relative Error } (\hat{\psi}_m) = \frac{1}{r}\left(\frac{\sum_{l=1}^{r} MSE\sqrt{\hat{\psi}_{lm}}}{\psi_m}\right)^2$$

Maximum Likelihood method for estimating the PERT (a,b,c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n*=20(20)200 are considered. The MLEs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a=5, b=6 and c=7 in Table 3.1.

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	3.5487	0.8775	0.9568	0.7051	1.9194	0.9754	0.7338
	b	4.2514	0.8001	0.91245	0.5926	1.9213	0.9457	0.6549
	c	5.6247	0.8834	0.9087	0.0379	1.9417	0.8954	0.6617
40	a	3.6598	0.8567	0.9457	0.6177	1.90549	0.9612	0.669
	b	4.3568	0.7587	0.8964	0.5854	1.9172	0.8954	0.6132
	c	5.7754	0.8766	0.8886	0.1199	1.8316	0.8996	0.5960
60	a	3.7698	0.866	0.8854	0.6069	1.8244	0.8547	0.6447
	b	4.4698	0.7095	0.8546	0.4683	1.8146	0.8412	0.5677
	c	5.8965	0.8547	0.8569	0.447	1.8099	0.8524	0.5764
80	a	3.9658	0.7987	0.8364	0.5739	1.8081	0.8102	0.5657
	b	4.721	0.6187	0.8136	0.3934	1.7646	0.7984	0.558
	c	5.9987	0.8405	0.8069	0.1691	1.8101	0.8146	0.5053
100	a	4.2586	0.7695	0.7963	0.4689	1.8038	0.7846	0.5196
	b	5.1047	0.5341	0.7732	0.3117	1.7313	0.7548	0.5354
	c	6.1548	0.7568	0.7568	0.1190	1.7585	0.7956	0.4403
120	a	4.4568	0.6755	0.7435	0.4089	1.7039	0.6987	0.4194
	b	5.5876	0.404	0.7398	0.2147	1.7011	0.6625	0.5066
	c	6.6548	0.7199	0.7069	0.0898	1.7033	0.7736	0.3096
140	a	4.6356	0.6174	0.6987	0.3313	1.7008	0.6147	0.5335
	b	5.6329	0.3627	0.6788	0.1998	1.6034	0.5983	0.4116
	c	6.7245	0.6552	0.6587	0.3023	1.6034	0.7196	0.2970
160	a	4.7546	0.601	0.6447	0.2125	1.6128	0.5987	0.5323
	b	5.7986	0.3285	0.6358	0.1911	1.5035	0.4968	0.3119
	c	6.8321	0.5759	0.6298	0.2832	1.5037	0.6987	0.2101
180	a	4.8856	0.53967	0.6087	0.1375	1.6107	0.5214	0.2295
	b	5.8247	0.3173	0.5987	0.1543	1.5003	0.4587	0.2146
	c	6.8769	0.4905	0.5746	0.2900	1.5004	0.6478	0.1230
200	a	4.9658	0.50921	0.5687	0.1124	1.5151	0.4982	0.2351
	b	5.9867	0.3013	0.5532	0.0775	1.4053	0.3245	0.1145
	c	6.9245	0.3547	0.4987	0.2567	1.4137	0.5966	0.2122

Table 3.1: Maximum Likelihood Estimation of PERT Distribution (a=5, b=6, c=7)

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A. Observations:

1) Average Estimate of PERT parameters by MLE are increased when sample size is increased.
2) Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Maximum Likelihood method for estimating the PERT (a, b, c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n* = 20(20)200 are considered. The MLEs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a = 2, b = 2.5 and c = 3 in Table 3.2.

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	0.9125	0.6970	0.7862	0.5963	1.9444	0.7950	0.7770
	b	0.9948	0.6840	0.7654	0.5435	1.9844	0.7750	0.7376
	c	1.2546	0.5742	0.7568	0.5770	1.9732	0.6558	0.6465
40	a	1.0201	0.5678	0.6215	0.5914	1.9222	0.6298	0.6157
	b	1.3232	0.4631	0.6722	0.5365	1.9198	0.6861	0.6706
	c	1.3742	0.3857	0.5956	0.5129	1.9161	0.4168	0.7049
60	a	1.2082	0.5607	0.5735	0.4959	1.8801	0.6128	0.6070
	b	1.9899	0.5129	0.5234	0.52697	0.8172	0.5332	0.4647
	c	2.0458	0.3069	0.5072	0.4435	0.8403	0.3108	0.2696
80	a	1.3893	0.4546	0.4687	0.4850	1.6670	0.4821	0.4014
	b	1.7462	0.5128	0.5139	0.5089	0.7323	0.5206	0.5091
	c	1.9667	0.2385	0.4465	0.3514	0.8133	0.2755	0.2694
100	a	1.4311	0.2538	0.4939	0.4108	1.5428	0.3572	0.3057
	b	2.0524	0.4155	0.4987	0.4799	0.6755	0.5195	0.4923
	c	2.5837	0.2329	0.3985	0.2345	0.8017	0.1388	0.1083
120	a	1.5306	0.2452	0.4356	0.3934	0.5049	0.3080	0.2817
	b	2.0309	0.4133	0.4635	0.4255	0.6468	0.5086	0.4915
	c	2.6154	0.1870	0.3836	0.1440	0.7932	0.1262	0.1030
140	a	1.7845	0.2349	0.4189	0.3293	0.5003	0.2754	0.2455
	b	2.1456	0.3223	0.4068	0.2905	0.6124	0.3695	0.3063
	c	2.5634	0.1335	0.3587	0.1362	0.7101	0.1657	0.1241
160	a	1.8539	0.2309	0.3956	0.3241	0.4962	0.2648	0.2573
	b	2.2457	0.1160	0.3854	0.2795	0.5033	0.2764	0.2678
	c	2.7854	0.1925	0.3247	0.1299	0.7095	0.1922	0.1852
180	a	1.8687	0.1269	0.3875	0.3179	0.3854	0.2262	0.2138
	b	2.2001	0.1490	0.3759	0.2565	0.4956	0.2001	0.199
	c	2.8365	0.1561	0.3145	0.1254	0.6061	0.1734	0.1616
200	a	1.8921	0.1104	0.2968	0.2106	0.3521	0.2089	0.1994
	b	2.3510	0.138	0.3524	0.1656	0.4548	0.1734	0.1602
	c	3.106	0.1237	0.2954	0.1156	0.5051	0.1313	0.1082

Table 3.2: Maximum Likelihood Estimation of PERT Distribution (a=2, b=2.5, c=3)

B. OBSERVATIONS:

1) Average Estimate of PERT parameters by MLE are increased when sample size is increased.
2) Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

Maximum Likelihood method for estimating the PERT (a, b, c) Newton-Raphson simulation for a three parameter combinations and the process is repeated 10,000 times for different sample sizes *n* = 20(20)200 are considered. The MLEs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters a = 3, b= 4 and c= 5 in Table 3.3.

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	a	1.4275	0.5945	0.6854	0.9932	1.0089	0.9872	0.9869
	b	1.5624	0.3634	0.5938	0.9373	1.0355	0.9375	0.9437
	c	3.0029	0.5746	0.6733	0.9258	1.1112	0.9598	0.9487
40	a	1.5227	0.4606	0.6527	0.9535	1.0072	0.9377	0.9244
	b	1.7462	0.2232	0.4932	0.9203	1.0259	0.9203	0.9145
	c	3.1279	0.4014	0.6105	0.8972	1.0141	0.9096	0.9038

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60	a	2.1883	0.4417	0.6535	0.9245	0.9898	0.9118	0.9037
	b	2.2451	0.2685	0.5064	0.7872	0.9909	0.9186	0.9073
	c	3.4899	0.1937	0.4398	0.7061	0.9742	0.9102	0.9022
80	a	2.3655	0.2824	0.5188	0.8476	0.9938	0.9069	0.8935
	b	2.5189	0.1416	0.3822	0.7362	0.9836	0.8685	0.7957
	c	2.7376	0.1367	0.3587	0.5917	0.9677	0.8907	0.8098
100	a	2.5432	0.1183	0.3432	0.668	0.9258	0.8892	0.7882
	b	2.6243	0.1083	0.3164	0.4911	0.9677	0.7551	0.6755
	c	3.8038	0.0941	0.3102	0.3551	0.9434	0.6654	0.6065
120	a	2.6243	0.1097	0.3296	0.2922	0.7027	0.8727	0.7644
	b	2.7035	0.0536	0.2268	0.3224	0.8549	0.6632	0.5657
	c	3.9419	0.0727	0.2598	0.2052	0.7576	0.6233	0.5018
140	a	2.7736	0.1058	0.3157	0.2041	0.895	0.7792	0.6508
	b	2.8692	0.04523	0.2054	0.2943	0.6979	0.6166	0.5614
	c	4.3938	0.052	0.2135	0.1668	0.6981	0.5822	0.5055
160	a	2.8356	0.1034	0.3198	0.2015	0.8015	0.6446	0.5448
	b	2.9332	0.041	0.1998	0.235	0.5838	0.6056	0.4564
	c	4.5636	0.0919	0.2969	0.1345	0.5651	0.4546	0.4036
180	a	2.843	0.1052	0.3121	0.1925	0.7258	0.6365	0.4402
	b	3.4493	0.0101	0.1051	0.2311	0.4684	0.5304	0.3529
	c	4.6176	0.1013	0.2908	0.1254	0.4265	0.4421	0.3004
200	a	2.9576	0.1007	0.3052	0.1667	0.6323	0.6217	0.5361
	b	3.8587	0.001	0.0298	0.0826	0.3987	0.4275	0.3501
	c	4.7283	0.1136	0.3296	0.0433	0.2509	0.4358	0.3064

Table 3.3: Maximum Likelihood Estimation of PERT Distribution (a=3, b=4, c=5)

C. Observations:

1) Average Estimate of PERT parameters by MLE are increased when sample size is increased.
2) Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

We calculate the Maximum Likelihood method for estimating the PERT (α, β). Newton-Raphson iterative procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are considered. The MLEs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α =3 and β = 2.5 in Table 3.4.

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	1.6954	0.9987	0.9954	0.9056	1.9856	0.9987	0.8964
	β	1.6045	0.9965	0.8832	0.8765	1.8956	0.9979	0.8542
40	α	2.0056	0.9856	0.8732	0.8569	1.9564	0.9951	0.8215
	β	1.6142	0.9764	0.8524	0.8432	1.8547	0.9923	0.8105
60	α	2.1457	0.942	0.8169	0.8009	1.8954	0.9863	0.7953
	β	1.7658	0.9258	0.8004	0.7932	1.7852	0.9854	0.7523
80	α	2.2478	0.9181	0.7968	0.7754	1.7954	0.9102	0.7721
	β	1.9854	0.9018	0.7966	0.7698	1.6987	0.9653	0.7432
100	α	2.4214	0.8994	0.7564	0.7436	1.6985	0.8932	0.7132
	β	1.9965	0.8965	0.7149	0.6998	1.5987	0.9258	0.7098
120	α	2.5147	0.8753	0.6998	0.6532	1.6148	0.8799	0.6958
	β	2.0154	0.8632	0.6913	0.6739	1.5584	0.8963	0.6423
140	α	2.6879	0.8269	0.6687	0.6522	1.5964	0.8637	0.6733
	β	2.1149	0.8083	0.6619	0.6403	1.4963	0.8215	0.6098
160	α	2.7034	0.7985	0.6302	0.5969	1.5016	0.8168	0.6498
	β	2.1391	0.7752	0.6289	0.6198	1.4198	0.8096	0.5698
180	α	2.7378	0.6984	0.6268	0.6009	1.4956	0.7998	0.6266
	β	2.1657	0.7025	0.6198	0.5998	1.3987	0.7932	0.5441
200	α	2.8145	0.6589	0.5769	0.5632	1.4098	0.7515	0.6089
	β	2.2987	0.6986	0.5684	0.5503	1.3276	0.7413	0.5229

TABLE-3.4: Maximum Likelihood Estimation of PERT Distribution (α =3, β =2.5)

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D. Observations:

1) Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
2) Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

We calculate the Maximum likelihood Estimation (MLE) method for estimating the PERT (α , β) Newton-Raphson simulation procedure for a two parameter combinations and the process is repeated 10,000 times for different sample sizes n = 20(20)200 are taken. The MLEs and their Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the parameters are unknown population parameters of PERT distribution. Population parameters α = 4.5 and β = 4 in Table 3.5.

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	α	2.3856	0.9947	0.9956	0.7854	1.9862	0.9985	0.6958
	β	2.3954	0.9457	0.9658	0.6985	1.8956	0.9758	0.8654
40	α	2.4573	0.93254	0.9887	0.7765	1.9521	0.9562	0.6477
	β	2.4956	0.8956	0.9498	0.6721	1.8526	0.8965	0.8431
60	α	2.6731	0.8754	0.9754	0.7564	1.8963	0.9142	0.6054
	β	2.7546	0.8569	0.8954	0.6587	1.7956	0.8126	0.8098
80	α	2.7452	0.8137	0.9542	0.7299	1.7586	0.8965	0.5932
	β	2.8998	0.7968	0.8187	0.6251	1.7453	0.7984	0.7951
100	α	2.8736	0.7764	0.8987	0.709	1.6958	0.8764	0.5542
	β	2.9643	0.7781	0.8047	0.6054	1.6548	0.7098	0.7421
120	α	2.9268	0.7589	0.8253	0.6955	1.5962	0.8654	0.5187
	β	2.8931	0.6548	0.6996	0.5547	1.5236	0.6587	0.6465
140	α	2.8546	0.7456	0.7798	0.6587	1.4968	0.8059	0.4965
	β	2.8213	0.6183	0.6587	0.5247	1.3968	0.6287	0.5025
160	α	3.2951	0.7154	0.7256	0.6198	1.2965	0.7998	0.4563
	β	2.9035	0.5964	0.5987	0.5187	1.2658	0.5987	0.4958
180	α	3.9654	0.6988	0.7058	0.6097	1.1597	0.7564	0.3954
	β	3.5089	0.5746	0.5782	0.4987	1.1058	0.5798	0.4251
200	α	4.3263	0.6587	0.6987	0.5988	1.0053	0.7249	0.3784
	β	3.8679	0.4987	0.5465	0.4545	0.9867	0.5398	0.4157

Table 3.5: Maximum Likelihood Estimation of PERT Distribution (α =4.5, β =4)

E. Observations:

1) Average Estimate (AE) of PERT parameters of estimated α, β by MLE are increased when sample size is increased.
2) Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) by MLE is decreased when sample size is increased.

V. CONCLUSIONS

Method of Maximum Likelihood Estimation is the better one for estimating the parameters of the PERT distribution; Since Sample size increases Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters are decreases.

The Method of Maximum Likelihood Estimation has the smallest Variance (VAR), Standard deviation (SD), Mean absolute deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both parameters, proving to be the efficient method.

VI. OBSERVATIONS FOR THE SIMULATION RESULT

1) Method of Maximum Likelihood estimators are less biased.
2) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error (RE) of the estimators are dependent on the sample sizes.
3) The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error(RE) of the estimators are independent on the population parameter values.
4) The Average estimate (AE) of the Method of Moments $(\hat{\alpha})$ and $(\hat{\beta})$ estimators are increased when sample size increased.
5) The Variance (VAR) of Method of Maximum likelihood Estimators$(\hat{\alpha})$ and $(\hat{\beta})$ are decreased when sample size increased.
6) The Standard Deviation (SD) of Method of Maximum likelihood Estimators$(\hat{\alpha})$ and $(\hat{\beta})$ are decreased when sample size increased.
7) The Mean Square Error (MSE) of Method of Maximum likelihood Estimators$(\hat{\alpha})$ and $(\hat{\beta})$ are decreased when sample size increased.

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8) The Relative Absolute Bias (RAB) of Method of Maximum likelihood Estimators$(\hat{\alpha})$ and $(\hat{\beta})$ are decreased when sample size increased.
9) The Relative Error (RE) of Method of Maximum likelihood Estimators$(\hat{\alpha})$ and $(\hat{\beta})$ are decreased when sample size increased.

REFERENCES

[1] Azaron, A., Katagiri, H., & Sakawa, M. (2007). Time-cost trade-off via optimal control theory in Markov PERT networks. *Annals of Operations Research*, *150*, 47–64.
[2] Blom, G., 1958. Statistical Estimates and Transformed Beta Variables, John Wiley & Sons, New York.
[3] Berkson (1957) developed the tables for the maximum likelihood estimate of the logistic distribution. Vol. 13, No. 1 (Mar., 1957), pp. 28-34 (7 pages).
[4] Balakrishnan and Kocherlakota (1985) developed estimation of Parameters in the double Weibull distribution. Vol. 47, No. 2 (Aug., 1985), pp. 161-178 (18 pages).
[5] Balakrishnan. N and Leung (1988) have obtained the best linear unbiased estimates for the location and scale parameters of type I generalized logistic distribution. Volume 17, 1988 - Issue 1, Pages 25-50.
[6] Beckman, R. J. and Tietjen, G. L. (1978). 'Maximum Likelihood Estimation for the Beta Distribution,' *J.* Statist. Cornput.- Sinul., 7, 253-258.
[7] Clark, C.E. (1962).The PERT Model for the Distribution of an Activity. Operations Research, 10, 405–406.
[8] Chapman, D. G. and Robbins, H. (1951). 'Minimum Variance Estimation without Regularity Assumptions,' Ann. Mach. Statist., 22, 581-586.
[9] Cheng R. C. H. and Amin, N. A. K. (1981). 'Maximum Likelihood Estimation of Parameters in the Inverse Gaussian Distribution, with Unknown Oripin,' Tech-rometrics, 23, 257-263.
[10] Cohen, A. C.and Whitten, B. J., (1982). 'Modified Moment and Maximum Likelihood Estimators of the Three-Parameter Gamma Distribution,' Comm. Statist.-Simul., 11, 197-216.
[11] Farnum, N. R., & Stanton, L. W. (1987). Some results concerning the estimation of beta distribution parameters in PERT. Journal of the Operational Research Society, 38(3), 287–290.
[12] Fielitz, B. D. and Myers, B. L. (1976). 'Estimation of the Parameters in the Beta Distribution - Reply,' *Decision Sciences*, 7, 163-164.
[13] D. L. Keefer and W. A. Verdini, "Better estimation of PERT activity time parameters," Manage. Sci., vol. 39, no. 9, pp. 1086–1091, 1993.
[14] Dimitri Golenko-Ginzburg. On the Distribution of Activity Time in PERT. The Journal of the Operational Research Society 39, No. 8, 767-771 (1988)
[15] J. J. Moder and E. G. Rodgers, "Judgment estimates of the moments of PERT type distributions," Manage. Sci., vol. 15, no. 2, pp. B76–B83, 1968.
[16] Gjeddebaek (1949) considered the estimation of parameters of normal distribution by maximum likelihood method from grouped data. Volume 1949, 1949 - Issue 1, Pages 135-159.
[17] Hill, J., Thomas, L. C., & Allen, D. E. (2000). Experts' estimates of task durations in software development projects. International Journal of Project Management, 18, 13–21.
[18] Kamburowski, J. (1997). New validations of PERT times. Omega, The International Journal of Management Science, 25(3), 323–328.
[19] Lindley D.V, (1950) suggested some grouping correction for maximum likelihood estimate of the parameter from grouped samples. Vol. 4, No. 2 (May, 1989), pp. 164-179 (16 pages).
[20] Li Yang, Yu Haishan, Shen qin and Zhang Shouzhong, "An Estimating Method for Aviation Project's Duration Based on Improved-PERT" Industrial Engineering and Management, China, vol. 4, pp. 38-42, April 2007.
[21] Lloyd E.H, (1952) obtained estimators of location and scale parameters of order statistics using least squares method of estimation. Vol. 39, No. 1/2 (Apr., 1952), pp. 88-95 (8 pages).
[22] Mac Grimmon, Kenneth R. and Charles A. Ryavec, "An Analytical Study of PERT Assumptions," Operations Research, Jan.-Feb., 1964, pp. 16-36.
[23] N. R. FARNUM and L. W. STANTON (1987) Some results concerning the estimation of beta distribution parameters in PERT. J. Opt Res. Soc. 38, 287-290.
[24] Ramamohan, C. H. and Anjaneyulu, G.V.S.R. (2011). How the Least Square Method be good for Estimating the parameters of Two-Parameter Weibull Distribution from an optimally constructed grouped sample. International Journal of Statistics and Systems, Vol. 6, No. 4, PP. 525-535.
[25] Ranneby, B., (1984). The maximum spacing method: an estimation method related to the maximum likelihood method. Scandinavian Journal of Statististics 11, pp: 93–112.
[26] Smith, R.L., Naylor, J.C. (1987): A comparison of maximum likelihood and Bayesian estimators for the three-parameter Weibull distributuion. Journal of Applied Statistics, 36, pp: 358–369 .
[27] Vijaya Lakshmi, M. Rajasekhram, O. V. and Anjaneyulu, G. V. S. R. (2018). Estimation of Scale (λ) and Location (μ) of Two-Parameter Rayleigh Distribution Using Median Ranks method, International Journal of Science and Research (IJSR),1(1), PP. 1948-1952.
[28] Vijaya Lakshmi, M. Rajasekhram, O. V. and Anjaneyulu, G. V. S. R. (2019).Estimation of Scale(θ) and Shape (α) Parameters of Power Function Distribution By Least Squares Method Using Optimally Constructed Grouped Data, Journal of Emerging Technologies and Innovative Research (JETIR), 6(6), PP. 320-328.
[29] Vijaya Lakshmi, M. and Anjaneyulu, G. V. S. R. (2019), Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Least Square Regression Method, International Journal For Research and Applied Science and Engineering Technology (IJRASET), 7(8), PP. 2321-9653.

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[30] Vijaya Lakshmi, M. and Anjaneyulu, G. V. S. R. (2019), Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method, International Journal of Research and Analytical Reviews, 6, 2, PP. 558-567.
[31] Woolsey, R.E., 1992. The fifth column: The PERT that never was or data collection as an optimizer. Interfaces 22, 112-114.
[32] Zhang Yun-ning and Sun Xi-xi, 2010, Research on improved PERT model in analysis of schedule risk of project, International Conference on E-Business and E-Government.

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PERT DISTRIBUTION AND ITS PROPERTIES

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ABSTRACT

The PERT distribution is one of the most popular probability continuous distributions with applications to real life data. In this paper some structural properties of this distribution such as Moments, moment generating function, Characteristics function, Cumulative distribution function, Survival function, Hazard function, and also derive the Information matrix. The plots of the Probability density function, cumulative distribution function, Survival function and hazard function of the PERT distribution were constructed for ease of understanding of its shapes under different parameter combinations.

Keywords: PERT Distribution, Moment Generating Function, Survival Function, Hazard Function, Information Matrix.

I. INTRODUCTION

The PERT distribution is widely used in risk analysis to represent the uncertainty of the worth of some quantity where one is counting on subjective estimates, because the three parameters defining the distribution are intuitive to the estimator. In PERT analysis the activity-time distribution is assumed to be a beta distribution, and therefore the mean and variance of the activity time are estimated on the idea of the 'pessimistic', 'most likely' and 'optimistic' completion times, which are subjectively determined by an analyst. In this paper, on the idea of the study of the PERT assumptions, we present an improvement of those estimates. It is also shown that, by means of additional reasonable assumptions, the activity-time distribution in PERT analysis could also be essentially simplified.

PERT helps within the management of the projects by forming a network diagram. The analysis of the network diagram in PERT helps in scheduling the activities associated with the project. Furthermore, using PERT forces the managers involved in the construction industry to organize and quantify project information and allows them to present a graphic display of the project.

In addition, this technique helps in identifying the critical activities involved in the project and in keeping check on those activities that need to be closely monitored. Especially in the case of construction projects, the PERT method would allow the management to have clear views regarding the utilization of processes to maximize the usage of available resources. PERT serves as a key tool to determine the cost, material, time, and requirement of capital related to a specific project.

In probability and statistics, the PERT distribution may be a family of continuous probability distributions defined by the minimum (a), presumably (b) and maximum (c) values that a variable can take. The pdf of PERT Distribution is

Where $\alpha = \frac{4b + c - 5a}{c - a} = 1 + \frac{4(b-a)}{c-a}$

$\beta = \frac{5c - a - 4b}{c - a} = 1 + \frac{4(c-b)}{c-a}$

$$f(x) = \begin{cases} \frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}} & if \ \ a \le x \le c \\ 0, \ \ otherwise \end{cases} \qquad (1)$$

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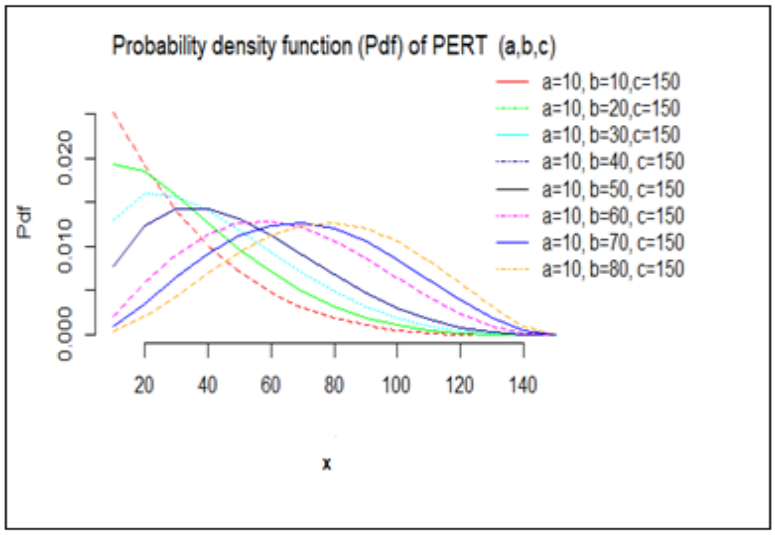

CHART-1.1: Probability Density Function of PERT Distribution (a,b,c)

MOMENTS OF PERT DISTRIBUTION

The Probability density function of PERT Distribution is

$$f(x)=\frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}}, \qquad a\le x\le c$$

The movements of PERT distribution are given by

$$E\left[X^{r}\right]=\int_{a}^{c}x^{r}f(x)dx$$

$$=\frac{1}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}}\int_{a}^{c}x^{r}(x-a)^{\alpha-1}(c-x)^{\beta-1}dx$$

$$E\left[X^{r}\right]=\sum_{k=0}^{r}\left[\binom{r}{k}a^{r-k}.c^{k}\frac{\Gamma(\alpha+k)\Gamma(\beta+r-k)}{\beta(\alpha,\beta)\Gamma(\alpha+\beta+r)}\right] \quad \text{Where} \quad r\ge k \qquad \ldots (2)$$

Putting r = 1, in equation (2), we get

$$\therefore \quad E(X)=\mu=\left(\frac{a\beta+c\alpha}{\alpha+\beta}\right)^{2}=\frac{a+4b+c}{6}$$

Which is the mean μ of the PER distribution **Putting r=2, in the question (2), we get**

$$\therefore E(X^{2})=\frac{a^{2}\beta(\beta+1)+2ac\alpha\beta+c^{2}\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$$

$$Now Varnance\ \sigma^{2}=E(X^{2})-E[(X)]^{2}$$

$$=\frac{a^{2}\beta(\beta+1)+2ac\alpha\beta+c^{2}\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}-\left(\frac{a\beta+c\alpha}{\alpha+\beta}\right)^{2}$$

$$\therefore Variance\ \sigma^{2}=\frac{(\mu-a)(c-\mu)}{7} \quad or \quad Variance\ \sigma^{2}=\frac{(c-a)^{2}}{36}$$ **Putting r = 3, in the question (2), we get**

$$E\left(X^{3}\right)=\frac{a^{3}(\beta)_{3}+3a^{2}c(\alpha)_{1}(\beta)_{2}+3ac^{2}(\alpha)_{2}(\beta)_{1}+c^{3}(\alpha)_{3}}{(\alpha+\beta)_{3}}$$ **Putting r = 4, in the question (2), we get**

$$E\left(X^{4}\right)=\frac{a^{4}(\beta)_{4}+4a^{3}c(\alpha)_{1}(\beta)_{3}+6a^{2}c^{2}(\alpha)_{2}(\beta)_{2}+4ac^{3}(\alpha)_{3}(\beta)_{1}}{(\alpha+\beta)_{4}}$$

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MOMENT GENERATING FUNCTION OF PERT DISTRIBUTION

Hyper geometric function: The Poehhammer symbol $(\alpha)_r$ is defined by

$$(\alpha)_r = \alpha(\alpha+1)(\alpha+2)(\alpha+3).....(\alpha+r-1) \quad (3) \quad = \frac{\Gamma(\alpha+r)}{\Gamma(\alpha)}, \ where\ r \in Z^+$$

$$and \ \ (\alpha)_0 = 1$$

The general Hypergeomentric function is

$${}_mF_n(\alpha_1, \alpha_2, ... \alpha_m ; \beta_1, \beta_2, ...\beta_n ; x) =$$

$$\sum_{r=0}^{\infty} \frac{(\alpha_1)_r (\alpha_2)_r .....(\alpha_m)_r}{(\beta_1)_r (\beta_2)_r .....(\beta_n)_r} \frac{x^r}{\angle r}, \ where\ m,n \in Z^+$$

If m=1=n, we get

$${}_1F_1(\alpha, \beta, x) = \sum_{r=0}^{\infty} \frac{(\alpha)_r}{(\beta)_r} \frac{x^r}{\angle r}$$

Which is also denoted by M(α, β, x) and is called confluent Hypergeomentric function

Result:

$$M(\alpha, \gamma, x) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \int_0^1 e^{tx}(1-t)^{\gamma-\alpha-1} t^{\alpha-1} dt$$

$$\Rightarrow \int_0^1 e^{tx}(1-t)^{\gamma-\alpha-1} t^{\alpha-1} dt = M(\alpha, \gamma, x) \frac{\Gamma(\alpha)\Gamma(\gamma-\alpha)}{\Gamma(\gamma)} = M(\alpha, \gamma, x)\beta(\alpha, \gamma-\alpha) \quad \text{... (4)}$$

$$Now\ let\ \ find \ \ \int_a^c e^{tx} f(x)\, dx$$

$$\int_a^c e^{tx} \frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}} dx$$

$$= \frac{1}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}} \int_a^c e^{tx}(x-a)^{\alpha-1}(c-x)^{\beta-1} dx \therefore \int_a^c e^{tx} f(x) dx = e^{at} \sum_{r=0}^{\infty} \frac{(\alpha)_r}{(\alpha+\beta)_r} \frac{(c-a)^r t^r}{\angle r} \quad \text{...(5)}$$

Which is the moment generating function of PERT Distribution

CHARACTERISTIC FUNCTION OF PERT DISTRIBUTION

$$\therefore \int_a^c e^{itx} f(x) dx = e^{ait} \sum_{r=0}^{\infty} \frac{(\alpha)_r}{(\alpha+\beta)_r} \frac{(c-a)^r (it)^r}{\angle r} \quad \text{... (6)}$$

CUMULATIVE DISTRIBUTION FUNCTION OF PERT DISTRIBUTION (CDF):

The CDF of PERT Distribution is

$$F(X) = \frac{B_z(\alpha,\beta)}{\beta(\alpha,\beta)} \equiv I_z(\alpha,\beta)$$

$$Where\ \ z = \left(\frac{x-a}{c-a}\right) \ and\ B_z\ is\ an\ incomplete\ Beta\ function \therefore CDF = \sum_{r=0}^{\infty} \frac{(-1)^r\ z^{\alpha+r}\ \Gamma(\alpha+\beta)}{\angle r\ (\alpha+r)\Gamma(\beta-r)\Gamma(\alpha)} \quad \text{... (7)}$$

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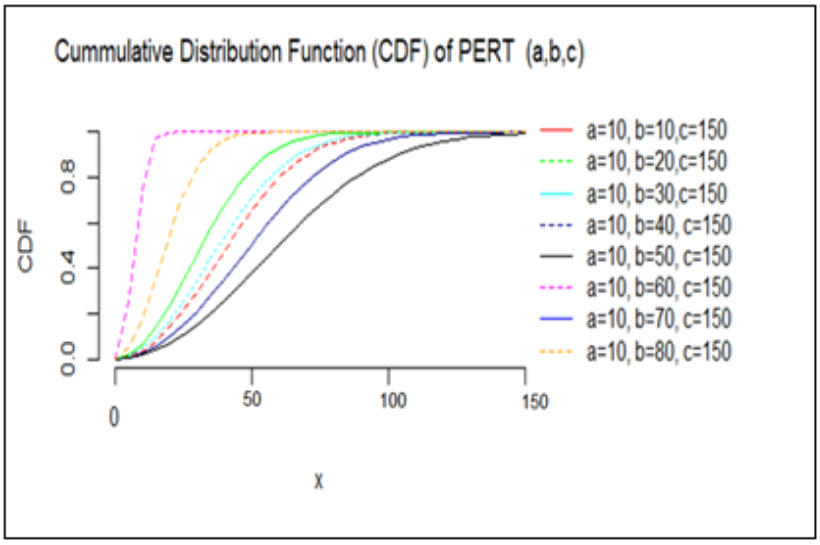

CHART-1.2: Cumulative Distribution Function of PERT Distribution (a,b,c)

SURVIVAL FUNCTION OF THE PERT DISTRIBUTION:

The Survival function = 1-CDF $\quad Survival\ function = 1-\sum_{r=0}^{\infty}\frac{(-1)^r\ z^{\alpha+r}\ \Gamma(\beta)}{\angle r\ (\alpha+r)\Gamma(\beta-r)\ \beta(\alpha,\beta)}$

$$= 1-\frac{\Gamma(\beta)\,z^{\alpha}}{\beta(\alpha,\beta)}\sum_{r=0}^{\infty}\frac{(-1)^r\ z^r}{\angle r\,(\alpha+r)\Gamma(\beta-r)}$$

$$Survival\ function = \frac{\Gamma(\alpha)-\Gamma(\alpha+\beta)\sum_{r=0}^{\infty}\frac{(-1)^r\ Z^{\alpha+r}}{\angle r\,(\alpha+r)\Gamma(\beta-r)}}{\Gamma(\alpha)} \qquad \text{... (8)}$$

$$Where\ \alpha=1+4\left(\frac{b-c}{c-a}\right),\beta=1+4\left(\frac{c-b}{c-a}\right)\ and\ z=\left(\frac{x-a}{c-a}\right)$$

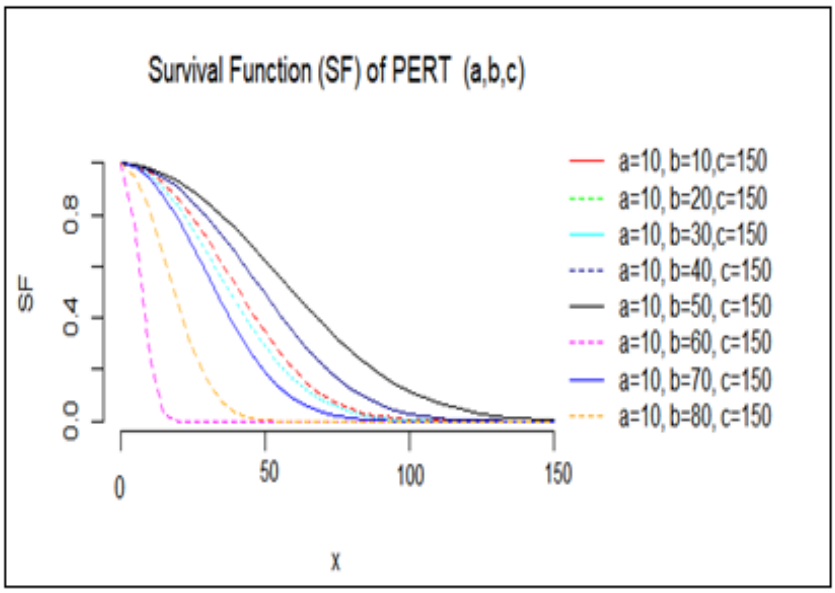

CHART-1.3: Survival Function of PERT Distribution (a,b,c)

HAZARD FUNCTION OF THE PERT DISTRIBUTION:

The Hazard function of the PERT Distribution is

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$$Hazard\ \ function = \frac{pdf}{Servival\ function}$$

$$Hazard\ \ function = \frac{\frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}}}{1-\frac{\Gamma(\beta)z^{\alpha}}{\beta(\alpha,\beta)}\sum_{r=0}^{\infty}\frac{(-1)^r z^r}{\angle r(\alpha+r)\Gamma(\beta-r)}} \quad \ldots(9)$$

$$Where\ \alpha = 1+4\left(\frac{b-c}{c-a}\right), \beta = 1+4\left(\frac{c-b}{c-a}\right)\ and\ z = \left(\frac{x-a}{c-a}\right)$$

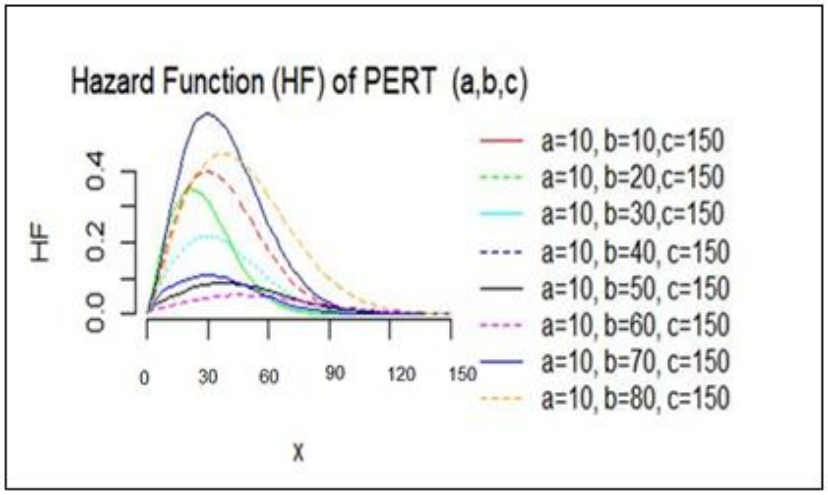

CHART-1.4: Hazard Function of PERT Distribution (a,b,c)

LIKELIHOOD ESTIMATION:

Assume that a random samples $x_1, x_2, x_3. \ldots\ldots x_n$ has been collected from a random variable X which is follows the four parameters of PERT Distribution is

$$f(x,a,c,\alpha,\beta) = \begin{cases} \frac{(x-a)^{\alpha-1}(c-x)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}} & if\ \ a \le x \le c \\ 0, \ Otherwise \end{cases} \quad \ldots (10)$$

Where $\alpha > 0$, $\beta > 0$ and $-\infty < a$, $c > 0$

Taking logarithms on both sides form the above likelihood function for a sample of size n for this distribution is

$$\log L(x,a,c,\alpha,\beta) = \log\left[\prod_{i=1}^{n}\left\{\frac{(x_i-a)^{\alpha-1}(c-x_i)^{\beta-1}}{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}}\right\}\right]$$

$$= \log\left[\frac{\prod_{i=1}^{n}(x_i-a)^{\alpha-1}(c-xLi)^{\beta-1}}{\{\beta(\alpha,\beta)(c-a)^{\alpha+\beta-1}\}^n}\right]$$

$$= \log L(x,a,c,\alpha,\beta = (\alpha-1)+$$

$$\sum_{i=1}^{n}\log(x_i-a)+(\beta-1)\sum_{i=1}^{n}\log(c-x_i)-n\log[\Gamma(\alpha)]-n\log[\Gamma(\beta)]$$

$$-n\log[\Gamma(\alpha+\beta)]-(\alpha+\beta-1)\log(c-a) \quad \ldots (11)$$

Differentiating (11) partially w.r.t. 'a' and equating to '0' as follows

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$$\frac{\partial \log L}{\partial a} = 0$$

$$\Rightarrow (\alpha-1)\sum_{i=1}^{n}\frac{(-1)}{(x_i-a)}+0+n(\alpha+\beta-1)\frac{1}{(c-a)} \Rightarrow (\alpha-1)\sum_{i=1}^{n}\frac{1}{(x_i-a)} = \frac{n(\alpha+\beta-1)}{(c-a)} \quad \text{... (12)}$$

Again differentiating (11) partially w.r.t. 'c' and equating to '0', we get

$$\frac{\partial \log L}{\partial c} = 0 \Rightarrow (\beta-1)\sum_{i=1}^{n}\frac{1}{(c-x_i)} - n(\alpha+\beta-1)\frac{1}{(c-a)} = 0 \; (\beta-1)\sum_{i=1}^{n}\frac{1}{(c-x_i)} = \frac{n(\alpha+\beta-1)}{(c-a)} \quad \text{... (13)}$$

Now Differentiating (11) partially w.r.t 'α' and equating to '0', we get

$$\frac{\partial \log L}{\partial \alpha} = 0$$

$$\Rightarrow \sum_{i=1}^{n}\log(x_i-a) - n\phi(\alpha) - n\phi(\alpha+\beta) - n\log(c-a)$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\log(x_i-a) - \phi(\alpha) - \phi(\alpha+\beta) - \log(c-a) = 0 \quad \text{... (14)}$$

Again Differentiating (13) partially w.r.t. 'β' and equating to '0', we get

$$\frac{\partial \log L}{\partial \beta} = 0$$

$$\Rightarrow \sum_{i=1}^{n}\log(c-x_i) - n\phi(\beta) - n\phi(\alpha+\beta) - n\log(c-a) = 0$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\log(c-x_i) - \phi(\beta) - \phi(\alpha+\beta) - \log(c-a) \quad (15)$$

Where $\phi(\alpha)$, $\phi(\beta)$, $\phi(\alpha+\beta)$ *are the digamma functions*

INFORMATION MATRIX: The solution of the four equations (12), (13), (14) and (15) gives the maximum likelihood estimator for a, c, α and β. Now we take second partially derivatives of L w.r.t. a, c, α and β, we get. The solution of the four equations gives maximum Likelihood.

Estimation for a,b,α, and β. The asymptotic variances and covariance of these ML estimates can be obtained from the information matrix, if certain regularity conditions are satisfied, as will be discussed. Taking the various second partial derivatives of L with respect to the parameters gives

$$\frac{\partial^2(\log L)}{\partial a^2}$$

$$= (\alpha-1)\sum_{i=1}^{n}\frac{1}{(x_i-a)} - \frac{n(\alpha+\beta-1)}{(c-a)^2} = \frac{-n\beta(\alpha+\beta-1)}{(\alpha-2)(c-a)^2}$$

$$\frac{\partial^2(\log L)}{\partial c^2}$$

$$= (\beta-1)\sum_{i=1}^{n}\frac{1}{(c-x_i)} - \frac{n(\alpha+\beta-1)}{(c-a)^2} = \frac{-n\alpha(\alpha+\beta-1)}{(\beta-2)(c-a)^2}$$

$$\frac{\partial^2(\log L)}{\partial \alpha^2} = -n\phi(\alpha) - n\phi(\alpha+\beta)$$

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$$\frac{\partial^2(\log L)}{\partial\beta^2} = -n\phi(\beta) - n\phi(\alpha+\beta)$$

$$\frac{\partial^2(\log L)}{\partial a\ \partial c} = \frac{-n(\alpha+\beta-1)}{(c-a)^2}$$

$$\frac{\partial^2(\log L)}{\partial a\ \partial\alpha} = -\sum_{i=1}^{n}\frac{1}{(x-a)} + \frac{n}{(c-a)} = \frac{-n\beta}{(\alpha-1)(c-a)} \qquad \frac{\partial^2(\log L)}{\partial a\ \partial\beta} = \frac{n}{(c-a)}$$

$$\frac{\partial^2(\log L)}{\partial c\ \partial\alpha} = \frac{-n}{(c-a)}$$

$$\frac{\partial^2(\log L)}{\partial c\ \partial\beta} = \frac{n\alpha}{(c-a)(\beta-1)}$$

$$\frac{\partial^2(\log L)}{\partial\alpha\ \partial\beta} = -n\phi'(\alpha+\beta)$$

Where ϕ'(x) is the trigamma function. The information matrix with elements that are negative of expected values of second partial derivative of L is

$$I = Negative\,of\ \ Expected\ values =$$

$$-\begin{bmatrix} \frac{\partial^2(\log L)}{\partial a^2} & \frac{\partial^2(\log L)}{\partial a\ \partial c} & \frac{\partial^2(\log L)}{\partial a\ \partial\alpha} & \frac{\partial^2(\log L)}{\partial a\ \partial\beta} \\ \frac{\partial^2(\log L)}{\partial c\ \partial a} & \frac{\partial^2(\log L)}{\partial c^2} & \frac{\partial^2(\log L)}{\partial c\ \partial\alpha} & \frac{\partial^2(\log L)}{\partial c\ \partial\beta} \\ \frac{\partial^2(\log L)}{\partial\alpha\ \partial a} & \frac{\partial^2(\log L)}{\partial\alpha\ \partial c} & \frac{\partial^2(\log L)}{\partial\alpha^2} & \frac{\partial^2(\log L)}{\partial\alpha\ \partial\beta} \\ \frac{\partial^2(\log L)}{\partial\beta\ \partial a} & \frac{\partial^2(\log L)}{\partial\beta\ \partial c} & \frac{\partial^2(\log L)}{\partial\beta\ \partial\alpha} & \frac{\partial^2(\log L)}{\partial\beta^2} \end{bmatrix}$$

II. CONCLUSION

The PERT distribution produces a bell-shaped curve that is nearly normal. The PERT distribution with unknown end points was investigated as regard maximum likelihood estimation of its parameters. The maximum likelihood equations are derived along with the information matrix. With some assumptions, the information matrix I can be used to establish a minimum variance bound for an unbiased estimator by means of Cramer-Rao inequality. Also, under suitable regularity conditions, consistency and asymptotic normality and efficiency can be claimed for the ML estimates. Then the diagonal elements of I^{-1} are the asymptotic variances of the parameter estimates; however, they turn out to be quite a task to obtain in closed form. Consequently I was inverted numerically for specific values of a, c, b, α, and β.

III. REFERENCES

[1] Azaron, A., Katagiri, H., & Sakawa, M. (2007). Time-cost trade-off via optimal control theory in Markov PERT networks. Annals of Operations Research, 150, 47–64.

[2] Ash, R.C. and Pittman, P.H., 2008. Towards holistic project scheduling using critical chain methodology enhanced with PERT buffering. International Journal of Project Organisation and Management 1(2), 185-203

[3] Ben- Yair A., Upon implementing the beta distribution in project management, Department of Industrial Engineering and Management, Negev academic college of Engineering, Israel. 2010. http :// braude. ort. org. il/industrial/13th conf/html/%5 cfiles% 5c113-p. pdf.

[4] Cheng and Chan, L.K (1988) proposed some simple estimates of Weibull parameters for grouped data. Volume 17, 1988 - Issue 2, Pages 325-341.

[5] C. E. Clark (1962) the model for the distribution of an activity. Opns Res. 10, 405-406.

[6] Cottrell W; Simplified program evaluation and review technique (PERT), Journal of construction Engineering and Management, 1999; 125(1):16-22.

[7] Garcia CB, Garcia PJ, Cruz SR; Proposal of a new distribution in PERT methodology, Annals of Operations Research, DOI 10.1007/s10479-010-0783-1, 2010.

[8] Ginzburg GD; On the distribution of activity time in PERT, Journal of operational Research society, 1988; 39:767-771.

[9] Herrerias VJM, Herrerias PR,Van Dorp JR; Revisiting the PERT mean and variance, European Journal of Operations Research, 2011; 210: 448-451.

[10] Keefer DL, Verdini WA; Better estimation of PERT activity time parameters, Management science, 1993; 39, 1086-1091.

[11] Malcolm D, Rose boom J, Clark C, Fazar W; Application of a technique for research and development program evaluation, Operations Research, 7, 646-669, 1959.

[12] Mohan S, Gopalakrishnan M, Balasubramanian H, Chandrashekhar A; A lognormal approximation of activity duration in PERT using two time estimates, Journal of the Operational Research Society, 2007; 58:827-831.

[13] Premchandra IM; An approximation of the activity duration distribution in PERT, Computers & Operations Research, 2001; 28:443-452.